C1B Stress Analysis (4ME06) Prof A.M. Korsunsky, HT Jan-08

Recommended reading

S.P. Timoshenko, J.N. Goodier, Theory of elasticity, Prentice Hall, 1990.

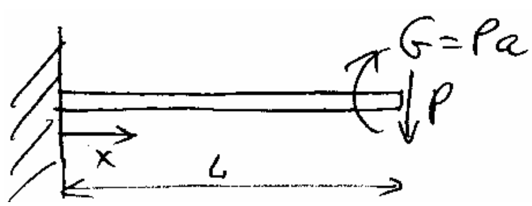
T. Mura, T. Koya, Variational methods in mechanics, OUP 1992.

A. Constantinescu, A.M. Korsunsky, Elasticity with Mathematica, Cambridge University Press, 2007.

S. Wolfram, The Mathematica Book, 4th ed., CUP 1999.

Problems

1. Determine the deflected shape of the above beam using the Rayleigh-Ritz method, assuming the following displaced shapes:



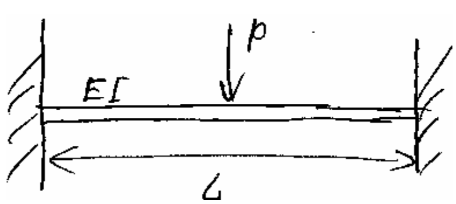
(a) $y = A + Bx + Cx^2 + Dx^3$

(b) $y = A\left(1 - \cos\left(\frac{\pi x}{2L}\right)\right)$

Compare the resulting values of the tip deflection. Are natural boundary conditions satisfied?

2. Choose a suitable trial function for the Rayleigh-Ritz method and estimate the central deflection and central bending moment in the above beam using a one- or two-term approximation.

Demonstrate that using the Galerkin method with the same trial function produces an identical result.



3. Consider the buckling problem shown.

First, show that the load displaces vertically by

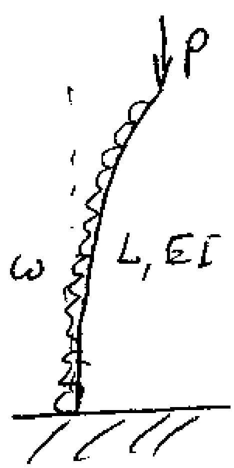
$$\Delta = \frac{1}{2}\int \left(\frac{dy}{dx}\right)^2 dx$$

Hence estimate the buckling load based on trial functions

(a) $w = A \left(1 - \cos\left(\frac{\pi x}{2L}\right)\right)$

(b) $w = A + Bx + Cx^2 + Dx^3$

(c) Re-evaluate using representation (b), but imposing natural boundary conditions.



4. The deflection at point (*x,y*) on a plate produced by a unit load at (ξ, η) is called the influence function. For a simply supported rectangular plate, of side *a* x *b,* and assuming a deflected shape of the form

$$w = \sum\sum a_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

Find the corresponding influence function.

5. For a symmetrically loaded circular plate the strain energy of bending is given by

$$u = \frac{D}{2}\int_o^r 2\pi\left[\left(\frac{d^2w}{dr^2}\right)^2 + \frac{1}{r^2}\left(\frac{dw}{dr}\right)^2 + \frac{2\nu}{r}\left(\frac{dw}{dr}\right)\left(\frac{d^2w}{dr^2}\right)\right]rdr$$

A simply supported circular plate of radius *a* carries a uniform load p_o. The thickness of the plate varies with radius such that

$$D = D_o\left(1 - \frac{r}{2a}\right)$$

Assume that the deflected shape is given by

$$w(r) = w_o\left(1 - \left(\frac{r}{a}\right)^2\right).$$

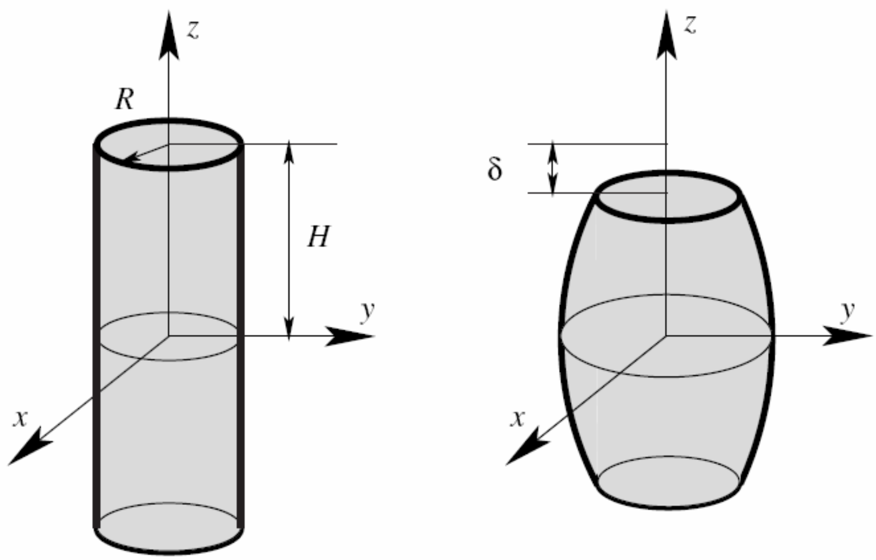
Find the central stiffness.

6. For a solid in a state of plane stress, show that if there are body forces p_x, p_y per unit volume in the direction of the axes *x, y* respectively, the compatibility equation can be expressed in the form

$$\nabla^2(\sigma_{xx}+\sigma_{yy}) = -(1+\nu)\left(\frac{\partial p_x}{\partial x}+\frac{\partial p_y}{\partial y}\right).$$

Hence deduce that the stress distribution for any particular case is independent of the material constants and the body forces, provided the latter are constant.

7. Consider the compression of a cylinder of radius R and length $2H$ that is rigidly attached to the top and bottom platens.



(i) Introduce the barrelling function as the ratio between the deformed external radius at height *z* and the initial radius, $f(z/H) = R'(z)/R$. Show that the displacement field (u,v,w) with respect to cylindrical polar coordinates (r,θ,z) given by $u = \frac{\delta}{H}\nu\frac{r}{R}f(z/H)$, $v = 0$, $w = -\frac{\delta}{H}z$

is kinematically admissible.

(ii) Compute the strains $\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}$ using the formulae from HLT, p.110.

(iii) Compute stresses σ_{rr}, σ_{zz} using the formulae from HLT, p.108 (under "plane stress"). Do the resulting tractions satisfy traction-free boundary conditions on the lateral surface of the cylinder?

(iv) Write the integral expression for the complementary energy potential $U_p^* = -\frac{1}{2}\int_{\Omega} \sigma_{ij}\varepsilon_{ij}\, dV + \int_{\partial\Omega^d} t \cdot u^D dS$ in terms of the barrelling function.

(v) Considering the simple triangular form of the barrelling function, $f(z/H) = q(1-|z|/H)$, evaluate the integral expressions for the complementary energy potential. Maximise it with respect to *q* and find the approximate solution. Comment on the possibility of improving the accuracy of this result.