1. Bernoulli to Leibniz[1]

[Groningen, 5 July 1698]

When I went to Leiden recently I met De Volder quite often, and once he invited me to lunch where I also got to know, and became friendly with, a number of other Leiden professors. . . . Later I met him several times at the houses of other professors and I got the sense that he is a great admirer of yours. . . . I also gathered that he is on Papin's side concerning the measure of force,[2] saying that you assume the elasticity of bodies gratuitously since it is only accidental to bodies, and that, if perfectly hard bodies are considered, your arguments no longer apply. For when perfect hardness is assumed, it follows, for example, that two bodies of equal size colliding head on with each other with equal speeds should not rebound but stop at the very moment of collision and that, generally, two perfectly hard bodies of whatever bulk and whatever speed should not separate after a collision, since their separation is produced by elasticity, which he says must be assumed in addition. Since shortness of time did not permit me to talk him out of his opinion, I promised him lots of extracts from your letters and from mine so that he could see what drove me to embrace your position.[3]

2. Leibniz to Bernoulli[1]

[Hanover, 22 July 1698]

In a little book on the laws of motion that I wrote some time ago as a young man, I had the opinion that Mr. De Volder now has that, from their nature, two bodies of equal size and equal speeds should not rebound after a direct collision, but should bring each other to a halt instead.[2] This also follows from the common notion of matter, namely when nothing other than extension and antitypy, i.e., impenetrability, is conceived in it. But later, from these and similar

considerations, I recognized that the nature of the matter formed into the universe is very different than is commonly believed and that elastic force is essential to every body, not in such a way that the force is like some inexplicable quality, but because every body, however small, is a machine from whose structure a recoil must arise when this is required to conserve force.[3] Moreover, this should not seem surprising to anyone who considers the actual division of the parts of this matter into parts exceeding every number. I leave it in your hands whether you would like to show this to Mr. De Volder with my greetings.

Anyway, please do let me know what you have written, or are writing, to Mr. De Volder, or if you get any reply from him.

3. Bernoulli to Leibniz[1]

[Groningen, 23 July 1698]

I have already told [De Volder] in person what you now write that I should point out to him, namely that a third thing beyond extension and impenetrability is required for the essence of body that consists in an innate power for the conservation of force, whence elastic force necessarily belongs to all bodies from their nature. But he retorted that you are postulating something that you cannot conceive: Is this third thing a substance or a mode? If it is a mode, it is nothing new; if it is a substance, is it spirit or body? Or perhaps a third thing? But you cannot explain this third thing unless you want to take refuge in the long since rejected substantial form of the ancients. I replied that it is enough that experience teaches us that, to the extent that they are harder, bodies have a more perfect elastic force, moreover that, given this, it can easily be demonstrated that the quantity of force is conserved but not the Cartesian quantity of motion, except in certain cases.

4. Leibniz to Bernoulli[1]

[Hanover, 29 July 1698]

There is no need to appeal to souls or forms or spirits in order to satisfy the most celebrated De Volder about elastic force. As I already noted in my last letter, even if these are left out of consideration, the universe is such that any portion of matter, however small, gets – from another, even smaller, portion of matter permeating it – a mechanism for restoring itself after deformation, in so far as that is necessary to obey our laws of motion. And so elastic force will be essential to every body on account of the structure of the universe. It is no more necessary for us to appeal to something greater for the conservation of power than for the Cartesians, who maintain the conservation of the product of size[2] and speed. For neither can be derived from extension and impenetrability alone. Moreover, it is insufficiently philosophical to appeal to the bare will of God like Descartes.[3] And however we measure the power that is conserved, it follows from the fact that force or action is not lost that there is something in body other than those two things, namely extension and impenetrability. For otherwise, as I noted in my last letter, two bodies of equal size colliding head-on would bring each other to a halt, and many other things would happen entirely foreign to experience and even to reason. These things would certainly follow with geometrical necessity from the simple composition of conatus[4], as I remember explaining somewhere in the Journal des Savants, and a long time ago in the theory of motion that I published as a young man.[5]

As for Mr. De Volder's objection that, "we are thus forced to postulate something in body that we cannot conceive,"[6] you responded well that is enough that experience forces us to admit something besides extension and impenetrability, whether it can be conceived by us or not. Furthermore, in order to show that this thing cannot be conceived, he asks whether, "that which we admit beyond extension and impenetrability is a substance or a mode," and he

adds, “if it is a mode, it will be nothing new; if it is a substance, it will be a spirit or a body or a third thing, and this third thing cannot be explained, unless we want to take refuge in the long since rejected” (so he thinks) “substantial form of the ancients.” But we could in turn ask what definition he assigns to substance or to mode. Besides, there are things that are neither substances nor modes, such as primitive attributes. Thus, determined magnitude is essential to any given matter and it is not a mode like shape or motion are, and nonetheless magnitude is not a substance but an attribute. Nor does it matter whether what we postulate is “new”, only that it is true. Furthermore, when it is asked whether there can be a substance that is neither spirit nor body, again there is a need for a definition, about which perhaps we do not agree with him. For he will put the essence of body in extension, but I postulate something additional. If he thinks that every spirit is endowed with thought and understanding, I will hold that there exist souls or forms that are not spirits. I do not see what prevents there being various grades of monads, so that some are endowed with understanding and other, inferior, ones with sense. And so, if we conceive substantial forms as things analogous to souls, it may be doubted whether they are rightly rejected. . . .

If it seems appropriate, excerpts from this letter or anything from previous ones concerning the measure of force and the nature of body, and likewise concerning the infinitesimal calculus, could be passed on to Mr. De Volder.[7]

5. Bernoulli to Leibniz[1]

[Groningen, 26 August 1698]

So far I have received no response from Mr. De Volder. . . .

I know that De Volder will persist in asking for a clear explanation of that third thing beyond extension and impenetrability that you consider necessary in body. Perhaps he will

retort: If you set up monads of this sort belonging to bodies, whether you call them substantial forms or things analogous to souls (intelligent or sentient, it doesn't matter to De Volder), the monad must be attributed to either the whole body or part of it. But it cannot be attributed to the whole body, since the body can be divided into mutually independent parts, and it cannot be attributed to a part, since that part is divided into many independent parts as well. However, if you say that the body is composed out of infinite monads, then each monad must be characterized as either extended or not extended. If they are extended, then, even if they are infinitely small, the previous difficulty returns, unless you are willing to resort to atoms. If they are not extended, they are, therefore, useless, since an extended thing cannot be composed from unextended things.[2] Thus, as a result, you will perhaps be forced to say that every point of the body (I mean an indivisible mathematical point) is given its own monad or such a soul.

6. Leibniz to Bernoulli[1]

[Hanover, 22 August 1698][2]

I have often said (and you seem not to disapprove) that it belongs to the nature of body that all the phenomena in bodies, even elastic force, can be explained mechanically, but that the principles of mechanism, i.e., of the laws of motion, cannot be derived from the consideration of extension and impenetrability alone. And so something else must be judged to be in bodies from whose modification conatus and impetus arise, as figures arise from the modification of extension. By monad I understand a substance truly one, which, of course, is not an aggregate of substances. Matter itself, intrinsically, i.e., bulk, which you can call primary matter, is not a substance; indeed, it is not an aggregate of substances, but something incomplete. Secondary matter, i.e., mass, is not a substance, but substances. So not the flock

but the animal, not the fish pond but the fish, is one substance. But even though the body of an animal or my organic body is, on the contrary, composed of innumerable substances, they are not parts of the animal or of me. But if there were no souls or analogous things, then there would be no I, no monads, no real unities, and so there would be no substantial multitudes. Indeed, everything in bodies would just be phantasms. Whence one can easily judge that there is no part of matter in which monads do not exist.

7. Leibniz to Bernoulli[1]

[Hanover, 30 September 1698]

P. S. to my letter to Mr. Bernoulli of Groningen written on 20/30 September 1698[2]. . . .

Of greater importance is the question of the origin of elastic force that has arisen between you and [De Volder] on the occasion of my dynamics. So it seems worth explaining my view to you both. When I insist that elastic force is essential to the bodies existing in nature, I do not mean that it should be sought immediately, as it were, from souls or forms. Rather, it arises from the structure of the system of the whole universe. Divine wisdom, and hence the laws of things imposed by God and the principles of dynamics deduced from the real metaphysics, and so, on that account, the forms created by God (i.e., the divinely impressed forces), taken together required this. And so, however small a body may be, there is a much subtler fluid surrounding and permeating it, from which the elasticity of the body comes. For otherwise that great, and it seems to me inviolable, axiom that governs nature, which I was perhaps the first to notice, would not be observed. I recently made an original use of it in the Nouvelles de la République des Lettres,[3] and I call it the law of continuity. When I objected to Huygens' support for atoms, a year or so before his death, he admitted

that it was worth consideration.[4] It says, in changes there are no leaps, and, consequently, that there is no assignable change in an instant, and likewise that it is impossible to pass from motion to rest or to motion in the opposite direction, or vice versa, except through intermediate stages.[5] Hence, those who maintain (as indeed everyone commonly does) that motion never happens through leaps or that a body cannot pass from one place to another place except through intermediate places, have seen the truth but not the whole truth. For the same thing is observed no less in changes of degree than in changes of place. Moreover, this avoidance of leaps in the changes of bodies is due to an elastic force existing in them. This is how it happens in collision that, by gradual movement, bodies that compress each other and then restore themselves yield to each other little by little and conserve their direction and force and, as you have seen demonstrated, the quantity of their motive action (which is very different from quantity of motion as commonly understood). You will also see that the same principle of nature eliminates Democritus' atoms and the first, and even the second, of Descartes' elements.[6] Similarly, as you know, with this as a touchstone, I rejected the laws that the followers of Descartes, Malebranche, and others have ascribed to nature as if I had directly observed that they were errors.[7] As I said, I do not deduce particular phenomena from souls or forms, but only the nature of body and of force in general. I hold that gravity, elastic force, attractions, repulsions, lines of magnetic force, and other things of that kind must be explained mechanically. But I derive the mechanical principles themselves from that which is dynamic, i.e., from forms, as if they were laws imposed by God and now contained in the nature of body. For it should not be thought that nature obeys God's prescriptions as subjects comply with an edict that has been promulgated, or that God himself is always forcing something deviant back on track and correcting his work, as bad marionette-makers are wont to do. Rather, by giving things laws he simultaneously gave them the force and striving to observe them, which the nature of entelechies consists in. Meanwhile, although

this is the case, the entelechies themselves and everything that is real in things subsist, and are conserved continuously by divine emanation.

8. De Volder to Bernoulli[1]

[Leiden, 21 November 1698]

To the most distinguished gentleman, Mr. Bernoulli.

I am most grateful for both your letters, which I have received safely.[2] I did not reply to the first one because I did not have anything to write, except to urge one who needs no urging to continue to extend the boundaries of mathematics with new discoveries. As for the second letter, in which you encourage me to write to the most renowned Leibniz about those worries that prevent me from agreeing unhesitatingly with his laws of motion, I am very pleased indeed that you think it worthwhile to put me in contact with the most illustrious gentleman. However, although I do not doubt the gentleman's friendliness, I shall not take advantage of it on this occasion, since I would not want the time that he spends so fruitfully on the advancement of learning to be wasted on any of my trifling objections, especially since, while looking back at what there is about these things in the Acta Eruditorum[3], I have discovered that my main difficulty has been anticipated by the most distinguished Papin. Indeed, considering the arguments set out there, I find that I cannot free myself from all the difficulties on both sides, and so <underline> I dare not embrace either side as certain </underline> .[B1 4] However, so that this letter does not come to you completely in vain, here, briefly, are the things that came to mind concerning this matter.

The illustrious gentleman's argument assumes this Cartesian axiom: the same force is required to raise a body of four units to a height of one foot as to raise a body of one unit to a height of four feet. If it is conceded that this is universally true, there can be no doubt that the

conclusion he derives from it is the appropriate one. But, in order to counter Papin's claim that things that can overcome an equal number of gravitational <underline> impressions </underline>[B2] can overcome an equal resistance, the renowned Leibniz adds that the reason he denies this is because it begs the question. But for me this would also be as good a reason to deny the universal truth of the axiom. And if the illustrious gentleman appeals to the self-evidence of the axiom (which seems to me, however, to depend upon knowledge of the action of the cause that produces gravity), it seems to me that this could be done just as legitimately in relation to the self-evidence of the proposition denied. For if we think of the resistance of gravity as consisting of percussions by which every ascending body is pushed downwards, <underline> and that the percussions are all the same size and act on a body in the same way, whether it is already moving or at rest </underline>[B3] (since the speed that a heavy body receives when descending or that it loses when ascending has no relation to the speed of the cause of gravity, or certainly none that needs to be counted here), <underline> and, finally, that the number of these percussions is proportional to time </underline>,[B4] does it not follow from this that whatever resistance the first percussion produces the same will arise from the second, the third, etc., and so that the quantity of this resistance should be measured by the number of the equally resisting blows? Hence the forces of two bodies that are the same size, one of which is moving with twice the speed of the other, will be in the ratio 2:1, even though the first ascends to four times the height of the other, since during that quadruple ascent it meets with only twice as many percussions of resistance. However the cause of gravity is supposed to act, it will push a descending body down with the same force as that with which it holds it back when it is ascending. It follows that that which it gives to the body when it is descending, whether it be quantity of motion or force, will be taken away from the body when it is ascending. In short, twice the quantity of motion must be lost when ascending four

feet, and to suppose four times the force in this ascent is to suppose the very thing that was being sought.

Next the illustrious gentleman contends that the force of a body of four units that has one degree of speed transferred to a body of one unit will only produce twice the speed, with which it can rise to a height of four feet, and not four times that speed, as the Cartesians would have it, since that would give it the power to rise to sixteen feet. But I do not really understand how this can be the case. Let there be five bodies of equal size, one of which is at rest with the others each having a speed of one. I believe that these four taken together will have the same force as a body that is four times their individual size and that has the same speed. But in each of the individual bodies the forces are equal. Therefore, however much change one of these bodies will bring about by transferring its force to the fifth body, it seems that each of remaining three will bring about just the same. But, with the transfer of the force of the first one, the fifth body will acquire one degree of speed. Now, if the force of the second is added on as well, <underline> surely it will give another degree? Why will it give less, I ask? For, since the size remains the same, surely all the change that is to be brought about will be in the speed alone </underline>?[B5] Is it really the case that the remaining three, even though they are each equal in force to the first, <underline> will not be able to bring about more change combined together than that other one on its own </underline>?[B6]

Furthermore, the renowned gentleman says[D1] that he does not concede that "a body that has three degrees of speed consists of a body of the same size that has one degree of speed three times over, <underline> or that it has three times its power on account of this </underline>.[B7] For even if it contains the degree of speed three times over, it does not also contain the quantity of body three times over, but only once."[5] It seems to me that it follows from this[B8] that a body that has four degrees of speed has less force than four bodies of the same size that each have one degree. And so, if the force of all four of these bodies is

transferred to another of the same size at rest, it will have not two degrees of speed but actually more than four. But perpetual motion will arise from that. I will not repeat what Papin says, but I will say this one thing: Although I believe that perpetual motion is among those things that can never be made to happen through mechanical means, I still am not persuaded that every perpetual motion involves a contradiction. For the structure of the universe itself gives rise to perpetual motion. Furthermore, if we ignore all impediments, a ball that descends along any inclined plane, e.g., a cycloid, will surely have a force by which it may reach <underline> the same </underline> [B9] height as it descended from when it reaches the lowest point. But if this is so, it will give rise to perpetual motion.

However, I cannot hide the fact that there is something from which I cannot free myself completely in the example that the illustrious gentleman employs, in which a force that can raise a body of four units to a height of a foot raises it to a height of four feet subsequently. In fact <underline> I can only counter it with this fact, namely, that it follows from the opinion of the most renowned Leibniz that the forces of bodies of completely different sizes are in equilibrium with each other on a balance </underline>.[B10] But I do not know why the illustrious gentleman calls this a paradox, as this paradox seems just as absurd as the one that I just raised. As for what he adds about the distinction that must be made "between the absolute force that is necessary to produce some persisting effect and the force for advancing along a certain path or for conserving its direction,"[D2 6] I really do not understand what he means by these things.

Moreover, the illustrious gentleman points out that he calls <u>force</u> that which always continues to have the same quantity, even when the quantity of motion is increased or decreased. This agrees with Huygens' <u>Laws of Motion</u>, which can be found in the <u>Philosophical Transactions</u>,[B11 7] in which he maintains that although the quantity of motion may be increased or decreased, nonetheless, "the sum of the products of the sizes of any hard

bodies and the squares of their speeds is always the same before and after their collision."[D3 8] Huygens derived these laws from the hypothesis that hard bodies of the same size colliding with the same speed both rebound after the collision with the same speed as before. However, given the hypothesis, how is this consistent with what Law 5 says, namely, "<underline> The quantity of motion in a given direction always remains the same after the quantity of motion in the opposite direction has been subtracted </underline>"?[B12 9] What concerns me is that I do not have anything to say with certainty about whether the quantity of force is the same or different. I find no reason why the force would increase, and I do not see what would prevent the contrary forces that oppose each other from <underline> canceling each other out </underline>.[B13] Soft bodies of the same size and with the same speed lose their motion when they collide with one another. I am strongly inclined to believe the same thing about hard, inelastic bodies, whatever Huygens supposes. Indeed, it seems likely to me that a body that is impelled in the opposite direction with equal force will come to rest whether it received this force just now or earlier from another source.[10] But, if this is true, bodies that are the same size colliding with one another with the same motion will both come to rest. And it is not clear in this case why it should not be said that both the quantity of motion and the force are lost. <underline> I suspect </underline>[B14 11] that the illustrious gentleman holds that elastic force is essential to all bodies in order to deal with this.

Behold the things that until now have held me back from accepting the thoughts of the illustrious gentleman without any worries. However, if I reflect on the keenness of the illustrious gentleman's intellect, the solidity of his judgment, and his excellent reputation in the world of letters for so many new discoveries, I have little doubt that these trifling objections, if they do seem to be so, will be easily cast aside – especially if I consider what he says about the nature of substance in the Acta Eruditorum.[D4 12] For if we had an a priori demonstration that every substance is active, I might easily persuade myself that from this

most fruitful source of truth follows not only the resolution of my worries but also of those difficulties that have burdened every natural philosopher up until now. Indeed, due to ignorance of this fact they have been compelled to conjecture that the cause of motion is in God, and some have even been compelled to summon Deus ex machina for any collision of bodies. For, since they learned from experience that matter is active, but did not observe this force to flow from the nature of body, they had nothing else in which to take refuge. But, if it is established that the nature of corporeal substance involves action, which remains the same as long as there are still bodies, the matter is resolved. And, unless I am mistaken, it will also be possible with this to explain those things about the cause of the continuation of motion that seem completely confused to many people. It will be clear what that striving is that persists in a body even if it has no effect because of obstacles (something that seems extremely difficult to comprehend to me), in what way contrary forces may follow from the same active force, and how the forces nonetheless remain the same even if the effects cancel each other out. Finally, lest I pile on too many things, perhaps it will be clear what that thing is that must be ascribed to matter besides extension. I confess I have no concept of this so far.

Would that it had pleased the illustrious gentleman to enrich the Republic of Letters with his thoughts on these matters and, with this explanation, free it from the primary difficulties that obstruct the path to a complete knowledge of physical things. As far as I am concerned, this would certainly be of the greatest moment, and I have no doubt others will agree. Indeed, I do not know whether anything would be greater. Because of this I must ask in earnest that, through whatever influence you have over the illustrious gentleman, <underline> you force this out of him </underline>[B15] for the benefit of the Republic of Letters; or if you perhaps know what his thoughts are on this matter, that you do not refuse to communicate this to me, who has the greatest regard for all the thoughts of the gentleman.

I do not remember what passed between us concerning elastic force.[13] It has always seemed absurd to me to suppose it in <underline> completely solid </underline>[B16] bodies. But, in fact, this is the very thing that the illustrious gentleman rejects with his explanation. Moreover, I do not understand that fluid that he supposes and which differs from Descartes' first element. "In changes there are no leaps"[14] is perhaps among the things that will be easily demonstrated when we are acquainted with the active force that is contained in the notion of substance. In the mean time it seems hard to accept that a body moving with a great speed, running into another that is immobile and incomparably smaller, <underline> is retarded </underline> by the smaller one <underline> in such a way that no leap occurs </underline>.[B17] But as I am finishing, I am reminded that I am stuck over whether I properly understand the things that the illustrious gentleman says about the common hyperbola. For when the sub tangents are equal to the abscissa from the asymptote in this [curve], another hyperbola will arise, exactly equal to the first, such that there is no difference between the area of each, <underline> except that in one of them a rectangle is added to the hyperbola, which is not in the other, and which has no ratio to the infinite area, since it is finite </underline>.[B18]

Goodbye most distinguished Sir, and please excuse me if I have been annoying by writing to you at such great length, indeed much greater length than I had originally intended. I am completely yours, B. De Volder.

I ask that you give my regards to the most distinguished Mr. Brun.[15] Goodbye again.

From Leiden, 21 November 1698 A. D.

9. Bernoulli to Leibniz[1]

[Groningen, 6 December 1698]

Here is a letter from De Volder in which you will find some objections to your dynamics, most of which were already made by Papin and me some time ago.[2] If you deem it worthy of an answer you could direct your reply to me, and I shall send it to him, although I think that it might be more advisable for you to write to him yourself, since this would give him the freedom to write to you in return. I also think that it would not be a bad thing if you published your dynamics (which you have suppressed, and which you have published only a specimen of so far) along with your responses to the objections made to you.[3] For, in this way, you would, once and for all, discharge the task of replying, which you would otherwise have to repeat ad nauseam. I hope for our sake that it might spark the desire to improve metaphysics, which we expect you to do first, if anyone, and which I also remember that you promised to do. We have two philosophies, the ancient and the modern, but both err in their extremism. What if you were to establish a third by preserving the middle-ground between the two? I know of no one more suitable for this task than you, and I have no doubt that if you constructed the system it would be very successful. Certainly, there are some distinguished gentlemen in Holland who would vigorously encourage and protect this brain-child of yours. Among these gentlemen is Mr. De Volder himself. Since he has admitted to me that he has long since abandoned the principles of Cartesianism as inadequate and mostly false, there is no doubt that once he properly understood your philosophy, and imbibed it, he would propagate it diligently and impress it upon his students (of which he always has an extraordinary number) in such a way that perhaps it might soon have dominion over the Cartesian and Aristotelian philosophies, as if reconciling the ancients and the moderns. I ask you, again and again, whether you are willing to consider the world of philosophy worthy of this. I beg you to consider us and posterity, and erect a memorial to your name. After all, do you think that you are inferior to Descartes?

10. Leibniz to De Volder[1]

[Hanover, 27 December 1698][2]

Gottfried Wilhelm Leibniz sends greetings to the gentleman most celebrated on account of learning and merits, Mr. B. de Volder.

I am taking the letter that you recently sent to my great friend the most clever Mr. Bernoulli, Professor at Groningen, as if it was written for me, since it is wholly concerned with setting out difficulties that you still have with the exposition of my views.[3] Not only is your humanity apparent in the letter, but your sincerity and devotion to the truth also shine through, and the power of your judgment is such that it cannot fail to provide me with something most fruitful, no matter what happens. So I did not deliberate for long over whether I should bother to reply.

I readily acknowledge that new ideas cannot be established except after extensive discussions and examination by distinguished men. The reason I freely take on the work of private discussion with people such as yourself (if only I knew more such people) is not so much that I seek praise for my work and the applause of the common folk (like those who publish immediately) as that I seek the confirmation and, as it were, the consolidation of the doctrine that is to be expected from legitimate judges, i.e., from the learned. But I come now to the matter at hand.

With good reason you seem to be undecided between two positions, one of which is, "the same force is required to raise a body of four pounds by one foot as to raise a body of one pound to a height of four feet,"[4] from which my measure of force follows. The other is, "things that can overcome an equal number of gravitational impressions can overcome an equal resistance," from which the commonly accepted measure follows. You add, not unjustly, that until I prove it with an argument, the former, which was affirmed by me, can be

denied with equal right on the grounds on which I denied the latter (i.e., because it begs the question). Of course, Descartes admitted the former and used it in his notes on mechanics[5], and Pascal did as well in the Treatise on the Equilibrium of Liquids.[6] So I could have used an ad hominem argument against the Cartesians. However, since I wanted to search out the truth for its own sake, I tried to prove my thesis by a reductio ad absurdum, namely to perpetual motion. In virtue of your sincerity, you acknowledge that it seems to you that that argument should not be rejected, when you say that you, "cannot hide the fact that there is something from which you cannot completely free yourself in my example in which the force that could raise a body of four pounds through one foot raises it through four feet subsequently." For perpetual motion follows from this; not the physical one that exists in the whole of nature, by which things return to the same state or an equivalent one, but a mechanical one, by which a body can not only rise to the same height by the force [that it acquires] from a fall from a certain height, but to an even higher place. This undoubtedly seems absurd; certainly it is contrary to all experience. But you think that I must, in turn, admit something no less incongruous, namely that forces of bodies that are completely different in size are in equilibrium on a balance.

Unless I am mistaken, excellent Sir, this contains the thrust of your difficulty. I will try to respond to this first, and then attempt to satisfy the others that you took the opportunity to raise. And, in fact, I think that it can be safely assumed as an axiom that the effect may not be more powerful than the cause, or, what comes to the same thing, that there is no perpetual mechanical motion.[7] I will not dwell on this now, especially since I am about to demonstrate the axiom itself below, which was originally posited by Descartes and only assumed by Pascal, and nothing prevents you from admitting that perpetual motion is impossible, except the difficulty of the opposite. So let us turn to this difficulty, so that we may see a little more clearly whether it is of such great moment. As an example, if body A of one pound endowed

with a speed of four and body B of four pounds endowed with a speed of one collide, I acknowledge that these bodies will bring each other to a halt, just as they are in equilibrium when placed on a balance. Given this, it seems absurd to you that their forces are unequal.

But, in fact, when the matter is considered more carefully with me, you will find that there is no difficulty in it at all, let alone an absurdity, but rather that everything comes together very beautifully. For an important distinction must be drawn between forces and actions. Either it is a question of the conflict of two things, as when there is equilibrium on a balance, or it is a question of the production of some absolute effect, such as when a weight is raised to a given height, or when a spring is stretched to a certain degree. And, in turn, forces are either dead, such as those that the first conatus of a descending weight has or that it acquires in any given moment, or they are living, such as those in the impetus[8] of a weight, which it receives in descending for some time. And the impetus of the living force is to the bare[9] solicitation[10] of the dead force as the infinite to the finite, or as the line to its elements in our differential calculus. For impetus is generated from the continued increase of solicitations. From here we also see that nature reconciles most elegantly the law of equilibrium of colliding [bodies], which is relative, with the law of equivalence of causes and effects, which is absolute, through the mediation of the law of gradual transition avoiding all leaps. For there is never a transition unless it goes through unassignable or infinitely small increments, and hence through dead forces. And the law of equilibrium never applies except with dead forces, whether the heavy bodies as yet lack impetus, – as happens on a balance, where both are merely striving to descend – or impetus has already been acquired – as in a collision. For they never act on each other, except through the mutual subtraction of an equal but infinitely small (or dead) quantity of motion. If there is a body A with a size of four and a speed of one and a body B with a size of one and a speed of four, then, if they come into opposition either on a balance, when they are merely striving, or through a blow or a

collision, when they impel one another with their own impetus, the speed of A is lost continually in infinitely small amounts and the speed of B is lost in infinitely small amounts in a ratio of 1:4, i.e., in a ratio inversely proportional to the [sizes of] the bodies. The quantity of the dead motion of A is of course 4□1 or 4, and in the same way the quantity of dead motion of B is 1□4 or 4. This follows from the Archimedean law of equilibrium.[11] Indeed, the demonstrations of Archimedes and others have no application except in the case of those first strivings cancelling each other through themselves. It happens like this when weights of four and one have been placed on a balance in such a way that they cannot descend unless the former begins to ascend or descend with a speed of one and the latter begins to ascend or descend with a speed of four. But, since the descent of the bodies is impeded by their equilibrium, the fulcrum of the scale supports whole weight of both, and it suffers some pressure and bending or stretching of its fibers. So whatever force is lost by the bodies is transferred to the elastic elements of those very bodies or to the bond by which they are joined together. Or if they are not elastic enough (like soft bodies), it is not returned to the motion of the parts of the wholes (which would happen if it were elastic) but is completely lost in relation to them. And this continues until the force of the collision is completely exhausted. On account of this, through the restoration of the elastic element according to the law of equilibrium, the bodies again receive forces in receding from each other to the same degree as those they lost in approaching each other (i.e., with increments of speed that are inversely proportional to the [sizes of] the bodies). It is a consequence of this that bodies A and B in the present case finally bring each other to a halt, not all at once as in equilibrium, but gradually. For, since they have the same quantity of (living) motion, and during the collision the diminutions in the quantity of motion of both are equal, it is necessary that the motion of both will be exhausted at the same time. This follows from the law of equilibrium through the continuous subtraction of dead force, i.e., of speeds inversely proportional to the

sizes.[12] But, if either of them has more motion, it should not come to a halt but continue moving. And it must be understood that, in those things that have the same quantity of motion, the force of the collision is equal to the force of all the bodies. It is different in other cases when two bodies taken together have a common force of progression besides the force of collision or relative force.[13] Meanwhile, just as it is found that the law of equilibrium is always observed in the differences or increments themselves, so it is found by the marvelous art of nature in the integral terms themselves, or the living forces, that, by taking into account both the force that remained in the bodies and all that was lost and transferred into the elastic elements, the same amount of living force is always conserved in accordance with the law of equivalence. This is most apparent after the restoration of the elastic element, when bodies receding from one another recover the entire force again. Then, if you imagine their impetus converted upward, as if they proceeded by being attached to a pendulum, the total quantity of the ascent is always found to be the same as that which could have been obtained originally, before the collision – assuming that nothing was absorbed by incidental things. But, if you suppose that the bodies convert their residual force upwards in the middle of the collision, it will also have to be imagined that the elastic element of the bodies, or something equivalent to the elastic element, will exert all the force that it received in the collision on any body that is introduced by hurling it upward. Again, the total quantity of ascent will be found to be the same as it was before the collision. And by means of these two laws, without communicating with each other, Mr. Bernoulli and I resolved a certain singular problem that had been proposed to me by Mr. Papin in exactly the same way.[14] Given the importance of the matter, it had to be explained at great length, so that it would be clear that we were not confused by the law of equilibrium but rather that we used it very successfully. It also happens in the same way with a falling weight that a measure of both dead and living forces is obtained, if it is imagined that at any moment it receives a new and equal infinitely small increase in speed.

Of course the speed increases in equal amounts according to time, but the absolute force itself increases according to distance or the square of the times, i.e., in accordance with the effect. So by analogy with geometry, or my analysis, solicitations are as dx, speeds are as x, and forces are as xx or ∫ .

Perhaps this could have been enough. But since, however, you bring up some other things most worthy of explanation, which unless I am mistaken I can explain easily, I shall try to deal with them as well. First, I shall demonstrate the axiom that Descartes and Pascal assumed to be evident, but that had seemed doubtful to you given the apparent conflict with another that seemed no less clear, and, indeed, in such a way that it is clear that one does not need to know the cause of gravity for this. And, at the same time, I shall show a reason for a difference with respect to the law of measurement, and why one of the axioms should certainly be relied upon and the other one is in error, in so far as it is opposed to the first. I hold that this law for evaluation, or rule of the truly universal mathematics, is most secure, so that a perfect repetition of some specific real measure may be employed. Thus, it is evident that a 2□8 rectangle is equivalent to a 4□4 square, since in each case the repetition of the same unit or measure, namely the little square, makes precisely sixteen.[L1] I proceed in the same way in dynamics, with the result that some effect may be taken as a real measure. So you will find that exactly as much is done when you raise one pound to four feet as when you raise four pounds to one foot. For, in both cases, there is a quadruple elevation of one pound one foot, which is an absolute, real, and self-standing effect.[L2] Suppose there is a weight ACEG of four pounds that is raised a height of one foot to BDFH, then one pound is raised one foot four times – pound A is, of course, raised a foot from A to B, pound C a foot from C to D, pound E a foot from E to F, and pound G a foot from G to H. In the same way, suppose there is a weight L of one pound that is raised to Q through height LQ of four feet, then it is clear that pound L is raised one foot through LM, and after that one foot through MN, and

after that one foot through NP, and, finally, one foot through PQ. And so there is no difference at all, except that what happened all at once in the first case happened successively in the second. This law for evaluation cannot be applied to the hitherto received methods of measuring opposed to mine. Obviously if weights A, B, C, etc. were equal, and A were given a certain speed by an initial impression, and afterward B were given a speed equal to the first in the same way, and then C likewise, etc., I admit that a real measure would be repeated exactly, since we have some body endowed with a given degree of speed three times over. But, if a new degree of speed is given to the first body alone, the repetition is not of a complete thing but only of something modal, from which a measure cannot be taken safely. And, in fact, three degrees of speed is much greater concentrated in one body than dispersed among three, and the common saying "a united force is stronger" happens to be true. The cause of this is the natural inertia of matter by which it is such that it resists motion.[15]

Suppose there are bodies L, with a size of one and a speed of one, M, with a size of three and a speed of one, and N, with a size one and a speed of three. It is evident that the power of M is in the ratio of 3:1 to the power of L, since what is in L is repeated exactly three times in M. But, furthermore, the power of N likewise seems to be in a ratio of 3:1 to the power of M, since the obstruction, i.e.,[16] the resistance, of N (on account of its smaller size, namely the inertia of its resisting matter) is one third of the resistance of M. Consequently N is nine times more powerful than L for two reasons, both because it has more speed and also because it has less resistance in proportion to the speed. For entelechies exert their action in matter, as Virgil says, "to the extent that sinful bodies do not slow them down."[17] Even though the result shows that it is absolutely correct, I would not dare trust this measure, unless I had demonstrated the same thing in another way by using the law for evaluating a real measure. At the same time, it can be gathered from this that what the incomparable Kepler first suspected about the inertia of matter is very true.[18] I arrive wonderfully at the

same measure of force by other demonstrations as well, which are a priori, i.e., from the inner nature of action and power. I omit these now to avoid being prolix and because I would think that what I have already said has quite some weight.[19]

Joachim Jung[20], Marcus Marci[21], Giovanni Alfonso Borelli[22], Huygens[23], Wren[24], Wallis[25], and Mariotte[26] had some good thoughts (in part) about these things, partly based upon actual experiments and partly upon specific hypotheses. However, I not only explain these things from their true sources, but also proceed further to cases that they have not touched on at all, or not satisfactorily. Mr. Huygens himself admitted that he had not thought about the conservation of force. After I pointed this out to him, he called the force that is conserved ascensional – not a bad idea, but nonetheless not entirely adequate. For the same thing is found in all living force, whether it is applied to gravity, or to elasticity, or simply to motion or any other effect. The law of equilibrium was presupposed by everyone, and from this they concluded that they should measure force simply by the product of the size of a body and the speed. And so (although the Cartesian conservation of quantity of motion has been refuted) they did not notice any conservation of force, which is still there nonetheless in many wonderful forms. Thus Wallis, in his book on motion, even while recognizing that quantity of motion is not conserved, nonetheless measured force by nothing other than that.[27] But according to the law of equilibrium, the measure proportional to the product of [the size of] a body and the speed has a restricted necessity, and should not be applied to absolute living force. Indeed, as I once noted in the Acta Eruditorum, the general rule for the measure of force through its effects applies no less in the case of the law of equilibrium (which concerns dead force) than in the case of the law of equivalence (which concerns living force).[28] For, in both cases, the measure of the force is proportional to the product of [the size of] the body and the distance or descent. And in the case of those first conatus that occur in bodies that are in equilibrium, the distance of the descents (which are then infinitely small)

are proportional to the speeds. But this is not the case with continued descent, i.e., when impetus is sought, and when living force arises on account of that. Nonetheless, those who are heedless have taken the one for the other. In this way, I think that I have revealed the sources both of truth and of error, and at the same time have solved the difficulty.

I will add something else that assured me of this in a wonderful way a long time ago[29], and after Mr. Bernoulli saw the truth of my measure, without any suggestion from me, he noticed through his own acuity that this measure of ours also agrees beautifully with the laws of the composition of motions.[30] Indeed, it is demonstrated by them in a certain way where it might have been feared that a divergence was possible.[L3] Let us suppose a sphere R that collides diagonally with two resting spheres S and T, which are equal size in size to it and to each other, in such a way that, at the moment of collision, their centers make a right-angled isosceles triangle SRT with R at the right angle. Let us complete the square V_1RW_2R, and let its sides around $_2R$ be extended, W_2R to $_3S$ and V_2R to $_3T$, in such a way that $_2S_3S$ (i.e., $_1S_3S$, for $_1S$ and $_2S$ coincide because sphere S is at rest before the collision) is equal to W_2R, and $_2T_3T$ (i.e., $_1T_3T$) is equal to V_2R. With these things assumed, I say that body R, after it goes from $_1R$ to $_2R$ along the diagonal $_1R\,_2R$ and collides with the two bodies there, comes to rest at $_2R$. But bodies S and T move along the sides, and S is transferred to $_3S$ with a speed proportional to $_2S_3S$ (equal to W_2R), and T is transferred to $_3T$ with speed $_2T_3T$ (equal to V_2R). This follows when two things are assumed. The first, which is known both from experience and by reason, is that one body colliding directly with another of the same size gives it all its force and direction, and for its own part comes to rest. The other, which is from the law of composition, is that S, situated at $_1S$ or $_2S$, is struck by R going obliquely along $_1R\,_2R$ in the same way as if R had gone directly along W_2R, and likewise T, situated at $_1T$ or $_2T$, will be struck by the same R going the same way, just as if R had gone along V_2R. After this happens it follows that R will come to rest and the motions of S and T are as I said.

If we compare this with our measure of force, we will see that precisely the same quantity of living power is conserved, since the power of R is proportional to the product of R and the square of $_1$R $_2$R, and the powers of S and T are the products of S and the square of $_2$S$_3$S, and T and the square of $_2$T$_3$T. Now given that R, S, and T are the same size, it is clear that the product of R and the square of $_1$R $_2$R, which is the power before the collision, is equal to the product of S and the square of $_1$S$_2$S + the product of T and the square of $_1$T$_2$T, which is the power after the collision. Furthermore, this also provides a very simple way of producing a method of the sort that Mr. Papin had demanded of me, by which the total force of a greater mass is transferred to a smaller one. He said he would concede if he were given such a thing, since I would thus have produced a perpetual motion, or an absurdity.[31] Let us suppose instead that the bodies S and T are in motion from $_3$S to $_2$S with speed $_3$S$_2$S and from $_3$T to $_2$T with a speed $_3$T$_2$T, both in the same time interval 3:2, and simultaneously strike R situated at $_3$R or $_2$R during the time interval 3:2. It is necessary that bodies S and T will come to rest at $_2$S and $_2$T after the collision and that R will go along the diagonal from $_2$R to $_1$R with a speed proportional to $_2$R$_1$R, or in the time 2:1, which is the same as the time 3:2. In this way the whole force of the mass S and T is transferred to half of this mass or to R. And so, if anyone wanted the same quantity of motion conserved, or that there was as much quantity of motion in R alone as there was previously in S and T, which have now been brought to rest, he would undoubtedly have a mechanical perpetual motion. Here Mr. Papin "became stuck, and withdrew for ten years."[32] And, although at first he almost conceded in a letter, nevertheless soon, after a few weeks, with his spirit revived[33], he denied that he allowed such a compounding of motion in this situation. He gave reasons, but I could get no sense out of them except that he was denying it. Finally, he proposed a future experiment that would perhaps not be so difficult for someone less busy than I am. I have no doubt that it would favor our position. He also denied other things that are still clearer.[34] Furthermore, it will be

worth noting here how neatly it turns out that the way geometers talk, e.g., when they say that the diagonal of a square can only be as much as its two sides, is also verified in dynamics. For if we suppose three bodies of the same size, the body moving at a speed proportional to the diagonal has the same power as the two moving at speeds proportional to the sides. So it could be said that Euclid, or rather the author (whoever it was) of this geometrical saying, in a way made a prediction about a science of dynamics that was unknown to him.

Now I shall deal at random with the remaining points in your letter, which do not really depend on the previous ones. Among other things, I observe that, although a certain number of gravitational impressions are overcome by an ascending weight, still these impressions do not act equally, nor do they encounter that on which they act in the same way. I have argued about this a great deal in correspondence with Mr. Papin, and have refuted his supposed demonstration. For even if the speed of percussion from the gravitational fluid were so great that the speed of the weight was incomparably small, and the weight could therefore be treated as at rest if you measured its speed relative to the speed of the fluid, nonetheless, since it is not a question of the effect that is produced in the fluid but that which is received by the weight, and the two states of the weight are not to be compared with the fluid but with each other, given that there are very significant differences between them, it is also reasonable that a significant difference will arise in the effects that the weight receives. In his letters, Mr. Papin offered some very subtle and seemingly strong opposing arguments. But finally, after detailed discussion, he had to look for other objections, since it was discovered that he had sinned against our infinitesimal calculus. Let me add this as well: It is one thing to overcome some force, such as an impression of gravity, another to consume it. And so, if we imagined equal forces of the fluid consumed in equal times by a weight, e.g., an equal number of particles of the fluid colliding with equal speed in equal times and brought to rest by the force transferred to the weight, then I would concede that the increments of the force

of the weight would be equal. But since this cannot happen[35], it is necessary that the particles of the fluid are deflected and retain a part of their power. So one would have to measure the power lost in order to know what the weight received from any given impact.

I also had a friendly disagreement with Huygens (alas no longer with us) concerning hard bodies, which, in my judgment, rebound and conserve force only through elastic force or by contracting and restoring themselves. If I remember rightly, he had some doubts about my principle that change never happens through a leap. You too, excellent Sir, seem to have some objection to this axiom, when you say, "It seems that a hard body moving with great speed, running into another which is immobile" (I believe you mean at rest, for properly speaking nothing is immobile) "and incomparably smaller, is retarded by it in such a way that no leap occurs." But however small a body may be, it nonetheless stands in some ratio to a big one, and there is some collision force, although a small one I admit. And the flexing[36] of each body through the resistance of its elastic element, through which the force of a collision is consumed little by little and transferred to the elastic element, must be returned by it in the same way, little by little. Furthermore, soft bodies absorb force not by destroying it but by receiving it into their small particles or by using it to break apart the threads that lightly connect them together, as when a bullet is shot through many sheets of paper. Again, the force expended breaking a thread does not perish, but is received by still more subtle matter. This is like the force that is used to pull apart two polished tablets, about which I have learned that you have performed some very beautiful experiments.[37] But, as for inflexibly hard bodies, such as we imagine atoms to be, I, like you, fully believe that there can be no compression in them and no conservation of force either. But, in turn, I think that there are no such things in nature, which I believe you will readily admit. I think that elasticity is essential to bodies on the basis of the order of things and metaphysical principles; although in nature it is accomplished by nothing other than a fluid moving around. In this I completely agree with

Descartes and Huygens.[38] But actually it follows from this that, so to speak, there are worlds within worlds, and hence that there is no first element. Although the elastic fluid itself seems uniform and simple with respect to the bodies to which it provides elasticity, in fact, however, considered with respect to itself, it consists again of bodies such as those we see. And so, for this reason, they also need other more subtle fluid for their own elasticity, and so on to infinity. Hence Descartes' second element[39] consisting of smooth spheres also does not exist,[40] and the bodies are no more spherical than our own Earth.

Would that I could explain my metaphysical meditations concerning the nature of substance and the things depending on it just as clearly, or that I might have the issues set out just like the mathematical part of the dynamics. Truly, there would be no delay in my communication, especially to men such as yourself, whose satisfaction is a good omen as far as truth is concerned. But, up to this point, it has been easier for me to reply to objections than to explain everything perfectly. And, while it seems to me that I have long been advancing at a walking pace, you easily understand on account of your utmost good sense how difficult it is to demonstrate clearly things that are sufficiently far removed from the common understanding and the prejudices of most people, and to present them so that they are safe from the treachery of ignorant men and often from the censure of the spiteful.

In the meantime, I have advanced something about the communication of substances in the Parisian and Dutch[41] journals, and lately I responded to the most distinguished Sturm, Professor at Altdorf, in the September Acta Eruditorum.[42] I also believe that my reply to the most learned and acute Bayle from Rotterdam, who had posed some objections in his Dictionary, has been published or is going to be published in the Histoire des Ouvrages des Savants.[43] He saw my reply to Mr. Basnage, which had been sent to him, and he admitted that it seemed to have some weight to it.[44] It is as if I am practicing with these skirmishes, and also preparing myself for a greater undertaking. As for the rest, I declare that, since matter is

always active and furnished with a striving, I would not want it to be believed that I think that it ever happens that that striving has no effect at all because of some obstacle. You quite sensibly write that this seems difficult to you. Thus, I am of the opinion that every striving, whatever obstacle may get in the way, has some effect but, so to speak, refracted and less full. And it is necessary that the obstacle itself suffers something from the obstruction. As for that which should be attributed to matter besides extension, I think it is clear enough that inertia (about which see above) among other things, is something that is not contained in extension alone. There is nothing for me to add about the hyperbola; I meant just what you say, as Mr. Bernoulli (along with a number of the other things which I am replying to you) has noted in the margin of the copy of your letter that he sent to me.

11. Bernoulli to Leibniz[1]

[Groningen, 7 January 1699]

I have sent your reply to Mr. De Volder. You explained the issues effectively, clearly, and elegantly. It was my duty to change as little as possible. However, based on the authority that you gave me,[2] I added some of the things that we have been discussing for a long time, namely those that struck me as likely to contribute to his being entirely converted, such as your a priori argument, "An action bringing about a double effect in a single unit of time is (virtually) twice an action bringing about a double effect in twice the time, etc."[3] Likewise, I added the demonstration by which you demonstrated that a body that has a force by which it can ascend to a height of four feet in one go can bring it about that the same body, or a body of the same size, ascends to the same height in stages.[4] I thought this was particularly necessary, since otherwise De Volder could have doubted, as I once doubted, whether raising a one pound weight L through four feet is exactly the same as raising the four one pound

weights A, C, E, G through one foot, "because in both cases there is a four-fold elevation of one pound one foot."[5] For he could still have been thrown into doubt by vacillating over whether these elevations are the same or not, i.e., over whether the one foot distances LM, MN, NP, PQ are covered in unequal times, while the one foot distances AB, CD, EF, GH are covered in equal times. But he will soon give up his doubts when he has seen that the same power really is required, whether the one pound is raised to Q in one leap, or whether it ascends through each of the individual one foot distances LM, MN, NP, PQ, by receiving a new impulse. . . . Finally, I have also told him about my way of explaining the action of gravitational matter, so that he might use it as a substitute if he has perhaps given up on the common explanation.[6] In the meantime, I hope that De Volder will come over to our side when he is willing to consider everything carefully. If this happens, I will consider it to be no ordinary gain, especially if fruitful success incites you to publish in full the thoughts that you have only given piecemeal, ignoring all the prejudices of the public and the judgments of ignorant men.

[Supplement]

Leibniz to Bernoulli[7]

[Hanover, 28 January 1696]

1. An action bringing about a double effect in a single unit of time is twice (virtually) an action bringing about the same double effect in twice the time; i.e., the traversal of two miles in an hour is (virtually) twice the traversal of two miles in two hours.

2. An action bringing about a double effect in two units of time is (formally) twice an action bringing about a single effect in a single unit of time; i.e., the traversal of two miles in two hours is (formally) twice the traversal of one mile in one hour.

3. Therefore, an action bringing about a double effect in a single unit of time is four times an action bringing about a single effect in a single unit of time; i.e., the traversal of two miles in one hour is four times the traversal of one mile in one hour.

4. If we had substituted triple effect, quadruple effect, quintuple effect etc., a nine-fold, sixteen-fold, or twenty-five fold action would have appeared; and, generally, it is clear that similar equitemporaneous motive actions of mobile things of the same size are proportional to the speeds squared, or, what amounts to the same thing, the force in the same body or a body of the same size is calculated by the product of the speed twice over. Q. E. D.

12. Leibniz to Bernoulli[1]

[Hanover, 23 January 1699]

I am very grateful that you added things to my letter to Mr. De Volder that seemed to augment it. You did so not only by the authority I had given you but also by right. Besides, you can see what might cause problems for others better than I can. I usually compare this with what I have often noticed in correcting proofs, where we generally correct other people's more easily than our own. In the same way, people other than ourselves can see more easily than we ourselves what is lacking in our work or what could usefully be added. But no one notices this better than someone who has experienced the same difficulties already and has now managed to free himself from them.

I ask you to send me what you wrote to Mr. De Volder on this matter, so that I might have the entirety of what he was given to consider – especially since it is likely that his reply will encompass all of it.[2]

Following your lead, I think that in future it would be better to leave out the word virtually in the a priori argument concerning the measure of action. It causes so much confusion that someone might question whether the different bases for measurement in each of the premises could be joined together as one in the conclusion.[3] Anyway, it is suffices to say simply, "A ducat is twice a thaler; a thaler is twice a half-thaler; therefore, a ducat is four times a half-thaler." There is no need to add, "A ducat is twice a thaler virtually, and a thaler is in fact twice a half-thaler formally."[4] And it is enough to proceed in the same way here. . . .

The way in which gravitational matter acts needs much further investigation than that up till now. And what you have said about the matter should not be disdained, even though many doubts arise in my mind on thinking about it. But this is not surprising in a matter that is difficult and far removed from the senses.

I have already thanked you, and I thank you now, for bolstering my reputation so much and for encouraging me to do things by which our view could be propagated and established to a greater extent.

13. Bernoulli to Leibniz[1]

[Groningen, 11 February 1699]

I have thus far received no response from Mr. De Volder. Perhaps he is preparing a formal response. I told you a little while ago, in my last letter, about everything that I added to your letter to him. I sent some of my objections, excerpted unchanged from our correspondence, which I thought could also occur to De Volder, along with your resolutions of them.[2] This was partly so that I might avoid the work of reformulating them, and partly so that he might know the story of how I came round to your view, so that he would not perhaps think that I had crossed over into your camp without good reason, or just to please you. I did

not omit the words virtually and formally in your a priori argument concerning the measure of action because I was not sure that you wanted them omitted. If this should cause difficulty for De Volder, you can respond to him as you once responded to me.[3]

14. De Volder to Leibniz[1]

[Leiden, 18 February 1699]

B. de Volder sends greetings to the gentleman most renowned and most excellent on account of many kinds of learning, Gottfried Wilhelm Leibniz.

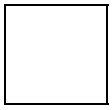
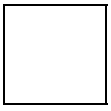
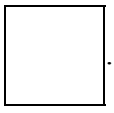
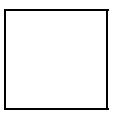
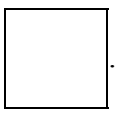
Your letter was delivered to me on 16 January. I had already heard a report of it a little while earlier, so it was not unexpected. However, your great kindness, which I happily saw there, was most pleasing to me. The only reason that I am responding to it so tardily is that my poor health forced me to abstain from all matters that require close attention. But, now that I have recovered, I did not want to delay any longer communicating some of my thoughts about the things that we have been discussing.

I admit that my difficulty concerning the balance – or, rather, as you rightly suggest, concerning the equilibrium that is attained by bodies whose speeds are inversely proportional to their size when they collide with each other – is now solved (provided that it is established that "the law of equilibrium never applies except with dead forces,"[2] or, rather, provided that your axiom "in changes there are no leaps"[3] is demonstrated, for I see that the former follows from the latter). In the case of the balance the facts are clear, since here one is concerned with the very beginning of motion. Although it is not so clearly apparent in bodies that have already received impetus, what you say in this case nonetheless commends itself to me all the more, since the persisting change of shape in soft things and the restoration of shape in elastic things show sufficiently clearly that contact is not completed in a single instant but requires a

certain length of time, albeit a very small one. I admit that, since they agree with the rest of your statements, these claims suffice for responding to the objection I made, even in the absence of a demonstration. Still, I believe that you will not deny that a full knowledge of this matter depends on the demonstration of your axiom.

Descartes' axiom is without doubt true, if we suppose that the body L (in your figure)[4] requires as much force to move through LM, as through MN, NP, and PQ, all equal to LM itself. But this is the very thing that is being investigated. If body L ascends from L to M with one degree of speed, and, having received another degree of speed at M, it ascends to N, and the same again to P and Q, then all the forces taken together, by which it ascends through LQ, will indeed be equal to the force of the body ACEG ascending through AB, equal to LM itself. If body L has two degrees of speed with which to ascend through LQ, someone who measures force by speed will maintain that the same amount of resistance will be met from the gravitational matter, and so as much force will be lost when it covers the distance LP – which is three times that of PQ – as when it covers PQ. But he will say that this follows from the fact that it takes the same time to traverse that distance.

Next, you add another argument taken from the composition of motion. I admit that the only thing I have against this argument, which I do oppose, is that it does not rely on the general nature of bodies, but the particular nature of perfectly elastic bodies. In these the conservation of what you call force is due not so much to the nature of the bodies as to the nature of elasticity. I remember, many years ago, when I was examining Huygens' laws of motion, which he had published without demonstration in the journals[5], that I derived the same laws for elastic bodies from these two hypotheses: [First,] the quantities of motion (which I was calling the forces) of two bodies going in opposite directions cancel each other out (which I thought was obvious from the equality of the opposing forces), but any remaining quantity, as well as any going in the same direction, is conserved and distributed

between each body in proportion to the size of the bodies. From this hypothesis I derived the laws of motion for soft bodies. The other hypothesis was that, since the force of the collision between bodies that are the same is the same whatever the ratio of their speeds (provided that the intervening distance is covered in the same time), the elastic force should be measured by this speed, and, therefore, restored in a ratio that is inversely proportional to the size of the bodies. With these things in place, I reasoned as follows: Suppose a body a, with speed c, and a body b, with speed f, collide with each other moving either in opposite directions or in the same direction.[D1] If the bodies are not elastic, after the collision they both proceed from A to B with speed  (if I suppose that ac is greater than bf in the case of a collision in opposite directions). But, if they are elastic, the elastic force will be c ± f and the speed must then be distributed between each of the bodies in a ratio that is inversely proportional to their sizes. Therefore, if it is the case that a + b is proportional to c ± f, as b is proportional to the speed which the elasticity gives back to body a, and as a is proportional to the speed which the elasticity gives back to body b, then a^6 will obtain from the collision a speed proportional to  from A towards B, and from the elasticity it will acquire a speed proportional to  . Therefore, the difference between these will be the speed of a itself, which is proportional to  , from A towards B, if this quantity is positive, and from B towards A, if it is negative. In the same way, b will have from the collision a speed proportional to  from A towards B, and from the elasticity it will also acquire a speed from A towards B, proportional to  . Therefore, the sum of these will be the speed of b

itself, proportional  , with which it proceeds from A towards B. When I noticed that Huygens' rules of motion follow from this conclusion, I had no doubt at all that these hypotheses of mine agreed with Huygens'. However, after the death of the illustrious gentleman, when it was possible to see his little book on motion, I discovered that I was wrong about this. For there, with no mention made of elasticity, he derives his whole position from this hypothesis: two bodies of equal hardness and with equal motion, colliding with each other directly, both rebound with the same equal speed conserved.[7] It is no surprise that the laws of elastic bodies follow from this hypothesis, since it would not be true unless it tacitly included the very nature of elasticity.

Moreover, something follows from these things that is of great importance for our discussion, namely that in the collision of perfectly elastic bodies the product of the sizes of the bodies and the squares of the speeds always remains the same. For if the square of  is multiplied by a, and the square of  is multiplied by b, the sum of these products will equal acc + bff. And, in the same way, it is confirmed that the force of elastic bodies should be evaluated in accordance with this measure, since if bodies a and b collide with each other again with the speeds acquired after the first collision, after this second collision body a will have a speed proportional to c, and b will have a speed proportional to f. And so here the cause will be exactly as powerful as the effect, since the one follows precisely from the other, and vice versa.

But, although this may be what happens with perfectly elastic bodies, nonetheless I do not see how you can apply this to the measurement of the force of all bodies. Indeed, it is even less applicable on my hypothesis, since in elastic things these very forces follow from some loss of force in inelastic things. Where the force of bodies is concerned, we should not consider that which follows from a particular property of bodies, but that which is the same in

all bodies, whatever their other conditions might be, since it arises from the generic nature of all bodies. And so, most renowned Sir, if you would like us to agree with you without any scruples, I think that it will be necessary to descend to the notion of substance, and to demonstrate that it is necessarily active from its nature, or certainly that the nature of corporeal substance is such that it is necessary that its forces are always conserved. For, unless this is demonstrated a priori, it will not be so easy, certainly for me, to conceive that the existence of mathematical body is a complete contradiction.

As for what you add about the inertia of matter, I do not see that that denotes anything distinct from extension. Each thing has from its nature a force for continuing in its state, which does not differ from the very nature of the thing, and in the case of extension this is inertia.

I have read through what you say in the Journal des Savants, the Acta Eruditorum, and the Histoire des Ouvrages des Savants concerning the nature of substances, the communication between them, and their forces.[8] But, for the most part, I see it treated so briefly and concisely that I dare not persuade myself that I have understood what you mean. You seem to me to deny that extension is a substance, when, nevertheless, that, if anything, is conceived through itself, i.e., it is conceived in such a way that one thing is represented by the concept. But in extension alone there is no real unity, but only an aggregate of many parts. For me, on the contrary, it is more difficult to conceive of really distinct parts in extension than unity. For if there is indeed no empty space, as you submit, it will not be possible for one part, which anyone might imagine for themselves, to be conceived without the others. From this it seems to follow that there is no real distinction between them, but that the distinction between parts that is imagined in these things consists not so much in a difference of substance as in a difference of modes. You add that, besides extension, a certain force is required that is like the soul, and that you think that it does not fall under the

imagination but is nonetheless clearly understood. For me it does not even fall under the intellect, as long as I do not know the cause or foundation of the force. When I talk about forces, I do not understand the cause of the forces. I am talking about nothing other than the effect. I would only understand the forces themselves when I knew what they were, where they arose from, and how they produced the effect necessarily. But I do not want to concern myself with delving into these things any further, since I am quite uncertain whether I understood what you mean correctly, and so I fear that the objections will completely miss the mark, and that I will exhaust both of us. However, there is one thing that I cannot pass over in silence. Your account of the communication, harmony as you call it, between mind and body pleased me a great deal. I shall try to explain how I have understood this with an example. Whether I have done this correctly or not will be for you to judge.

Let us imagine a machine put together from many parts and carrying out its operations by its own forces. Let us also imagine a mind in which there is an adequate idea of this machine and of all its parts. This will certainly represent that machine very exactly. Nothing will be represented in it that is not in the machine, and whatever change obtains among the parts of that machine, a representation of this will follow necessarily from the idea. And, vice versa, whatever follows from the idea will necessarily obtain in the machine, but in such a way that what happens in the machine happens from the nature of the machine and what happens in the idea happens from the nature of the idea, which is a representation of the machine. For if things obtained in the machine that did not follow from the idea of the machine, or, vice versa, certain things followed from the idea that did not obtain in the machine, it would follow that this idea was not the idea of that machine, which is contrary to the hypothesis. If we now imagine that there is a mind that has the idea of one of the parts of this machine, that idea will have the same relation to the adequate idea of the whole machine as that part has to the whole machine. And, just as everything that happens in the whole

machine follows very evidently from the adequate idea of the machine, in the same way from the idea of each part will follow everything that happens as a result of the force of this part alone. But a mind that has only the idea of this part will not perceive clearly, but confusedly, those things that obtain due to other parts of the machine acting on this part, i.e., it will sense them, since, although they will not in fact follow from the mind's idea, they will nonetheless have some relation to it. And, if things happen in the machine due to its other parts that do not affect this part, just as the part will not be changed at all by those things, so the mind that has the idea of this part will neither perceive nor sense anything of these things.

As I freely admit, it is a great task to demonstrate clearly things that are quite remote from the common grasp and the prejudices of most people. This is the very reason that I think that you are the right one for job. But I beg that the things that you say about criticisms will not deter you. For believe me, most renowned Sir, your reputation in the minds of all who cultivate letters and the sciences is such that you should fear nothing from either ignorant or malevolent people. If there is anything to be feared from those people, the danger arises more when paradoxes are proposed without the foundations on which they depend than when these same things are defended from everyone's scoffing with demonstrations derived from first principles. Even if there are perhaps few judges who understand these matters and love truth alone, the force of demonstration with these people is such that the rest will not even dare open their mouths. And if anyone should attack you they will harm themselves more than you, and, as they completely ruin their own reputation, such as it is, they will augment yours. So I would earnestly entreat you again and again not to deprive good people of your thoughts because of the stupidity or ill-will of evil people. Of course, it is up to you to decide on and choose a course of action. We must be satisfied with whatever you do. Goodbye.

From Leiden,

18 February 1699 A. D.

[Supplement][9]

The most distinguished Bernoulli has very kindly sent me some excerpts from the letters that were sent back and forth between you concerning this matter. In them I find an argument by which you conclude that the force of a body L is the same whether it ascends from P to height T by a continuous motion, or whether it ascends in four repeated stages PQ, QR, RS, ST, with a new stimulus each time.

<<set fig. 5 here>>

I do not know why I cannot see the force of this argument. But I can all the more easily persuade myself that I am at fault, since I see that the most wise Bernoulli accepts it. Still, I shall add what I am missing in it, or rather what I do not understand, so that from your kindness you can free me from this blindness more easily. I readily grant that if a weight L descending from T to S has the force by which a spike F overcoming a spring G arrives at 2F from 1F, then the same weight descending from T to P makes the spike starting at 1F arrive at 5F overcoming four springs, and that the same springs have a force by which they bring body L to T by four recoils. But I will deny that the forces by which it descends from T to P in this way taken together are equal to the force by which body L would be able to ascend from P to T in one go. For, since the impetus that the body acquires from T to S is removed by the spring F, that same impetus must be acquired four times so that it may arrive at P from T. Since, if the impetus acquired at S were conserved and not removed by an opposing thing, then body L would eventually acquire double the impetus at P, not four times. And this double impetus is enough for body L to ascend from P to T.

In the same excerpts I see another, metaphysical, argument that you use for deriving your measure of force a priori from the nature of action. I get completely stuck with this, particularly since the words virtually and formally that you employ confuse me, and I do not

clearly understand what you mean by them. You say: "An action bringing about a double effect in a single unit of time is twice (virtually) an action bringing about the same double effect in twice the time."[10] I understand this: The first action during any given period of the time through which it acts certainly traverses twice as much of its path as the other action traverses in an equal period of time. I would conclude from this that during those equal periods of time the one action is double the other, since the effect is double (although on your view the action will be quadruple). And hence, if you consider the total time and the entire action of each, the first action is equal to the second. If two men transported two equal bodies in such a way that one proceeded continuously, whereas the other divided his time – now moving, now at leisure, according to the rule that the time at leisure equals the time working – but during the time spent moving he used a force by which he covered twice the distance, then surely after both bodies had been moved through the same distance, the force of the man who had completed the task with no interruption would be equal to all the forces of the other taken together? The effect indeed would be one and the same. And 2 + 0 = 1+1, which cannot be said of 4 + 0.

15. Bernoulli to Leibniz[1]

[Groningen, March 4 1699]

Here is De Volder's reply, which I received ten days ago. It seems that his main desire is for your axioms to be demonstrated: that in changes there are no leaps; that elasticity is essential to bodies; that every substance is necessarily active from its own nature; that the inertia of matter is something distinct from extension; that besides extension a force is needed, which is like a soul; and that this force can be conceived without its cause and ground. As you foresaw, the words <u>virtually</u> and <u>formally</u> caused him difficulty.[2] He does not

seem to have understood the strength of the argument concluding that the force of a body is the same whether it ascends to a given height continuously or in stages. So you should reply to all these things, if you see fit.

16. Leibniz to Bernoulli[1]

[Hanover, 14 March 1699][2]

Here is my response to Mr. De Volder.[3] You may judge whether it is excessively long or not, and if it seems that something is left out do add it in your own letter. For I have no doubt that you can do this very well, and that you can easily see what may cause others difficulty. I have also dared to leave you with the burden of satisfying De Volder about the argument from the equivalence of continuous ascents and ascents in stages. For I cannot remember what was once at issue between us, and that letter is not to hand but buried among others.[4]

17. Leibniz to De Volder[1]

[Hanover, 3 April 1699]

Gottfried Wilhelm Leibniz sends greetings to the gentleman most celebrated on account of learning and merits, Mr. B. de Volder.

A more outstanding example could not easily have been found of your most keen judgment and exceptional love of truth than the one that you gave me in your excellent letter that was also most full of kindness. Would that I could answer it as satisfactorily as I would like. But it is something to advance to a certain point. The things that are not yet ready to be defended by rigorous proofs will meanwhile commend themselves as clear hypotheses that

are beautifully consistent with themselves and the phenomena. I also believe that most of these things will seem certain to anyone considering them carefully.

The axiom in changes there are no leaps, which I use, is like that. I believe that it follows from the law of order and depends on the same reasoning by which everyone recognizes that motion does not take place through leaps, i.e., that a body does not move from one place to another distinct place unless it goes through the intervening ones. I admit that once we have assumed that the author of things was pleased by continuity in motion, by this very fact, leaps are excluded. But how do we prove that this pleased him except through experience or the principle of order? For, since everything happens through the perpetual production of God and, as they say, by a continuous creation, why could he not have, so to speak, transcreated a body from one place to another at a distance from it leaving behind a gap either in time or space, e.g., by producing the body at A and immediately thereafter at B etc.? Experience teaches that this does not happen, but the principle of order, which makes it the case that the more things are analyzed the more they satisfy the intellect, establishes the same thing. This does not happen with leaps where in the end the analysis leads us, so to speak, to mysteries. Consequently, I think that the same thing obtains not only in changes from place to place, but also from form to form, or from state to state. Experience also refutes all changes through leaps. And I do not think that any a priori reason can be given against a leap from place to place that does not also militate against a leap from state to state.

I think that the way that you deduce the rules of motion for two colliding bodies from the fact that the motion is destroyed by the collision and restored by an elastic force is absolutely right, provided that something is added that makes it more comprehensible, namely, as we have already agreed, that each happens successively in accordance with the law of equilibrium through the intervention of dead forces. I have often been surprised that our Huygens hoped to deduce this from something other than elastic force, which, as you

rightly point out, he could not have done without tacitly presupposing it.[2] If it seems acceptable to those of you to whom he entrusted his posthumous writings, perhaps I could at some time add his little book on motion to my dynamics[3] and embellish my work with such an outstanding addition, since the connection between the topics and the similarity of the arguments suggest that it will be useful and pleasing to compare our different results. But where more than two bodies collide together, he does not seem to me to really get to the truth. I infer this from a certain proposition that he assumes in his book on light that does not seem true to me.[4] As for the claim that this measure of force can be applied to all bodies, I will now indeed confirm it by satisfying all the difficulties that can be raised against it. According to my hypothesis, to the extent that bodies are not perfectly elastic, the force is taken up by their internal parts, which are elastic themselves, and so it does not perish but just becomes insensible. You will not deny that these things are consistent with what is natural and orderly, i.e., with experience and reason. Without elasticity one could not obtain the axioms of avoiding leaps and of the conservation of forces, absolute or relative. Nor could one obtain the reconciliation of the laws of dead and living force and of the composition of motions with the quantity of force. And indeed these things cannot be demonstrated except from the supreme law of order, for they are not absolutely necessary, with contraries that imply a contradiction. The universe could have been constituted in innumerable ways, but the one that was supported by the greatest reason prevailed. However, the activity of substance is more of a metaphysical necessity and, unless I am mistaken, it would have had a place in every universe.

I do not think that there is a substance constituted from extension alone, since the concept of extension is incomplete. Nor do I think that extension is conceived through itself, but that it is a resolvable and relative notion. For it is resolved into plurality, continuity, and coexistence, i.e., the existence of parts at one and the same time. Plurality also belongs to

number and continuity, also to time and motion, while coexistence is only added in that which is extended. But from this it appears that something must always be assumed that is continued or diffused, such as whiteness is in milk, color, ductility, and weight are in gold, and resistance is in matter. For, in itself, continuity (for extension is nothing but simultaneous continuity) no more completes a substance than multitude or a number, where there must be something numbered, repeated, and continued. And so I believe that our thinking is completed and terminated more in the notion of dynamism than in that of extension, and no other notion of power or force should be sought than that it is an attribute from which change follows whose subject is substance itself. And I do not see anything that escapes the intellect here. The nature of the case (I think) does not allow anything more clear, such as a picture. I think that that which is extended has no unity except in the abstract, namely when we divert the mind from the internal motion of the parts by which each and every part of matter is, in turn, actually subdivided into different parts, something that plenitude does not prevent. Nor do the parts of matter differ only modally if they are divided by souls and entelechies, which always persist.

I have noted that, following the example of Kepler, in his letters Descartes also recognized inertia in matter.[5] You derive this from the force that any given thing has for persisting in its state, which does not differ from the very nature of the thing. Thus, you think that the simple concept of extension is enough even for this phenomenon. But the axiom concerning the conservation of a state itself needs modification, since, e.g., a thing that moves in a curved path does not preserve curvedness intrinsically but only direction. But, allowing that there is a force in matter for preserving its state, there is certainly no way that that force can be derived from extension alone. I admit that each and every thing remains in its state until there is a reason for change. This is a principle of metaphysical necessity. But it is one thing to retain a state until there is something that changes it, which even something

that is intrinsically indifferent to both does, and another, which is much more significant, for a thing not to be indifferent but to have a force and, as it were, an inclination to retain its state, and so to resist changing. And so once, in a certain little book published in my youth, assuming matter to be intrinsically indifferent to motion and rest, I inferred from this that a very large body at rest should be moved by a colliding body, however small, without weakening the colliding body, and from this I inferred rules of motion abstracted from the system of things.[6] And such a world could certainly be imagined, at least as possible, in which resting matter would obey that which moved it without any resistance. But such a world would be pure chaos. And so two things on which I always rely here, success in experience and the principle of order, brought it about that I later recognized that matter was created by God in such a way that there is a certain repugnance to motion in it, and, in a word, a resistance, by which a body intrinsically opposes motion.[7] And so a body at rest resists every motion, and a moving body resists one that has greater motion even in the same direction, so that it weakens the force of the thing impelling it. Therefore, since matter intrinsically resists motion by means of a general passive force of resistance, but is put into motion by a special force of action, i.e., an entelechy, it follows that, even with enduring motion, inertia constantly resists the entelechy, i.e., the motive force[8]. From this I, showed, in the preceding letter, that a united force is stronger, i.e., that the force is twice as great if two degrees of speed are united in a one-pound body as it would be if they were dispersed through two one-pound bodies, and, thus, that the force of a one-pound body moving with twice the speed is twice as great as that of two one-pound bodies moving with unit speed because, although the amount of speed is the same in both, in the single one-pound body the inertia of the matter hinders only half as much. This inequality of force between a single one-pound body and two of them having speeds inversely proportional to their sizes has been demonstrated in another way from our measure of force. However, it is elegantly derived

from the consideration of inertia as well. So completely does everything agree. And so the resistance of matter contains two things, impenetrability, i.e., antitypy, and resistance, i.e., inertia. And, since they are everywhere equal in body, or proportional to its extension, I locate the nature of the passive principle, i.e., of matter, in these, just as I recognize primitive entelechy in the active force exercising itself in various ways through motion and, in a word, something analogous to the soul, whose nature consists in a certain perpetual law of the same series of changes, which it runs through unhindered step by step. We cannot do without this active principle, i.e., the ground of activity, for accidental or changeable active forces, and motions themselves, are certain modifications of some substantial thing. But forces and actions cannot be modifications of a merely passive thing, such as matter is. It follows, therefore, that there is a primary active, i.e., substantial, thing, which is modified by the added disposition of matter, i.e., that which is passive. Hence secondary, i.e., motive, forces and motions themselves should be attributed to secondary matter, i.e., to the complete body itself that results from the active and the passive.

And so I come to the communication between the soul, or any entelechy of an organic body, and the machine of the organs. I am happy that my hypothesis concerning this does not completely displease you, a man of such keen understanding and judgment. And indeed, you illustrate the hypothesis very clearly, attributing to the soul an adequate idea of the corporeal machine. This is the very thing that I mean when I say that it is the nature of the soul to be a representation of the body. And so, it is necessary that the soul represent to itself in order whatever things follow from the laws of body, some distinctly, but others confusedly (namely those that involve a multitude of bodies). The first of these is to understand, the second to sense. However, I think that you agree with me that the soul is one thing, the idea of the body another. For the soul remains the same, but the idea of the body is constantly one way then another, according to how the body, whose present modifications it always exhibits, changes

itself. Of course, the idea of the present state of the body is always in the soul, but it is not simple, and so neither is it purely passive. Rather, it is joined to a tendency toward a new idea arising from the previous one, so that the soul is thus the source and ground of different ideas of the same body, arising according to a prescribed law. However, if you take the expression adequate idea in such a way that it signifies not that which is changed, but the persisting law of change itself, I do not object. And, in that sense, I will say that the idea of the body is in the soul and that the phenomena follow from it. For the rest, in all these things there are some questions that should be analyzed in more depth, which I shall not fail to do when the occasion arises. For, even though I cannot easily present everything in such a way that it is demonstrated a priori with geometrical rigor or explained deeply, even when I see the reasons, nonetheless, I venture to promise that no objection can be raised that I would not hope to answer. I think that this is something that should not be disdained in matters so remote from the senses, especially since the agreement of doctrines, both with the phenomena and among themselves, is among the most powerful marks of their truth. And objections that have any weight are always useful for making the nature of the matter more apparent. And so I recognize how very much I and all lovers of truth are obliged to you. For I feel that you illuminate things for me in such a way that I seem to understand my own thoughts better when I read yours. Perhaps with the help of you, Mr. Bernoulli, and other such people (would that there not so very few of them), I can progress to the point that I can secure what I am currently defending in various ways with clear demonstrations. If so, I shall not begrudge others my insight owed in large part to you people. I shall certainly fear the opinions of others less by relying on your judgment.

But I wish that I could demonstrate everything as fully as it seems to me that I have demonstrated, by the argument that Mr. Bernoulli recently sent you from our correspondence, that motive actions completed by the same body in the same period of time are proportional

to the squares of the speeds.[9] I foresaw there, and recently wrote to him, that I was afraid that the words formally and virtually would trouble you.[10] In fact, the demonstration proceeds just as well with them left out. However, so that their sense is clear here, I will explain with an example that is just like our case. A ducat is twice a thaler virtually (for it is equivalent to the value of two thalers, so I am supposing). A thaler is twice a half-thaler formally (for it actually consists of two half-thalers, and is equal to them in value on account of that). And so a ducat is four times a half-thaler. And it does not matter whether the one is contained in the other formally or virtually, provided that its value, i.e., its purchasing power, is contained, since the conclusion follows from the weaker condition. It is the same in our case. To take up the argument again: In a uniform motion of the same body, (1) "An action bringing about a double effect in two units of time is twice an action bringing about a single effect in a single unit of time."[11] For example, the action of covering two leagues in two hours is twice the action of covering one league in one hour. For the first action formally contains, i.e., exactly repeats, the second one twice, since it traverses one league in one hour twice. (2) "An action bringing about a single effect in a single unit of time is twice an action bringing about a single effect in twice the time." For example, the action of covering one league in one hour is twice the action of covering one league in two hours. Clearly, that which produces the same effect more quickly does more. I also assume that actions producing the same effect are proportional to the speeds, or inversely proportional to the times. And so an action that traverses a distance at twice the speed is twice as efficacious as an action traversing the same distance at unit speed, i.e., which amounts to the same thing, the latter is contained virtually in the former twice. From here the conclusion now follows, namely (3) "An action bringing about a double effect in two units of time is four times an action bringing about a single effect in the same double amount of time." For example, the action of covering two leagues in two hours is four times the action of covering one league in two hours.

In the same way, it may be shown that an action bringing about a triple effect is nine times an action bringing about a single effect in the same time. And generally, actions lasting the same time are proportional to the squares of the speeds. Q. E. D. But I ask that you give careful attention to every single claim, since they contain something hidden. Let the action of covering two leagues in two hours be called L; let that of covering one league in one hour be called M; and let that of covering one league in two hours be called N. It is clear that M is rightly interposed between L and N, the ratio of these actions (contained in the conclusion) being that of actions that last the same time but with different effects. And so the ratio of L to N is naturally composed from the ratios of L to M and of M to N. For the ratio of L to M (contained in the first premise), which is of actions of the same speed but with different effects (i.e., the same as the ratio of those effects), is absolutely simple and not resolvable any further, since it is resolved here, i.e., it is measured by a formal repetition. But it is also the case that no simpler ratio can be interposed between M and N. Therefore, I rightly hold from the second premise that the ratio of actions M to N having the same effect but different speeds is the ratio of the speeds – just as the ratio of the actions from the first premise that have the same speed is the same as the ratio of their effects. So actions that are different both with respect to their speeds and their effects are calculated by the products of the effects and the speeds, i.e., the action of covering two leagues with twice the speed is four times the action of covering one league with unit speed, where it is clear that the actions take the same time. So again you have another consistent statement of the demonstration at hand.

Moreover, a new and admirable conclusion follows from this: The same quantity of motive action is conserved in the world, i.e., (so it may be understood properly) the same amount of action is operating in the universe in any one hour as in any other hour. It should be explained thus, so that it is stated in a more accurate way than usual, and then the conservation of action fits together nicely with the measure and conservation of force. This

could also have been predicted for other reasons, since action is nothing other than the operation of force over time, i.e., proportional to force multiplied by time. So actions are calculated by the products of times and forces, and thus actions taking the same time are proportional to their forces. But, as we show here, actions taking the same time are proportional to the squares of their speeds. Therefore, things are as they were before. Forces are indeed proportional to the squares of their speeds, and everything fits together most beautifully. I have always regarded the current demonstration to be of greater importance because it does not employ weight, or elasticity, or any other hypotheses or accidental features, but arises most clearly from primary and maximally abstract notions. The true solution to a problem that has confused others is also clear from this, so that we should no longer be surprised that the same quantity of motion is not conserved. Certainly Descartes set himself a good goal, but he got lost along the way. He saw, as through a fog, that the quantity of action and force is conserved. But taking one thing for another, as they say, he took what he calls quantity of motion for motive action and the measure of force. This gave rise to an intolerable opposition between measuring force from the degree of speed and measuring it through the effect, and the absurdity of perpetual motion and of an effect that is more powerful than its cause followed from this. Now that the darkness has dissipated, all these things vanish away. It will no longer be a paradox that the same effect is always produced by different quantities of motion, since the action is always equal when the time is the same. As for the argument taken from the simultaneous or successive ascent of a weight, since I cannot remember well enough what Mr. Bernoulli and I once said about this, and so what he might have sent you from our correspondence, I am now asking that most intelligent man if he is willing to explain the matter to you himself.[12]

Goodbye, excellent Sir, and take care of your health as well as you can. For when you said that it has not been at its best lately you filled me with anxiety, as anyone would be who

knows how much the world of learning still has to gain from your remaining with us as long as possible. From Hanover, 24 March/3 April 1699.

18. De Volder to Leibniz[1]

[Leiden, 13 May 1699]

B. De Volder sends greetings to the most excellent gentleman Gottfried Wilhelm Leibniz.

Words cannot express how much pleasure your letter brought me. I received it later than I would have done otherwise, since I was out of town. Certainly, there is genuine cause for me to be extremely grateful to you for spending so much time on me when you are distracted on all sides by so much of your own business. I would really like to agree with you about everything, but, whether it is because of the obscurity of the matter or the dullness of my mind, I do not properly understand everything that you say. And so, not only can I not agree with you without hesitation, but, because of this, I also hesitate over whether I am giving objections that are to the point. However that may be, to the extent that I can, I will indicate briefly what is primarily causing me difficulties.

First of all, I really do not understand how we can determine things that are not absolutely necessary from a law of order. For those that are known by reason certainly cannot be otherwise. Nor can experience do anything in this respect. First, since we are concerned with a universal property of bodies, how can this be known by experience, which is entirely of particular things? Second, since the things that are deduced from an experience have a necessary connection with it, to that extent they also show that this world, which includes this experience, could not have been arranged in any other way than that it include in it all the things that follow from the experience.

Indeed, since the cause is always equal to the effect, I believe that it follows that there will be as much diversity in causes as in effects, and vice versa. I do not think that a certain effect can be produced by different causes, unless those causes that are said to be different agree completely in all the things that are required for producing the effect, in which case they are not really different. If, however, we suppose that it was possible for this same universe to be constructed in many different ways, in that case I find no basis on which I would have preferred one over another. Indeed, I do not understand that which you are calling the law of order. We are accustomed to making a kind of order with our intellect, by which things may be conceived or retained by us more easily, but what has this to do with the actual universe?

These things hinder me, which is why those arguments that are derived from order, particularly Malebranche's,[2] have little effect on me, as do those that are derived from what is pleasing to God – i.e., from an extremely obscure principle it seems to me, which very often has no foundation except our ignorance. Someone who knows nothing about a triangle other than that it is a figure with three sides will no doubt imagine that, if he had so desired, God could have made a triangle that has angles equal to two right angles and another that has more or less. For he finds no contradiction. However, he would have found it easily if he had had better knowledge of the nature of the triangle.

Given these things, most renowned Sir, I believe that you will easily see why I do not understand the argument that you derive from order. And I do not really follow what you mean when you say, "the principle of order makes it that case that the more things are analyzed the more they satisfy the intellect."[3] Nor do I see what the intellect could find wanting in the following state of affairs: two perfectly solid bodies, in a word mathematical bodies, which are the same size and moving in opposite directions with any equal speed, collide with one another and are transferred from motion to rest in a single leap. I ask you,

what is there here that is not understood completely, given the hypothesis that there are bodies like this? What is there in what was said here that is against order and reason, which commands me to alter the hypothesis that I think I have understood clearly? Certainly nothing occurs to me that does not seem reasonable, nothing that leads me to mysteries.

But "experience refutes all changes through leaps." Most of them. I do not deny that. I do not believe that even you yourself meant all of them, for experience does not reach to insensible bodies. But you add that no a priori reason can be brought against a leap from place to place that does not militate against a leap from state to state. This filled me with a certain surprise, not so much at what you said, but because I realized that I am not sure whether I would have ever investigated this matter, which may perhaps have happened with other things as well. However, here are the things that come to mind as I am thinking about it now. If a break in time were allowed, a body would be at rest during that time and at rest afterwards without a new cause that brought from it from rest to motion. If there were a break in place, there would have to be a cause of the interruption and of just that amount of interruption. But there is no such thing in motion, for I cannot conceive of motion without direction, nor of direction except in a straight line (for curvature is continuous change in direction) that is continuous from its nature. Therefore, if an interruption did ensue, the cause of the interruption would be something other than the motion, and it would be external to the thing that was moving. And so, since it is agreed that no body can act at a distance (unless perhaps it seems to you that this should be demonstrated as well), how can some principle be conceived in a body that is in a given place from which an immediate translation to a distance of a foot or a mile might follow? Moreover, in our case, in the example that I brought up earlier, I think that it is obvious why they come to rest in a single leap. But it is not clear to me what kind of cause there is in motion for an interruption in place.

But if God had intended it in this way, i.e., if it had happened in this way, I have no doubt that, in that case, bodies would have had some other foundation in them from whose concept the interruption would have followed necessarily. And it could not be known that God had willed this universally (though in particular cases experience could show it), unless there was a concept of this foundation and the interruption was known to follow from it. For the same reason, it cannot be known that in changes there are no leaps, unless we derive a foundation from the nature of bodies from which every leap may be ruled out. As long as this does not happen this conclusion will be uncertain for me.

But, if you intend this claim to be regarded as a hypothesis, not only do I not object, but I am also most willing to grant that this hypothesis of yours is not subject to the same difficulty as the one that postulates inelastic bodies. For, on that hypothesis, unlike on your hypothesis, there are causes that destroy force and quantity of motion but none that increase them. But, if I might be allowed to argue in the following way: How do I know that God did not will that the force of inelastic bodies disappears by degrees? However this may be, for my part, if it could be obtained, I would like to know something more certain, rather than employing a mere hypothesis as this universal foundation.

Nevertheless, I am very happy that you consider the activity of substance absolutely necessary. So this will necessarily follow from the nature of substance, which is one of those things that I would very much like to see demonstrated for many reasons.

You deny that extension alone constitutes a substance. So that this is not just a disagreement about a word, I will say what I mean by the word substance. I form its notion not from things but from concepts: first, because all our knowledge depends on these alone; and second, because the notion of substance is a mere concept, or, as they say, a being of reason, since it can even apply to things that have no common attributes, should there be any. Moreover, I find this difference among my concepts: sometimes they represent some one

thing to me, from which I can separate nothing without the whole thing perishing, sometimes two or more things. And, in the second case, there is also a difference, in that I can conceive one without the other, and this either reciprocally or only in one direction. If reciprocally, as with the concepts of extension and thought, where the one does not involve the other, and vice versa, I do not say that this one concept but two. But if it only happens in one direction, I say that this is the concept of an accident or mode, and its object an accident or mode – as when I think of motion, whose concept contains both extension (which I can conceive without motion) and translation (which one cannot conceive without extension). If, in fact, only one thing is represented to the mind, as when I think of extension, from whose concept I can separate nothing without the whole thing perishing, I say that this is the concept of a substance and its object a substance. In this sense extension alone will indeed be a substance. If you understand anything else by the word substance, I beg you to explain what you mean.

I am most surprised that you say, "no other notion of power or force should be sought than that it is an attribute from which change follows, whose subject is substance itself." You add, "I do not see anything that escapes the intellect here." I myself see nothing that the intellect may grasp. Certainly subject of changes is a mere logical notion, explaining nothing at all. I ask you then, what do these words signify, besides the fact that a thing is changed and that there is some cause of its change?

You are of the opinion that, "that which is extended has no unity except in the abstract." But I think that I perceive it even if we suppose that that which is extended is divided into parts that are moving around in different ways. As far as I am concerned, where one thing can neither exist nor be conceived of without another, and vice versa, they are one thing. Moreover, since it is inconsistent for a vacuum either to exist or be conceived of, it is inconsistent, if we are willing to speak this way, for one part of matter to be conceived of or to exist without all the rest.

But, “the idea of extension is resolved into plurality, continuity, and coexistence.” I do not think that it is resolved into plurality. The continuity of extension is extension itself, which is only in motion because motion involves extension. And it is not in time, because, although the continuity of time may be signified by the same word, it is nothing but a being of reason, like time itself. Moreover, existence adds nothing to the nature of existing things, and so coexistence does not either. And although it follows from the nature of extension, it adds nothing over and above that to it.

Next you judge that the inertia of matter differs from extension, because through extension matter is indifferent to motion and rest, but through inertia it resists changing. This confuses me. I recognize no other indifference in extension than that through which it receives motion and rest, if there is a cause that produces the one or the other. And I do not think that it follows from this that any force at all, even the least, may produce any motion at all, even the greatest. If no inertia distinct from extension were ascribed to matter, do you think that it would be the case that the smallest body endowed with any motion whatsoever would set the largest in motion without any loss of its own motion? The very nature of cause and effect, which maintain a certain fixed proportion between one another, seems to be inconsistent with this state of affairs. Certainly, it requires a greater efficacy to move a greater distance than a lesser one with the same speed. But this is because the effect is greater. Besides, resistance itself does not seem to be mere passivity. For I believe that to the extent that it resists it strives that much in the opposite direction.

“Changeable active forces are modifications of some substantial thing.” Without doubt. “But they cannot be modifications of a merely passive thing.” But there will be no such thing, if it is indeed demonstrated that every substance is active. My greatest difficulty rests with whatever truth there is in this. I conceive of extension and of the infinite variety of its modes. I seem to be able to conceive of it equally well with all of its parts resting or

moving variously. Nevertheless, I know from what happens that it is changed and so moves. And it is clear to me from what happens that I cannot do without an active principle. But, if I had a demonstration that every substance is active by nature, I believe that at the same time I would have a demonstration that all extension moves necessarily. And I would have the true cause of the motion and, therefore, of all change, whose explanation has remained closed off to all natural philosophers up till now. But it seems to me that for this to happen it would not just be required that I was aware that there was a certain active principle, i.e., some cause of the change, but also that I knew what it was and whence it arises. And I do not understand how the concept of force differs from the concept of power that the Scholastics fashioned (which I am certain that you will not agree with), unless it is explained what the foundation from which force arises is in reality and it is shown at the same time that this force that I am supposing, i.e., that which produces the effect, necessarily follows from that foundation. I might have said the same about the expressions active principle, entelechy, inertia, etc. But if these things were explained we would not worry about terminology.

May I ask, is the active principle extension itself, a mode of extension or, in fact, some other substance distinct from extension and, therefore, having nothing in common with it? If it is either extension or a mode of extension, extension will not be a merely passive principle. If it is another substance, how can it act on extension? And, unless I do not understand your communication theory properly, it will not be of any use in this matter. For, if I was close to what you mean with my example (which I am glad is not entirely displeasing to you), it follows that the soul is not an active cause of matter but only a representation, and that another force, i.e., an entelechy, must therefore be conceived of in matter, which belongs to the matter. If you distinguish this from extension, it will have to be explained how it can act on extension. If you do not distinguish it, it will have to be derived from the nature of extension. I conceive of the adequate idea, which I ascribed to the mind, as consisting of

many ideas, as a machine consists of many parts, and among these there will be an idea of the active principle that is in that machine. Hence, I do not think that there is one idea that intrinsically tends towards change, just as there is not one part in the machine that changes intrinsically. But, just as the parts of the machine act on one another by moving, so these ideas (which taken together make up the adequate idea of the machine) act on one another by representing. Would that I could understand the active principle that is distinct from extension that you seem to recognize in body, and know how change in matter follows from it, as clearly as I think that I understand ideas and the fact that the changes that arise from the things that are represented necessarily arise from them.

You correctly judge that the last demonstration by which you conclude that actions lasting the same time are proportional to the squares of their speeds is to be preferred to the rest because it is general and assumes nothing in particular. But something still stops me from being able to agree to it. Besides what I wrote about this in my recent letter, it should be added that the more that I think about it the more it seems that actions should be compared with one another only with respect to their effects, and that no account at all should be taken of the time, since that is irrelevant to the production of the effect. It is indeed true that actions performed at the same speed are in the same ratio as the times, not because time contributes anything, but because in that case the ratio of the times and the effects is the same. Indeed, since the whole nature of action consists in the producing of an effect, and the force of an action, given that it is the force of the cause, is equal to the effect, it seems clear to me that actions are proportional to effects. Consequently, the second proposition is assumed incorrectly, with the result that it is not the case that, "actions producing the same effect are proportional to the speeds, or inversely proportional to the times." Nor do I admit that, "that which produces the same effect more quickly produces more force."[4] For whether it takes a

shorter or a longer time, it will use precisely the force, from which the same effect follows. Therefore, in both cases it seems to be neither more nor less than the effect, but equal to it.

Forgive me, most renowned Sir, for having detained you with such a lengthy discussion. I ask this all the more since I am afraid that perhaps I may not have understood your intentions correctly at all, and that, as a consequence, I may have introduced some irrelevant things. Whether this has happened or not, as with all my thoughts, I most willingly submit to your most solid judgment. Goodbye.

From Leiden,

13 May 1699 A. D.

19. Bernoulli to Leibniz[1]

[Groningen, 3 June 1699]

Here is Mr. De Volder's response.[2] He presents many objections thoroughly and intelligently, but he seems to be too strict and fastidious in demanding proofs, even of things that I judge cannot be proved exactly. The things demanded from the law of order are like this, and it is the same with your active foundation for substance. So I do not see how you will convert him completely, unless you omit anything that is not certain and demonstrable. I had predicted almost all the objections that De Volder would make, and I certainly approve of most of what you had written to him. Moreover, I did not think that you had very much need of my opinion on this, otherwise I would have replied to you without delay, even though I was expecting a letter from you. Nevertheless, there were things that still caused me difficulty, though I do not think that I should raise them. It is better for De Volder to do that. For then I profit from each side without being fully committed to it. I am filled with delight when I read your most profound thoughts, which are alive with a certain profound

metaphysic, as I said to De Volder himself when I sent your last letter on to him. But conversely, I am also filled with no small admiration for the soundness of De Volder's objections and the delight with which he expresses them. It will be pleasant and fruitful for me to see the outcome of the skirmish that has begun between you.

20. Leibniz to De Volder[1]

[Hanover, 6 July 1699][2]

Gottfried Wilhelm Leibniz sends greetings to the gentleman most celebrated on account of learning and merits, Mr. B. de Volder.

I am happy that my letter did not completely displease you. I confess that I did not expect everything in it to satisfy immediately. Few men can say things – even true things – in such a way that they are accepted immediately, and the truth does not always seem like the truth. Besides, we all have prejudices that are very hard to eradicate.

I must admit that I was surprised that you seem to call into question an axiom whose truth I thought was established[3] by the natural light and by the testimony of the whole of nature, namely, an action that produces the same thing more quickly is greater, or it is more to cover a league with uniform motion in one hour than in two.[L1 4] Furthermore, I do not say that the effect is greater if you measure it by the distance traveled, but that there is more reality and perfection in the quicker motion itself. For could anyone deny that it differs from the slower motion or that it differs in that it is greater?

But why am I offering arguments for this claim? Could I hope to produce anything clearer? Would anyone dare to complain if someone supported it without providing his own foundations? I can be satisfied as well, since the claim is demonstrated without another postulate. But I think that by now whatever ambiguity was hindering you should have been

removed. For this reason, I do not know whether it is advisable to immerse ourselves in more obscure issues – such as those concerning the nature of substance and extension, where I see that our notions are different – before we come to an agreement about the measure of force. It is better to complete a few things than bring up many.[5] However, let me touch on some other matters, lest my intentions be further mistaken.

Assuming the existence of solid bodies, I grant that changes through leaps would follow from their collisions. Although the intellect wants for nothing in the reasoning here, it finds much to be desired in the premises and the conclusion. It does not yet seem that we are more certain – either by reason or by experience – that motion does not happen through leaps, than that other changes are continuous. For even if it is conceded that the direction of motion is always in a straight line, still, nothing would prevent God – were he to transcreate bodies from place to place through leaps – from ensuring that the places in which a body was re-created were themselves in a straight line.

I certainly had not believed that plurality could be denied in that which is extended, especially if we admit actual parts, as you do – unless we were to deny plurality even in a herd and an army, i.e., everywhere. Nor is any one part of matter absolutely[6] necessary for another, and, even if it were, this connection would not produce the unity of substances. In a substance that is truly one in my sense there are not many substances. And where there are many substances they do not constitute one substance but an aggregate.

You say that subject of change is nothing but a logical notion. You could have said metaphysical. But it is enough that it is true. We pour scorn on the obvious things, but, nevertheless, things that are not obvious follow from them. I think that we should start with nominal definitions. This is what I had in mind when I said that we should seek no other definition of power than the one that I have already introduced. The next consideration – how change comes about – is causal, and perhaps here there are things that escape the intellect.

In fact to my mind, it is inconsistent with the laws of power, i.e., of cause and effect, and, as I might say, the rules of the real metaphysics, that a greater body be impelled by a smaller with impunity. Nevertheless, this is not inconsistent with common notions of body sought in pure geometry, i.e., those that can be derived from extension and impenetrability alone. And, from this and other considerations, I conclude that, as well as that which is geometrical and mathematical in bodies, there are the sources for a dynamical and (I might say) metaphysical entity. For, as I showed some time ago when I was young, if we accept the common notion of matter, we must also accept that in a collision a body receives a conatus, which the other body strives to give to itself, when it strives to go forward.[7] And thus, we must accept that the body is carried along by a conatus composed of its conatus and the conatus given to it. For nothing prevents this, since all conatus[8] are compatible with each other. And it follows from this that a small body might carry away with impunity a very large body that is completely at rest or moving more slowly than it.[9]

It only remains for me to respond to the two questions that you asked me, excellent Sir, at least as I understand them. The first is, "whether the active principle is extension, or a mode of extension, or a substance distinct from extension."[10] I reply that it seems to me that something is prior to extension and constitutive of the very substance that is in that which is extended. Furthermore, for me extension is nothing but an attribute of an aggregate resulting from many substances. And so the principle of acting can be neither extension nor a mode of it, and it does not act on extension but within that which is extended.

The second question is, "whether an animate body has entelechies that belong to it distinct from its soul."[11] I reply that it has and that they are innumerable. For it consists, in turn, of parts, each of which is separately animated or actuated.

These, most celebrated Sir, are the things that seem to me to require a response at this time. I wish that it had been possible in the present circumstances to explain everything more

distinctly or to prove it more soundly. But it is something in the infancy of our philosophy to be able to say some things that seem irrefutable and to derive the remaining things from a few hypotheses that ought not to be scorned. Perhaps someday it will possible to go even further, especially if the rays from your light are again shone on me. Goodbye. From Hanover, 23 June 1699.

[Supplement][12]

Gottfried Wilhelm Leibniz sends greetings to gentleman most celebrated on account of learning and merit, Mr. B. de Volder.

I am happy that my letter did not completely displease you. I confess that I did not expect everything in it to satisfy. Few men can say things – even true things – in such a way that they are accepted immediately, and the truth does not always seem like the truth. Besides, we all have prejudices that are very hard to eradicate. However, I will try, if I can, to say something helpful about your comments, so that I may use the occasion to make further progress.

[13] I think that you will concede that not everything possible exists. Whether the possible things in novels can be imagined or not, I do not believe that they all exist, unless we think that More's Utopia or Barclay's Argenis are histories of somewhere.[14] When this is admitted, it follows that some possible things come into existence rather than others not from absolute necessity but from some other reason (namely, of goodness, order, perfection). The agreement of experience with the laws of order provides something like a test and, although it may not have the force of a universal demonstration, it can provide great confirmation nonetheless, and certainly, many things are not known in any other way. In short, I believe that the very universe before us, in all its detail, could only have been produced in one way, and I do not accept different possible causes of it. In my opinion, there is room for a choice

between different possible universes, but not between different ways of producing the same complete universe. I think that goodness, perfection, and order are established by reasons that are no less evident than numbers and figures. Imagine that a triangle must be produced, but that there is no other reason that determines the kind of triangle. Undoubtedly, an equilateral triangle would be produced.

I admit that, if there were such things as solid bodies, a change through a leap would arise on their collision. Although the intellect wants for nothing in the reasoning here, it finds much to be desired in the premises and the conclusion.

Even if we concede that the direction of motion is always in a straight line, I do not see how its continuity follows from this. For transcreation could take place in such a way that the places in which a body was produced successively through leaps by God were in a straight line. Indeed, as I already noted in my last letter, if it is supposed that everything is always being created by God, and we leave the laws of order[L1] aside, nothing prevents a body from being transcreated from place to place through leaps, so that in alternating moments it leaps and is at rest for a time. What can be said about leaps can be said with equal right about other gaps, such as the vacuum and rest.

Related to this is what certain philosophers believe, namely that motion is slower or faster depending on whether it is interrupted by more or fewer little rests.[15] So I think that we are no more certain, either by reason or by experience, that motion does not happen through leaps than that any other change is continuous.

The things you say, excellent Sir, about the notion of substance are subtle and ingenious, as is usual from you. Anyone can freely assign names to concepts, but such concepts will not always correspond to actually existing things, or even to received usage.

You say that, “the notion of substance is not formed from things but from concepts.” But are not concepts themselves formed from things? You say that, “the notion of substance

is a mental concept, or, as they say, a being of reason." But, unless I am mistaken, the same can be said[L2] of every concept. And we say that beings are real or of reason, not in connection with concepts, but in connection with the objects of concepts. Moreover, I believe that a substance is a real being, indeed the most real. You say that "concepts are of two kinds: sometimes they represent one thing from which nothing can be separated without the whole thing perishing." For you, "this is the concept of a substance, and the concept of extension is like that. And sometimes two or more things are represented by a concept." This is a little obscure to me. Certainly, every notion, i.e., definition, is such that you can remove nothing from it without the entire defined thing perishing. But then some other defined thing can nonetheless arise, as when you take away the notion of equilateral from a square. The square perishes but a rectangle remains. A notion from which nothing can be taken away must be simple and primitive. But I do not think that the notion of substance should be defined in this way or that the notion of extension is like that. Furthermore, you say that, on the contrary, those two or more things "are so related that one can be conceived without the other,"[16] and thus that, "perception and extension are related so that neither involves the other and so that extension is involved in motion, but not vice versa." And so motion is an accident or mode. But I strongly disagree with all this, and I believe that perception is involved in extension, and even motion, and that a substance can involve or be involved in other things just as much as an accident. Extension is an attribute. That which is extended, i.e., matter, is not a substance, but substances. Moreover, duration, time, and the enduring thing on the one hand correspond analogously to extension, place, and the located thing on the other. It does not appear that there can be things that have no common attributes. Nor do I think that the concept of extension is primitive, or that nothing can be taken away from it, since it is resolved into plurality (which it has in common with number), continuity (in common with time), and co-existence (in common even with things that are not extended). I had not

believed that plurality could be denied in that which is extended, especially if we admit actual parts – unless we were to deny it even in a herd and an army, i.e., everywhere. Continuity in motion is different from continuity of place, for in the former there is both continuity of time and of variation in change of degree of speed. Time is no more or less a being of reason than[L3] space.[17] To coexist and to exist before or after are real things. But I admit that they are not real in the way that matter and substances are generally assumed to be. But it is easier to show what these things are not than to explain in words what they are and to demonstrate this through reasons.

You say that subject of change is nothing but a logical notion. But it is enough that it is true, although you could with equal right have said metaphysical. We pour scorn on obvious things, but, nevertheless, things that are not obvious sometimes follow from them. We should start with nominal definitions. And I spoke of things of this kind when I said that we should seek no other definition of power than the one that I had advanced. The next consideration – how change comes about – is causal, and perhaps here there are things that escape the intellect.

You say that the unity of that which is extended is perceived even if it is divided into parts moving around in different ways, because given parts can neither exist nor be conceived without the others. And so you assume two things that I could not bring myself to concede: that one part of what is extended cannot exist or be conceived of without the others, and that things of this sort are one. From this you show that a vacuum is impossible. But your arguments did not accomplish this. If it is conceded that a vacuum is impossible, it indeed follows that one part of matter cannot exist without some other part, but it does not follow at all that it cannot exist without this part or that part. Besides, unless I am mistaken, this argument proves too much. For according to it, things that are scattered here and there will also be one thing. As I understand unity, such things are more properly called many and do

not constitute one thing except as an aggregate when they are grasped with one thought. In a substance that is truly one there are not many substances. I myself recognize neither inertia nor motion in extension. In extended matter I recognize both, but not on account of the extension.

You note very well, and this is also my view, that it is contrary to the laws of power, cause, and effect that a greater body be impelled by a smaller with impunity. But from this very fact, I prove that there is something dynamic in body[18], by whose force the laws of power are observed, and, on account of this, that there is something besides extension and antitypy. No such thing can be proved from these two alone. I responded to someone in the same way several years ago in the Journal des Savants.[19] I recognize that there is something involved in resistance beyond being acted on. I thought that it was worth noting, for the sake of those who do not yet hold that every substance is active, that secondary motive forces are not modifications of a merely passive thing, and that there is, therefore, an active substantial principle. I think that an extended thing that is at rest is among those things that cannot be distinctly conceived[L4], like the fastest motion.

You ask, most celebrated Sir, whether in my opinion, "the active principle is extension, or a mode of extension, or a substance distinct from extension." I reply that it seems to me that that principle is substantial and constitutive of the very thing that is extended, i.e., matter, i.e. of the thing that has not only extension and antitypy, but also action and resistance. For me, extension itself is an attribute resulting from many substances existing[L5] continuously at the same time. And so primitive force can be neither extension nor a mode of it, and it does not act on extension, but within that which is extended. You also ask, "whether an animate body has entelechies that belong to it distinct from its soul."[20] I reply that it has innumerable ones. For it[21] consists in turn of parts, each of which is separately animated or as if so. In the soul there is an adequate idea of matter, but for me the

soul itself is not the idea of matter, but the source of ideas for itself – ideas that arise in itself from its own nature, and by which the different states of matter are represented in order. An idea is, so to speak, something dead and intrinsically unchangeable, like a figure. By contrast, the soul is something living and full of activity, and in this sense I do not say that it is some one idea, which tends towards change from itself, but that various ideas succeed one another, one of which can, nonetheless, be inferred from another. Indeed, in another sense of the word, I could say in a way that the soul is a living, i.e., substantial, idea, but it would be still more proper to call it an idea-forming substance. And I do not think that you intend anything else when you claim that ideas act on each other by representing. For I do not believe that you think that ideas are substances running into each other like bodies.

I assumed that, "it is more to produce the same thing more speedily," and that there is also a great profit in the nature of time, which every experience indeed confirms. And so I thought this postulate was reasonable. If it is rejected, I admit that my most recent demonstration, and almost every measure of the forces in nature, is useless. In the meantime, it may be enough that my measure is demonstrated from this hypothesis and that all the phenomena are derived from it.

These, most celebrated Sir, are the things that I thought could be said in reply. I wish it had been possible to explain everything more distinctly and to prove it more soundly. But it is something in the infancy of our philosophy to be able to say some things that seem irrefutable, and to derive the remaining things from a few hypotheses that ought not to be scorned. Perhaps someday it will possible to go even further, especially if the rays from your light are again shone on me. Goodbye. From Hanover, 23 June 1699.

21. Leibniz to Bernoulli[1]

[Hanover, 26 June 1699]

When debating with Mr. De Volder I need your help very much because it is quite difficult to defend new things. And so, whenever you are on my side, it is right that you support me and act as confessor and defender of the truth. As for your claim that it is less trouble for you to act as a spectator, this could have been said more plausibly by me in my advancing years.

However, the objections seem to me mostly to originate from prejudices. His notion of extension is very different to mine. It is the same with substance and many others. Furthermore, it is one thing to object, another to demand proofs. I do not avoid replying to objections. But, even if I had them ready, there would not be the space and time to give proofs of all of this.

Above all, I am surprised that he denies that, "it is more to bring about the same thing more quickly."[2] Who would entertain this obsession for demonstrations? And who would hope to satisfy someone who does not even concede things confirmed by the testimony of the whole of nature? Meanwhile, were my views taken as a hypothesis, I believe that this would be enough for reasonable judges – as I think Mr. De Volder himself will be when he has considered the matter more. Such things could have been more appropriately said to him by you than me. But I am not so much seeking to persuade as to make progress by considering both sides. The truth always prevails sooner or later.

22. De Volder to Leibniz[1]

[Leiden, 1 August 1699]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

You are surprised, most renowned Sir, that I deny that, "an action that produces the same thing more quickly is greater."[2] In fact, I am amazed that you take this axiom to be so certain that you do not hope to produce anything clearer than it, especially since the entire explanation consists in this very thing. I would certainly like to have been able to see from your reply why the reasons that I gave for denying it in my last letter do nothing to undermine its self-evidence. If nothing else, they still seem to me to have the power to make your proposition rather uncertain. Since time is a mere being of reason it produces nothing in so far as it is distinguished from action. Only actions, i.e., causes of effects, should be compared with one another. Since these are in fact equal to their effect, surely it follows that the ratio of the causes, either of actions or forces, is the same as the ratio of the effects? However, I certainly do not deny on the basis of this that, "there is more reality and perfection in the quicker motion itself," or that, "it differs from the slower one in that it is greater."[3] Rather, I would think that what is more real in the quicker motion is compensated by the longer duration of the slower one.

Indeed, it seems to me that everything depends upon the same principles that you use in the response to Abbé de Catelan in the Nouvelle de la République des Lettres 1687, p.137, where you say expressly, "that there is always a perfect equality between the full cause and the entire effect,"[4] and where these things are further considered. In our case the effect is one and the same. And so the causes, which consist entirely in actions of the same forces, will also be the same.

If there were an interruption in motion, since it would follow neither from the speed of the motion nor from the direction, it would have a cause other than the motion. And I cannot determine whether anything might prohibit God from transcreating bodies from place to place through leaps while taking care that the places are in line with each other. Indeed,

since I have no notion of the kind of cause that might have brought about these leaps, I can conclude nothing about it.

I acknowledge plurality in a herd and in an army because they have parts joined together by a connection that is not necessary. The connection in that which is extended, where I certainly think that one part of it is necessary for another, is different. I do not admit a substantial distinction between the parts of extension, but only a modal one. In this I think I am like most, though not all, of those who ascribe all the variety in bodies to motion alone, seeing that it produces only a difference of modes.[5]

It seems to me to make little difference whether you say that subject of change is a logical or a metaphysical notion. I said that it was logical because I found nothing about it that that did not explain when it is taken in this general way and not restricted further.

Every body has a conatus for persisting in its state, and it will not be disturbed from that, except by an external cause. You are right that this gives those things a conatus toward motion. But since more of a cause is required to move a larger body than a smaller one, the motive force will indeed remain the same but its speed will necessarily be less.

I have nothing to add about the active principle that is prior to extension itself. Since I am so confused, I understand nothing about these things.

It seems to me that I understand your response to the other question. But I asked this question for the same reason as the first one about the entelechy of an animated body, insofar as it is regarded as one thing, when the parts are disregarded, and insofar as they are individually animated.

I was asking about both of these things because I judged that it follows that neither the one nor the other can do anything to extension if this active principle, i.e., the entelechy, differs completely from extension.

I am pained, most renowned Sir, that I have not been able to assent wholly to your opinion. But I would be pleased if you could persuade yourself that it in no way arises from a fondness for confrontation. Indeed, on the contrary, I would have been moved by your authority alone, which in my view is the greatest, had I not judged that you yourself would not approve if I assented to something that I had not understood adequately. Goodbye.

From Leiden,

the first of the month, August 1699.

23. Bernoulli to Leibniz[1]

[Groningen, 7 August 1699]

Here is Mr. De Volder's response. He told me about the contents of your letter that reached me sealed, so there is no need to send it again.[2] He replies to your complaint about his being too rigid, that he does not object for the sake of contradicting but solely from the love of truth.[3] He freely accepts that, "it is one thing to present objections, another to demand demonstrations,"[4] but that, on the other hand, it is no less certain that it is one thing to reply to objections, but another to prove that which is being asserted. He has not troubled himself over your measure of force so much as elicited from you, if it is possible on this occasion, a demonstration of your claim that every substance is necessarily full of activity. He has repeatedly asked for this demonstration, which still seems to be of the greatest importance to him for many reasons. But now hope has disappeared, and he does not want to press this further, lest he seem rigorous again. Furthermore, he complains that you think that he is so devoted to Descartes that he casts his vote in favor of Descartes' axioms without considering them. He has no doubt that Descartes was mistaken about many things.[5] However, he does not think that all the objections that have been made against Descartes are real objections.

Moreover, he says that he is very surprised that you complain so about his dissent – which contains the very thing that you are disputing – and still do not give a word of response to the reason he gave for his dissent. He wonders whether you did not pay attention to the reason, or whether you thought that it was too frivolous, or did not consider it worthy of a response, or, finally, whether some other reason, which cannot be divined, may have intervened.

24. Leibniz to De Volder[1]

[Hanover, 1 September 1699][2]

Gottfried Wilhelm Leibniz sends greetings to the gentleman most celebrated on account of learning and merits, Mr. Burchard de Volder.

It is good that you acknowledge that the whole account depends on the axiom that states that more is involved in bringing about the same thing more quickly, and, thus, that my measure of force is demonstrated when this one thing is conceded. As things stand, I think that I can have confidence in it, since the axiom is confirmed by all the evidence, and up till now everyone has accepted it in the form in which I use it. And, unless I am mistaken, it reduces the whole matter down to the simplest and clearest notions, which has not been done so far. I admit that I have not responded to the objections that you have brought against it, though, in fact, nothing pleases me more than to be able to discuss objections. However, since you have now set them out quite elegantly, and added to them, I will answer them with pleasure and, unless I am mistaken, show that the difficulties arise from nothing other than taking the axiom in a different sense.

You say that time is a mere being of reason. This is true, although I think that it is no more or less real than motion. Then you add that time brings about nothing. To be sure, but you know very well that there is some cause or perfection in the agent, which makes it the

case that a free action is produced more quickly. And we measure this perfection a posteriori, as it were, by the time, and there appears to be no other method by which it may be known. So, even if the axiom is in a certain sense universally true, as I shall explain below, nevertheless, in order to remove every concern, it should be understood with respect to the underlying matter, namely with respect to intrinsic action, i.e., action that is pure and depends solely on that which is acting. This kind of thing occurs when something moves without obstacle and exercises its force freely. Other things being equal, the only basis for measuring here[3] is the time in which the thing completes what it does. So, when I said that a quicker action is greater, or that it is more to cover a mile in one hour than in two, I made it clear enough which action was under discussion and that an intrinsically quicker one was intended, i.e., one that is quicker even when it is free, as opposed to one that is quicker only because it is impeded less. The action that I call free here I sometimes call formal as well, since it is natural to that which acts, i.e., it proceeds intrinsically from the nature or state of the thing. It follows that it must be continuous, since any interruption is produced by another. And so, it must also be understood to be an absolutely free action, involving nothing violent, i.e., an action that encounters no external force or resistance that it must overcome.[4] For in the other case, it is clear that the force of a thing that is acting, which is qualified by something resisting, cannot be measured from its duration alone. So a violent action cannot be taken as a simple exercise of power like a free action, and, consequently, it is not calculated by the product of the power and time through which it operates. There is a great difference between an operation that conserves power and arises from it spontaneously, and a use that consists in its consumption, i.e., its destruction, on account of a conflict with something external. But, even if there is no action in nature that is actually free from obstruction, nevertheless, that which belongs to an object intrinsically may be separated through mental abstraction from

what is added in by accidents, especially since the former is measured from the latter, as if a priori.

So the solution to the difficulty that, unless I am mistaken, has held you up so far consists in this, namely that, in the case of an action using up its force (because it must overcome some obstruction to which the force must be transferred), the force should not be measured from the duration but simply from the effect. For however much time is available, a greater effect cannot be produced by a given force. Indeed, by acting more quickly it only consumes itself more quickly. But when the operation of a power is instead conservative, despite acting more quickly it is productive without any cost. I once said some of these things to Catelan, pointing out at the same time that, far from causing me difficulty (as he thought), the consideration of time supplied a demonstration of my view.[5] But my argument was quite different from what he was expecting, namely the one that I had revealed to you, as you appear more suited to what seems to me a profound and elegant analysis. So far I have kept it from him and from the public. A remarkable harmony is seen there, in that the same measure emerges whether the moving thing acts by conserving its force while acting and producing nothing else (except what I would call its own formal effect, or operation, namely change of place), or whether it produces some other effect by violent action, thus consuming the force of that which is acting, as when it raises a weight or stretches a spring. Either way, it follows that the forces are calculated from the squares of the speeds.

I still consider what I said to Catelan, namely that the entire cause and full effect are always equal, to be absolutely true.[6] However, I do not mean a formal effect, which is only modal, but a real one, namely an effect that could in turn be the cause of some effect. Examples of such real effects are: a body having a given speed; a weight being raised to a given height; a spring being stretched to a given degree.[L1] They cannot be produced in something else by a free action, but only by a violent one. Nonetheless, even in the free or

formal action of a mobile thing, we can conceive of a sort of real effect by analogy, namely when it is thought of as acting on itself. This will not be a change of place (which I consider to be something merely modal), but the mobile thing itself proceeding into the next moment at a given speed, having arisen from itself at the previous moment proceeding at the same speed. In this sense, the axiom of the equality of the full cause and the entire effect is verified even in formal, i.e., free, action. But in this case, it is rather that the previous force is conserved than that a new one is produced. It is also clear from this that a new force cannot be produced unless the previous force is used up at the same time. Otherwise the force in the universe would increase. But, in general, it remains true that it makes no difference how long a real effect takes to be produced, either in a free action or in a violent one, although in a free action the time is already intrinsically determined, whereas in a violent one it varies according to the circumstances. However, since the power exercising itself through a free action would in vain be measured by its real effect (given that it is identical with the cause, i.e., the same in every respect), in such a case it is useful to appeal to the modal effect and its speed. In fact, it is the only recourse. And thus, the quantity of free action must be measured from this a priori. In summary, as I say, no account is taken of time in a real effect, but it always is in a formal effect, which is also modal. And force is correctly and appropriately measured either by quantity of action, when freely exercised, or by quantity of effect, when violent. And, as I said just now, the outcome is always the same in such cases.

Since things are like this, and they are very clear to anyone who considers them with reasonable care, I cannot not be surprised that it seems to you, endowed with intelligence and discerning as you are, that there is a conflict among my positions here. They are born from meditations over a long time, and so the more they are examined, the more they seem to cohere. So much so that, whenever new objections are raised, there is new material for fine reflections (as here, unless I am mistaken). Indeed, in a certain sense, more is involved in

actions completed more quickly, even with violent actions, so to that extent the axiom could be taken as completely universal. For saving time always matters a great deal, not only for us mortals who are limited to a short time as far as this life is concerned, but even for the immortal author of things who, doing everything in the most reasonable way, did not neglect a tract of space or time, or ground, so to speak, even though he has an infinite amount, or pass on anything that is not sufficiently cultivated. And, even though it appeals to a final cause, I do not consider this argument less solid on account of that. However, one cannot always conclude from a quicker action (as one cannot with a violent action) that there is greater power in the agent, but only that there are more favorable circumstances – as when a weight that has gathered impetus moves in a straight line rather than obliquely, which happens in an ascent where the curve AB described in a vertical plane along which a ball advances upwards finally ends in the straight vertical tangent BC.[L2]

Furthermore, you have overlooked here (as I have done rarely on other occasions) the reasons that I gave. As I said recently, a quicker motion differs from a slower one, and it differs in being greater, i.e., in there being more reality in it. They are indeed different, but they are not of distinct kinds. They differ in size. I do not see what could be denied here. I also thought that it would be of some note to you that it follows most beautifully from my method of measurement (provided an intrinsically quicker action is thereby considered more perfect) that the quantity of motive action in the world always remains the same. Thus, we have finally attained the goal that Descartes had sought in vain, mistakenly wanting to conserve quantity of motion. But, if all this has no effect on you, we may have little hope that the other things that I say about the argument will. For, although those beautiful multiple results in the conclusions concerning motion may seem to have a greater influence on the imagination, nevertheless, in its free association, it will always have something that takes over that reasons do not touch. And the extreme patience that would be needed to reduce

everything to formal arguments should not be demanded from either of us, although we would always obtain equivalent results from each other if we adhered to the laws.

But I would like to cover some other things as well. You were insisting that I should prove that the law of continuity applied to changes a priori.[7] I said that it might just as well be demanded that the law of continuity as applied to motion be proved a priori.[8] You offered a reason based on direction, namely from the fact that moving things always strive to follow a straight line.[9] I replied that I did not see that the conclusion followed, for, if there was a cause that led to bodies being transferred through leaps, it could nonetheless do this in such a way that it always transferred them in a straight line.[10] So, unless I am mistaken, you had to show that it could not do this in order that your proof be absolute. I added the hypothesis of transcreation as an illustration, speaking with philosophers – especially the Cartesians – who say, not improperly, that God continually creates everything.[11] According to them, for a thing to be moved is nothing other than for it to be reproduced successively in different places.[12] So it would have to be shown that reproduction cannot happen through leaps. Or rather, this could not be shown except by appealing to the reason that I gave for the universal law of continuity. But, even if you do not accept the reproduction of things, the same will have to be said whatever the cause of motion. Besides, even if it is not accepted by an opponent, a hypothesis can be assumed by one who is responding until it is refuted. You rightly say that an interruption is inconsistent with both the speed and direction of motion, namely if first you take motion to be something continuous in nature. But someone who utterly rejects continuity in things will say that motion is essentially nothing other than successive leaps across intervals, proceeding not from the nature of the thing, but from the action of God – in other words, that it consists of reproductions in discrete places –, and philosophize almost like someone who compounds matter from mere discrete points and who supports that opinion with the labyrinthine difficulties that surround the nature of the continuum, from which leaps

certainly do not follow, but from which other things that are usually not understood very well do. However, this hypothesis of leaps cannot be refuted, except by the principle of order, through the influence of the supreme reason that does everything most perfectly.

Since every extended body as it actually found in the world is, in fact, like an army of creatures, a flock, or a confluence like a cheese made of worms, the connection between the parts of any body will be no more necessary than that between the parts of an army. And, just as some soldiers can be substituted for others in an army, so in every extended body some parts can be substituted for others. Therefore, no part has a necessary connection with another, even though in matter in general when one part is removed it is necessary that some other be substituted.[13] And in the same way, it is necessary when soldiers are enclosed in a small place (i.e., one not holding many people) that someone take the place of one who is leaving. But I had already intimated this sufficiently well in my last letter, and I do not see what could be said about any body, with the soul taken away, that would not apply equally well to an army or a machine. Accordingly, I understand there to be a true unity (not merely a sensible one), or monad, where there is something in which there are not many substances.

I also ascribe all the variety of bodies to motion (including in that the causes of motion), but I nonetheless recognize a substantial distinction among the parts of matter. Everyone else who puts a rational soul in man that is something substantial lacking in extension recognizes this as well, though, certainly, they do not admit such a mind in all the parts of matter.

You seem to call notions that explain nothing logical or metaphysical. But I do not believe that such things are notions at all. Moreover, the one that I mentioned is so far from explaining nothing that I think that demonstrations of the greatest importance may be derived from it and similar notions. But we are so made and predisposed that, although we properly distinguish intelligible things from imaginable things in theory, so to speak, and publicly

declare that we distinguish them, nevertheless, in practice, we do not observe this and we take those things that are not imaginable to be almost nothing.

When you say that you understand none of the things said about an active principle that is prior to extension, I take this to mean nothing other than that you can imagine none of them. Meanwhile, for a proposition to be said to be understood, it is enough that it[14] follow necessarily from things that are understood. Unless I am mistaken, you certainly understand something when the Cartesians speak of the human soul, and, as far as I am concerned, this does not differ in kind from other entelechies. And so to your first question, what the principle of activity is, the same response will be given as to the question of what the soul is, although I myself might respond in a little more detail. However, I notice that preconception and authority are so powerful that many people think that they understand things in Descartes that they deny understanding in others. As for your other question, whether an animated body has an entelechy distinct from the soul, you interpret this as being a question about the animated body as a whole, and not about its separately animated parts. I reply that such a body does not have another entelechy apart from the soul and the entelechies of the separately actuated parts. Indeed, it would not be the soul of the whole thing, but only the soul of a separately animated part, if it were not at the same time a soul dominating in the whole thing on account of the structure of the whole.

When you say that more of a cause or force is required for a larger than for a smaller body to be moved at a given speed, you already tacitly presuppose that body resists motion. For, if it does not resist, but is indifferent and in a state of equilibrium, as it were, I do not see why its size would oppose something impelling it. And, however much such indifferent things increase in size, resistance will never arise. Likewise, if it is not required that things pass through lesser degrees, any reason or impulsion will be enough to incline or determine a given speed and direction of motion. Consequently, any moving body, however small, will be

enough to carry off with it any body, however large, without resistance or a weakening of its motion. Since this does not happen, and, on the contrary, a greater force must be expended and consumed for a greater motion of a body to be produced, we see that matter resists motion. It also follows from this that bodies are compressed before they allow themselves to be carried off, which is also a reason why the law of continuity is observed in changes and why a greater motion can never be produced except by a transition through a smaller one.

You infer that if an entelechy is completely different from extension, it can do nothing to extension. But is motion not different from extension and nonetheless able to do something to it? Besides, accurately speaking, extension is merely something modal, like number and time, and not a thing, since it abstractly designates a possible continuous plurality of coexisting things. But matter is the many things themselves, and, thus, it is an aggregate of things that contain entelechies. So, if by the word extension you mean matter itself, I do not allow an entelechy that is completely separated from extension. Finally, in metaphysical rigor, for one thing to be able to do something to another is nothing other than for one thing to correspond spontaneously to the other (as we have agreed in connection with the communication between soul and body).

There is no need for you to apologize for your dissent, for that is not a voluntary thing. The desire for truth, care in investigation, and candor combined with moderation in speech should be enough for us both, and they cannot be anything but useful and pleasing to men of good will. Goodbye.

P. S. I have learned from our friend Mr. Bernoulli that it seems more important to you that light is shed on the activity of substance than that force of bodies be measured. I agree, and I approve of your judgment. But, nonetheless, it has always seemed to me that this is the gate through which to pass to true metaphysics. The soul is gradually freed from the false notions of the populous, and even the Cartesians, concerning matter and motion and

corporeal substance, when it has come to understand that the rules of force and action cannot be derived from these notions, and that now one must either take refuge in Deus ex machina or understand there to be something higher in bodies. But if the mind is led[15] unprepared into this holy of holies, where the completely unexpected nature of substance and body can be viewed from its origins, then there is a fear that its darkness will be overwhelmed by the excessive light.

25. Leibniz to Bernoulli[1]

[Hanover, 11 September 1699]

Since Mr. De Volder has complained that I have ignored some of his objections, I am responding very fully. And, if I am not mistaken, I am removing every kind of difficulty concerning the measure of action. And I am explaining the reason why in free actions, i.e., those merely exercising their own power, the action and force for acting should be measured not only from the effect but also from the quickness of the effect, whereas, in actions that are violent and use up their own power through acting, only the quantity of the real effect should be measured, not the quickness. And so, what I now say is not only compatible with what I said before, but also fits in with it most beautifully, so that no account of the time is taken for a real effect, but account is taken for any kind of formal effect.

I have no doubt that if Mr. De Volder continues with the fairness that he seemed to show initially, he will acknowledge that he is satisfied with this argument.

Since the doctrine of the activity of substances is abstracted still further from matter and the senses, I thought that it would be pointless to discuss it until we agree about the easier issue. It is not important whether he says that he already despairs of a demonstration of my view concerning substance or not. And it is not right for me to rush into things far from

practical concerns, while omitting other things that are in fact necessary, and to pour forth immature thoughts that are not yet worked out sufficiently. So I am very far from the ridiculous vanity that I could present myself as such a demonstrator.

And, I ask, can I embark on this with any hope, if it is clear that he is not satisfied in the mathematical matters that I judge to be most certain? In order to convince people in metaphysics it would be necessary to adopt another kind of writing, one whose form I conceive in my mind rather than have explicitly.

For the rest, I do not want you to ask him about his rigor on my behalf. How does it harm me if he continues with his rigor, or, I might say, his obstinacy? What else is he doing besides closing the door to his advancement? I freely acknowledge the difficulties of the objections, and in fact I resolve them. And when I do this, I shed much light on the matter. I admit that it is one thing to respond to difficulties, another to demonstrate what is sought. But do you think that the whole of my gift should be forced upon someone who has shown contempt for part of it? Besides, it is a very great thing to solve difficulties, and I would dare say that up until now no one has achieved this in philosophy. I can easily believe that it appears greater to uncover the inner nature of substance than to measure the force of bodies. But this very measure of force prepares the mind so that it may more easily bear that great light that dulls and, as it were, fills with darkness those who are limited by the common, or even Cartesian, notions concerning the nature of matter, motion, and force. And so, I think that one must pass through this door from mathematics to metaphysics. Therefore, we should not complain so much as show others the right way and urge zeal toward it. If they fail, they should not complain about us, or we about them, but they should complain about themselves.

26. Bernoulli to Leibniz[1]

[Groningen, 26 September 1699]

Your note to De Volder pleased me greatly. It will no doubt influence him, even if it does not persuade him.[2] First of all, the distinction between a formal and a real effect is very elegant, as is the one between free actions and violent ones, where you show for sure that we must take account of speed in the former but not in the latter. I have always thought that the measurement should be taken this way. Hence, the force of ascending bodies should be measured only by the height, not by considering the time taken to reach it. For, since a body uses up its power uniformly by ascending, it is clear that it makes no difference whether the distance to be covered is covered more quickly or more slowly. Therefore, the objection that a body that takes more time to ascend to the same height also meets more resistance from gravity, since the number of blows by which the weight is forced down will be greater, is pointless.[3]

27. De Volder to Leibniz[1]

[Leiden, 12 November 1699]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

I do not know why but, although I agree with almost everything that you produce to support the truth of the axiom that you assume, I still cannot completely agree with your conclusion. What you say about free and violent action pleases me very much. I have never doubted that, "there is a cause or perfection in the agent, which makes it the case that a free action is produced more quickly",[2] and I agree completely with that which you add: "this perfection should be measured by the time." So what is it that holds me back from agreement at this point? Here it is. Action is the exercise of power. But, since action happens over time, unless I am mistaken, the strength of an action should be measured by consideration of both

the power that is the cause of the action and the time over which the action takes place. So actions will be calculated from the products of powers and times. It follows from this that actions of the same power are proportional to the times, and that actions completed in the same time are proportional to the powers.

I do not believe that there will be any disagreement between us about this. But I fear that you will not accept that which seems to me to follow immediately from this. A free action considered in itself (not with respect to a certain determinate time, but with respect to its quickness or productivity), which is the same during all the time in which it acts, certainly appears to me to correspond to the force from which it arises, and thus to be equal to it. I admit that we should measure its perfection by the time in which it covers some distance. But, although this may be the case, still, it does not in fact depend on the time but only on the ratio of the forces, since the perfection is the same during any period of time. If actions of this kind. taking a certain length of time to be completed, are to be compared with one another, first the intrinsic productivity of the actions should be considered, and then the time in which they are completed. A consequence of this is that an action that covers a certain distance in an hour is equivalent to one that covers the same distance in two hours. For, insofar as the first action is twice as quick, i.e., as perfect, as the second, the perfection of the first will be twice that of the second. But, since the one is exercised for twice the time, it will in turn be twice the other as far as time goes. Without doubt the first action is in the ratio of 2:1 to the second as far as productivity or quickness of action is concerned. But, as far as time is concerned, it is in the ratio of 1:2 to the second. Hence, the actions themselves that are calculated from the product of the intrinsic productivity and the time over which this is exercised, will be equal to each other. It is also the case that the following rests on the same principles: The power by which two miles are traversed in two hours is the same as the power by which one mile is traversed in one hour. For, since it is free, it remains unchanged. However, the power from

which the action that completes one mile in one hour arises is twice the power from which the action that completes one mile in two hours arises. For the intrinsic productivity of the first action is twice that of the second. Thus, the power from which the action by which two miles are traversed in two hours arises will be twice the power from which the action by which one mile is traversed in two hours arises.

I will also add the following, which I think will make clearer the source of the difference in our calculations: You say that actions producing the same effect are inversely proportional to the times. I agree if you are talking about the productivity of an action considered in itself, but not if you are talking about the strength of an action completed in a certain length of time. For I calculate that in the following way, and you will be the judge of whether I am right or not: The effect of the actions is the same in both cases. Therefore, the productivity, or efficacy, or perfection of the action, or whatever name one wishes to give that which is always the same in an action, multiplied by the time is the same. So the efficacy of action A multiplied by a time of one is equal to the efficacy of action B multiplied by a time of two. Therefore, the ratio of the efficacy of action A to the efficacy of action B will be 2:1, i.e., inversely proportional to the times. But the efficacy of the action corresponds to the power. Therefore, powers producing the same effect are inversely proportional to the times.

These, most renowned Sir, are the things that prevent me up to this point from agreeing with your view absolutely. Indeed, I suspect that you will deny that the productivity of an action corresponds to the force. For if this is true how can you defend your thesis? But, since this productivity of an action is the immediate and full effect of the force, I do not see how it could not correspond to it. Perhaps you will say, “this effect is formal, not real.” But I do not understand how this formal effect can correspond any less to the power than the real effect that is produced by virtue of this formal effect. Furthermore, how can it be known how

much of a real effect this or that power produces except through experience, which must be beyond the reach of all a priori demonstration?

I am surprised that you say that I did not mention the reason by which you maintained that, "a quicker motion differs from a slower one, and differs in being greater," since I quoted those very words in my last letter and gave them my full assent. But I added that which I have explained in more detail in this letter, that that which is more real in a quicker motion is compensated by the longer duration of a slower motion.

I maintain this difference between that which is extended and an army: the individual members of the army can perish while the rest remain; however, the least part of that which is extended cannot perish without the whole thing perishing at the same time.

I acknowledge that, "for a proposition to be said to be understood, it is enough that it follow necessarily from things that are understood," if it follows evidently from things understood a priori, but not from things understood a posteriori. Consider the nature of gravity, so that what I mean may be clarified by an example. I understand this as well as possible if I have understood that the descent of bodies necessarily follows from the universal structure of the world or from the particular structure of our earth. But I do not understand it at all if I infer from the fact that I see bodies descend that some explanation of why this happens, which I call gravity, is required.

I already gathered from your previous letters, illustrious Sir, what this most recent one certainly confirms. You suspect that I am so devoted to Descartes that I assent entirely to his authority and think that I understand in him that which I would deny that I understand in others. I make no apology for this, since it would deservedly accomplish nothing. For those who cling tightly to him would do that as well. However, I will add that I recognize his many and great errors concerning basic principles, as much in physics as in metaphysics, although I agree with him on many things, especially general ones. I will mention this as an example

that is relevant to our concerns. I do not understand at all what Descartes and the Cartesians mean when they speak about the human soul being joined to the body and acting on the body, and, in turn, being acted upon by the body.[3] In my view these are meaningless words, unless they show how motion follows from thoughts and thoughts from motion. But I think they are so far from doing this that, on the contrary, I believe nothing is more absurd. This is also why your explanation of this conjunction – which does not of course explain how the soul acts on the body and the body on the soul, but how the changes in the soul and the body happen at the same time, even though they each arise from their own principles – pleased me so much.

So if your entelechies do not differ in kind from the soul, does it not follow that, just as the soul has no power over the body and the body has no power over the soul, entelechies have no power at all over matter and matter has no power over them? So, just as some force distinct from the soul must be posited in the body by which the functions of the body are carried out, there will be some force for change in matter distinct from the entelechy itself. Indeed, this was what I was gesturing towards when I said that, if an entelechy differs from extension, or rather, from that which is extended, it has no power over it. Of course, if an entelechy is neither extension nor a mode of extension, it will produce no change in that which is extended. So are you right to respond that an entelechy does not differ from matter? Certainly, I think that it differs from what you call primary matter, and whose nature consists in inertia and antitypy. Therefore, either the primary matter's entelechy brings about some change or none. If none, it is not in fact the entelechy of that matter. But, if it does bring some change about, as it does since it moves the matter, there is still the doubt over how an entelechy, which is neither primary matter nor a mode of it, but something substantial that is distinct from primary matter, could act upon primary matter and bring about a change in it. And I do not think that we will be allowed to say that the one corresponds to the other spontaneously. For this implies both that change arises in the entelechy from the entelechy's

force and in the primary matter from the primary matter's force. However, since you identify the latter with inertia, I believe that there are no active forces in it.

In my argument about the continuity of motion I assumed that the cause of motion is in body and definitely not that God moves body as if by a miracle. Nor did I assume that any bodily cause can ever be conceived to act at a distance. But we should not dwell on that point, since whatever the truth of it is, it makes little difference for the crux of the matter at hand. I will add just this on the present occasion: I have never liked the opinion of Malebranche and certain Cartesians that motion does not arise from the force of the collision of bodies, but from the immediate power of God moving this or that body on the occasion of the collision.[4] Indeed, as a result of this effect, I have never doubted that there is some motive force in body itself through which, as through a secondary cause, the motion that arises from the collision of bodies is produced. The things that you say about this in your Specimen Dynamicum[5] (which I reread on this occasion, thinking that I might be able to be get some illumination from there) pleased me a great deal. The fact that I had not found a cause for this in body is one thing that always bothered me. I believe that this is the very reason why many people, after denying all action to bodies, called on God in support and ascribed all active force to him. This is why I jumped for joy when I read in the Acta Eruditorum that you, who affirm nothing without the best reason and from whom, if anyone, I most expected something of this kind, assert, "There is active force in all substances."[6] For I saw that, if this was demonstrated from the notion, i.e., the nature, of substance, many things about which we are now perplexed would be explained. And no doubt innumerable things would follow from this that would be of the greatest use in physics. Indeed, if it is demonstrated that every substance is active from its nature, and so, that the active force that originates from the nature of substance is necessarily conserved, it will be clear that all bodies that collide with each other moving in opposite directions are elastic. For if some of them were not elastic, their

combined quantity of motion would disappear and so also the force that is associated with the quantity of motion. Certainly, the force of bodies colliding with each other with equal speeds would disappear, since all the motion would disappear. Moreover, we now agree that that which is conserved in the collision of elastic bodies is proportional to the squares of the speeds multiplied by the sizes. Indeed, all these things, and the other things that you bring forth in this matter, have so much force with me that I marvel at the way they harmonize with one another beautifully. And I am already on your side in heart and soul, and would like nothing more than to be able to be on your side intellectually as well. However, I would not demand from any mortal, and much less from you whom I respect so much, that everything be reduced to formal arguments. For they are not at all necessary for those with intelligence, and of no use to those without intelligence. So, if you see anything missing among those things that I have now explained more fully, I would be happy if you are willing to supply the cure for my dullness. But if you judge the situation to be hopeless, I will lament my misfortune. Perhaps I should not dare claim that my mind is free enough of the false and commonly held notions of the Cartesians concerning matter, motion, and corporeal substance. However, I think that I do understand well enough that the rules of force and action cannot be deduced from their principles, whatever laws we may follow. And so we must either have recourse with them to Deus ex machina, which I do not approve at all, or deduce the force of bodies from corporeal substance, which I only wish I could do. Frankly, I admit my ignorance in these matters, and so you would make me very happy if you were willing to shed light on this for me. However, I ask this from you in such a way that I will always acknowledge that I am extremely obliged to you if I obtain it, but that I have no complaints against you if I do not. It is entirely up to you. Goodbye.

From Leiden,

12 November 1699 A. D.

28. Bernoulli to Leibniz[1]

[Groningen, 21 November 1699][2]

You will see from the enclosed letter by De Volder that he liked your last letter to him well enough. Meanwhile, however, he is still presenting his objections. In particular, he insists that you must give a demonstration of the activity of substance, since he is convinced that a remarkable number of consequences can then be elicited from it. This is not without merit. Of all the difficulties that he raises, the most significant-seeming to me is the one that he finds with your way of explaining the communication between soul and body, according to which the two substances can do nothing to each other, even though their changes arise at the same time from their own particular principles. For De Volder thinks that it also follows from this that entelechies (since they do not differ in kind from souls) can do nothing to matter, and that matter can do nothing to them. I remember that De Volder once made this same objection to you before.[3] I am surprised you did not answer it then. Certainly, I think it is worth some kind of response. In addition, should De Volder come back to you or me again, I have not yet been able to convince him to acknowledge the equivalence of continuous and interrupted ascents. He thinks that more force is required for the latter than the former. However, there is still hope of convincing him. He has commanded me (since he thinks I am more in your favor) to ask for the demonstration of the activity of substances that you have denied him. He adds (I believe as a joke) that, as I ultimately side with you, you certainly have room to communicate this demonstration to me, since I already have a purified mind, and that he cannot possibly believe that I am any less eager to know about this matter than he is. I certainly agree that I am eager enough about the matter. But I would not want to press

you if I would seem troublesome to you.

29. Leibniz to De Volder[1]

[Hanover, 20 January 1700]

Hanover, 9/20 January 1700

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of merits and learning, Mr. Burchard de Volder.

I had some trouble understanding the difficulty that you show that you still have about the measure of action. But, nonetheless, I think that I have finally found its source and hope to block it. We are agreed that "actions are calculated from the products of powers and times."[2] Sometimes I also express this by saying that actions are calculated from the products of intensions (which you call productivities) and extensions, i.e., diffusions. But this diffusion, as I shall soon explain, can be understood in two ways, with respect to time and with respect to place. Furthermore, we speak of the perfection or power of an agent as measured by time in one sense, and the extension of an action as measured by time in another sense. Actions by which the same distance is traversed are calculated from the reciprocals of the expended times, but the powers by which the same distance is traversed are calculated from the reciprocals of the squares of the expended times. Moreover, actions whose power or intension is the same are calculated simply from the times.

And in this last sense (which you accept as I do), it is most true that the intension, i.e., "the productivity of an action does not depend on the time, but only on the ratio of the forces." I also concede that, "if actions are to be compared with each other, first their productivity can be considered" (the intension, the powers from which they arise), "then the time" over which the productivity is exerted or the extension. But what you also propose is

far from clear, namely, "a consequence of this is that an action that covers a certain distance in one hour is equivalent to one that covers the same in two hours." Let us look at your proof of the inference. You present it in this way, "Insofar as the first action is twice as quick or perfect as the second,[3] the perfection of the first will be twice that of the second. But since the second is exercised for twice the time, it will in turn be twice the first." And so, by balancing them out, they will be made equal to each other. But I would think that the inconsistency of the conclusion is enough to conclude that some problem must be hidden in the premises. So that we may examine the inference clearly, let there be two actions, the first covering one mile in one hour, the second covering one mile in two hours. You say that these are equal and assume one thing that I concede, namely that the time of the second is twice that of the first. But you also assume another thing that I deny, namely that the power, or intension, or productivity of the first is twice that of the second. According to me, it is four times, since, for me, the powers are calculated from the squares of the speeds. So power is being confused with speed. In order that this might be clearer, I propose two different analyses of action: on one, actions are calculated from the products of powers and times; on the other, they are calculated from the products of effects (i.e., distances traversed) and speeds. So that you may appreciate the difference between these analyses better, it will be worth considering something that is commonly overlooked. Just as extension of action, i.e., diffusion, can be understood in two different ways, so you show that intension can be taken in as many ways, and that, together with the corresponding extension, this is sufficient for the total measurement. So, if the extension of an action is taken in terms of the time, then the intension is the power. But, if the extension of the action is taken in terms of the distance, then the intension is the speed. This should be properly noted and emphasized in every such case. And, unless I am mistaken, its neglect has brought about much confusion in this area. For it is quite clear that intension taken in the one sense must be different from intension taken in the

other sense.[4] Both are correct, of course. Actions are calculated from the products of powers and times expended, and are calculated from the products of speeds and distances traversed. So, since distances are, in turn, calculated by the product of speeds and times, it follows that actions are calculated from the products of times and squares of speeds. But, according to the first measure, the same actions were calculated from the products of powers and times. Therefore, powers are calculated from squares of speeds. In this way you again see how beautifully everything fits together, and is connected by an undeniable relationship. But what if someone persisted in saying that only the analysis of action in terms of time and productivity over time should be allowed (and not the other analysis in terms of distance and productivity exerted over distance), and so wanted the quantity of action to be the same provided that the distance covered was the same? Then he would be preferring one analysis over the other without a reason, since someone else could claim, with equal right, that the only analysis was into distance and productivity exerted over distance. Both these positions are unreasonable. So, it is clear that each analysis has an equal right to be admitted. Furthermore, it is necessary that these productivities differ from each other. For if actions were calculated from the products of productivities and times, and, at the same time, calculated from the products of the same productivities and distances, it would follow that times are calculated from distances, or that actions lasting the same time also cover the same distance, i.e., that every motion has the same speed.

You give another argument as well, which goes like this, “The power by which two miles are traversed in two hours is the same as the power by which one mile is traversed in one hour.” This I agree with. But you continue, “The power that completes one mile in one hour is twice the power that completes one mile in two hours.” I do not agree with this, since, according to me, it is four times as much. Your ratio works with actions and speeds, but not with powers. Therefore, I do not have to accept your inference that, “the power that

completes two miles in two hours is twice the power that completes one mile in two hours." On the contrary, according to me, it will be four times as much.

You also give a third argument, which goes like this, "I maintain", you say, "that actions producing the same effect are inversely proportional to the times." I agree. You say that you accept this proposition, if it is about the productivity of the action considered in itself, but not if it is a question of the strength of an action that has been going on for a certain period of time. These things are rather obscure. My claim is absolute and involves everything in free action that[5] can be measured in any way. But the argument by which you try to support your distinction sheds light on this for us. You say that, "the effect of the actions is the same in both cases," ex hypothesi. This I agree with. Therefore, "the productivity of the action multiplied by the time is the same." I agree with this too, if by the productivity of the action you mean the speed. For, as I have often pointed out, and as is self-evident, effects i.e., distances traversed, are calculated from the products of speeds and times. So, when the same distance is traversed, the product of the speed and the time is also the same. But I do not agree with what you propose, if by the productivity of the action you mean what we meant above, namely the power. And to add here what has been said before, I agree, if by the productivity of the action you mean the intension that comprises quantity of action together with extension over distance, i.e., if you mean speed. But I do not agree, if by the productivity of the action you mean the intension that we assumed initially that comprises quantity of action together with extension through time, i.e., if you mean power. So, you see again the way in which confusion arises from this source. But let us follow your argument in the case where the distance traversed is the same. You say, "So the productivity of action A multiplied by a time of one is equal to the productivity of action B multiplied by a time of two. Therefore, the ratio of productivity of action A to the productivity of action B will be 2:1, i.e., inversely proportional to the times." This is all very well, if by the productivity of

the action we mean the speed. But here yet again you change the sense when you finally conclude, "But in fact the productivity or efficacy of the action corresponds to the power." I deny this for the productivity of the action that you have relied upon in your argument so far, since that was the speed, which does not correspond to the power, as I have already shown. And so, I do not accept your further claim that, "powers producing the same effect are inversely proportional to the times." For me, they are inversely proportional to the squares of the times, whereas actions, and likewise speeds, are inversely proportional to the times when the effect is the same.[6]

So now you see, eminent Sir, that the great paradox that uniform actions that produce the same effect in very different times are equivalent[L1] has no rational support. Rather, it depends on a confusion between power and speed, which I have shown are things that differ from each other as much as time and distance do. Now, I think that you will very willingly abandon this position. "And certainly if you suppose that a certain quantity A consists of, or is determined by, two factors l and t, in the whole universe you will never find two quantities A that are equal if they have the same value for l but differ for t. Nonetheless, this would have to be the case if that paradox were true."[7] For, since an action is determined by a given time and distance (namely the one that the time is expended traversing), if there were equal actions covering the same distance in unequal times, then there would be two equal As, each of which was determined by its own l and t, and which had the same value for l but differed for t. Since this contradicts the most general and, so to speak, metaphysical mathematics, whose axiom has already been accepted, it is again conclusively demonstrated that powers should not be calculated from speeds. Nevertheless, it is also common sense to everyone that doing the same thing more quickly involves more. And often those who are quicker are rewarded, since it is agreed that they have actually done more work – namely, if the action is understood to be uniform and so uninterrupted. And so, I hope that just as you have accepted

my distinction between free action and violent action, you will also be satisfied with my distinction between two intensions, namely the one that constitutes quantity of action together with extension in time, and the one that constitutes quantity of action together with extension in space – i.e., between power and speed. In fact, in this whole business, the account is balanced and everything adds up so that, whether you measure free actions more metaphysically or violent actions more physically, no objection (of which I am now aware) can be raised to which I cannot guarantee a definite answer.

I now move on to other things, which I wish I could treat as satisfactorily, and touch on certain others briefly in passing. It seems to me that the parts of that which is extended can be lost no less than those of a machine or an army (which are themselves examples of extended things), even if it is necessary that another extended thing be substituted for the one that has been taken away. And, in this way, the extended thing perishes but not extension, just as men perish but not human nature. I do not think that we disagree about things that follow from things that are understood in such a way that they themselves are also understood. I acquitted you of the charge of slavery some time ago, on account of your intelligence and judgment, and I readily admit that, even in matters relating to Descartes, you conduct yourself with the freedom proper to a philosopher.[8] But there are things that have been implanted in us as a result of habit that get stuck there unnoticed. I often see this in myself.

When I say that the soul or entelechy has no power over the body, I understand by body, not the corporeal substance whose entelechy it is, which is one substance, but the aggregate of other corporeal substances constituting our organs. For one substance cannot influence another, let alone an aggregate of others. What I mean is this: Whatever happens in a mass or in an aggregate of substances in accord with the laws of mechanics is expressed in the soul or entelechy [or, if you prefer, in the monad itself, i.e., the[9] one simple substance consisting of both activity and passivity][10] through its own laws. But the force of change in

any substance is from itself or from its entelechy. And this is true to such an extent that, even in an aggregate, whatever will come to exist can be inferred from the things that are now in the aggregate. However, since there are so many individual entelechies in the mass of our body, it obviously follows that not everything that happens in our body is to be derived from our entelechy, even if it agrees with it.[11] Without a doubt, entelechy, i.e., force, i.e., activity differs from resistance, i.e., passivity[12]. You could take the former for form and the latter for primary matter. However, they do not differ in such a way that they should be regarded as two distinct substances, but as constituting one. And the force changing the primary matter is certainly not a force that is proper to it, but is the entelechy itself.[13]

If we grant that[14] one substance cannot influence another, as many concede, it follows from this that any substance whatsoever is intrinsically active. For it is unreasonable to call in God, and it does not explain or change anything. One can even argue that there is no influence of one substance on another from the inexplicability of influence itself[15]. You may take some time to consider these things carefully, until we can examine everything in greater depth. Goodbye.

[Supplement 1][16]

(1) Actions are calculated from the products of powers and times.

(2) Actions are calculated from the products of effects and speeds.

(3) Effects, i.e., distances traversed, are calculated from the products of times and speeds.

(4) Actions are calculated from the product of times and the squares of speeds.

(5) The action of covering two miles in two hours is twice the action of covering one mile in one hour.

(6) The action of covering one mile in one hour is twice the action of covering one mile in two hours.

Therefore, (7) the action of covering two miles in two hours is four times the action of covering one mile in two hours.

(8) If the distance or effect is the same, the actions are calculated from the speeds.

(9) Therefore, from (1), if the effects are the same, the powers are calculated from the speeds and the reciprocals of the times.

(10) But, if the effects are the same, the speeds are calculated from the reciprocals of the times.

(11) Therefore, if the effects are the same, the powers are calculated from the reciprocals of the squares of the times.

(12) Generally (from 1 and 2), powers are calculated from the products of distances and speeds and reciprocals of times.

(13) From (3), speeds are calculated from distances and reciprocals of times.

[Supplement 2][17]

| I hope at some point[18] to show that every substance has a force of acting, indeed that they always act. But since my thoughts on this issue consist of many things completely abstracted from sense and remote from common usage, and since I am now busy with a great number of very different things, I dare not promise that which I do not know that I can adequately deliver. For I see that I must ascend to the highest principles in connection with which we have scarcely anything in order so far. In fact, aside from you, there is hardly anyone with whom I would prefer to discuss these things. So, when I get chance to do this, I will make sure that you are one of the first to learn what progress I have made. And I think that this will be very much in my interest, since there promises to be great benefit to me in what you show me. Meanwhile, we can use the things that we intend to demonstrate as hypotheses or postulates – whose truth can already be seen dimly – with profit and success.

However, I, like you, would very much prefer to have things explained a priori – though it is not even done like that everywhere in Geometry, and we are a long way from being able to do this easily in metaphysics. But, even though I recognize that an analysis perfect in the last detail is beyond my powers, with the help of your efforts, I hope to at least approach it more closely. I feel that I am encouraged by nothing more than by the thoughts of those like yourself, whether they are in agreement or even appear to be contrary. I always derive new light from them, whether I am trying to accommodate them or even oppose them. If only I could benefit from such things now. But other things burden me so much that, if you knew how completely overwhelmed I am, perhaps you would not only forgive me for the involuntary delay, but offer what I might almost call a feeling of commiseration that I am not able to do the things I want. Goodbye and excel in whatever way you can. From Hanover, 1699. |

30. Leibniz to Bernoulli[1]

[Hanover, 22 January 1700][2]

Here at last is my reply to Mr. De Volder. I predict that it will succeed in his case just as it will satisfy you. I hope that he will now recognize how much difference there is between the different ways of measuring the productivity of action. I cannot foresee what other difficulty could remain for him. I cannot think of anything further from reason that one could defend than that two actions producing the same thing at different rates are, nonetheless, equal to each other. This is not only incompatible with the notions[3] in our minds but with every experience as well. . . .

The difficulty about the communication of soul and body seems to me to have been resolved by my response to Mr. De Volder. I do not remember my having noticed it before in his letters. If it is in there, I overlooked it by accident. It is not my habit to ignore things. . . .

Mr. De Volder demands a demonstration of the activity of substances. You already see how difficult it is to demonstrate mathematico-physical things, such as those in mechanics. How do you think it will be in metaphysics? A demonstration is reasoning whose force is evident and by which you could expect the indubitable conviction your adversaries. Even when we already have everything assembled in our minds it is difficult to express it in words. How much more difficult it is when we have not yet arranged our thoughts satisfactorily. And so, someone who does not give what he does not have should not be said to be withholding it. Meanwhile, I have included a first attempt at a proof, more so it not seem that his wishes were being neglected than because I hope to satisfy him.

I am amazed that someone might think that continuous ascents require less force than interrupted ones. Does he think that four pounds descending one foot and one pound descending four feet are not the same, when each case consists of one pound descending one foot four times? The only difference is that the descent is successive in one case, in the other simultaneous. Moreover, the successive one can bring about the simultaneous one, and the simultaneous one can bring about a successive one, and neither can bring about anything more. But things that can produce one another other are always equivalent.

31. Bernoulli to Leibniz[1]

[Groningen, 10 February 1700]

I have sent Mr. De Volder your letter,[2] but I have not yet received a reply to it. You have answered his objections well, exactly as I predicted. But I doubt that he will admit to

being satisfied about everything. I fear that he will complain that the attempt at a demonstration, as you call it, of the activity of substance, is too concise and incomplete, since it does not seem to follow that, "there is no influence of one substance on another, therefore every substance is intrinsically active."[3] Unless, perhaps, you understand the conclusion to be restricted as far as the predicate is concerned, namely by understanding it as: Therefore every substance that is active is intrinsically active. But then the original question, "whether every substance is therefore active" would recur.[L1] De Volder could regard the difficulty about the communication of soul and body as solved, provided that he is willing to recognize the distinction that you make between body and matter.[L2] But what if he says: If every entelechy acts on matter, but our souls do not act on our bodies, then what is this matter which our souls act on?[L3]. . .

When I sent your letter to Mr. De Volder I also presented to him a new demonstration for showing the equivalence of continuous and interrupted ascents.[4] I hope that he will finally be convinced.

32. Leibniz to Bernoulli[1]

[Hanover, 8 March 1700]

If, like all the scholastics, we mean by substance that which can act or be acted upon, and, moreover, nothing is acted upon unless it also acts, it follows that every substance can act. For, if it is already established that every substance that can act is intrinsically active, it follows that every substance is like this. If anyone introduces a substance that can neither act nor be acted upon we must wait for a new definition of the term from him, and then we will see what follows from that. . . .

I would very much like to see the demonstration of continuous and interrupted ascents that you sent to Mr. De Volder.[2]

33. De Volder to Leibniz[1]

[Leiden, 5 April 1700]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

You rightly note that my objections arise because I assume that the power and the speed in the same body is the same thing. I thought that I was allowed to do this as long as it was not yet clearly demonstrated to me that power should not be measured by speed alone. For this is the hypothesis that I was maintaining, and not only I, but Descartes, Borelli[2], and all those who measure force by what is commonly called quantity of motion. As long as I was reasoning from this hypothesis, I thought that it clearly followed that the strength of actions (by which I understood whatever belongs to the power) should be measured by the speed multiplied by the time, i.e., by the distance covered. In fact, I am not denying that you reply correctly given your hypothesis. But I did not use these arguments so that I might demonstrate the falsity of your opinion, but so that I might show the basis, supported by the hypothesis that I was maintaining, on which I required a further demonstration of the axiom that you assumed. This is why, in my last letter, guessing that you would respond somewhat as you did, I added that I suspected that you would deny that the productivity of an action corresponds to the force. And, at the same time, I added why I thought that this could not rightly be denied, since this efficacy of an action would be the immediate and full effect of the force.[3] Indeed, it seems to me that, up until now, everything here turns on whether speed (to avoid the ambiguity in the expression productivity of an action) is just the power of bodies of the same size or whether it is an effect of the power, and of such a kind that it does not

exhaust the power or is not its full effect. It follows from my original reasoning that, if indeed the first hypothesis is assumed, "an action that covers a given distance in an hour will be equal to an action that covers the same distance in two hours,"[4] although the power is twice as much. If the second hypothesis is assumed, the action itself will be twice as much. For the effect of the power operating for that time, and hence, the strength of the action, will not be the distance covered but the distance traversed with that speed, since now the speed is not the power itself but an effect of the power. From here what you want follows. When the distance traversed is the same, actions are proportional to speeds.

I believe that this is what you mean when you say that the true strength of an action consists of intension together with extension over time, as well as of another intension together with extension over place. For, as you show, in the first case the intension is power, whatever that may be in the end and whatever productivity is contained in it. In the second case, the intension is speed, which is taken not to involve all the productivity of the power. It does not determine the power until it is combined with distance and involves all the productivity of the force. While I was turning these things over in my mind, I read through the letter that the most clever Bernoulli sent to me along with yours.[5] There I saw it shown that, through the composition of motions, a body with two degrees of speed can flex four springs, each of which would require the force of the same body endowed with a speed of one degree in order to be flexed individually. He supposes that this body strikes three of the springs obliquely, each separately, such that the speed of the perpendicular component of the motion striking the spring is half of the total that the motion has initially. Then he shows that, after already flexing three springs, the body still has one degree of speed that it may expend flexing the fourth, which it strikes perpendicularly. It follows from this motion (which can take place on a horizontal plane and, thus, does not involve any notion of gravity, which leaves me and others with doubts) that when the speed is doubled the force is quadrupled. I

am not worried about the springs here, both because we suppose that no force is communicated from the spring to the body, since it is merely flexed, and because it would be the same if we supposed any other body put in the place of the spring and impelled with the speed required to consume the one degree of speed with which the first body struck perpendicularly.

However, since excessive caution does not do any harm, I wanted to apply the same reasoning to the hypothesis that I have followed up until now. It is certainly acknowledged by all who measure force from the quantity of motion that, if two bodies of the same size that are inelastic collide with one another, one moving and the other at rest, then after the collision both will proceed in the direction of the moving body with half the speed that the moving body had before the collision. Indeed, this is what they mean by the conservation of the same quantity of motion.

<<set fig. 7 here>>

So body A strikes body B of the same size obliquely with speed AB = 2, such that when the right triangle ABC is constructed from them the side BC perpendicular to body B is half AB. After the collision, therefore, A will retain the same speed through BD, which is parallel and equal to AC, and a speed in the direction DE, which is parallel to and equal to half the speed BC. And so, it will tend in the direction E with speed BE = √3¼. Here again at E there is a body of the same size as A, perpendicular to which is the straight line EF = 1. And so, after the collision, it proceeds along EH with speed EH = √2½. In the same way, after the third collision at H, with the same law holding, it will proceed along HL with speed = √1¾ , and, after the fourth collision at L, it will proceed along the straight line LO with speed 1. And so, when this body, A, has communicated a quarter of the speed it originally had before all the collisions to each of the bodies of the same size placed at B, E, H, and L, it still retains half of its speed. Although this does not yield the same proportion of forces as there was with the

springs (the reason for this can be easily deduced on your view), it does, however, make it clear that the hypothesis on which force is measured by quantity of motion does not accord with the truth and is not internally consistent. For, if force is to be measured by quantity of motion, body A has given to those bodies a total of four lots of force each equal to its own force, and, nonetheless, retained half its own force. I believe that no one will deny that this is manifestly absurd. Furthermore, I easily deduced from this that in bodies that are the same or of the same size, power should not be measured by speed alone or action by speed and time alone, and so, that you are right to assume that it is distance covered with a certain speed. Given this, your measure of force follows.

I now come to the most important matter, namely the one concerning the activity of substance. When this has been analyzed, I believe that the rest of the things that have been discussed occasionally will be explicable. I am very pleased that you were willing to enter into this debate and to present, as you put it, an attempt at a proof.[6] I will not dwell on the obscurity of what is involved in your claim that one substance cannot influence another. For I have always been most convinced by what I think you mean by these words, namely that there is no intelligible passage from the properties of one substance to the properties of another, and so, none from the operations of one to the operations of another. I think that this is also the reason why those who deny the intrinsic activity of substance flee to God. Since it has always seemed to me that this is a retreat, I have never doubted that the truth of your conclusion is established a posteriori. Even so, I have searched for an a priori demonstration from the notion of substance itself but thus far in vain. Most renowned Sir, if you are willing to communicate this demonstration to me, I declare that I will be extremely obliged to you. Goodbye.

From Leiden,

5 April 1700 A. D.

34. Bernoulli to Leibniz[1]

[Groningen, 6 April 1700]

I am sending a letter from Mr. De Volder that I received a week ago. It seems that he is relaxing his grip somewhat. I praise the frankness of the man, wanting not to convince but to be convinced. If you still have something like a demonstration for convincing him of the activity of substance, I ask that you not keep it from him who he is demanding one so persistently. Indeed, you add the following in your letter to me, "If like all the scholastics we mean by substance that which can act or be acted upon, and, moreover, nothing is acted upon unless it also acts, it follows that every substance can act."[2] He could not deny that the inference is entirely valid, but I fear that he might say that you take it for granted that, "nothing is acted upon unless it also acts", since he (along with many others) might be convinced that there are purely passive substances. For the rest, my demonstration of continuous and interrupted ascents has finally completely convinced De Volder of the truth about the measure[3] of force, as you will gather from his letter. For, based on this, he has himself elegantly demonstrated the falsity of the common hypothesis that depends on quantity of motion. I once gave you a similar demonstration based on the oblique collision of one body into another. Moreover, De Volder subsequently accepted this but only for elastic bodies, arguing that it was not yet sufficient for perfectly hard ones.[4] In fact, this argument, as it stands, goes through for perfectly hard bodies (if there are any, or rather, if they can be conceived) no less than for elastic ones. Thus, it can survive all opponents as it stands, since it is pure and abstract. Since you ask for it, at the end of this letter I will transcribe what I sent to De Volder verbatim, although you could grasp it well enough from the other letter from De

Volder.[5] For instance, an absurdity follows if it is held that when there is a collision between two inelastic bodies of equal size, one of which is in motion and the other of which is at rest, both will proceed in the same direction as the moving body after the impact and at half the speed that the moving body had before the collision. Based on my argument, De Volder shows that a body can give a quarter of its speed (which would then determine its quantity of motion) to four bodies of the same size as itself, and yet still retain half its speed. This would, therefore, increase its original quantity of motion, and so, a measure of force involving that would not even be internally consistent, let alone agree with the truth. If, on the contrary, one holds (as we do) that when there is a collision between two inelastic bodies, one of which is in motion and the other of which is at rest, both will proceed after the impact not with half the speed but with a speed which is in a ratio of √1/2:1 to the original speed, then, according to us, the same quantity of force is truly conserved. Then, from my reasoning, it follows that the body can move seven bodies the same size as itself, and give each of them a speed that has a ratio of √1/2:1 to the original speed, and that, after the seven impacts, it will itself retain the same speed it gave to the other bodies. This agrees beautifully with the true measure of force, for that is what should have emerged. And, thus, our rule is general and wonderfully self-consistent in all the special cases. . . .

At first I thought I would have enough room for the excerpts taken from my letter to De Volder, but, since this letter is longer than I expected, here they are enclosed on a separate sheet.

[Supplement][6]

I will demonstrate that the force by which a body ascends to a certain height in one leap is equal to the force of the same body descending again in four stages, and that it is equal to all the force that it acquired at each stage by flexing a spring, i.e., equal to the force of four

springs flexed in this way. Suppose a body that has two degrees of speed when it begins its ascent, so that, as you will agree with me, the speeds are proportional to the square roots of the heights. And since, by hypothesis, the four springs are each at a distance of one quarter of the total height, it is plain for all to see that the body strikes each of the springs with one degree of speed, and that the whole degree of speed is used up in flexing it. Therefore, there is now only one thing that it is incumbent on me to prove, that I show (with no attention paid to gravity) that a body with two degrees of speed (which I conceive as perfectly hard, lest you suppose it only applies to elastic bodies) can flex precisely four such springs, where the body's having one degree of speed is required for each to be flexed separately.

<<set fig.8 here>>

Imagine a body, C, obliquely striking a spring, L, at a speed of CL proportional to two and at an angle of inclination CLP of thirty degrees whose sine[7] CP is thus half the radius CL. Now what will happen after body C collides with spring L? Obviously, since the motion along CL is composed of the two adjacent lines CP and PL, and since CP, in virtue of which the body hits the spring L directly, expresses half of the body's speed along CL, this motion along CP will be used up by the flexing of the spring (for it is just as if body C hit the spring perpendicularly with speed CP, because by hypothesis it can consume precisely this amount), with the speed of the body remaining in the direction PL. Then PL is produced to M such that LM = PL = $\sqrt{3}$ (for it is assumed that CL = 2) and another similar spring is placed at M, forming the angle LMQ with LM whose sine LQ = CP = 1. By the same reasoning, it is clear that, after body C has flexed spring L, it flexes spring M losing the motion along LQ, and retains the motion along QM. And then QM is produced to N, such that MN = QM = $\sqrt{2}$, and a spring is placed there, forming a forty-five degree angle MNR with MN, namely so that MR = CP = 1 as well. In the same way, it is clear that the whole of the motion along MR is for flexing spring N and that, in the meantime, the body will move in the direction RN and with

speed RN = 1. Finally, if it strikes spring O perpendicularly with this remaining speed, it will give up all its remaining force by flexing it. Given these premises, I argue in the following way: The power of body C by itself, without any help from gravity or from anything else, is such that it could flex exactly four such springs. And, for each of these flexings, it is required that the body fall a quarter of the height to which the body can rise by its initial speed. But that same power is enough for the body to rise the total height in one leap. Therefore, as much power or force is expended to ascend the total height in one go as in flexing the four springs during the body's descent. Q. E. D.

35. Leibniz to De Volder[1]

[Wolfenbüttel, 6 September 1700]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of merits and learning, Mr. B. de Volder

Several months have passed since I received your last letter, but I could not reply any sooner because of the many distractions brought about by a trip to Berlin. For her Most Serene Highness the Electress of Brandenburg had visited her mother and brother, and I accompanied her back to court where I happily spent almost the whole summer, restrained by the most pleasant of fetters, I must admit. For, besides the great kindness of such a noble lady towards an old retainer, her husband the most mighty Elector himself has for some had in mind the founding of a scientific society, which he has charged me in particular with endeavoring to make a reality in accordance with his orders.[2] Following the example of the Royal Society of England, distinguished men have been enlisted, not only from among the Elector's own subjects, but also from a long distance away. There will be no fixed number of members and nothing more is required from those who live outside Berlin than that they

write to the secretary of the society roughly once every three months, if they cannot do so more often, and pass on as much information as they conveniently can about their own work or anyone else's that will be of benefit to the advancement of learning. In return, the society will ensure that their studies are supported, and that everything appropriate is made public. Those who are from Berlin will meet on a regular basis, and some of them will seek useful jobs for themselves. We will not neglect the investigation of works of nature and art, nor astronomical, mechanical, and chemical observations. On his own initiative, the most serene Elector wanted the German language and culture to be included as well. So that this might be of greater use to the sciences, I have persuaded him to establish dictionaries of technical terms used by artisans and those in various other walks of life, in which a huge store-house of things and words lies hidden[3]. Besides this, it will also have an interest in history and in more refined studies. To echo a phrase, nothing will be overlooked that might lead to "the advancement of sounder learning."[4] I have been so busily engaged with these matters, along with daily court duties, meeting friends, and working during the night to draft various documents for promoting, forming, and equipping the Society, that there has been almost no time left for myself, and none for tending to correspondence with friends. I have also frequently followed the court to neighboring country houses, and often I have scarcely got home by one in the morning or later, and, as a consequence, I am not a little tired. But I console myself with my success, and I hope this will enable me to be excused by my friends. I did not leave Berlin before almost everything was sorted out, the Electoral charter for the foundation signed, the appointment of the president of the society approved by the most important people, and, not least, work begun on an observatory building, and certain ordinances set up, which will be valid throughout all of the Elector's very extensive lands, and from which money will flow in time to the society that will be able to fund physical and mathematical experiments, and scientific instruments, to set up a printing press, and to

support the most necessary or beneficial work. Furthermore, the most serene founder wanted me to have charge of the society even when absent, and to report to him if I needed anything, so he has given me letters that confer a title. So there, most celebrated Sir, you have what I have been doing. I hope that it will be received with approval, given your ardent concern for the public interest and the pursuit of truth. Indeed, I am expecting a great deal from your help and advice. For, you see, it concerns, not so much us or Brandenburg as all the human race, that those who are in power are encouraged toward such noble pursuits, and attention should be paid not to let them come to nothing if any of these happy impulses arise. Furthermore, among those who could offer useful advice and help, I would hardly put anyone before you, and I have no doubt that you are willing, given that your love of truth has been established by a great deal of evidence. So, please accept these things not only so that I may be excused by you, but also so that you are not ignorant of things that, by Hercules, seem to be no more relevant to me than to you. Now I come to the contents of your letter.

First of all, nothing brings me more pleasure than congratulating you on your discovering the truth and accepting my laws for measuring force. Mr. Johann Bernoulli has shown most elegantly how a body with a speed of two, i.e., by our reckoning a force of four, uses it up by striking four springs at an angle.[5] But if, instead of the springs L, M, N, and O in his diagram, we suppose, with you, that there are four bodies that are the same as C in every respect and at rest, and which C strikes at the same angle as it struck the springs, then the resulting motion of the body C will be exactly the same. Moreover, bodies L, M, N, and O, which are in place of the flexing springs, will move with a speed of one in such a way that their respective directions are parallel to CP, LQ, MR, and RN. And so the whole force of body C will have been transferred to four bodies of the same size as itself, each moving with half its speed. The only thing that is assumed by Mr. Johann Bernoulli is the law of oblique collision, which, unless I am mistaken, nearly everyone accepts. In addition, I assume that, if

an object collides head on with another of the same size that is at rest, it is brought to rest and gives up all its speed to the other. And these things are confirmed no less by experience than by common assent. But, in fact, you, excellent Sir, have clearly shown that, whether a spring is used, or, in its place, a body of the same size, it is impossible to avoid an absurdity on the common hypothesis that measures the force of a body simply by the speed. And I would certainly dare to contend that nothing other than our hypothesis (which is the one true hypothesis) will satisfy the various, more or less complicated, cases that I have tried out in the past in many ways, involving oblique collision and other things.

For the time being, these things establish the truth very well and reduce the contrary to absurdity. However, since they require special assumptions about such things as weight, elasticity, and oblique collision, they do not uncover the sources and causes sufficiently, which can be done only with my measure of action. I have no doubt that you already find this pleasing, since it makes manifest that most beautiful fact that, when the same quantity of force is conserved, the same quantity of motive action is also conserved in the universe over equal periods of time. Descartes had seen this obscurely and pursued it, but he did not follow through well, and substituted speed for force, and regarded the time of action as instantaneous.[6] You will recognize, I think, that nothing is more reasonable than that an action is considered greater to the extent that it produces the same thing more quickly, and that, just as the action of covering two miles in two hours is (extensively) double the action of covering one mile in one hour, so the latter is (intensively) double the action of covering one mile in two hours. Consequently, the first action is four times the third. And so, actions that are completed in equal times are proportional to the squares of their speeds, in other words, to their forces. For actions are calculated from the product of forces and times. But measurement in terms of intension must not be left out here, since it is clear that two actions producing the same thing in different times are different, and that they differ in quantity, i.e.,

in degree, in as much as they are distinguished by one thing alone, namely time, which differs by simple magnitude. Hence, there is no room for a compensating factor, as there could be if other distinguishing features happened to be present as well. And so, there is not a general, or, so to speak, metaphysical, basis of measurement, according to which such things can be considered equal. I had intimated all this previously, lest it seem that I had assumed my thesis in an entirely arbitrary way, even though it should be immediately accepted by most people. And I bring these things up again not because I want to revive earlier discussions, but so that the sources of something very beautiful may be understood more deeply, and so that it may be established that the principles of nature are no less metaphysical than mathematical, or rather, that the causes of things lie hidden in a certain metaphysical mathematics, which measures perfections, i.e., degrees of reality.

Finally, most celebrated Sir, you ask that the intrinsic activity of substance be derived not only a posteriori (as they say) – from the fact that it is proved that it can receive nothing from another created substance[7] and that experience shows that it acts nonetheless – but also a priori, i.e., from the very nature of substance. So, you will see that we must first establish a notion of substance and agree upon it. Even if this will perhaps take some effort, I think that it will be worthwhile nonetheless. And so, I gladly step onto the field, provided that you are willing to help with this investigation (which you are most capable of given your very keen intelligence), and to say what you understand by substance, whose activity you want demonstrated, or give a nominal description[8] of it, so that we are not fighting over different things like blindfolded gladiators.[9] For a long time, philosophers have not been much concerned with defining things. You know that Spinoza recently gave a definition of substance that no one accepts, and so he did not prove anything pertaining to this matter.[10] I do not know whether those who have responded to him have given anything that is more correct, since I have not been able to examine their writings carefully. As for myself, even

though it seems to me that I have established something, I would, nonetheless, like to know how much it can agree with your opinion. For the rest, since I have mentioned refutations of Spinoza, I should like to ask you what it was that I read (perhaps in Bayle's dictionary, which is stuffed with many excellent things) about your countryman Bredenburg first having refuted Spinoza in a little published book, then meditating more deeply, and, finally, coming to think that Spinoza's reasoning had the force of a true demonstration, and, being so disturbed by this that he begged his friends' help in escaping from it, but in vain. Then Kuyper published what Bredenburg had written solely for his friends, which gave rise to disputes among them that were conducted in Dutch.[11] I would like to know where Bredenburg's argument appears or how it can be obtained.

For the rest, goodbye and be well. From Wolfenbüttel, 6 September 1700.

36. De Volder to Leibniz[1]

[Leiden, 18 October 1700]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

I was delighted to see from your last letter that the most serene Elector is thinking of founding a scientific society, or rather, that, with your advice, he has founded it. It is indeed a good thing that, while most princes think that it brings them glory to squander their wealth, and not only their subjects' wealth but also their lives in war (i.e., on the destruction of the human race), someone emerges who thinks about the sciences and those things that may be of benefit to humanity. But, just as nothing else is to be expected from the Elector himself than that he would devote his will and authority, and perhaps some of his abilities as well, in such a useful endeavor, so I fully approve of the fact that he chose you to be president of the society. As for you, most renowned Sir, who have embraced in your mind all that is

praiseworthy in the sciences, it can certainly be expected that you will prescribe suitable and, in the future, productive rules and methods for conduct and ways of acting and inquiring. And, with your plans, you will establish a society that will be of great benefit to the Republic of Letters for a very long time. From your letter, I see that many names already have been submitted to the society and that more will be. So, will everyone be admitted who wants to be? Or will this be your choice, or the choice of the initial members of the college? If the former, one perhaps will have to worry about there being too many people; if the latter, one will have to worry about grudges. Certainly, if it is the latter, I think you should be careful not to forget Mr. Fatio, unless you wish to rouse new accusations against yourself.[2] But why do I talk such nonsense about this to you, when I am absolutely convinced that whatever you have put in place will have been put in place rightly and wisely, and out of the best interests of the society.

As for the laws for measuring force. After I recognized that speed must be distinguished from force, unless I am mistaken, I reasoned in a way that agrees with your principles, or rather, I set out your reasoning for myself in the following way: Without a doubt, action is nothing other than the exercise of force for a certain time. By the exercise of force I mean just whatever may arise from force and is produced by it. I only bring in time because we are concerned with the action to be determined, and this determination does not follow from the forces alone, since in themselves they would persist forever. But, since the product of the forces comes into existence successively and homogeneously, it is best measured by a homogeneous succession, i.e., by time. These things are indeed common to all actions. But, if we now examine corporeal action, it will have to be considered what the exercise of force consists in there, i.e., what the full effect of this force is. But it is just a distance covered with a certain speed. For just this is produced and nothing more. Speed itself indeed determines force, but it does not determine the action, since the speed, just like the

force, will stay the same forever. Therefore, that action is determined by the distance. Time should not be considered here, since it is not an effect of the force, but only a measure from which the speed for a given distance is determined or the distance for a given speed is determined. In fact, since speed is considered as an effect of power (not as power), and, thus, contributes to the magnitude of an action, it is clear that actions covering the same distance at different speeds are proportional to the speeds. So, given this (if I designate distance by d, speed by s, and time by t),[3] the magnitude of the action will be proportional to ds, or, since d is equal to st, the action will be proportional to s^2t, which is just what you maintain.

At last, I turn to the most important matter, namely the intrinsic activity of substance. You quite rightly observe in connection with this that a definition of substance must be agreed upon before activity can be demonstrated of it. Moreover, since it seemed that you were demanding this from me, behold the way in which I explain the notion of substance. I assume as a starting place that all our thinking depends upon concepts alone, and that words are their signs. So first the concepts will have to be distinguished, then the words allotted to them individually as signs. If I consider my concepts, I seem to find this difference among them: either the concept represents one thing to me, and I can remove nothing from the representation without the whole thing perishing, or it represents two things to me, one of which I can conceive separately, the other of which I cannot. If the first occurs, I say that the concept is the concept of a thing, i.e., a substance, and I call the object corresponding to it a thing, i.e., a substance. However, if the second occurs, I call those concepts the concepts of modes, and the objects of those concepts I call modes. I use this denomination rather freely because, as is generally acknowledged, whatever things exist or are conceived are either substances or modes. So I think that all our concepts are covered by this distinction. For if more than two things are involved in a concept, say A, B, and C, either it will be possible to conceive each of them separately, and they will be three concepts not one, or it will be

possible to conceive A without B and C, but neither B nor C without A, though B can be conceived without C, and C can be conceived without B. In this case, there will be two concepts of distinct modes of one substance. If, however, A can be conceived without B and C, and B cannot be conceived without A, but can be conceived without C, and C cannot be conceived without B, C also will be a mode, but it will be a mode of the mode B. This can be illustrated with the example of any right angled cone. The right angle cannot be conceived without the cone, and the cone cannot be conceived without a body. But the body can be conceived without the cone, and the cone without the right angle. And it is the same with other cases. I will clarify them with an example. I find within myself a concept of motion. Two things are clearly involved in this concept: first body itself; second the translation of body, which I call motion. They are such that I can conceive of body without motion, but cannot conceive of motion without body. Motion will therefore be a mode of body. Furthermore, if I want to form the concept of body, it is necessary that I conceive of some attribute of it that represents its essence. Now this will represent either one or two things. If it represents one, I will call this thing, from which I can separate nothing, and to which I can ascribe motion, corporeal substance, and motion will be a mode of it. If however, it represents two things, either I can conceive each of them without the other, and then I will say that the body is composed of two substances, or I will be able to conceive one without the other, but not vice versa, in which case I will call the first the substance of the body, the second a mode of it. I would have used the example of extension, which I have always found the most fitting for this purpose in the past, except that I was worried you might disagree with the fact that I say extension represents one thing. However, I will use it now, given that the truth is not necessarily required in examples, and since I think that you will see what I intend more clearly from it.

When I conceive of extension one thing is represented to me. I can indeed conceive of it as greater or less, but, whatever the magnitude, I conceive of extension of one kind, and I either conceive of the whole of it, as it were, or none of it. However, in figure and motion I also conceive of extension, which I can conceive without motion or figure, and I conceive of either a certain boundary or a translation of extension, which cannot be conceived without extension. They will therefore be modes. But, if you deny that extension represents one thing to the mind, so be it. In that case the thing itself that is extended and the extension that belongs to it are conceived of in the concept of extension. If things were this way, I would say that extension was a mode of the thing that is extended. But, as for that thing of which extension is a mode, I would either conceive of nothing, or if I conceived of something, the same question would return concerning it – namely whether it represents one thing that is conceived of on its own or in fact two, of which one is conceived of on its own and the other is not. It is clear that, on this reasoning, we will finally reach something that is not conceived of at all or that is conceived of on its own. This final thing will be called a substance, since all the remaining things belong to it, and we will have either no concept of it, and it will be a mere word signifying nothing, or we will have a concept that represents one thing. This is my notion of substance, which I think agrees very well with the common, albeit more obscure, definitions of substance. It exists through itself, i.e., it needs no subject in which to exist, and it will sustain accidents. And, except for an efficient cause, it will need nothing else in order to exist, which, in brief, is Descartes' view.

I would not dare promise myself, most renowned Sir, that you are going to approve of these things. For I suspect from the things that have passed between us that you form a different and far more abstruse notion of substance. If you give it to me, it will, I hope, shine some light on the differences between your notion and mine. And then we must also examine whether activity follows from your notion and whether the thing that was at issue can be

accommodated. Truly, whatever you give to me, you may be certain that I will never argue over words.

While I am on these issues, I happened to come upon a passage in the Nouvelles de la République des Lettres, for the month of October, in which the author, Mr. Bernard, after having mentioned Malebranche's opinion, which makes God the sole cause of motion, offers this, "All that has been said up till now is as reasonable as could be, but there are some men who suspect that one should not cut a knot that one cannot untie. It is said that one of the great philosophers of our times has found a demonstration to prove that created beings have some efficacy. Everyone should beseech him to bring such a beautiful and useful discovery to light."[4] I have no doubt that he is talking about you. It is certainly true that there are many people who, like me, very eagerly desire a demonstration of this claim.

Finally, as for your request about Bredenburg, I have heard almost the same concerning the matter as Bayle says.[5] This Bredenburg was one of the principal teachers of his group, which meets three or four times each year at Rijnsburg, in a district near this city, with its members coming together from all over. This group has two peculiar features: they admit all those who profess Christ to the sacrament of the Eucharist, whatever opinion they may have on those controversies that have so lamentably scattered Christians into sects; furthermore, they grant every man freedom of speech, of preaching, of praying publicly, and of singing hymns. As I said, Bredenburg was prominent among them. It is not necessary to repeat Bredenburg's refutation of Spinoza or the demonstration that he then wrote down and communicated with friends, since Bayle has written about it.[6] But his demonstration (which I believe you care about the most) contains four propositions. The first of these says that there is a necessarily existing being; the second, that all the attributes of that being exist necessarily; the third, that working, i.e., being in motion, or rather, active force, or, perhaps even better, genuinely and necessarily acting force, is an attribute of this being; and, the last,

that the effects of this being necessarily exist, and exist just the way they do. The axioms from which he deduces these things are: 1) Nothing is produced from nothing; and 2) That which is, i.e., is something, never passes away into nothing, since there is an infinite distance between being and nothingness; and, as a result, 3) No more is contained in the effect than in the cause.

In summary, the reasoning is as follows: There is something. Therefore, it is certain from axioms one and two, that a real, necessary being exists. For if you deny this, you are required to acknowledge that there is nothing. For, when this necessarily existing being is denied, everything disappears. From this, he concludes, secondly, that the attributes of this being necessarily exist, seeing that they express its nature. Third, because the greatest conceivable active force is required for it to conserve itself in being eternally, it is just as impossible for God, i.e., the necessarily existing being, not to be active as to not exist. Indeed, this active force is an attribute of God. Finally, from the first and second axioms, it follows that a necessarily active force necessarily exists. For there is an active force. He adds this here in roughly this way, rather confusedly as is clear. The fourth follows easily from these, namely that all effects are such as they are necessarily.

Here, for you, are the main points of the demonstration, which I took from the little book, which I finally found at a friend's house, for I looked in vain for it at the booksellers'. If it had been possible to get hold of the little book itself, I would have sent it to you with pleasure. As Bayle rightly says, this little book was discovered after the death of Hartigveld.[7] You will find everything that pertains to morals, piety, and what they call devotion to be excellent. But, with good reason, you will find firmness of judgment and subtlety of reasoning to be lacking. Kuyper published it, an utterly worthless man, unless I am very much mistaken, who had insinuated himself into Hartigveld's friendship with some outward show of piety.[8] Indeed, he had lost the friendship after he was convicted of two or three

serious charges, and had regained it again with some feigned show of repentance. He also added a refutation, to which Bredenburg replied.[9] After that a great controversy arose among the members of the Rijnsburg group, to which both Hartigveld and Kuyper belonged. Some accused Bredenburg of being an atheist and a despiser of all religion (on the grounds that, as even Bredenburg admitted, it could not be made consistent with God's acting necessarily). For it cannot be the case that someone has a demonstration, which he reckons was correctly constructed, and, nonetheless, convince himself of the opposite of what had been demonstrated. But, in fact, Bredenberg, while acknowledging that he recognized that this demonstration was correctly constructed, and, indeed, explicitly stating that it could not be refuted unless it was established that something could come from nothing, nonetheless asserted that he believed the opposite, adhering not to reason but the revealed word of God.[10] Bayle seems to approve of this means of escape, but Limborch, who was called a professor of theology among the Amsterdam Remonstrants, did not approve.[11] A dispute took place publicly between him and Bredenberg as to whether anyone could believe the opposite of something of which they had seen an evident demonstration.[12] The matter continued in this way, with some favoring Bredenburg, others denouncing him, right up to Bredenburg's death. These are the main points of the affair. Goodbye.

From Leiden,

18 October 1700 A. D.

37. Bernoulli to Leibniz[1]

[Groningen, 15 October 1700]

I am now sending the distinguished De Volder's reply, which I received the day before yesterday, brought by the last post. If he is also invited to join the new Brandenburg

Society[2], I believe that he will not refuse, since, in his letter to me, he declares that he is particularly pleased that you will be in charge of the society, and that he is very optimistic that it will be a great success as a result. You will find out for yourself whether you have great reason to worry about Mr. Fatio, as De Volder jests. De Volder now seems to like your laws for measuring force and action more, for he not only accepts them, but has also produced a demonstration of them based on your principles. But I fear that there is a world of difference between the two of you over the concept and definition of substance, since I remember that, for you, extension is not a substance, but substances. Bredenburg's demonstration, by which he intends to show that everything exists necessarily, is pathetic, since, even if it were really true, nonetheless, it would not follow by the force of his argument; otherwise the world would have existed from eternity.

38. Leibniz to De Volder[1]

[Hanover, 31 December 1700]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

Having barely returned from my journey to Berlin, I had to set off on another from which I got back just yesterday.[2] I did not want to put off my reply into the next century, which will begin tomorrow. I hope that you will have a good share of it, along with a happy new year.

I am glad that you approve of the founding of the new society. Given your prudence, you will easily understand that we can avoid both jealousy and commotion in selecting members by showing a certain moderation. This is, for the most part, dictated by the nature of

the thing. For the society will not be an academy, which would have been bound by stricter rules.[3]

I do not think that Fatio, about whom you jest, would want our favor. Although, of course, if he wanted to contribute to this publicly useful institution, he should not be turned away, for I have no doubt that he could make a contribution. I do not disparage his intelligence, even though I wonder about his equanimity. It is from friendship that you attribute all those things to my plans. I am quite aware that I am not a significant factor, and, since I am busy beyond belief, I would like to hear other people's advice, and no one's more than yours.

I am also glad that you now not only approve of, but have also clarified, my metaphysical arguments derived from basic notions[4], concerning the measure of motive action. The method of calculation that you try out is absolutely identical, down to the notation itself, to one I already communicated to a Florentine friend twelve years ago.[5] I will set it out in brief. Let power be p, action a, time t, speed s, and distance d.[6] Now a is proportional to pt, i.e., actions, i.e., the exercise of powers, are calculated from powers multiplied by the times during which they are exercised. Or, to put it another way, since powers cannot be known except through actions, 1) p is proportional to a:t (i.e., a/t), i.e., powers are calculated from the actions that they produce and the reciprocals of the lengths of time during which they produce them. And actions should be measured by their perfections, which consist in the quantity of the whole effect, if that is taken in a broad sense, or in the effect taken more strictly combined with its quickness. And so, motive actions will be measured both by the distance covered and the speed with which it is covered, i.e., 2) a is proportional to ds, which signifies that motive actions are calculated from the products of the distances traversed and the speeds with which they are traversed. Finally, 3) d is equal to ts, i.e., distances are calculated from the products of speeds and the times in which they are traversed. This is a

geometrical truth, whereas the previous two are metaphysical. Many elegant things can be derived from these foundations, but the following will do for now: (from 2 and 3), that 4) a is proportional to tss, i.e., motive actions are calculated from the products of the times and the squares of the speeds. Then from 1) and 4) it follows that p is proportional to tss:t i.e., eliminating the times, that 5) p is proportional to ss, i.e., motive powers are calculated from the squares of the speeds with which they act. And so, from 5) it follows that 6) if the same quantity of motive force is conserved, either in the whole universe or among bodies that are assumed to be interacting with one another, the sum of the squares of the speeds multiplied by the masses of the bodies will stay the same, and conversely. Moreover, from the first claim it could already have been concluded that 7) if the power also remains the same during equal periods of time, then the quantity of motive action in the universe stays constant, and conversely. These things apply only to uniform motion, but they can be adapted to non-uniform motion as well, just as I am accustomed to say, when multiplying a figure by a figure as Gregorius a Sancto Vincentio[7] conceived of it, that they are properly calculated from the products of the figures, in which products they in turn appear.

Your inquiry concerning the notion of substance is most beautiful. Since you have embarked on this, I will join in. Even if we sometimes try out different paths, we will still be aiming at the same target. I get stuck, however, when you say that "the concept of a substance is that which represents one thing in such a way that nothing can be removed from that representation."[8] For you seem to describe it in another way soon after, so that a substance is that which can be conceived separately. This definition does not seem to coincide with the previous one. And, since I do not understand the first sense well enough, I will follow the latter for the moment, which you explain beautifully as follows: If we had A, B, and C, and each could be conceived without the following one, but not the other way round, A would be a substance and B its mode, and C a mode of a mode, provided, of course, that A did not

require yet another thing in order to be conceived. These things are certainly very clever. However, two things to be shown arise, one concerning the notion itself, the other about its application. As for the notion itself, the difficulty is that it does not prohibit there being two things A and B, which each can be conceived separately, and also a third thing C that requires both of them. It follows from this that there could be something that was a mode of two substances at once, or that was in two subjects at once. Furthermore, it could happen that C required A and B unequally for its concept. Not to mention the fact that modification seems to imply something more than the simple need for a concept. Moreover, I think that the concept is to be understood as distinct. To turn to the other thing to which I was wanting to draw attention, namely the application to substance. Surely for the concept of a substance we also need attributes. And, if you insist that at least no other substance is required, we would be guilty of a circular definition. Furthermore, except for the first substance nothing can be conceived through itself, or so I believe. But I think that you acknowledge that this is not the only substance, or rather, that people have understood the word substance in such a way that there are many substances in the universe. Matter is certainly not conceived through itself, but through the parts from which it is constituted. And I judge that minds are no more conceived through themselves, since they have a cause. And, in general, creatures have a connection arising from a common cause. To me extension seems to be nothing other than a continuous order of coexisting, as time is a continuous order of existing successively. Hence neither one is a substance any more than the other, but only the very things that exist in this way. Besides, modes are changeable, but substances, attributes, and properties persist. And, unless I am mistaken, the sort of definition that is required is one by which substance is distinguished not only from modes, but from other predicates as well. So I submit all these things to you to be examined, and I hope that in this way we will penetrate more and more into the heart of the matter.

Thank you for pointing out the quote from Bernard, the author from the Nouvelles de la République des Lettres. I had taken this to be referring to anyone but me, given what was added at the end. Thank you also for the exposition of Bredenburg's demonstration, which is not very satisfactory [and suffers from the common fallacy that the determination, i.e., the infallibility, of what will happen is confused with necessity. The truth of future contingents is determined by causes, but there is nonetheless a reason why those future contingents should not to be judged necessary].[9] But I was under the impression that this material was concerned not so much with the question of necessity – where the difficulty arises not from the issue itself, but from confused notions – as with the doctrine of substance, which is why I was asking for information about it. For the rest, goodbye and be well. From Hanover, 31 December 1700.

39. De Volder to Leibniz[1]

[Leiden, 13 February 1701]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

I am happy that I have understood you correctly. You should not be surprised that I have used almost the same notation as you in the calculation, for I owe this to you. In several issues of the Acta Eruditorum, I saw that you use the initial letters of words to stand for the things that the words signify, wherever this is appropriate.[2] I have imitated this, since I found that in this way it is easier to remember the things that we use signs to designate. I surveyed with the greatest pleasure the consequences that you derive from your calculations. No doubt the same principles could be applied to non-uniform motion, provided that the ratio between time and increase or decrease of speed, whatever it is, is fixed.

Concerning the notion of substance, I said that, for me, the concept of a substance represents one thing in such a way that nothing can be removed from the representation without the whole thing perishing. You complain about the obscurity of this, and you do not think that it agrees with my other account, where I said that a substance is that which can be conceived separately.[3] I do not see why you find this obscure, but I shall try to explain my intentions as well as I can. It seems obvious to me that when I conceive of one thing, either the whole thing, as it were, is represented to me, or none of it. For it is not composed of anything. If I conceive of a thing that has no parts (or, if it has parts, they are all of one and the same nature), is it not true that this thing, or this one and the same nature of all the parts, is conceived of either as a whole or not at all? Since the concept represents nothing but this one thing, what is clearer than that this one thing can be conceived of independently of everything else? And this is also what I had attributed to a substance. But you object that we can conceive of two things, A and B, separately as individuals, and then also conceive of a third thing C, which requires them both. I do not think so. For A and B are either substances of the same nature or substances of entirely different natures. They cannot be partly of the same nature and partly of different natures, since they each represent one single thing. If they are of the same nature, then the third thing C could indeed involve a common nature of A and B in a general concept, but it could not involve either A or B in particular unless it were determined in a more particular way. But this would make C a mode of that substance whose nature it involved particularly. On the other hand, if A and B are substances of different natures, then one and the same thing could not involve both their natures. Let us suppose that A and B are two bodies that are conceived of separately as individuals, and that C is, e.g., a spherical figure. C involves a common nature of A and B in its genus, and without this common nature it can neither be conceived nor exist. But C does not by any means involve the determinate nature of either A or B. For if C is determined specifically according to the

nature of A – because of the producing cause or some other reason – then it will not require the nature of B. But if A is a body and B is a mind, there could not be any one mode common to both of these substances, unless we say that C is composed partly from a mode of the body and partly from a mode of the mind, i.e., unless we say that C is not one mode but two modes, and that these are modes of different things.

You say, "modification seems to imply something more than", what you rightly include, "the simple need for a distinct concept."[4] I do not see this, most renowned Sir, especially since I can reduce whatever difference I notice between my concepts to the difference between substances and modes.

You ask next whether we need attributes for the concept of a substance. Without a doubt, if one is concerned with the concept of a determinate substance. For since a determinate substance is nothing but its nature, its concept represents nothing but an attribute. And we call whatever is in the concept of a thing and represents its nature an attribute. I think that it is true that there is no place for attributes when the general notion of substance is sought, since they necessarily determine one substance or another. The general notion of substance is the mode of conceiving we use to distinguish in concept what we call a substance from all its modes. Thus, I think that the general notion of substance should be defined through the mode of conceiving.

Then you object that nothing except the first substance can be conceived through itself, since everything else obviously has a cause. I think that conceiving of the existence of a substance necessarily requires a cause, but not conceiving of the essence, which alone is the issue here. Moreover, you rightly say that, "modes are changeable, substances, attributes, and properties persist." This is easily derived from my definition. Finally, you add that we should seek a definition that would allow us to distinguish substance not only from modes but from

other predicates. I do not fully understand this, since I do not consider anything to be real except modes and substances or their attributes.

If only it had pleased you to include your notion of substance. For if I had discerned your intentions better, I would have understood your objections more clearly. And, at the same time, by combining both definitions, I think I would have uncovered what is lacking in my definition more easily.

I am not at all surprised that Bredenburg's demonstration did not please you. But I am perplexed by my failure to understand your further comment that the question of necessity is difficult because of confused notions, not because of the subject matter. I would be very grateful if you would consider explaining this mere word, especially since the notions of cause and effect seem to incline very strongly toward necessity. Certainly it seems that nothing happens without a cause, and every cause seems to produce its effect necessarily. But this is only a passing thought. Goodbye.

From Leiden,

13 February 1701 A.D.

I wish you good luck and happiness in your work during the coming year and many thereafter.

40. Leibniz to De Volder[1]

[Hanover, 6 July 1701]

[L1] Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

[2] I am returning to my meditations and to the duty of writing, which has been interrupted for a long time, and I ask your forgiveness for not having responded to your most

recent letter more promptly. It often happens that the things that I want to do more carefully are put off for longer than they deserve. If only I could now satisfy you that much better by how long I have delayed. But I turn to your letter. You say that a substance is that whose concept represents one thing in such a way that nothing can be taken away from the representation without the whole thing perishing. But consider whether something that adds new things to the perfections of others can be a substance. Moreover, if I am not mistaken, each and every perfection is the content of the representation. It is in this way that Democriteans conceive of space as a substance and body as another more perfect substance that adds resistance to extension (as you rightly observe, examples need not be true when they are used only to clarify).[3] [4] Some of the moderns even think that a new perfection can be added to certain bodies, namely the perfection of thinking.[5] And, for a long time, besides simple animated bodies of various kinds, the Schools have had the more perfect sentient body, and, most perfect of all, the rational body. I say this so that I may observe that your notion of substance does not seem to apply to those things that are commonly so called, but only to a most simple substance. The same is true when you say that substance is that which is conceived through itself,[6] to which I opposed the maxim that the effect cannot be conceived better than through the cause, but that every substance apart from the first has a cause.[7] You reply that a cause is required to conceive of the existence of a substance, but not to conceive of its essence.[8] But I reply again: the concept of a possible cause is required to conceive of its essence, and the concept of an actual cause is required to conceive of its existence.[9] I can foresee an elegant surrejoinder for you based on a geometrical example: The essence of an ellipse, for example, does not depend on a cause, since different causes can produce the same ellipse, namely a conic section, a cylindrical section, or the motion of a thread. But the existence of an ellipse cannot be conceived unless some determinate cause is assumed. My further reply to this is twofold: first, even if it is not necessary to conceive of a

determinate mode of generation in order to conceive of the essence of an ellipse, nevertheless, the essence of an ellipse, or of anything else, cannot be conceived perfectly unless its possibility can be demonstrated a priori through a formal cause that exists in every particular mode of generation, and, for this, simpler lines must necessarily be used; the second further reply is that I have long since established that with incomplete things, such as lines or shapes, it is possible for one to be similar to another even if they are generated by different causes, as an ellipse formed by a conic section is similar to an ellipse described by a motion in a plane. But this cannot happen with complete things, and so one substance is not perfectly similar to another, nor can the same substance be generated in many ways. And, a long time ago, I also concluded from this (and other arguments besides) that there are no[10] atoms, that space is not a substance, and that primary matter itself, i.e., matter separated from all activity, should not be included among the substances.

I now turn to modes which, like you, I distinguish from other predicates, i.e., from attributes and properties. Yet if we define modes solely in terms of their needing the concept of another, then properties also will be modes. Moreover, it is common to modes and properties that they are in something. But the same definition will also apply to things that are not in something, such as effects, which need causes in order to be conceived, as I have already said. Thus, all effects will be modifications of their causes, and it will be possible for the same thing be a mode of many things at the same time, since the same thing can be the effect of many concurrent causes. Who will deny, moreover, that one substance is modified by the intervention of another, as when a body rebounds because of an obstacle in front of it? Therefore, the concept of both bodies will be necessary in order to conceive of the rebound of one of them distinctly, and yet the rebound can be a mode of only one of them, since it can happen that the other continues on its path and does not rebound. So, something more is required for a modification than needing the concept of another, and to be in (which is

common to properties and modes) is more than to need the concept of another thing. In my opinion, of course, there is nothing in the universe of created things that does not need the concept of every other thing in the universe for its perfect concept, since each thing influences every other in such a way that, if it were imagined that that thing were removed or different, everything in the world would be different than it is now. For the rest, I admit that if A and B are two substances of the kind that you define, i.e., most simple ones, then[11] they cannot have a common predicate. However, it does not necessarily follow from this that there cannot be some third thing C, which needs both of them for its concept. For we see that, just as relations result from many absolute things, so also qualities and actions result from many substances. And, just as a relation is not composed of as many relations as there are things that are related, so other modes depending on many things are not resolvable into many modes. And so, it does not follow that a mode that needs many things is not one thing but composed of many. Besides, it is not clear how it is possible for modes to arise on your notion of them. For a substance such as you define, whether as that which has only one simple representation or that which has one attribute, will have only one mode. The source of diversity is not apparent since only one comes from one. Whence the mode will be invariable, contrary to the hypothesis. And indeed, or for that reason, the thing will have no mode, since it is not even clear how the mode will be different from the attribute. So certainly if, as is commonly done, a body is said to contain nothing other than extension, and extension is conceived as some simple and primitive attribute, then no reason can be given for how any variation arises in bodies, or how many bodies exist. I have demonstrated elsewhere (in my reply to Mr. Sturm published in the Acta Eruditorum) that unless matter is heterogeneous (which happens through entelechies), no variety of phenomena can arise, and equivalent things would always be substituted for one another.[12] For the rest, I do not distinguish here between the general notion of a substance and the notion of a determinate substance. For

every substance is determinate, even if different substances are determined by different modes. As for my own notion of substance, I would prefer for it to arise from our mutual considerations[13] (and certainly I think that a good start has been made), rather than for it to be produced by me alone and as if it were forced upon you.

On returning to your letter before last,[14] so that I might get a better idea of everything, I noticed a few things that still need comment. It will be difficult to give[15] examples of concepts from which no representation can be removed. Primitive notions lie concealed in derivative ones, but they are distinguished only with difficulty. I doubt whether a body lacking motion can be conceived. I admit that motion cannot be conceived without a body. The concept of motion does not involve just body and change, but also the reason and determination of change, which cannot be found in a body if it is merely passive, i.e., if you locate its nature in extension alone or in extension and impenetrability. I conceive of more than one thing in extension, namely both continuity (which it has in common with time and motion) and coexistence. So it is not necessary that either the whole of extension be conceived or none of it. But it is clear that for there to be extension there must be some thing that is repeated continuously, or many things whose coexistence is continuous.[16] You ask, excellent Sir, what we conceive of besides extension in the thing to which extension is attributed. I reply, things to add action and passion to extension. So (you say) extension will be a mode of that which is extended. I reply, in turn, that extension, in my sense, will not be a mode of the substances from which it results, since it is invariable[17] and designates a numerical determination of things, which remains the same under any change whatsoever. And surely you agree with me that modes must be variable. In the meantime, I grant that not only extension but also action and passion cannot be conceived of on their own. And certainly, it is neither necessary nor easy to arrive at[18] notions of the greatest simplicity, as I have already noted. So, if we required this to conceive of a substance, I am afraid that we

would have to do away with all created substances, which would be to cut the knot and not untie it.[19]

I had mentioned necessity in passing, talking about Bredenburg, and I had said that the difficulty concerning necessity that people discuss arises from confused notions.[20] You say that these few remarks aroused your curiosity, and that you would be grateful if I explained my meaning a bit more clearly, since it seems that every cause produces its effect necessarily. Of course, I agree that nothing ever happens without a reason, i.e., without a reason why it happens this way rather than another. But the reasons are usually inclining rather than necessitating, so that the contingency of things remains unharmed.[21]

I understand from a letter written in Leiden that you propose to publish a description of Huygens' planetarium.[22] By doing this you will leave both the Republic of Letters, and the memory of that great man indebted to you – and the fuller the description, the greater the debt. I would like to know how Huygens' planetarium differs from Rømer's, both in its construction and in what it does.[23] For, unless it was supposed to do something else, such a distinguished man would certainly not have embarked upon a time-consuming project already undertaken by another. I beseech you not to begrudge us information about other things of yours or Huygens' that you are preparing for publication, or, generally, about what you are working on in more esoteric areas of study (in which you are capable of so much). The letter also says that Mr. Hartsoeker is making considerable advances in grinding object-glasses out of a huge sphere without the help of dishes.[24] If I can find out from you about these and similar scientific advances, I will add that to the other favors. Goodbye. From Hanover, 6 July 1701.

41. De Volder to Leibniz[1]

[Leiden, 7 October 1701]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

Since it seems that you are quite set on this, though I fear that it will not succeed, let us proceed in an examination of the definition of substance that I gave. Here, in brief, is my reply to your objections.

Given my definition of substance, it follows that every single perfection of a substance (which you correctly call the content of the representation) represents something separate to me, which has nothing in common with what is represented by any other perfection of the same substance, if there can be any. It seems proper to conclude from this that every single perfection denotes a different substance, or at least, that if we say that one substance has many attributes, we are really saying that those attributes are the same, and that all that is missing is for us to understand the unity or identity and that, on the contrary, we perceive the utmost diversity among them that can be found in thought. In which case we seem to say something in words other than what is understood by the mind.

You add that my "notion of substance does not seem to apply to those things that are commonly so called, but only to a most simple substance."[2] I believe that the first part is not opposed to my definition, since it is known that people commonly speak of these notions in a very confused way, and that the second part supports it. For, if my definition applies to a simple substance, it certainly includes the general notion of substance, which is necessarily contained in the notion of a simple substance. And what if every substance were simple insofar as it is a substance?

Concerning what you say about the cause of a substance, I would think that if the essence of a substance cannot be conceived without its possible cause, then no such essence is ever conceived. It seems to me that I do indeed conceive of the possible causes of modes,

but not of substances. This is why we commonly say they are created, i.e., that they have a cause by which they are produced, but who knows how.

I would never have objected to what you add concerning the different ways in which ellipses are produced, because, for a long time now, I have been completely persuaded that your previous rebuttal entirely answers the difficulty. But I do not know whether I understand your subsequent reply correctly. I suspect, however, that by incomplete things you mean purely passive things to which we arbitrarily attribute whatever motion we wish, and in which we locate some or all of the forces. By complete things, I suspect that you mean things that have a determinate force of acting in them, through which (indeed through all of which) they seem to produce a determined effect by necessity, and necessarily in the way in which they are determined to action.

I distinguish properties from modes by the fact that properties are convertible with the essence of a thing, and so do not differ from it except in our way of conceiving. Modes are not at all like this. I do not think that properties require the concept of another thing. Let us suppose that extension is conceived by itself. Being in a place is a property of it. They are different words, I admit, but the same concept. For what is it to be in a place than to be extended?

Effects will be modifications, or, as I prefer, properties, of a cause. For they are convertible with one another. And certainly, if we understand cause in the proper sense, as that which causes insofar as it is a cause and insofar as it acts with all its force (for it is not a cause in virtue of that by which it does not act), it will be difficult to distinguish cause from effect at all. Let us imagine an ellipse drawn by means of a thread fixed at its foci. The proper cause of the ellipse is the circular path of the tip of the pen that draws it. But surely this is the ellipse itself.

I think that it is true that the same thing can be the effect of many concurrent causes, if we speak loosely, as we are commonly accustomed to. But it is false with a more precise notion of causes and effects. For the various causes will either be causes of the same operations – as when two equal weights acting together lift some body, in which case the effect of each will be similar rather than the same, for each weight raises half the body – or causes of different operations, from which it necessarily follows that the effects are different.

I do not deny that a substance is modified by the intervention of another, if you are talking about substances of one specific nature. In fact, then, insofar as they are substances, they are not different. For this reason, I think that you are right when you add that a substance of a given nature may influence any other of the same nature, in such a way that if one of them were imagined as being removed or changed, everything would be different from what it is now. Indeed, perhaps this opens up even broader issues.

If A and B have no common predicate, they will have no common operation, and hence no common effect. So a third thing C, which is supposed to need both of these for its concept, will not need anything that is common to them, since there is no such thing. Rather it will need predicates different from both of them. But since they are different, the one does not involve the other, and so they can be separated from one another, as A can from B. So it will follow that the two predicates contained in C can be separated from one another, i.e., it will follow that this third thing C, is not one thing but is composed from two different things.

However, I would not want to deny that a mode that results from many substances of the same nature can be one thing, provided that the formal cause is one and the same, as in the example of the ellipse that you mention. I will illustrate what I mean with an example. Let there be two bodies, A and B, acting on a body, C, in such a way that, at a given time, A is pushing it along the straight line CG and B is pushing it along the straight line CE. The body will be moved along CF just as if it had been impelled along the straight line CF by one body

D.[D1] This is not surprising. For, since the two determinations along CE and CG are equivalent to the one along CF, the same effect necessarily follows. And I do not think that it can happen otherwise.

Next you say that no modes will arise given my notion of substance. I do not deny this. Indeed, this is why I have asked so eagerly for a demonstration that would make it clear that every substance is active by nature. Certainly, the facts themselves show that all substances that are subject to a variety of modes involve action and active force, either in themselves or in the substance that produces the variety. For this reason, I readily allow that every created substance has power from an active force, since experience shows that it is subject to infinite variety. All I require is an a priori demonstration of the fact that this active force arises from the nature of a substance and is inseparable from it.

No matter how difficult it is to distinguish primitive notions from derivative ones, nevertheless, I think that it is a requirement of a perfect science that it distinguish and deduce the derivative from the primitive ones.

You maintain that you conceive of many things in extension, namely continuity and coexistence. But I regard these as the same thing. For things between which there is a necessary and reciprocal connection are the same to me. I can indeed conceive of continuity without coexistence, as in time. But it does not follow from this that these are distinct in an extended thing, since I can conceive of a species without an individual, even though what I call the species in the individual does not really differ from the very individual to which I attribute it. You say that for there to be extension there must be many things whose coexistence is continuous. This holds me back. I would not so readily say that they are many things, since, on the definition just given, there is a necessary and reciprocal connection between these things (if you wish to speak of them as things). For whatever plurality you

suppose here, it is certain that part A of the extension cannot be conceived and cannot exist without part B, and vice versa.

These are the things, most celebrated Sir, which came to mind when considering your letter. I submit them willingly to your most solid judgment.

Furthermore, as for what you ask about Huygens. You know, I believe, that the most distinguished Fullen and I were entrusted by the late, lamented Huygens with printing all his writings that were suitable for publication. This has been executed rather slowly, both because the writings were rather slow in reaching us and because we do not live in the same place, and on account of various other things that have caused delay. We have it in mind to publish his Dioptrics, the treatise On Parhelia, and his Planetary Automaton just as he left them, as well as some of his things on motion.[3] The Dioptrics is already with the printer, but work is proceeding slowly. I do not know Rømer's [planetarium] well enough to say how it is different. Huygens' version shows both the longitudinal and latitudinal motions of the planets in a plane around the sun at a given time. This is achieved by a clockwork mechanism that moves an axle that is common to all the planets and fitted with different teeth for each planet in such a way that, when the planets are put in their proper place, the machine indicates the places of the planets for many years, by its motion alone.

I had not seen Hartsoeker for a long time, nor had I heard anything about him. So I wrote to him, lest I report back to you without sufficient information. He sent his greetings to you and replied that, "it is true that he has been busy, and with great success, grinding object-lenses from a large sphere; that he has made the best-known lenses [with focal lengths] of 100, 200, 300, 400, 500 and 600 feet; and that he did not accomplish this without dishes, but by using glass dishes, as he explains in his Dioptrics."[4]

You will see the replies that Oudin and Bayle gave to your last letter attached to this one.[5] Goodbye.

From Leiden,

7 October 1701 A.D.

42. Leibniz to De Volder[1]

[Berlin, 27 December 1701]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

As I was preparing to reply to your last letter, both of our previous letters, which were left at home, would have been very useful to me. And so my reply was delayed while I was hoping to return to Hanover. But finally, as my absence has continued, I decided that this should come to an end, and that it would be preferable to give you some kind of response than to stay silent any longer. Anyway, I hope you will be kind enough to take both my silence and my response in good spirits.

Turning to your points, I do not think that one can give an example of a perfection that has nothing in common with another perfection, and that does not involve any relationship to another, and so can constitute a substance on its own.

If the notion of substance in general does not apply to anything except a most simple substance, i.e., the primary substance, then this alone will be a substance. I admit that you are within your right to understand the word <u>substance</u>, so that God alone is a substance and other things are called something else. But it is my intention is to look for a notion that will apply to others as well, and that agrees with ordinary ways of speaking, according to which you and I and others are counted as substances. You will not deny that I am allowed to do this, and that it will be useful if it succeeds. Besides, not everything that applies to one substance contains the general notion of substance, just as not everything that applies to a

square contains the general notion of a parallelogram. I concede that every substance is simple in a certain sense. I say substance, not aggregate of substances.

And, if it were supposed that every substance existed from eternity, nevertheless, all substances aside from the primitive one would have had to have causes, evidently extraneous reasons, for why they existed rather than not.

If properties B and C differ from each other and from essence A by way of conception, then certainly other things than A must be assumed in their concepts.

If an extended thing were conceived of on its own, it would not be in a place.

I think that it is clear that neither a pen moving in a path nor the path of the pen is an ellipse. The ellipse is on the paper; the pen is outside of it. The ellipse is permanent, the path of the pen is transitory.

I do not know whether it can be said that when two equal weights raise the same body at the same time they have no common effect, but that there is a separate effect of each, namely a half. For one cannot assign the particular half of the body that it is ascending to either of the two weights. They act as if they were undivided.

As for what I said about the connection and sympathy of substances. I think that it pertains to all of them, even if they are not of one specific nature.

Indeed, I do not think that an A and a B that have no predicate in common are possible things. However, it does not follow that if two predicates that concur for the concept of C itself are separable, then the concept of C itself is not one. For example, a square is a rectangular equilateral, but rectangular can be separated from equilateral (as in that which is called an oblong), and equilateral can be separated from rectangular (as in a triangle, a pentagon, etc.). But nonetheless a square is one figure and has one concept.

As I praise your penetration in other matters, I praise your frankness in acknowledging that no modes will arise on the notion of substance that you proposed. In fact,

I think that it can be demonstrated that such a thing does not exist, and that an existing substance must be either the subject of modes, or certainly the cause. Nonetheless, that which is a subject is always the cause of modes as well, but not the converse.

This very thing is what you desire to be investigated in primitive notions, or at least is the sort of inquiry that you want to be undertaken with respect to them.

You concede that existence and continuity, which make up the notion of extension, differ formally. I do not claim anything else. But that whose notion is composed of different formal concepts is not primitive.

It is among the principal errors of the Cartesians that they conceive of extension as if it was something primitive and absolute and that constitutes a substance. This error must be discarded in order to philosophize correctly, or the nature of body and of substance will not be understood, in my judgment at least. Such a mistaken adoption of primitives renders philosophy worthless, as in the case of occult qualities, and to sin in such things is common today. Not so long ago, in the Journal des Savants, I criticized those who think that the perception of heat, cold, etc., does not represent what is in motions, but is something inexpressible and arbitrary that God impresses on the mind on the occasion of motions.[2] But to return to the matter at hand. We will never find extension without many things whose coexistence is continuous, nor will we discover anything more in it than the fact that such things are understood. Also, the connection of these things is not necessary, for some of them can be removed. And the fact that others are substituted for them makes no difference. However, if you distinguish extension from that which is extended, then it is something abstract, like duration, or like number separated from things, where the connection of parts is necessary in the same way as it is in extension. Thus, in a triad, three intelligible unities are bound together by an eternal bond, even if the connection between the three posited things is perhaps not necessary. Yet, if some things are taken away others suffice, and things are never

lacking for the numbers. And there is no more a vacuum in numbers than in place, in time, and in other orderings, unless we suppose that the universe is destroyed, so that only possibilities remain. For that is what extension, duration, and numbers without things are, although, after the fashion of Platonic ideas, they are often conceived of as substances. Furthermore, extension is relative to some nature whose diffusion it is, just as duration is relative to a thing that persists. However, it is peculiar to extension that different successive, extended things fit one and the same place, i.e., they take up the situation of other things in the order of coexisting things. But it is peculiar to time that many things exist together at the same time.

Thank you for passing on to me the most distinguished Hartsoeker's greeting (which I ask you to return when you have the chance) and his response, and for taking the care to send me the letters from the most distinguished gentlemen Bayle and Oudin. I venture to ask if you would pass on the enclosed to them. If [the focal lengths of] Hartsoeker's lenses are growing so long, this is why we may expect important new astronomical discoveries from them. I would hope that the microscope, which is even more useful, will also be advanced in this way. The most celebrated Tschirnhaus is also grinding lenses of this sort, and has advanced the art a great deal.[3] As you may know, he recently went to France, via Holland. Unfortunately, I was away when he passed through Hanover. I hope that he will get everyone's attention at the Royal Academy of Sciences with the excellent specimens that he told us to expect. I am greatly looking forward to the great Huygens' Dioptrics, along with all his other works.[4] For the rest, I wish you a happy new year, and many more to come. From Berlin, 27 December 1701.

43. Leibniz to Bernoulli[1]

[Berlin, 27 December 1701]

Not least of our most acute De Volder's prejudices is that he conceives of extension as something primitive that constitutes the essence of body, when extension brings nothing to mind except a certain diffusion (i.e., a continuous repetition) of some common nature throughout a plurality of coexisting things. Moreover, it does not describe what in the world this nature might be, but presupposes it. It follows that extension is something relative rather than absolute, and that the actual nature of corporeal substance (i.e., the nature that is understood to be diffused) is something prior to extension. Unless I am mistaken, I have already indicated this to him in previous letters, but in such a way that he has not paid enough attention to it.[2] I have not repeated this here, but have confirmed it. If I manage to satisfy you with it, it would be appropriate if you would also help him understand this better.

44. Bernoulli to Leibniz[1]

[Groningen, 14 January 1702]

I shall today send off the letter that you wrote to De Volder[2], along with the ones enclosed for Mr. Oudin and Mr. Bayle.[3] You certainly do not need my help making your thoughts more intelligible to De Volder, unless you would like me to hold a candle to the Sun. I see that he has a notion of substance very different from yours. It is also true that he considers extension as the primitive essence of body. You, on the other hand, say that it is only something relative, and that it presupposes some common nature throughout many coexisting things. He says that he cannot conceive of what this common nature of body might consist in. And no doubt he will demand a clear and distinct idea of it from you, and also now your definition of substance, which, unless I am mistaken, you promised, if he gave his first.[4]

45. De Volder to Leibniz[1]

[Leiden, 3 April 1702]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

I am very much afraid that when the topic of primary notions is brought up between us (certainly when it concerns the one that we have tied to the word substance, and which we apply to almost everything) we both discuss many things that are not understood properly by the other because we do not associate the same notions with the same words. However, unless I am mistaken, it would have been possible to be rid of this unfortunate situation for the most part, if, just as I gave my notion of substance, it had pleased you to give yours. For I believe that in this way I might have understood your intention more clearly, and been able to respond more precisely to those things that you offer. Now I worry that I often miss the point in many cases by not adequately grasping what you mean. If this should happen, I hope that you will pardon a poor man for not understanding what you are hiding. Indeed, I confess that I am Davus, not Oedipus.[2] But, in brief, here are my thoughts as far as this confusion permits.

Whatever constitutes the notion of a most simple substance contains within it the general notion of substance, since that is necessarily applied to all substances. That which makes it the case that I, you, etc., are called substances, is necessarily common to all substances, and so to a most simple one as well. And the difference that is commonly proposed between the primary substance and secondary substances does not so much involve what people like to regard as the notion of substance strictly speaking, namely, not being in any another thing. Rather, it involves the notion of a cause, in that the primary substance requires no cause except itself, whereas secondary substances require some other cause distinct from themselves. It is indeed true that not everything that belongs to a square contains the general notion of a parallelogram. But it is also true that the notion of a square

contains everything that is contained in the general notion of a parallelogram. In a not dissimilar way, if I form the notion of a most simple substance, the general notion of substance is certainly involved in it. And, since only one perfection is being considered in this notion, it is sufficient on its own to constitute a substance. Besides, even if many attributes can constitute a substance together, it, nonetheless, does not seem that a plurality of attributes necessarily enters into the general notion of substance, since a most simple substance with one attribute can be conceived. Certainly, the notion of a substance will involve some attribute, whether one or many. But however this may be, it is not the case that many things are necessarily involved in the general notion of substance, unless, perhaps, you want some attribute not considered in itself, but instead, as one might say, as modified, to enter into the notion of a substance. I might indeed have suspected this, except that by this reasoning it would seem to follow that the causes of modes are being taken to be like properties of the substance, whereas this is the very thing that is being questioned.

You concede that every substance is simple in a certain sense. If I understand these words, they favor my view. For I interpret them thus: The attributes of a substance flow from a principle and source in such a way that they have a necessary and reciprocal connection between them. If I am interpreting your words correctly, they say the same thing as I intended. For I consider things that have such a connection between them not to be many in reality, but to be the same. Moreover, we are seeking the notion of a substance, not of an aggregate of substances.

I happily grant that everything that exists has a reason for existing rather than not existing.

I said that properties do not differ from the essence of a thing except by our way of conceiving (by which I distinguished them from modes). I shall clarify what I mean by this with an example. I conceive of a plane figure bounded by three straight lines. I call this a

triangle and suppose that its essence consists in this. I then demonstrate of this triangle that the longest side necessarily subtends the greatest angle, that its angles are equal two right angles, etc. I call these properties of the triangle. They are certainly distinct from the essence of the triangle in the way that they are conceived, but they by no means differ from it in reality. Rather, they represent one and the same nature of a triangle to me through distinct ways of conceiving. I judge that this follows from the fact that these properties are convertible with the nature of the triangle.

I do not understand what you mean when you say, "if an extended thing was conceived of on its own, it would not be in a place."[3] Indeed, this seems to me to be the same as if someone had said, if an extended thing were conceived on its own, it would not be extended. I do not understand what being in a place adds to an extended thing except an external denomination.

If you separate from the path of a pen all the things that are external to it and that, in the end, are employed only so that the tip of the pen is led around in accordance with a certain law, nothing will remain except the path of that tip. This is clearly an ellipse, whether it leaves behind its trace on the paper or not.

I admit that no one can determinately assign the part of a weight that is raised by each of two weights that are raising it with equal force. But, nevertheless, it cannot be denied that there is a similar action in each, that a similar effect proceeds from each, and that each one raises half the weight.

Besides parallel opposite sides and right angles, sides of a certain length are also contained in each rectangle. None of these can be separated from a rectangle without destroying the rectangle, nor from its concept without that perishing. Certainly, when the nature of a rectangle is considered generally, any ratio at all between the sides can be taken. However, in any given rectangle there is a certain ratio that cannot be separated from it

without it perishing. And, in this respect, the square does not have any privilege, except that it pleased us to call that figure that has a ratio of equality between the sides by the special name square, and to include all the rest under the general name rectangle. Therefore, it is not surprising that, if this or that actually existing rectangle is considered (rather than the concept of a rectangle in general), it has one nature and one concept. Indeed, if opposite parallel sides are joined to each other by right angles with a certain ratio between the sides, or even with the sides having a certain length, these cannot be separated from this rectangle or from each other within this rectangle, without the rectangle itself being thereby utterly destroyed. But, if I add motion to this rectangle, the concept of the moving rectangle now involves two things, because motion can be separated from a rectangle while that same rectangle remains. It is consistent with this that, if two separable attributes come together for the concept of a thing C, this is not one thing, and nor is its concept unitary. For the separation in question here is not one that is brought about solely by an abstraction of the mind, but one that holds according to nature. Since the attributes constituting the concept of the thing C are separable, they will certainly be distinct, will require distinct causes, will produce distinct effects, and so will not constitute one thing (unless perhaps you want to use one with the completely loose meaning to which we are commonly accustomed, as when we say one house, one city, etc.). This is especially the case here, since we are talking about attributes of substances, which, for that very reason, are not in any subject. For nothing prevents many modes from belonging to one subject, or the concepts of modes from including one common concept.

You do not think that it is possible for there to be two things A and B that have no common predicate. So does a distinction between such things therefore imply a contradiction? I do not see that. But, in case there is some ambiguity lying hidden in the words here, let us suppose (assuming that this could be brought about) two things that exist with completely different properties. It is correctly predicated of both that they exist, that they

are things, that they are substances or modes, that they have causes of both their essence and their existence, and everything besides that can be generally ascribed to all things. In this sense, they will, without doubt, have certain common predicates. However, unless I am mistaken, I will surely have been right in saying that there is nothing common between these two things. For although I designate what I call thing and substance with the same name, indeed, although I apprehend it with the same general concept, nevertheless, it is not in fact the same in the thing itself, since the things differ just as much as the very natures of the things. The Cartesians distinguish corporeal from thinking substance. They hold that the nature of the first consists in thinking, of the second in extension, and they claim that these two differ from each other in such a way that they have no properties and no accidents in common between them. I am not going to discuss now whether or not this is true. However, they call them both things, they name them substances. But, since these general terms fix the nature of the thing quite neutrally, and only express our concept by means of which we apprehend those things, which is utterly external to the things themselves, they do not qualify as common attributes. I believe that there is nothing wrong with this, if those things that we assumed earlier on the basis of their position are indeed true.

You add that it can be demonstrated that that from which no modes arise does not exist etc. If these things were to be demonstrated a priori, the matter would be settled. Certainly, I agree with you that it is evident from experience that bodies are in no way devoid of all force. But I seek a demonstration of this fact that is known through experience, from the very nature of substance. You insist that, if I distinguish extension from that which is extended, it is abstract, like duration, number, etc. But, even if this is entirely true, nonetheless, it will still remain to be explained what those things are that are extended, which attributes they have, and by which concept they are represented. I seem to understand no

more about the matter, if these things are indeed taken to be this way, than I understand what the things are that are denoted by the number three when that number is perceived.

Of the letters that you enclosed with your last letter, I immediately sent the one to Mr. Oudin, and I made sure that the other was sent to Rotterdam. Goodbye.

From Leiden,

3 April 1702 A.D.

46. Leibniz to De Volder[1]

[Hanover, April 1702]

G. W. Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

I wish that it were possible for me to return adequately to your earlier points and, being in possession of most of what you ask for, to set out and connect the notions with each other properly. But perhaps you yourself will settle the matter more satisfactorily. I do not think that one unique perfection is observed in the general notion of substance. But you seem not to admit a substance unless it is a substance of one attribute, and this is a consequence of your notion of substance. Let there be a substance of attributes A and B, then, since for you there can be another substance of attribute A, it is clear that substance AB is not conceived through itself but through substance A, and so, it will not be a substance for you. And if one absolute, simple attribute can constitute a substance by itself, no reason can be given why another could not do that as well. But, if one supposes that every substance is of one simple attribute, then it is certain that nothing of the origin of modes and changes can be understood in the nature of things, for whence can they come except from substances? In fact, I hold that there is no substance that does not involve a relation to all the perfections of all the others. So

a substance of one attribute cannot be conceived, nor, as far as I know, can we conceive of any simple and absolute attribute or predicate separately. I know that the Cartesians felt differently about the former, and Spinoza about the latter as well, but I also know that this arises from lack of sufficient analysis, the touchstone of which is the demonstration of predicates from the subject. For in every demonstrable proposition of which we do not have a demonstration, there must be a term that is not sufficiently analyzed.

When I say that every substance is simple, I understand by this that it lacks parts. To be sure, if all the things that have a necessary connection with each other were one substance, it would follow, at least on the assumption that the vacuum was excluded, that all the parts of matter would compose one substance, because they have a necessary connection. But, through this very [reasoning], a substance would be confused with an aggregate of substances. However, you very rightly point out that the notion of substance is being sought, not the notion of an aggregate of substances.

I do in fact accept what I believe that you hold up as something that I ought not to accept, namely, "if an extended thing was conceived of on its own, it would not be extended".[2] For such an extended thing implies a contradiction. I also think that it is certain that that which is conceived of on its own cannot be in a place. For to be in a place is not a bare extrinsic denomination. Indeed, there is no denomination so extrinsic that it does not have something intrinsic for a foundation. This itself is also among my fundamental doctrines.

I am surprised that you want the actual path of the tip of the pen to be an ellipse. A path is a successive being, like motion and time, whereas an ellipse is a permanent being, like a line. I have already touched on this in the preceding letter.[3] If the motion does not leave behind a mark on paper, it will leave one in the world. And, even if it did not leave anything behind, when the motion was completed we could still combine all the positions of the tip in

our mind. Besides, the same ellipse can be described by different motions, if the same points are passed through.

When two weights acting in the same way raise one thing, the half that is to be assigned to each is half of the power, not half of the thing, as when in the law there are two joint owners of the same thing. And so it remains impossible to assign separate effects to each of the weights.

Unless I am mistaken, you were saying that, if two predicates that come together for the concept of a subject C are separable – i.e., as I understand it, that they can be found in other subjects without each other – , then the concept of C itself is not one. I brought up the example of a square, whose predicates are being rectangular and being equilateral, two predicates that are not together anywhere else. And this separation is certainly not just one made by the mind, but exists in nature, since an oblong is a non-equilateral rectangle, and there is a kind of triangle that is equilateral but not rectangular. I cannot satisfactorily relate this to what you say in your latest reply. I admit that at least some ratio between the sides cannot be separated from a rectangle, but some ratio between the sides is one predicate and the ratio of equality is another. They differ as genus and species. It does not matter whether the square has any privilege of simplicity over other rectangles, if indeed it does have it. These two predicates also require completely different causes.[L1] For let AB be moved with respect to CD. The cause that makes the angle at A or C a right angle is one thing, the cause that makes it the case that AB is equal to CD is another. From this it can easily be understood that the effects produced in this way are also different. But the concept of the square is no less one because of this.

Any two things A and B not only have in common that they are things and substances, but they also have a certain sympathy, as I remember saying in a previous letter. And you did not seem to object.[4]

The Cartesians think that some substance can be constituted by extension alone because they conceive of extension as something primitive. But, if they undertook an analysis of this notion, they would see that extension alone can no more suffice for that which is extended, than number can for the things that are numbered. I agree with you that, just as the idea of the number three is not sufficient to understand three things, so the idea of extension, i.e., diffusion, is not sufficient for understanding the nature whose diffusion it is. This is the very nature that I think we should investigate. And I leave it to your judgment whether that can be anything other than dynamism, from which there is action and passion.

Finally, suppose that the a priori demonstration of everything that you wish for cannot be had. Will the hypothesis agree with things any less because of this? Even if it can only be proved a posteriori, it will be worth more than a hypothesis. And can anything more powerful be adduced against that other notion of substance than the one that you yourself acknowledge, that on this view modes and changes do not arise? So, even if its impossibility were not demonstrated, it would be enough that the notions are understood in such a way that they agree with what is known and with what is useful, and that difficulties are resolved, whence the road to higher reasons is also revealed. Accordingly, if any doubts are brought against what I have said, I shall try to answer them willingly and candidly, just as I have always been accustomed to do up until now, which is something you will perhaps hope for in vain on the other hypothesis. All that remains is to say goodbye and be well. From Hanover, April 1702.

47. De Volder to Leibniz[1]

[Leiden, 25 July 1702]

B. De Volder sends greetings to the most renowned gentleman Gottfried Wilhelm Leibniz.

I remain uncertain whether I understand your view correctly. This hesitation has been the reason for my silence until now, and perhaps it still would be if I had not thought it better to respond with something off the point rather than nothing.

If I understand your words correctly, when you say that dynamism is the nature that is diffused, you mean that corporeal substance is composed from matter, to which, unless I am mistaken, you ascribe impenetrability and inertia, and from active force, i.e., entelechy. I am quite willing to admit that it follows straightforwardly from the innumerable changes that experience teaches us happen to corporeal things that there is a force that is the cause of these changes in bodies themselves. And I do not deny that when this force is assumed many difficulties in physics are solved that would perhaps be insoluble otherwise. As an example of this kind of thing, I adduce the fact that it will be evident from this assumption that that force will produce translation until it is prevented by another stronger opposite force. And so, it will easily be explained why moving bodies keep themselves in motion, which I might say, in the very least, has always seemed to me very difficult to explain on the Cartesian hypothesis. For, since motion for them is not a kind of enduring force from which translation follows, but just translation itself, which consists in a perpetual change of position, the same rule that they employ to solve this problem will require a new cause at each moment for this change in place, just as they also claim, correctly, that in curvilinear motion a new cause is required at each moment that changes the determination.[2] And, if this is taken away, just as this body will no longer change its determination, so the previous body will no longer change its place. The mutual action and passion of bodies, and many things of the same kind, follow from that same force without the intervention of Deus ex machina.

But, if you are looking for something that I dislike about all this, here it is: I am certain that one and the same thing cannot be separated from itself, i.e., that those things that

can be separated from each other are not one and the same. And since they are different, they require different causes.

However, I do not deny that many distinct modes can come together in one and the same subject, if, of course, those modes contain that one subject in their concept. But as straightforward as this is in the case of modes, it seems just as difficult to me in the case of substances, which have no subject distinct from themselves. For, although we can conceive of substances through attributes or properties, still I believe that it cannot be said that those attributes are in the substances, but rather that they are the substances themselves. For let us suppose a substance conceived through an attribute A. This attribute will be either the substance itself, or it will have a subject distinct from it that it is in. In that case, I believe that everyone will agree that this subject, which is distinct from the attribute, will be properly what we call the substance. Therefore, it will be either utterly unknown or known through another attribute that represents the nature of this subject. Since the same question may reappear concerning this attribute, this will go on to infinity unless an attribute is found that does not differ from its subject. But, if I may speak in this way, this attribute will be its own subject, and for that reason it will be properly a substance.

Next let us imagine there is a substance that has two attributes A and B, but of such a kind that they can be separated from each other, and, on account of that, do not have a necessary and reciprocal connection. For if they have this connection, they will not be two attributes but one. And so, since each of these attributes is the attribute of a substance, the subject of attribute A will be the attribute A itself, and the subject of attribute B will be the attribute B itself. Since the attributes differ, the subjects will differ as well. And since they are the same as the attributes, the substances will therefore differ. So this substance will not be one thing, but will be composed from two substances that differ from one another very greatly, just as the attributes themselves do.

From this reasoning, you can easily tell that I desire it to be shown that, on your notion of corporeal substance, there is a necessary and reciprocal connection between matter and the active force that is ascribed to it, which together constitute one substance. Or at least that it be shown that, if perhaps matter can be conceived without active force, it nonetheless cannot exist without that force. As long as one or the other of these things is not shown, it seems to me that these two things are conjoined gratuitously, especially since active force seems to be related to matter just as a mode is related to a substance, since I cannot conceive it without matter but can conceive matter without it.

I believe that those who regard every distinction between bodies as a mode of extension (as so many do today, though not everyone), will not deny that, when the vacuum is excluded, all bodies indeed differ only modally and are one and the same with respect to substance.[3]

Being in space, i.e., occupying a space with a certain magnitude and figure, represents nothing in my view besides being of that same figure and magnitude. I do not find any difference in concept when I contemplate a mathematical body and the space in which it is said to be. If whatever is not absolutely required to make an ellipse is taken away from the path of the pen, nothing will remain except a point describing the circumference of the ellipse. And, therefore, when the path is taken in precisely this way, it will not differ from the circumference of the ellipse itself. All the points through which the pen is led are points on the ellipse. If we conceive of the path partially, we conceive of part of the ellipse, and if we conceive of the whole thing, we conceive of the whole ellipse. But, in truth, I think that little turns on this, as with the issue of whether half of the power, or half of the things themselves, should be ascribed to two causes that raise a weight with the same force. I believe that we really agree.

Since the notion of a square is the notion of a mode, indeed perhaps of a mode of a mode, it is not surprising that it is composed of many things, because that is characteristic of modes.

Behold, most renowned Sir, this is not so much a reply to you as a description of my malady, which impedes me from grasping the force of your reasons. If you provide a remedy for this you will certainly do something for which I will be grateful for a long time. Goodbye.

From Leiden,

25 July 1702 A. D.

48. De Volder to Leibniz[1]

[Leiden, 7 October 1702]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

Nothing more unexpected has happened to me than that I have received from the most distinguished Bayle, after a few days' delay, what you had written against his objections along with the letter that you see attached to this one.[2] In it he seems to avoid a response, either because he thinks that his remarks still retain their original force, or because something else has hindered him.[3] I lost much hope because of this. For I had hoped that a great light would be shone on things that are obscure to me by the materials that are about to be returned to you, with which I might be able to find my way around the twists and turns of your hypotheses more easily. Perhaps it would be better to follow this fine example, especially since I very much doubt whether I understand your view in such a way that I am capable of forming objections that strike directly at the heart of your position. However, I will try to explain how I understand your hypothesis and what I have found lacking in it, so that, if

perhaps I stray from your opinion in the end, or I have not penetrated it sufficiently due to the weakness of my mind, you may lead me back on track in accordance with your kindness.

A mass of matter, as long as it is divisible to infinity and there is no empty space, has no unity. As long as it is merely passive, impenetrable, and resists motion, it can do nothing unless there is added a motive force, an entelechy, which, together with the mass, will make a corporeal substance, a unity, a monad, since you maintain that it is indivisible. However, every single corporeal substance contains in it infinite machines, each equipped with its own forces, and each of these again contains infinite others, and so on in this way to infinity. Given this, it seems to me that the unity that you had established before disappears again. For by this reckoning, every single corporeal substance will be not so much a substance as an aggregate of infinite substances, and it will never be possible to reach a simple substance that has not been joined together out of many, just as in the division of matter it is never possible to arrive at a little mass that is not composed of many others. Perhaps you will judge that this aggregate of machines becomes one through an entelechy arising from all of them. But, just as I acknowledge that, without a force added to matter, the change that I recognize on a posteriori grounds will not come about, so (as long as no clear necessity compels you to it, which I do not see so far) it seems hard to me to accept the hypothesis that something else arises from an aggregate of many substances, i.e., from an aggregate of parts of matter endowed with a certain force, except that which results from the connection of the matter and from the composition of the forces, and which is produced from those things on that account. Moreover, many will perhaps think that the unity, which you seem to me to suppose as the foundation of your hypotheses, does not differ from the arbitrary things that we assume in mathematics.

Moreover, with respect to these entelechies, you want all the changes that will ever take place in a mass that has an entelechy to be pre-established, so that substances certainly

do not seem to act on each other. You illustrate this by appeal to the opinion adopted by Malebranche and others that all plants and animals pre-existed in the first plant and first animal, and arise from there in the future.[4] In fact, it seems to me that they assumed this hypothesis concerning plants and animals for no other reason than that they despaired of finding the causes through which an organic constitution might arise from an inorganic[5] mass. So, in order that they would not seem to be saying nothing, they took refuge in the first act of creation, i.e., in an entirely unknown cause. I admit that forces from which everything else follows would have to be sought from that act. But, simply adopting a certain kind of force without demonstration seems to me to involve a very big assumption. Your explanation is better, provided that it may be demonstrated that these substances of yours do not act on one another. I shall come to this matter soon.

I acknowledge, however, that, if things are assumed to be like this, every single body is influenced by every other body, and that there is a certain mutual relation between everything and everything else. I also acknowledge that the present state of each one of the substances involves that of all the rest. Indeed, I acknowledge that all its future states follow from this state. Nevertheless, I do not accept that there will be anything in any aggregate of these substances that does not follow from the nature of the whole mass, which you seem to call <u>spontaneous</u>.

But, even granting that things are like this, there are still things that I do not fully understand. First, I do not understand why you say that the motive force that, together with matter, composes a corporeal substance, is indivisible. For, since that force is derived from the mass and speed of bodies, and the speed in a given mass can be increased and decreased, the force also will be increased and decreased. Perhaps you will say, since you like to speak in this way, that the primitive force remains the same, without opposing such a change in the derivative force. I can certainly understand this in the following way, namely, that the total

force that is obtained by considering all bodies at a given time always remains the same, although it may be distributed in different ways. But I do not really understand how the force remains the same in some particular body when the organs through which the force is exercised are united. Let us suppose that there is nothing to be found in individual bodies except the forces that they now have, derivative forces if you so wish. Surely everything that will happen in the universe subsequently will follow from this alone? So why is it necessary also to postulate primitive forces and those indivisible things?

Next, if a corporeal substance is a mass endowed with a certain motive force, I do not see why you shy away so much from the action of these substances on one another. For such a mass might run into another mass, also endowed with a certain striving. And surely one can understand and deduce what may happen after this conflict from a consideration of the masses and speeds of each of them, in the same way as an effect is deduced from the action of a cause? So what reason is there for me to deny that these substances act on one another, when I seem to understand their action and the effect of this action so clearly? The other case, when we deny that the mind acts on the body or the body on the mind, is of no help here. As I understand it, we do not deny it in this case because they are substances, but because they are totally different kinds of substances, so that the properties of one cannot be deduced from the properties of the other, which does not happen in the case of these bodies. You do indeed seem to me to put every entelechy in a different species, or certainly something close to that, though, granted you do not put it in that way. But I would like to know why you do this, if the explanation of the entelechy is indeed grounded in the nature of these forces. But if things are otherwise and you mean that the entelechy is distinct from these forces, I will ask what it is, and how it might set a mass in motion. For, at present, this appears just as obscure to me as that which I just said about the mind setting the body in motion.

From the other things that I wrote to you in my last letter, you will easily conclude that I will not readily treat a mass and its entelechy as one thing or a substance, unless a necessary connection is demonstrated between the mass and the entelechy. Indeed, I will not take your entelechy, in which such a variety of strivings is contained, to be one substance, unless it is demonstrated that this necessary connection is to be found as much among all the strivings that it has at present as among those that it will have in the future. Besides, I seriously doubt whether a thing that is unified and simple can be subject to any change from its own nature, since every change always follows from another.

There appears to be greater clarity in those things that you say next about souls. For if your entelechies are souls (which you will perhaps not regard as inappropriate, since you ascribe a certain kind of life and perception to them), allowing that they are very much inferior to human souls, I think I see how the same change occurs among them with respect to their perceptions as arises with respect to their bodies. For, since the soul of each one of the bodies would have had in it a perception of the present state of its body, it would also have included in this perception the present state of the other bodies acting on it and, at the same time, all the changes that followed from the bodies that were determined in this way; and this by virtue of perception – distinctly, if we imagine this perception to be distinct (or rather, adequate), or confusedly, if it should be this kind of perception. This is why I very much approve of what you bring up concerning distinct and confused concepts. But even if things are taken to be this way, the same difficulty returns, namely how this soul move might a mass.

But it is time that I stopped, especially since I have reason to fear that, if I have perhaps not understood exactly what you mean, the more I go on, the more I will stray away from the goal of truth. However, I will add this one thing. If it is true, "that extension is the order of possible coexistences, as time is the order of inconsistent possibilities,"[6] I am

surprised that time and number agree with all things, bodies as much as spirits, but that extension agrees with bodies alone. But let me take my hand from the writing tablet. Goodbye.

From Leiden,

October 7, 1702 A. D.

Because Bayle wanted it this way, you will see that your essay is enclosed with this letter, though I doubt whether this corresponds to your intentions. Furthermore, in this essay I saw mention made of certain letters to Arnauld. I would really like to know what these are, and where might they be found.[7] Goodbye again.

49. Bernoulli to Leibniz[1]

[Groningen, 28 October 1702]

De Volder concedes that there are entelechies of bodies, if nothing is to be understood by this other than an original impressed force. But he denies that this force is essential to a body because, if it was impressed, it could be taken away from it again and destroyed. He also says certain things that do not differ much from what I have warned you.[2]

50. Leibniz to De Volder[1]

[Berlin, Early 1703/Hanover, 20 June 1703][2]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merit, Mr. B. de Volder.

I am responding to your two letters full of profound thought with one and the same effort[3], and I hope that the things that I said in response to the most distinguished Bayle have

also made my opinion on many things more apparent to you. Unless I am mistaken, you have shown this clearly enough. Mr. Bayle himself also writes that he has now examined my hypothesis more deeply, and it seems that there is no sticking-point except the possibility of a spontaneous progression of thoughts in the soul.[4] But, there is no difficulty for me here[5] from experience, since we often perceive such a progression. Why then may we not also believe that it is possible in other cases even a priori, since I judge that it is necessary from the very nature of every substance, which must act, i.e., have a tendency to act.[6] Also, add the fact that everywhere (in complete things that is) the present is pregnant with the future, so that all future states are pre-established in the present state. Your difficulties arise from other sources. I turn first to your earlier letter, in which you ask for a necessary connection between matter (i.e., resistance) and active force, so that they are not joined together gratuitously[7]. But the cause of the connection is the fact that every substance is active and every finite substance is passive, and passivity is connected to resistance. Therefore, the nature of things demands such a conjunction. It cannot be so impoverished that it lacks a principle of action, and it no more allows a vacuum in forms than in matter. I will ignore for now the fact that these same things are the sources of action and unity.

I do not approve at all of the doctrine of attributes that they formulate these days, as if some one, simple, absolute predicate (which they call an attribute) constitutes a substance. For I do not find among notions any predicates that are entirely absolute, or that do not involve a connection with others. Certainly thought and extension, which they commonly propose as examples, are anything but these kinds of attributes, as I have often shown. And, unless it is taken concretely, the predicate is not the same as the subject. And so, the mind indeed coincides with the thinking thing (even if not formally), but not with thought. For the subject must involve future and past thoughts as well as the present one.

You judge that those who rest the distinction among bodies solely on what they think

of as modes of extension (as almost everyone does today) do not disavow the view that bodies differ only modally by excluding the vacuum. But two individual substances must be distinguished more than modally. Furthermore, as the matter is commonly conceived, they will not even be found to be distinguished modally. For if you assume two bodies A and B, of equal size and with the same shape and motion, it follows from such a notion of body, namely one derived from the putative modes of extension alone, that they have nothing at all by which they may be distinguished intrinsically. Is it not the case, therefore, that A and B are not different individuals? Or is it possible for there to be[L1] different things that nonetheless cannot be distinguished intrinsically in any way?[L2] This and innumerable things of this sort clearly show that the true notions of things are turned completely upside down by that new philosophy that forms substances from things that are only material or passive. Things that differ must differ in some way, i.e., have a specifiable difference in them, and it is surprising that people have not employed this most evident axiom, along with so many others. But, content to satisfy the imagination, people commonly do not attend to reasons, and, hence, so many monstrosities that are contrary to the true philosophy have been introduced. Clearly they have made use only of incomplete and abstract, i.e., mathematical, notions, which thought[L3] supports but which nature does not recognize in their pure form, such as the notions of time, of space, i.e.,[L4] a purely mathematically extended thing, of merely passive mass, of motion considered mathematically, etc. In such cases people can imagine things to be different without a difference,[8] e.g., two equal parts of a straight line, since, of course, a straight line is something incomplete and abstract, which it is appropriate to consider for the sake of theory. But, in nature, every straight line is distinguished from every other by what it contains. Hence, it cannot happen in nature that there are two bodies that are perfectly equal and similar at the same time. Also, things that differ in place must express their place, i.e., the things surrounding them, and so, they must be distinguished not only by place, or by an

extrinsic denomination alone, as such things are commonly conceived. Hence bodies in the way that they are ordinarily assumed, like the atoms of the Democriteans or the perfect globes of the Cartesians,[9] cannot exist in nature, and they are nothing but incomplete thoughts of philosophers who examine the natures of things insufficiently. Besides, in my very recent response to the most distinguished Sturm, I demonstrated by another invincible argument that, with a plenum assumed, it is impossible that matter as commonly conceived – as formed from modifications of extension, i.e., (if you prefer) passive mass, alone – is sufficient for filling the universe, but that it is clearly necessary that something else be supposed in matter, from which the principle of the variation and distinction of the phenomena may be obtained.[10] And so, besides augmentation, diminution, and motion, there must be alteration and, therefore, heterogeneity in matter. However, I will not admit the generation and corruption of substance.

I now proceed to your other letter.[11] When I say that a substance, albeit a corporeal one, contains an infinity of machines, at the same time I think that it must be added that it includes[L5 12] the one machine composed from them, and that it is actuated by one entelechy, without which[13] there would be no principle of true unity in it. But, I think that it is clear from what I have said that an evident necessity forces us to admit entelechies. And I also do not see how true unities can be avoided if one is to have real beings and substances. Arbitrary unities, which are used by us in mathematics, have no place here, rather they are fit for apparent beings, such as all beings through aggregation are – like a flock[14] or an army, whose unity is from thought. And it is the same in any aggregate, so that you will find nothing that is truly one if you take away the entelechy.

[15] Properly and rigorously speaking, perhaps one will not say that the primitive entelechy impels the mass of its body, but only that it is joined with a primitive passive power that it completes, i.e., with which it constitutes a monad. However, it cannot influence other

entelechies and substances, even those existing in the same mass. But in the phenomena, i.e., in the resulting aggregate, everything is indeed explained mechanically, and masses are understood to impel each other. And in these phenomena, nothing is needed except the consideration of derivative forces, once it is agreed where they result from, namely the phenomena of aggregates from the reality of monads.

In my judgment, there never arises a new natural organic machine, since it always has an infinity of organs so that it expresses the whole universe in its own way. Indeed, it always involves all past and present times, which is the most certain nature of every substance. And it is settled that what is expressed in the soul is also expressed in the body. Whence, both the soul and the machine animated by it, and the animal itself, are as indestructible as the universe itself. Because of this, such a machine cannot be put together by some mechanism any more than it can be destroyed. And no primitive entelechy whatsoever can ever arise or be extinguished naturally, or ever lack an organic body. Certainly, as far as my consideration of these matters goes, these things should not be otherwise. And they are not derived from our ignorance concerning the formation of fetuses, but from higher principles.

I regard the substance itself, endowed with primitive active and passive power, like the I or something similar, as the indivisible, i.e., perfect, monad, not those derivative forces that are continually found to be one way and then another. And, if there is nothing that is truly one, then every true thing will be eliminated. The forces that arise from mass and speed are derivative and belong to aggregates, i.e., phenomena. And when I speak of the enduring primitive force, I do not mean the conservation[L6] of total motive power with which we were once concerned,[16] but the entelechy always expressing that total force as well as other things. And, indeed, derivative forces are nothing but modifications and echoes of primitive forces.

Hence you understand, excellent Sir, that corporeal substances cannot be constituted from derivative forces alone joined with resistance, i.e., from vanishing modifications. Every

modification presupposes something lasting. Therefore, when you say, "let us suppose there is nothing to be found in individual bodies except derivative forces,"[17] I reply that the hypothesis is not possible, and that from it error again arises, because we take incomplete notions for complete concepts of things.

I do not admit the action of substances[18] on each other in the proper sense, since there appears to be no way by which a monad may influence a monad. But does anyone deny collision and impulse in the appearances of aggregates[19], which are certainly nothing but phenomena (though founded and regulated)? Meanwhile, I also find it to be true in phenomena and derivative forces[20] that masses do not so much give other masses new force, as determine the force already existing in them[21], so that a body drives itself away from another by its own force, rather than being propelled by the other.

It is necessary that entelechies differ, i.e., that they not be completely like each other. Indeed, it is necessary that they be principles of diversity, for they express the universe in different ways, each according to its own way of being situated toward it. And that is their function, to be so many living mirrors of things, i.e., so many concentrated worlds. However, we rightly say that the souls[22] of animals with the same name (for instance human ones) are of the same species, not in the mathematical but in the physical sense, in which father and son are held to be of the same species.

[L7] If you take a mass to be an aggregate containing many substances, you can, nonetheless, conceive of one substance that is preeminent in it, if indeed that mass constitutes an organic body animated by its primary entelechy.[23] For the rest, in the monad, i.e., the complete simple substance, I do not unite anything with the entelechy[24] except a primitive passive force, which is related to the whole mass of the organic body. Indeed, the remaining subordinate monads placed in the organs do not make up a part of the organic body, although they are immediately required for it, and they come together with the primary monad for the

organic corporeal substance, i.e., the animal or plant. I therefore distinguish: (1) the primitive entelechy, i.e., the soul; (2) matter, namely, primary matter, i.e., primitive passive power; (3) the monad completed by these two things; (4) the mass, i.e., the secondary matter, i.e., the organic machine, for which innumerable subordinate monads come together; and (5) the animal, i.e., the corporeal substance, which the monad dominating in the machine[25] makes one.

You doubt, excellent Sir, whether a thing that is one and simple is subject to changes. But, since simple things alone are true things, the rest are only beings through aggregation, and therefore, phenomena, and, as Democritus used to say, exist by convention not in reality.[26] So it is clear that without change in simple things, there would be no change in things at all. Nor indeed must all change must come from without, since, on the contrary, an internal tendency to[L8] change is essential to a finite substance, and it could not arise naturally in monads from any other source. But in phenomena, i.e., aggregates, every new change is derived from collision, according to laws prescribed partly by metaphysics and partly by geometry. For one needs abstractions in order to explain things scientifically. Hence, in a mass, we regard the individual parts as incomplete things, each contributing something of its own, but we regard the whole mass as completed by the coming together of all of them. And so any body whatsoever is understood to tend intrinsically in a tangential straight line, even if curvilinear motion follows from the continual impressions of other things. But, in the substance itself, which is intrinsically complete and involves everything, the maintenance[27] of the curved line itself is contained and expressed, since everything that will happen is also predetermined in the present state of the substance. For there is as much difference between a substance and a mass as there is between complete things, as they are intrinsically, and incomplete things, as they are comprehend by us through abstraction. By abstraction one can assign what must be ascribed to any part of a mass in the phenomena, and everything can be

distinguished and explained rationally, something that necessarily requires abstractions.

You seem to have grasped beautifully my doctrine of how any body whatsoever expresses everything else, and how any soul or entelechy whatsoever expresses both its own body and, through it, everything else. But as soon as you have considered the force of this, you will see that I have said nothing else that does not follow from it.

I had said that extension is the order of coexisting possible things and that time is the order of inconsistent possibilities[L9]. If this is so, you say that you wonder how time pertains to all things, spiritual things as much as corporeal ones, but extension to bodies alone. I reply that the reason is the same in both cases and for both sorts of things, namely that both spiritual and material changes have their home, so to speak, in the order of successive things, i.e., in time, as well as their[L10] place in the order of coexisting things, i.e., in space. For, even if monads are not extended,[L11] they nonetheless have a certain kind of situation in extension, i.e., they have a certain ordered relation of coexistence to other things, namely through the machine[28] over which they preside. I do not think that there exist any finite substances that are separated from every body and, therefore, that lack situation or order in relation to the other coexisting things in the universe. Extended things involve intrinsically many things endowed with situation. But things that are simple[L12], even if they do not have extension, must, nonetheless, have a situation in extension, although it may not be possible to designate it precisely, as with incomplete phenomena[29]. Goodbye. From Berlin, 1703.

P. S. If I remember correctly, I sent this letter to you last Winter. Now, since our most celebrated Bernoulli, to whom I had entrusted it, says that it did not reach him, I am sending it again, copied from my draft. I am very grateful to you for the gift of Huygens' posthumous writings, which I have not yet received from Leipzig.[30] I hope they will reach me nonetheless. From Hanover, 20 June 1703.

51. Leibniz to Bernoulli[1]

[Hanover, 20 June 1703]

It is many months since I wrote to either you or Mr. B. de Volder. I found a draft of a letter to De Volder among my papers and I enclose a new transcript of it. I beg you to take care to send it to him.[2] You will see from it how I respond to his difficulties and, unless I am wrong, it can cover yours as well.[3]

Since there is almost nothing in body but entelechy, I do not see how it could be deprived of it. Certainly there cannot be a substance without an entelechy. When I say that, at creation, force was impressed on body, I mean nothing other than that it had no existence before it had force. I add that it could not have had any existence before it had force. Derivative and variable forces are modifications of a substantial and permanent thing. Nothing is impressed on a thing by God that does not arise from its nature, and it is unintelligible for an impression to be made by anything else.

52. De Volder to Leibniz[1]

[Leiden, 30 October 1703]

B. De Volder sends greetings to the most honorable gentleman Mr. Gottfried Wilhelm Leibniz.

The more I ponder your letters, the less I am surprised by the fact that you have quite often written that you have anticipated everything in such a way that you are ready to respond to any objections whatsoever. It seems to me that you include whatever can follow from the laws of mechanics in your hypothesis, and so, it is pointless to prepare defenses against you based on those. Indeed, I think that I dimly see a way (whether this is really the case will be

for you to judge) in which someone might be able get to the point where they could support your hypothesis in this way. For, if I assume that the various things in this universe are substances and that they do not act on each other, the rest seems to follow without difficulty. Since experience teaches that change happens, but this cannot happen without action and passion, and, moreover, substances can neither act on nor be acted on by each other, it will be necessary that we either summon Deus ex machina with Malebranche,[2] or assume, with you, that in each substance there is a certain force of acting, and that it contains all its changes intrinsically, and that they unfold successively. One consequence of this fact will be that, since no substance can perish from itself or from another (seeing as nothing can act on it), each substance always remains one and the same, indeed, that each substance envelops past, present, and future times intrinsically, and many other things like this in your hypothesis that appear paradoxical.

As a matter of fact, I have long preferred your hypothesis to Malebranche's, if only because he requires God to perform separate actions, whereas you, more naturally, require him only for the first act of producing things. However, this seems to be common to both; that they leave completely obscure the reason for the original production of the entelechies or for the divine action on individual things. Furthermore, neither of them can be overturned by the laws of mechanics, since both assume them.

Besides that, what concerns me most of all in your account is that you assume gratuitously that all substances are active, since a principle of action enters into your definition of the word substance. So that there not is controversy over a word, it seems to me that that which was initially at issue between us must still be demonstrated, namely that nothing can exist that does not have a power of acting.

Indeed, I do not even really understand what that which you call substance is. If I understand it at all, it is surely nothing other than an active and resisting force. Certainly I do

not at all understand the subject of these forces, which, for you, I believe, is in fact the substance itself. Now I return to your letter, which, to be brief, I understand as follows.

You suggest that the fact that every substance is active and every finite substance is passive is the reason for the connection that I had sought between active force and matter. But this is the very thing that is at issue and whose demonstration was being sought. And I do not really understand why you require a passive power in a substance. For, since the substance cannot be affected by anything else, this resistance will serve no other function except resisting its own active force.

Bodies A and B of the same size, figure, and motion have no intrinsic differences. However, it is true that a reason can be given why body A occupies this place rather than that. But, this is not derived from the intrinsic nature of the body A itself, as much as from the system of the whole universe. It follows from this that, at any given time, body A occupies this place and body B occupies that place. Indeed, all the places that those bodies will each occupy in the future follow from this as well. Moreover, I am convinced that the intrinsic difference that you seem to suppose here is due not so much to the nature of things as to your hypothesis.

I admit that there will be no difference in bodies, however they are altered, if all their different forces, or else the different effects of the forces, are removed. But, if it is supposed that they have those forces that you call derivative or the effects of these forces called in from wherever (as it is on Malebranche's hypothesis), then it seems to me that the same phenomena will follow as now.

You add, "true unities cannot be avoided if one is to have real beings and substances,"[3] and that, "if there is nothing that is truly one, then every true thing will be eliminated." Either I do not understand this or it is assumed on the basis of your hypothesis.

I believe that it is on the same basis that you deny that derivative forces could be present in bodies on their own. But, if we take it that a corporeal mass has a certain kind of motive force built in, to be deduced from the size and speed, we will certainly have the same phenomena as now. It does not appear that anything can be deduced from the primitive and stable forces in different parts of the mass except that, according to you, they can be called particular substances; whereas this whole corporeal universe may perhaps be just one substance.

When I said in my previous letter that each body is affected by another, I had not deduced this from entelechies but from derivative forces. I have no doubt that a simple thing can be changed, but I do not think that it can be changed by itself. For I have always been convinced that whatever follows from the nature of a thing is always in the thing in an invariant way and cannot be removed from it, certainly as long as the nature of the thing remains the same, since there is a necessary connection between it and the very nature of the thing. Therefore, a change that happens while the nature of a thing remains the same must necessarily be due to an external cause. You also say, "an internal tendency to change is essential to a finite substance." In fact, this is the very thing that demands a demonstration, and which, unless I am mistaken, is overthrown by the argument just given. Goodbye.

From Leiden,

30 October[4] 1703 A. D.

53. Leibniz to De Volder[1]

[Hanover, 19 November 1703]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

Without a doubt you have hit the nail on the head when you judge that, according to my view, what are truly called substances (i.e., the monads, i.e., the perfect substantial unities from which everything else necessarily results) have no influence on each other, and that we are of the opinion that anything that happens otherwise belongs to the phenomena. However, you seem to depart from my intention because you think that only a hypothesis is being advanced, whereas I think that the doctrine is necessary. I do not assume gratuitously that substance is active, as can be understood well enough from many of the things that I have already written. If, after arbitrarily inserting activity in the definition of substance, I had inferred from this that every substance is active, and then applied my arbitrary definition to the things that are really in nature and commonly called substances, I would have been begging the question or committing the fallacy of smuggling the conclusion into the definition. [I recognize intrinsically active monads[2], in which nothing else can be understood besides perception, which certainly involves action].[3]

And so, when it is asked what we understand by the word substance, I point out that aggregates must be excluded before everything else. For an aggregate is nothing other than all the things from which it results taken together, which clearly have their unity only from a mind, on account of those things that they have in common, like a flock of sheep. For it is one thing for two bodies to have no distance between them (in which the appearance of the continuum[4] composed of these things consists), or for one to be impelled against another (in which their connection consists), but it is another thing for them to be truly one, since no real foundation for unity can be found in these things. Were I unable to convince someone of this fact, I would be setting forth all the rest in vain. And so, I thought that we should move forward in stages and that this issue must be examined with care before anything else.

You also seem to want a cause from me when you do not yet admit the effect. So the fact, i.e., the question, must be agreed upon first, namely whether every substance, at least

every substance known to us, should be considered active – which can be established from the phenomena.

You think that the resistance in a substance can bring about nothing other than the fact that the substance opposes its own active power. But this should not seem absurd to you, since in quasi-substances, i.e., bodies, it is also the case[5] that the bulk restricts the speed that another tries to impress. Certainly there must be a principle of limitation in limited things, just as there must be a principle of action in acting things.

You attribute the intrinsic difference among bodies not so much to the things as to my hypothesis. But I had added a demonstration[6], namely from the phenomena.[7] Because if things were otherwise, one state of the plenum could not be distinguished from another, for equivalent things would always be substituted for each other. This applies primarily to the Cartesians, who do not recognize qualities, i.e., forces, in motion but only translation, as if God placed bodies first here and then there, and then gave the mind arbitrary sensations not corresponding to the states of the body. It also applies especially to Malebranche, Sturm, and other occasionalists, who attribute all force or active power to God alone, so that there is no principle of distinction in corporeal things. Dissenting from both in this matter, you admit derivative forces like I do, and in this way you hope to be able to save the differences in the phenomena. But then you should have addressed my other argument, that derivative, i.e., accidental, forces are mere modifications, and that that which is active cannot be a modification of that which is passive, since in modification there is only a variation of limits. And so modes only limit things and do not increase them, and, therefore, they cannot contain an absolute perfection that is not in the thing to be modified. Otherwise indeed, these accidents would have to be conceived in the way that substances are, as things depending on themselves. So you must either accept my position, or take refuge in the misguided idea that you suggest – that perhaps the whole universe is only one substance. Anyone who uses the

word substance in this way twists it from the sense that others give it. And I do see any plausible argument for such a paradox. Those that B. de S. offers for it do not contain any shadow of a demonstration (if I am any judge).[8] Besides, if the dispute concerning the word substance is set aside, it is enough that you admit distinct subjects, i.e., things in which there are modes. If this is supposed, the argument that I have already brought up – that a mode only limits and does not increase – still remains. It can be seen from this that motion does not suffice without forces, and that derivative forces do not suffice without primitive entelechies.

In order to attack what I said – that an internal tendency to change is essential to a finite substance – you say, "whatever follows from the nature of a thing is in the thing in an invariant way and cannot be removed from it, certainly as long as the nature of the thing remains the same, since there is a necessary connection between it and the very nature of the thing."[9] But it would follow from this that nothing at all is active by its own nature, since action is certainly a variation of a created thing that is acting. I reply that a distinction must be made between properties, which are permanent, and modifications, which are transitory. Whatever follows from the nature of a thing can follow either permanently, or at a time; and, if at a time, either at once and immediately (namely in the present), or with something prior mediating, so that it is in the future. You find an image of this in quasi-substances, i.e., in bodies that have force, or have been in put in motion. When nothing external is assumed, it follows from the nature of a body moving in a given straight line with a given speed that it will arrive at a given point on the straight line when a given time has elapsed. Does this mean that it, therefore, arrives at this point always and permanently? You should, accordingly, conceive in the primitive tendencies that which must be recognized in the derivative. The situation is as it is with laws of series or the natures of curves, where the entire progression is fully contained in the very beginning. The whole of nature must be like this; otherwise it

would be absurd, and unworthy of wisdom. I do not even see a semblance of a reason for doubting this, except that we are frightened off by unusual things.

Furthermore, if you agree with me that the system of occasional causes is not worthy of a philosopher, and if you think that the influence of substance on substance (I am speaking about true ones) is inexplicable, I do not see how you could have doubts about the internal tendency to change in things, since we are taught that there are changes in things by our experience of the phenomena, as well as from the inside where the operations of the mind themselves exhibit changes. Therefore, I think the fact is demonstrated a posteriori, and your objections are also satisfied.[10] Nor can you safely avoid an internal principle of change whatever you try to do, even if you have recourse to the system of B. de S. For, even if you suppose that the change in any individual is due to its being acted upon by another, and that this again is due to another, a principle of change still cannot be found in this way, since the difficulty is only postponed, not solved. So the reason will either be internal to the whole universe of things, and so to the parts as well (for what is such a universe other than all the particular things?), or it will be in an extramundane substance, to speak as Martianus Capella does,[11] or rather, a supramundane one, namely God. The latter position is the true one if you seek the ultimate principle. But since God (because he is[12] most perfect) works in a natural way, in which there must be reason and order[13], he must be said to have produced principles of change in things, with the result that subsequent things can be inferred from prior ones. When you allow this once, you will admit it with equal right in all things. Indeed, it must be said that if he has not produced these things, he has produced nothing lasting at all and no subject of change. Goodbye. From Hanover, 19 November 1703.

P. S. The most clever Bernoulli sent me his comments on Huygens' De Motu,[14] and that gave me the occasion to reply to him at length. I have asked that this be sent to you as well. For it seems to be of some moment.[15]

54. De Volder to Leibniz[1]

[Leiden, 5 January 1704]

B. De Volder wishes all things fortunate and happy, for now and for many years following hereafter, for the illustrious gentleman Mr. Gottfried Wilhelm Leibniz.

I recognize that it is one thing for two bodies to have no distance between them or for one to be impelled against the other, and another for them to be really one thing, i.e., as I understand it, to be one thing in such a way that it cannot be divided into parts. Really, I think that what is being asked is whether such a unity is found in bodies. You appeal to entelechies so that you might establish this. Indeed, I noticed that you said somewhere, if there is not something that is truly one, there is not a true thing.[2] But I did not understand what the argument was there. For, if by true things you just mean bodies that are truly one and indivisible, I see nothing absurd in the fact that things that are really like those you call bodies are not found. Certainly, I have no doubt that this needs a demonstration.

I had judged that all the phenomena could be saved if derivative forces were assumed to be in bodies. But yes, as you rightly suggest, I "should have been addressing the other argument, that derivative forces are mere modifications and that that which is active cannot be a modification of that which is passive etc."[3] However, I very much fear that I may not be able to do that, not because the force of the argument deters me, but because the obscurity does. For, if truth be told, I do not understand fully what you mean. I do not conceive of any modes in a simple body except determinate magnitude, figure, and either motion or rest. There is nothing active in magnitude and figure alone. But I would hesitate to say the same about motion, i.e., that which results from size and speed, i.e., about derivative force. I see no difference between the forces that you call derivative and primitive, other than that the former

are continually changing from one moment to the next, whereas the latter are supposed to be constantly the same. But, whether a thing has more or less duration does not change its nature. Therefore, I do not see why the former are not active.

When I said that that the universe is perhaps only one substance, I was clearly speaking about the corporeal universe, in this way following the common opinion of almost everyone of regarding bodies as substances, not in so far as they are these bodies or those, but in so far as they are bodies. I do not understand why that moved you to mix in the opinion of B. d. S. here, as if it were similar to mine. For what is peculiar to that view is not what I said about the corporeal universe, but the fact that it treats thought and body as the same substance, which has always seemed most absurd to me.

You also say that it follows from the considerations that I had brought against an internal tendency to motion that absolutely nothing is active by its own nature. This is true, unless the action is always of exactly the same kind, and so does not lead to any change in the thing that is acting. You add, "whatever follows from the nature of a thing can follow either permanently or at a time." I reject the latter. It follows from the nature of a triangle that the angles are equal to two right angles, and it follows permanently. And I do not see how any succession (which is required for action) can follow from the nature of a thing considered intrinsically, unless the thing itself is successive, which agrees perfectly with derivative forces, but not so much with primitive forces. Nor am I moved by the examples that you provide. It follows from the nature of a moving body that it reaches a given point at a given time, and it will always reach it, unless something external intervenes. Moreover, in laws of series it is just the same. All the terms are contained in the very nature of the series in a unique and invariant way, and nothing successive can be conceived in that. So these examples seem to count in my favor rather than against me.

So, if we suppose that, in the beginning of things, God introduced into matter as much

derivative force as it has now, will it be necessary to have recourse to an intrinsic tendency of things for change? It is like this: either particular bodies will not be substances, if you demand your unity or primitive force for substances, or if they get called substances, they will be substances of the same nature, which nothing prevents from acting on each other. I admit that refuge must be taken in God to set up these derivative forces, but you do the same for the primitive ones. I say no more than that God produced a principle of change in things, so that later things can be inferred from prior things. For he produced derivative forces, from which all change follows. Experience teaches us that changes happen. I do not deny this. However, we were seeking not what experience would teach, but what would follow a priori from the very nature of things. And no experience that I know of teaches that those changes have their origin from within. Moreover, the ways in which the mind acts are too obscure to seek an argument from there. Consciousness teaches that change happens to the mind, and, indeed, that it happens in such a way that, in many cases, it is not conscious of the action of an external cause. But to conclude from this that the change depends on the intrinsic nature of the mind seems to me to be going too far.

It was a pleasure to read through what you had written to the most distinguished Bernoulli.[4] Goodbye.

From Leiden,

5 January 1704 A. D.

55. Leibniz to De Volder[1]

[Brunswick, 21 January 1704]

Gottfried Wilhelm Leibniz wishes the gentleman distinguished on account of learning and merit, Mr. B. de Volder, many happy and favorable years.

We should see whether anything can be settled between us about monads. By truly one you understand that which cannot be divided into many.[2] I admit that my unities are like that. But you say that what is being sought is whether there are such unities in bodies, and that I appeal to entelechies so that I may prove this. In fact, I do the opposite. In order that I may prove entelechies, I appeal to unities. Although it is also true that, if entelechies were demonstrated by some other means, true and real unities would be had.

For the rest, I had undertaken to prove that these unities exist from the fact that there would otherwise be nothing in bodies. I gave the following argument: First, that which can be divided into many is aggregated, i.e., constituted, from many. Second, things that are aggregated from many are not one thing except from a mind, and they have no reality except that which is borrowed, i.e., that is from the things from which they are aggregated. Therefore, third, things that can be divided into parts have no reality, unless there are things in them that cannot be divided into parts. Indeed, they have no reality other than that which is from the unities that are in them. I do not see what could trouble you, especially since you concede that many things that are adjacent to each other or that are impelled against one another, do not constitute one real thing on account of that. So, from where will you derive their unity, or what reality will you give the whole beyond that of the things from which it is composed? It is for this reason that I think the following: Bodies, which are commonly taken for substances, are nothing but real phenomena, and are no more substances than perihelia or rainbows, and this is not something that is overturned by touch any more than by sight. A monad alone is a substance; a body is substances, not a substance. And the difficulties concerning the composition of the continuum, and others of this kind, cannot be made to disappear any other way.

You speak as if you do not understand what I mean when I say that derivative forces are mere modifications, and that the active cannot be a modification of the passive. So do

you not understand what modification means, or active, or passive? In the meantime, finding I know not what obscurity in my argument, you have touched on what I said so cursorily that you even attribute to me things that I did not say, or rather, the opposite of what I said. You maintain that I deny that derivative forces are active, when you say: "therefore, I do not see why the former are not active" (i.e., derivative forces).[3] But, in fact, I am so from denying that they are active, that from the fact that they are active and, nonetheless, modifications, I conclude that there is some primitive active thing of which they are modifications.

You say that motion, i.e., that which results from size and speed, is derivative force. I do not take motion to be derivative force,[4] but I do think that motion (namely change) follows from it. Moreover, derivative force is the present state itself in so far as it tends toward a following state, i.e., preinvolves a following state[5], just as everything in the present is pregnant with the future. But the persisting thing itself, insofar as it involves all cases, has primitive force, so that primitive force is like the law of a series, and derivative force is like a determination that designates some term in the series.

I do not remember who before B. de S. said that there is one substance in the whole universe. So, please forgive the fact that he came to mind, especially since he only serves as an example.

If bodies are not substances insofar as they are this or that body, then they are not individual substances. It is as if you say that Peter is a substance only insofar as he is a man, i.e., that species are substances, but individuals are not[6]. Besides, the universe of bodies will not be an individual substance in this sense. For it is nothing other than an aggregate of individual bodies, unless you appeal to something else that persists, which is what the man that I spoke of was also forced to do. And this will ultimately be a substance, because it is also a monad. In fact, he could have acknowledged something analogous to what he

somehow granted to the whole universe in all of the parts. Substances are not wholes that contain parts formally, but total things that contain partial things eminently.

If nothing is active from its own nature, then nothing at all will be active. After all, what reason is there for activity, if it is not in a thing's nature? However, you add the restriction "that a thing can be active by its own nature, if the action is always of the same kind."[7] But, since every action contains change, we, therefore, have what you seemed to deny, an internal tendency to change and something temporal following from a thing's nature.

Of course, you deny that "that which follows from the nature of a thing belongs to it at a time,"[8] and you prove this from the nature of a triangle. But you do not distinguish between universal and individual natures. From universals there follow eternal things; from individuals there follow temporal things as well, unless you think that temporal things have no cause.

"I do not see," you say, "how any succession can follow from the nature of a thing considered intrinsically." Undoubtedly it cannot, if we are assuming that the nature is not individual. "Unless," you add, "the thing itself is successive."[9] [10] But all individual things are successive, i.e., subject to a succession. And so you fall back into my view. For me, nothing is permanent in those things except the very law that involves the continued succession, which in individual things corresponds to the law that is in the whole universe.

Besides, you yourself acknowledge (in what I call 'quasi-substances') that it follows from the nature of a moving body that, in a given a time, it will arrive at a given point if nothing impedes it. Therefore, you admit that temporal things follow from the natures of individual things. I do not see what you are objecting to.

You say that in a series (of numbers, say), "nothing is conceived as successive." So what? I do not say that a series is a succession, but that a succession is a series, and that it has

this in common with other series, namely that the law of the series shows where it must reach by continuing its progression, i.e., that, with the starting point and the law of progression given, the terms will be produced in order whether the order or priority is of nature only or also of time.

I do not agree when you say: "All the terms are contained in the series in a unique and invariant way." This is only true in a certain sense, in uniform series. But there are series in which there are maxima, minima, bend points, etc.

When you say: "in the beginning of things God introduced into matter as much derivative force," you already tacitly involve primitive force in matter, because it cannot be understood what matter is except through monads, since it is always an aggregate, or rather, it results from many phenomena, until we arrive at simple things.

You say: "nothing prevents substances of the same nature from acting on each other." But you know that philosophers have, in fact, denied action between similar things. And what prevents substances of different natures from acting on each other? When you have explained that, you will see that it prevents all finite substances from influencing each other. Not to mention the fact that all substances are of different natures, and that there are no two things in nature that differ in number alone. And those things that are believed to be like that are so only by a fiction of the mind that does not recognize the difference or ignores it, or abstracts from it. There is only one case of a substance acting immediately on a substance, namely the action of the infinite substance on finite ones – an action that consists in continually producing, i.e., constituting, them. For it is necessary that there be a cause for why these finite substances exist and agree with each other, and this must arise from an infinite substance that is intrinsically necessary. [But if anyone claims other substances that succeed prior ones are always produced by God, and that they do not remain the same, he would be quarreling about a word, for there will be no further principle in things for deciding the

question. The substance that succeeds is taken to be the same as long as the same law of the series, i.e., of the continual simple transition, persists that gives rise to our belief in the same subject of change, i.e., the monad. I say that the fact that there is a certain persisting law, which involves the future states of that which we conceive as the same, is the very thing that constitutes the same substance.

And if anyone concedes to me that there is an infinity of perceivers, in each of which there is a fixed law of the progression of their phenomena, that the phenomena of the different perceivers agree with each other, and that there is a common reason for their existence and for the agreement in the thing that we call God, I neither posit anything else in things nor think that anything else should be posited. I believe that other positions and questions arise from notions that are not analyzed well. And I will be surprised if anyone shows that anything else must be added. If we always had this in mind in the midst of our dispute, we would avoid much wrangling.][11]

You say: "experience teaches us that changes happen. However, we were not seeking what experience would teach, but what follows from the very nature of things." Do you think that I could, or would want to, demonstrate anything in nature unless changes were presupposed? But you say: "no experience teaches that they have their origin from within." But I did not recommend this on the basis of experience.

You say: "the ways in which the mind works are too obscure." I thought that they were most clear, and almost the only things that are clear and distinct.

I thought that you accepted that at least something in the mind comes from within, i.e., not from any other finite substance, and I perhaps wished to infer from this that my view is intelligible. But you take this to mean that I required that everything in the mind to be like this. I admit that this is my view, but I do not require this.

Admittedly, since my previous letter[12] is not to hand, it is possible that I may be wandering from argument to argument rather freely, since it is as if I am divining the context in which these things were said. But, even if we cannot agree (which perhaps is not even what you want), we are at least providing or receiving the opportunity for revising and advancing our thoughts. It is certainly enjoyable and profitable to me, and it will also be useful for anyone who looks back over everything at some time in the future. I will not say whether it is agreeable to you, and I would not want to presume this.

Goodbye. From Brunswick, 21 January 1704.

56. Leibniz to Bernoulli[1]

[Hanover, January 20 1704]

I think that you know a good deal of my metaphysical thoughts about motion, but Mr. De Volder less. Hence I would ask you to communicate them to him, but his letters indicate that you have done this. . . .

As you will see, perhaps to your surprise, I continue to discuss things with Mr. De Volder because it is pleasant and sometimes even useful for reviewing things more carefully.

57. Leibniz to Bernoulli[1]

[Hanover, 2 May 1704]

Perhaps Mr. De Volder has taken offence at some of the things that I may have said too freely, because he seems to have been writing without paying enough attention and to have shown himself to be insufficiently teachable.[2] This has made me afraid that we may be debating in vain.

But if we offend in this manner, others often see this better than ourselves. And so, since you have read my latest reply, I would like to know whether you think that it is such that it could have offended the man, or deterred him from continuing the debate. I am accustomed to disputations with learned men endowed with intelligence and judgment being easy and enjoyable. Indeed, since I have had my reasoning on these matters well organized for some time, it is hard to think of anything that might cause difficulty.

58. Bernoulli to Leibniz[1]

[Groningen, 31 May 1704]

I recently wrote again to Mr. De Volder and strongly urged him to reply to both of us. However, he has still not written back, which makes me very worried that he might be ill, especially since I know that he suffers an attack of jaundice nearly every year. Unless he is now perhaps suffering from something even worse, since jaundice would not prevent him from writing. In your latest letter to him[2], I do not remember having read anything at all that could justifiably have offended him, so this, at least, will not give him any reason to withdraw from the debate. Besides, why should he stop writing to me as well?

59. De Volder to Leibniz[1]

[Leiden, 31 May 1704]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

It seems to me that you argue in this way: That which can always be divided further and further has no reality, except from the things from which it is aggregated, and so none at

all, except from things that cannot be divided. The argument rightly concludes that indivisible unities cannot be assigned within the bulk of bodies. But, nonetheless, unless there is perhaps some ambiguity in the word reality, it does not show that mathematical body (of which I conceive of innumerable properties very clearly, in that I can conceive of existing and inhering in nothing else in it – which is commonly thought to be the notion of substances) has no reality.

A ten foot body is composed of ten feet, of 17280 points, and so forth.[2] This unity may be arbitrary; nevertheless, the magnitude can be a true one and truly existing.

Next, if we compare the view of bodies according to which they can be infinitely divided with your entelechy, I do not see as much difference as there appeared to be at first glance. Each body is composed of parts, and these again from others, and so on in this way to infinity. Each corporeal substance has an infinity of others under it, and these again have others under them, and so on in this way to infinity,[3] so that, in neither case, can we reach a substance not containing many substances in itself. There is only this difference: You add to each of these substances an indivisible entelechy producing everything successively. If I were able to form a clear and distinct notion of this entelechy, it would help me greatly in grasping the thing itself. Now, I seem to perceive nothing except the name, unless I consider the general notion of force that attends it.

Moreover, I have never understood force to be something substantial, but something inhering in a substance. Indeed, I have always regarded force as being like an external denomination when considered without the foundation from which it arises, and, by contrast, the foundation as that which is in the thing. Perhaps it is the same as that which you call primitive force, from which derivative force arises. But, because of the dullness of my mind, I understand nothing about these things, except that you insist that all the remaining changes arise from them.

When I said that the corporeal (for I expressly added this) universe is perhaps one substance, I said nothing other than that which Descartes and all who have followed the mechanical philosophy meant: the substance of all corporeal things is one. But I already pointed this out in the previous letter,[4] so I am surprised that the same thing is repeated. You say that temporal things follow from individuals. I do not object to this. But it remains to be explained how individuals differ from universals, and why temporal things follow from the former and not from the latter. Universals do not act on each other, individuals do, and, therefore, with regard to their action they are subject to change. How this could happen in substances that do not act on each other is certainly obscure to me.

I have reviewed everything briefly in this way not so much so that I might make a reply, but to prevent you from misunderstanding my silence if I had said nothing at all. For I would not want to disguise the fact that there were things in your letter that I found completely unacceptable, and I do not know whether you may harbor suspicions that I do not want us to reach agreement etc. Given this, I can only conjecture that what I wrote was disagreeable to you. But I should not wish to be wholly burdensome to friends. On account of this I shall leave off. Goodbye.

From Leiden,

31 May 1704 A. D.

60. Bernoulli to Leibniz[1]

[Groningen, 21 June 1704]

Here at last is a reply from De Volder. It was written quite a long time ago, but I received it just now. As you feared, it seems that De Volder has taken some offence, as he shows more explicitly in his letter to me than in the one to you. For he writes to me as

follows: “You ask why I have been silent so long. It is because I was unsure whether I should reply to the illustrious Leibniz at all. For his last letter contains some things that convince me that he does not like being contradicted, as is the way with great men. I would not want to do anything that might displease him, especially since I do not expect very great fruit from this debate of ours. Everything comes down to this: that instead of a demonstration of the activity of substance from its nature, I receive his terminology of entelechies, unities, and primitive force that contains all change intrinsically. However, I understand nothing of this, except in so far as I have some understanding of derivative force, i.e., quantity and speed. The sole reason that I wanted so badly to know that every substance is active from its nature was that I discovered something about the cause of action that I did not fully understand, but this does not seem to have become better known through the notion of an entelechy. Nonetheless, as you see, I have written some kind of a reply, since you seemed to want this. Without your letter, I am unsure whether I would have emerged so quickly from this doubt about whether to write back or not, especially since an exhaustion that takes away all my mental energy has seized me for some weeks. I am not sick, but I am not well either. The condition is more troublesome because there is no prospect of an end to it.” This is what De Volder says, but please make sure that he cannot tell that I have written it all out for you from your reply. I fear that the exhaustion he complains of may threaten some more serious disease, since I myself have been burdened by a severe exhaustion in the limbs some time before an illness.

61. Leibniz to De Volder[1]

[Hanover, 30 June 1704]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

I see that you have received some of my letters unfavorably. Obviously, we do not always write in such a deliberate way that something does not slip out that needs a charitable interpretation. We are aware of this, and so we ask for and give this favor in turn.[2]

Please excuse the fact that I am occasionally forced to express surprise at misunderstandings when replying. This is because we have lost the habit of restricting ourselves to the arguments. These are your words to me, "It seems to me that you argue in this way: That which can always be divided further and further has no reality, except from the things from which it is aggregated, and so none at all, except from things that cannot be divided. The argument rightly concludes that indivisible unities cannot be assigned within the bulk of bodies. But, nonetheless, it does not show that mathematical body has no reality," etc.[3]

I am forced to repeat your words (which, for the sake of brevity, I would not otherwise have done) so that there may be less room for error on my part. I reply to this first by repeating my argument more distinctly as follows: Anything that can be divided into many things (which already actually exist) is aggregated from many things, and a thing that is aggregated from many[4] is not one except from a mind, and has no reality except that which is borrowed from what it contains. Then I inferred from this that, therefore, there are indivisible unities in things, because otherwise there will be no true unity in things and no reality that is not borrowed, which is absurd. For where there is no true unity then there is no true multitude. And where there is no reality except that which is borrowed, there will never be reality, since it ultimately must be proper to some subject. I would have preferred it had you indicated whether you had a difficulty here, and with which of my words. Instead (second), you suggest a conclusion different to mine, though I do not understand how you mean to infer it from my words. For you think that, "It is rightly concluded from this that indivisible unities cannot be assigned within the bulk of bodies."

But I think that the contrary is to be concluded, namely that we must return to indivisible unities as the primary constituents in corporeal bulk, i.e., in the corporeal things to be constituted. Unless you think perhaps that it is rightly concluded that bodily masses themselves are not indivisible unities.[5] I admit this, but we are not concerned with it. Certainly bodies are always divisible - indeed, they are actually subdivided as well - but not their constituents. These annoying features of debating and the necessity of responding at such length would have been avoided if we had stuck precisely to each other's arguments.

[6] You go on to say to me, "But, nonetheless, unless there is perhaps some ambiguity in the word reality, it (your argument) does not show that mathematical body has no reality, for I conceive of innumerable properties of it very clearly." Here I reply in two ways: First, it does indeed follow from my views that mathematical body is not something real; and next, the urgency with which you say that you conceive of that body most clearly (as real), does not establish its reality.

The following is relevant to the first point: From the fact that mathematical body cannot be resolved into primary constituents, it may be inferred that it is certainly not real, but something mental, designating nothing other than the possibility of parts, not something actual. Indeed, a mathematical line is like an arithmetical unity, and in both cases the parts are only possible and absolutely indefinite. And a line is no more an aggregate of the lines into which it can be cut up, than a unity is an aggregate of the fractions into which it can be broken up. And just as a number that is being used for counting is not a substance without the things that are counted, in the same way, neither is mathematical body, i.e., extension, without[7] active things and passive things, i.e., motion. But in real things, namely bodies, the parts are not indefinite (as they are in space, a mental thing), but are actually assigned in a certain way, in accordance with the divisions and subdivisions that nature actually institutes according to different motions. And, although these divisions proceed to infinity, nonetheless,

they all result from certain primary constituents, i.e., from real unities, though infinite in number. But, accurately speaking, matter is not composed of constitutive unities, rather it results from them, since matter, i.e., extended mass, is nothing but a phenomenon founded in things, like the rainbow or the perihelion. And there is no reality in anything except the reality of unities, and so phenomena can always be divided into lesser phenomena that could appear to other more subtle animals, and the smallest phenomena will never be reached. By contrast, substantial unities are not parts, but the foundations of phenomena.

Now, excellent Sir, I come to your objection. You say, "I conceive of innumerable properties of mathematical body very clearly." I concede this, just as with number or time, which are nothing except orders, i.e., relations pertaining to the possibility and eternal truths of things, which must then be applied to the actual things in accordance with how they arise. You add, "I conceive of mathematical body as existing and inhering in nothing else." This I do not concede, except in the way that we conceive of time as existing or inhering in nothing. If you take mathematical body to be space, it must be compared with time. If you take it to be extension, it must be compared with duration. Certainly, space is nothing other than the order of existing simultaneously for possible things, just as time is nothing other than the order of existing successively for possible things. And, as physical body is to space, so the state or series of things is to time. Body and the series of things add to space and time motion, i.e., action and passion, and its origin. Certainly, as I have often reminded you (though you seem to have disregarded this), extension is an abstraction from that which is extended, and it is no more a substance than a number or a multitude can be considered a substance. And it expresses nothing other than a certain non-successive (like duration) but simultaneous diffusion or repetition of the same nature, i.e., what amounts to the same, a multitude of things of the same nature, existing together with some order among them. It is this nature, I say, that is said to be extended, i.e., diffused. And thus, the notion of extension is relative,

i.e., extension is the extension of something, just as we say that a multitude or duration is a multitude or duration of something. Moreover, this nature that is supposed to be diffused, repeated, or continued is that which constitutes physical body, and it can be found in nothing other than the principle of acting and being acted upon, since nothing else is suggested to us by the phenomena. But of what sort this action or passion is, I will say later. So you see that if we undertake an analysis of the notions, in the end we always reach the position that I am urging. It is really not surprising that the Cartesians did not understand the nature of corporeal substance and did not arrive at the true principles while they took extension as something absolute, ineffable, irresolvable, and primitive. Relying on the imagination, and perhaps also seeking the applause of people, they were willing to rest where the imagination stopped, even though they boasted otherwise that they had correctly distinguished the things that are imaginable and things that are intelligible.

You say, "I have never understood force to be something substantial, but something inhering in a substance." Rightly so, if you mean changeable force. But, when force is taken to be the principle of action and passion, which is therefore modified through derivative forces or what is momentary in[8] action, you can understand well enough from what has been said that this is involved in the very notion of extension, which is, in fact, intrinsically relative. So, in your own analysis of corporeal substance you will arrive here in the end. And the same thing is even more apparent[9] (as was shown above) from the consideration of the analysis of multitude and phenomena into unities and reality.

You add, "I have always regarded force as being like an external denomination when considered without the foundation from which it arises." I would prefer to consider derivative force with respect to "the foundation," like a figure is with respect to extension, i.e., as a modification. And you know that from my account, in which I demonstrated <u>a priori</u> the true measure of (derivative) force, that I said <u>force</u> multiplied by the time in which it operates

produces action, and that force is, therefore, that which is momentary in action, but with a relation to the following state. And this is what I have often said, and I do not remember having deviated from it: Unless there is something in us that is primitive and active, there cannot be derivative forces and actions in us, because everything accidental, i.e., mutable, must be a modification of something essential, i.e., perpetual. Nor can it involve anything more positive than that which it modifies, since every modification is only a limitation – figure a limitation of that which is changed, derivative force a limitation of that which causes change.

You continue, "By contrast the foundation is that which is in the thing. Perhaps it is the same as that which you call primitive force, from which derivative force arises." I believe that this is most true. And so, it appears that we agree here on this matter.

You add, "But because of the dullness of my mind, I understand nothing about these things, except that you insist that all the remaining changes arise from them." But you do yourself an injustice with this excessive modesty, since you understand the matter as far as its nature allows. Or do you wish to imagine things that can only be understood, to see sounds, to hear colors? And, in fact, I believe that you do not deny what I asserted (that changes arise from this source). Do you think that to know that is to know nothing?

Moreover, it is worth considering that this principle of action is most intelligible, because there is something in it that is analogous to that which is in us, namely perception and appetite. For the nature of things is uniform and our nature cannot differ infinitely from the other simple substances of which the whole universe consists. Indeed, considering the matter carefully, it should be said that there is nothing in things except simple substances and in them perception and appetite[10]. Moreover, matter and motion are not so much substances or things as the phenomena of perceivers, the reality of which is located in the harmony of perceivers with themselves (at different times) and with other perceivers.

When Descartes and others say that, "the substance of all corporeal things is one" they mean by that one similar nature. They do not (I think) mean that all bodies in fact make up one substance.[11] Certainly, reality itself shows that the world is an aggregate, like a flock or a machine.[12]

I had said that temporal things follow from individuals. You say that you, "Do not object to this, but that it remains to be explained how individuals differ from universals, and why temporal things follow from the former and not from the latter." But, unless I am mistaken, there is an essential ordering of individuals with respect to specific parts of time and place, from which universals are abstracted by the mind.

Finally, you add, "Individuals act on each other and, therefore, with regard to their action they are subject to change. How this could happen in substances that do not act on each other is certainly obscure to me." It seems that this is directed against my opinion concerning the pre-established harmony among simple substances, which cannot act upon each other. Nonetheless, they do produce change in themselves. And it is necessary that this happen on your view as well. You acknowledged an internal foundation of force, i.e., action, above. And so an internal principle of change must be acknowledged. And, if it were not acknowledged, there would be no natural principle of change at all, and so no natural change would occur. For, if the principle of change were external to everything and internal to nothing, it would exist nowhere, and, along with the occasionalists, we would have to have recourse to God as the only agent.[13] Therefore, it is, in fact, internal to all simple substances, since there is no reason why it would be in one rather than another. And it consists in the progress of the perceptions of any monad, and the whole nature of things contains nothing beyond this. You see how the matter is reduced to something simple when principles are reached that are clearly necessary and sufficient, so that it seems not only superfluous, but inconsistent and inexplicable, to add anything else. To go beyond these principles and to ask

why there is perception and appetite in simple substances is to seek something ultramundane, so to speak, and to call on God to explain why he willed something to exist from among those that are conceived of by us.

I was forced to be more long-winded in responding, repeating your words at length so that I might eventually come across a basis for agreement about something. For I have noted that before, when we proceeded more freely, we almost forgot what had gone before, and sometimes ran around in circles or wandered off to other things, which sometimes produced involuntary signs of impatience. As for the rest, our excellent mutual friend Mr. Johann Bernoulli, who has scarcely recovered from a serious illness himself, wrote to me that your health has not been its best. This is very upsetting to me, since I set great store in your reflections and expect much fruit from them yet. And so, I think that merriment, exercise, and, in a word, the sort of regimen that is adapted to the body, are needed. This is the true medicine for chronic illnesses and irregular dispositions, but we often neglect it when we are distracted by habit or business.

Goodbye. From Hanover, 30 June 1704.

62. Leibniz to Bernoulli[1]

[Hanover, 1 July 1704][2]

I believe that when he has attentively read my most recent letter, which I am now sending on, our De Volder, given his great intelligence and equanimity, will say to himself that my complaint was not entirely unjustified. It is certainly surprising how often even famous men drift away from what is at issue in a discussion and onto something different when they are doing something else or have something else on their mind. I have employed three or four arguments more than once to which he has never responded directly, namely

that all the reality of aggregates consists in simple things;[3] that extension is something relative, i.e., the extension, or diffusion, of something;[4] that force and action cannot be modifications of a thing that is intrinsically merely passive.[5] And now he himself recognizes under another name – the "foundation of force" – what I have insisted on so many times.[6] So what does he require me demonstrate? Whence this foundation in things comes perhaps? But then he would surely require a new foundation. It seems to me that this would be just like looking for deeper origins for number, space, and time than those that exist in their notions. Therefore, just as I explained the ultimate grounds of space and time in terms of orders of existence that are either simultaneous or successive, I have explained his "foundation of force" by analogy with the principle of activity that we experience in ourselves, namely as that which contains nothing other than perception and appetite. Nothing can proven or known in the universe that is more fundamental than the supreme and common reason for all perceiving substances and for the harmony between them, which we call God. And a new and extremely clear demonstration of his existence shines forth here.[L1] And so, I have not been piling on empty words in place of real things, as he twice insinuated in his responses to my letters.[7] Certainly, he will not find the arguments that I gave elsewhere easy or easily proven invalid. And I do not know whether there are any others to be found that are their equal and that would penetrate further into the heart of things. Unless I am mistaken, the things themselves show that the word entelechy, as well as the words derivative force and[L2] monad fit the things that the words are meant to signify.

May I ask you please to pass on to Mr. De Volder the things that I have written, but to do it as if it were of your own accord. This way he is more likely to understand that my admonition was not completely unreasonable.

63. De Volder to Leibniz[1]

[Leiden, 14 November 1704]

B. De Volder sends greetings to the illustrious gentleman Gottfried Wilhelm Leibniz.

There are two reasons that I am replying to you somewhat tardily: First, so that I might be less of a nuisance to you by making my letters less frequent; second, because I was afraid that by replying to the issues without understanding them properly, I might perhaps end up saying things that you would think are off the mark. I had to be even more wary of this, because I noticed many entirely new and unexpected things in your last letter. For instance, I have kept asking where the force of corporeal substance arises from. It now seems to me that you do away with bodies altogether, in as much as you place them only in appearances, and that you substitute forces alone for things; and not even corporeal forces, but "perception and appetite."[2] These things certainly filled my mind with confusion, so that, even now, after having read and reread your letters many times, I dare not say with any confidence that I understand satisfactorily the things that you say. However this may be, I shall attempt some kind of response to the things that you say, or, at least, to what I take you to be saying, in case you might perhaps take my silence other than I intend it, and I shall be happy if I am on target. If I accomplish less than this, I hope, given how fair you are, that you will forgive me, on account of the novelty and obscurity of the task in which I am engaged.

I admit, and I have admitted already in a previous letter, that indivisible unities are not to be found in a mathematical body.[3] But at the same time, I would add that I still get stuck over whether or not this unity is found in an infinite mass of extension, if, in fact, the parts that we conceive of as different in this mass seem not to be distinguished in reality, since no part can be either assumed or conceived of unless all of them are assumed and conceived of.

Surely those who say that the substance of all corporeal things is one, or rather, those who think that bodies themselves are only distinguished by modes of extension, are admitting that this corporeal universe is composed of one substance affected by an infinity of different modes.[4] But this does not really matter much.

If we take the notion of extension in general, both abstracted from the modes by which it is affected, and in such way that it is considered intrinsically, it exhibits possible and indefinite parts. When this same notion is applied to existing things, i.e., when it is affected by its modes, it has fixed divisions made in accordance with the actual varieties of motions. However, this means that there is no greater difference between the first kind of extension and the second one, which is already actually divided, than there is between a mass of ivory considered generally and the same mass formed into various statues.

You say, "Extension is an abstraction from that which is extended, and it is no more a substance than a number or a multitude can be considered a substance. And it expresses nothing other than a certain non-successive (like duration) but simultaneous diffusion or repetition of the same nature, or, what amounts to the same, a multitude of things of the same nature, existing together with some order among them. It is this nature, I say, that is said to be extended, i.e., diffused." Of the things you say here, the word <u>diffusion</u> appears obscure to me. I easily grasp the way in which the same indivisible thing may be repeated, as might happen in a number. I do not understand how it can be diffused, i.e., what seems one and the same thing to me, how it can be extended, i.e., how it can exist with different parts at the same time. Moreover, if those natures of things that you want repeated are not each intrinsically extended, I do not understand how extension arises from their repetition.[5] Points will not make a line, even if they are repeated an infinite number of times, nor will lines make a surface, even if they are repeated infinitely, nor, finally, will a surface make a body. On the contrary, it is necessary that the individual parts have extension for extension to arise.

Therefore, even if these indivisible unities of yours are repeated infinitely, they will never be diffused and never produce extension, unless they are each extended, considered intrinsically. It is commonly said that those things that have a place, as they say, penetrate each other, but that it is different with things that are in a place. And so, it is said that there can be innumerable little souls in a needle point producing no extension between them. Although I would not be willing to prove this claim, it does, nonetheless, suppose that extension cannot be produced from an unextended thing.

I remember that you have said quite often that, unless there was something active in us, there could not be derivative force and actions in us. But, as far as I know, you never brought forward a reason for this claim. Now at last you have and I approve of it.[6]

So that you may show how individuals differ from universals you say that, "There is an essential ordering of individuals with respect to specific parts of time and place, from which universals are abstracted by the mind." Without a doubt. But how does that order bring it about that a change follows from those things, which cannot follow from universals? Whatever that order is like, a thing, nonetheless, has nothing intrinsic except its nature, and nothing follows from it except that which is contained in that nature. A spherical body is the same wherever it is in time and place, and nothing follows from it except that which is contained in the nature of a sphere.

But, to be brief, it is certain that a principle of motion must be acknowledged that is internal to the things themselves, or else refuge must be taken in God. Still, this does not prevent me from failing to understand any less this internal principle, or the way in which it operates. The difficulty of understanding the internal principle has made most people take refuge in God. It cannot be resolved properly, in my view, unless we have a perfect understanding of this principle and its way of operating.

If you ask what I thought needed to be demonstrated to me, I would say first, of course, that the notion of corporeal substance be given to me, and then that active force be deduced from that. Let me clarify with an example. Suppose I had asked which figure it is that has angles equal to two right angles. Someone who first gave me the notion of a triangle, i.e., of a figure comprehending three straight lines, and then demonstrated that this property follows from it, would have satisfied me. Someone who said that it is a figure that contains this principle of equality intrinsically would not have. And so, I will not ask why there is perception and appetite in simple substances, nor why extended mass subjected to innumerable changes arises from them (for you seem to me to deny this), but why it appears from them and how perception and appetite produce this appearance. I must confess that I will neither imagine nor understand the internal principle of action as long as I am ignorant of these things. Would that it were possible to consider some example of a primitive force, and to identify from this the source from which innumerable changes follow successively while the nature of the primitive force, nevertheless, remain the same. If I understood these things, I would already have judged that I perceive some internal principle. Now I seem to perceive nothing except this one thing: that there is a certain cause of the changes. But as to what that cause is, and in what way it produces its changes, I am completely ignorant of all these things. Goodbye.

From Leiden,

14 November 1704.

64. Leibniz to De Volder[1]

[Berlin, January 1705?][2]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.[L1]

Your letters never fail to be most welcome to me, for they always either teach me something, or at least provide the occasion to consider or explain things. You say that you "noticed many very new and unexpected things" in my most recent letter.[3] But perhaps you will find that the same things were already adequately introduced in previous letters, and that only prejudice prevented you from reaching the same point before now, so that you are no longer searching for substance and the cause of force where it does not exist. As a result, I am forced to insist on certain things more explicitly, and to reply, if not to what was asked, at least to what should have been asked. You say that "you asked where the force of corporeal substance arises

from," and that I, "in fact, seem to do away with bodies altogether and to place them only in the appearances, and to substitute forces alone for things; and not even corporeal forces, but perception and appetite." I do not really do away with body, but reduce it to what it is. For I show that a corporeal mass that is believed to have something besides simple substances, is not a substance, but a phenomenon resulting from simple substances, which alone have unity and absolute reality. I relegate derivative forces to the phenomena, but I think that it is clear that primitive forces can be nothing other than the internal strivings of simple substances, by which they pass from[4] perception to perception by a certain law of their nature, and, at the same time agree with each other, representing the same phenomena of the universe in a different manner, something that necessarily arises from a common cause. It is necessary that these simple substances exist everywhere and that they are self-governing (each as far as itself is concerned), since the influence of one on another cannot be understood. Anything more beyond this in things is posited in vain and added without argument. For, since everything ought to be deduced from the phenomena, by what evidence, I ask, will you prove

that there is something real in them beyond these things or something substantial besides the substances from which appearances arise in themselves out of themselves in conformity with the eternal laws of metaphysics and mathematics? Whoever adds anything to these things will accomplish nothing, and will both work in vain in giving explanations and be thrown into inextricable difficulties. And, in this way, the Academics used to argue, not completely improperly (even if they were not received correctly, or used good arguments badly) against those things that are imagined to be outside of us, i.e., outside souls or simple substances.[5] Indeed, I suppose nothing everywhere and throughout all things except that which we all admit in our own souls on many occasions, namely internal spontaneous changes. And, in this way, I exhaust the totality of things with one act of the mind. Moreover, I consider bodies to be the same kinds of things as corporeal forces, namely to be among the phenomena, if, indeed, they are understood to super add anything to simple substances and their modifications, just as we say, not improperly, that a rainbow is a thing, even if it is not a substance, i.e., it is a [real, i.e., well founded,][6] phenomenon [that does not disappoint the expectations of one who proceeds rationally]. And, in fact, not only sight but touch has its phenomena [, and corporeal masses are like this, as are beings of aggregation, whose unity comes from a perceiver].

I do not say that, "A unity exists in a mass of extension," or rather, of extended things, or, as I would prefer, in a multitude of things, but rather, innumerable unities.

I would not say that, "the corporeal universe is composed of one substance affected by an infinity of different modes," even though it can be said that matter considered intrinsically (i.e., insofar as it is passive) is similar to itself everywhere. [For it can be said that matter is real to the extent that there is a reason in the simple substances for the passivity that is observed in the phenomena]. The true substance is not in the aggregative whole, but in the individual unities, just as in the ocean there is not one substance or thing, but every drop

contains other things, although all the drops are assumed to be made from similar stuff. However, the water is actually divided before it is formed into drops, as is the mass of ivory that you mention before it is formed into statues. And it is the same with any mass, even though in mathematical extension, by which possibles are understood, there is no actual division and there are no parts except those that we make by thinking. And there are no first elements, any more than a smallest number is found among the fractions as the element of the rest. [Hence number, hour, line, motion, i.e., degree of speed, and other ideal quantities of this kind, i.e., mathematical entities, are not in fact aggregated from parts, since the way in which someone may wish to assign parts in them is completely undetermined. Actually, it is necessary that they be understood in this way, since they signify nothing other than the mere possibility of assigning parts in any way whatever].

If there were no divisions of matter in nature, there would be no distinct things. Indeed, there would be nothing except the mere possibility of things. By contrast, the actual division into masses produces things that appear distinct, and it presupposes simple substances. Those people err greatly who believe (as I think many commonly do) that there was, or could have been, at one time a certain matter, i.e., mass, everywhere uniform and[7] at rest, and that things arose from the division of it, or that they could fall back into it by being returned to rest.[8] Those who have a mind that is possessed by that confused image fail to consider sufficiently other things as well as the fact that, on this view, no foundation would arise for a distinction in corporeal things (something I once demonstrated to Sturm), since whenever anyone imagines such a mass as being in motion, equivalent things may be substituted for each other.[9] How amazing that an opinion that overturns all the phenomena could have become so prevalent. But, it is much less plausible that from such stuff there might have arisen souls, i.e., perceivers, which are not only in us but everywhere in things, as nature's uniformity in variety will easily convince the wise. Those for whom matter is as I

have said imagine such an origin of things from matter, grasp at shadows rather than things, take extension – an intrinsically ideal thing, consisting in a relation like number and time – for a substance, and form things from ideas, as from Pythagorean numbers.[10] And, even if impenetrability is added to extension, it still does not follow that anything complete is brought about, from which a reason could be given for motion, and especially a reason for the laws of motion, not to mention the apparent difference between things.

I intend the diffusion that I conceive of in extension, and which seems to have put into you the suspicion of I know not what hidden paradox, to be nothing other than a continuation in which the part is similar to the whole, as we conceive of whiteness diffused in milk, and the same direction everywhere in a straight line, and equal curvature in the circumference of a circle. But, in fact, my unities, i.e., simple substances, are not diffused (as we commonly conceive of the flowing of a point), nor do they constitute a homogeneous whole, for the homogeneity of matter is produced only by an abstraction of the mind, when they are considered as only passive and, therefore, as incomplete.

I thought I had already given, "the reason why derivative force and actions presuppose something active". Certainly, I brought it forward publicly in the reply to Sturm.[11] I am glad that you now approve of it.

Individuals involve infinity. In forming universals certain circumstances only are abstracted by the mind while concealing innumerable others. And so, only in the individual is there a complete notion, and, because of this, it includes changes as well. A mathematically perfect spherical body exists nowhere in nature. It is formed by the mind by concealing aberrations. And it is the same for any other figure that a finite mind can grasp, namely that that figure can exist nowhere exactly. "The essential ordering of individuals," i.e., the relation "with respect to time and place," is to be understood from their relation with respect to the contents of time and place, both near and far, which is necessarily expressed by every

individual in such a way that the universe could be read in it, if the reader was infinitely clear-sighted.

Finally, you ask, "Why are these appearances are produced[12] in me or any other true substance?" I say that subsequent appearances are produced from those preceding them in accordance with the metaphysical and mathematical laws of eternal truth. But the reason why there are any such appearances at all is the same as the reason for the existence of the universe. For you can easily see that simple substances can be nothing other than the sources, i.e., the principles, [and at the same time the subjects,] of just as many series of perceptions unfolding themselves in order, expressing the same universe of phenomena with the greatest and most orderly variety. In this way, the supreme substance has poured forth his perfection as much as was permitted into the many substances that depend on him, which ought to be conceived of as individual concentrations of the universe and (some more than others) as imitations of the divinity. I do not think that any other reasons for things can be understood, nor (in short) can they be desired. Things ought to have existed either in this way, or in no way at all.

65. Leibniz to De Volder[1]

[11 October 1705][2]

From my letter to Mr. De Volder 11 October 1705

In the recent September Nouvelles de la République des Lettres I was surprised by the newly published Elémens de Géometrie de Monseigneur le Duc de Bourgogne, but I was all the more surprised by what there is in it about unities, which is exactly my view. Yet, for all that he writes, he has left unsolved the difficulty that, while geometers show that extension does not consist of points, metaphysicians show the opposite, that matter must result from

unities, i.e., simple substances.[3] In fact, matter is not continuous but discrete and actually divided to infinity, even if no assignable part of space is devoid of matter. Yet space, like time, is not something substantial, but something ideal, and consists in possibilities, i.e., the order of possible coexistents at any given time. And so, there are no divisions in it, except those that the mind makes, and the part is posterior to the whole. In real things it is the opposite, unities are prior to the multitude, and multitudes do not exist except through unities. [It is the same with changes, which are not really continuous].[4]

[5] Endpoints of lines and unities of matter do not go together. Three continuous points in the same straight line cannot be conceived of. But two endpoints are conceived of, the endpoint of one straight line and the endpoint of another out of which the same whole is constituted, just as there are two instants in time, the last one of life and the first of death.

One unity is not touched by another, but there is a perpetual transcreation in motion in such a way that, when a thing is in such a state that by continuing its changes for an assignable time there must be a penetration of a proximate thing after a time, each and every point is also in another place, just as the avoidance of penetration and the order of the changes demand.

Space can be understood as filled with matter that is actually divided into parts in infinite ways.[L1] For example, there might be three circles touching each other, with three others touching each other placed in between, and then three others between them. This will be one way of filling up the gap that is in the middle. Another way of filling it is if the greatest area is inscribed that touches all the circles that enclose the space. Time is also resolved into unities of duration through actual changes, i.e., into just as many creations, infinite in number. For nothing exists of time except instants.

66. De Volder to Leibniz[1]

[Leiden, 5 January 1706]

B. De Volder sends greetings to the most renowned gentleman Gottfried Wilhelm Leibniz.

The reason for my long silence is not so much my health (although that is also not as good as it might be, and may, indeed, have contributed something to it), as that I had nothing new to say in reply to your letters, and was worried that I might make you sick by repeating the same things. Indeed, I do not know what causes me still to get stuck on the same difficulties that I posed in my last letter. Certainly, I do not seem to understand primitive force, nor do I comprehend how extension can arise, or, if you prefer, appear from, indivisible things. Moreover, there has fallen upon me a despair of ever understanding correctly certain of your opinions. Since I have moved forward so little after so many attempts, I think that things are required for an understanding of them that you regard as ultramundane and not even to be sought. The departure from the area of our friend Bernoullli, who has facilitated my communication with you, is approaching.[2] All these things made me more reluctant to write to you, afraid that I might annoy you too much with the same old story. Indeed, except for the things that I wrote before and those that are contained in these very few remarks, I do not have anything further to add.

I can tell you the following concerning the things that you ask about the writings of Huygens and Hudde.[3] We have found nothing more among Huygens' papers that we judge appropriate for publication.[4] The papers themselves are being preserved in our university library, to which Huygens had bequeathed them. I also expect nothing from Hudde's, since, in accordance with his will, they all will have come into the possession of Dedel, his brother's son, who is a municipal official in Amsterdam, and ignorant of mathematical things. I remember that I often saw the deceased with quite a thick octavo volume, in which I very

often heard him say that many of his discoveries were written, but in such a way that he himself could scarcely understand them any more, let alone that they could be understood in any way by anyone else. But I would not have any hopes for any of his papers, given that they have fallen into those hands.

Perhaps the news has reached you that I have retired from my academic position and that I took care of things in so far as I could so that our mutual friend Bernoulli might succeed me. But this was in vain, since he has committed himself to the people of Basel. In the interim, they have given the responsibility for teaching philosophy and mathematics to our friend Mr. Bernard who edits the Nouvelles de la République des Lettres.[5] However, I believe that they will not give him the title of professor, unless things turn out successfully. The most illustrious Bernoulli recommended Mr. Hermann[6] to me, who introduced himself when he visited these parts a few years ago, and I have seen some excellent examples of his mathematical work in the same Nouvelles. Our friend appealed to your recommendation of the same Hermann, but he did not include anything that shows he is competent in philosophy and metaphysics. If you are willing to write up something about the matter for us, this might be able to count in his favor at some point in time. I do not think that it is possible for me to persuade our people to do anything immediately. However, your recommendation, which our people will no doubt take very seriously, will surely be most advantageous when the occasion arises. Nothing remains except to wish that the year that we have already begun will be happy and fortunate for you and that many more will follow. Goodbye.

From Leiden,

5 January 1706.

67. Leibniz to De Volder[1]

[Hanover, 19 January 1706]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

I am happy that you are well and that you remembered me. I congratulate you also on your retirement. I pray that it will be accompanied by long-lasting good health, so that your erudition may continue to be enjoyed by the public, and so that I too may continue to enjoy your singular judgment. For, whenever I seem able to satisfy your arguments, I satisfy myself.

You rightly despair of obtaining from me something for which I neither raise nor have the hope, nor even the desire. In the schools they commonly seek things that are not so much ultramundane as utopian. The clever French Jesuit Tournemine recently provided me with an elegant example.[2] After he had offered some praise of my pre-established harmony – which seemed to provide an explanation of the agreement that we perceive between soul and body – , he said that he still desired one thing, namely, an explanation of the union, which assuredly differs from the agreement. I responded that, whatever that metaphysical union is that the schools add over and above agreement, it is not a phenomenon and there is no notion of, or acquaintance with, it. Thus, I could not have intended to explain it.[3]

I fear that the force that is conceived of in extension or bulk as outside perceiving things or their phenomena is of this kind. For there can be nothing real in nature except simple substances and the aggregates resulting from them. However, in these simple substances, we know nothing other than perceptions or reasons for perceptions. Anyone who postulates more things needs to provide the marks by which they are established and made evident. I consider it to have been demonstrated (as I have written again and again, although I grant that I have not yet ordered everything so that I can easily provide a demonstration for

the eyes of others) that it is essential to a substance that its present state involves its future states, and vice versa, and that force, or the reason for the transition to new perceptions, cannot be derived from any other source.

It is also obvious from what I said that in actual things there is nothing but a discrete quantity, namely the multitude of monads, i.e., simple substances, which in any sensible aggregate, i.e., any aggregate corresponding to the phenomena, is, indeed, greater than any number however large. But continuous quantity is something ideal that pertains to possible things and to actual things in so far as they are possible things. Of course, the continuum involves indeterminate parts, but, nevertheless, nothing is indefinite in actual things. Indeed, any division that can be made in actual things has been made. Actual things are composed as a number is composed from unities, ideal things as a number is composed from fractions. There are actually parts in a real whole, but not in an ideal whole. Indeed, when we – confusing ideal things with real substances – seek actual parts in the order of possible things and indeterminate parts in an aggregate of actual things, we entangle ourselves in the labyrinth of the continuum and in inexplicable contradictions. However, the science of continuous things, i.e., of possible things, contains eternal truths that are never violated by the actual phenomena, since the difference is always less than any given assignable difference. And we have no other mark of reality in the phenomena – nor should we desire one – than the fact that they agree with each other and with the eternal truths.

I am sad to have lost the many excellent ideas of the distinguished gentlemen Huygens and Hudde. Not long before he died, in the Acta Eruditorum, Huygens enigmatically described a new invention for perfecting the use of the pendulum in ships[4], but I am afraid that whatever it is, it will have been completely lost. From Hanover, 19 January 1706.

P. S. That learned gentleman in Basel, Mr. Hermann, about whom you seek my opinion, is, as you know, an excellent mathematician. I have not examined what he has done in other fields well enough, since I have never spoken with him, nor corresponded with him, except about Mathematics. However, I do not doubt that anyone who can do a harder thing can do something that is less hard, and that, since he has devoted himself to theology, he has already cultivated a knowledge of philosophy.

[Supplement 1][5]

It is the same for the things at issue between us. I do not see what argument could prove that there is anything in[6] extension, bulk, or motion[7] beyond the phenomena, i.e., beyond[8] the perceptions of simple substances. And so, the active and passive force that is conceived of as a certain something in bulk outside perceiving things, is nothing but a phenomenon, like the rainbow, or an image in a mirror, or a dream, albeit one wholly consistent with itself. And the reality of sensible things consists in nothing other than the agreement of the phenomena.

The substances whose existence can be proven, therefore, are perceivers and their common cause, which contains the reason for all the perceivers, and the agreement. The cause is infinite in perfection, but the perceivers are infinite in number, and they are the simple substances, or monads, from which everything else results. Thus it appears that in real things there is no continuous quantity, just discrete quantity, i.e., the multitude of monads. In monads, there is a force of acting and being acted on, but only an internal one. For truly they cannot act on each other reciprocally.

[Supplement 2][9]

Gottfried Wilhelm Leibniz sends greetings to the gentleman distinguished on account of learning and merits, Mr. B. de Volder.

I am happy that you are well and that you kindly remembered me. I congratulate you also on your retirement. I pray that it will be accompanied by long-lasting good health, so that your erudition may continue to be enjoyed by the public, and so that I may continue to enjoy your singular judgment. For, whenever I seem to be able to satisfy your arguments, I satisfy myself.

You rightly despair of obtaining from me something for which I neither raise nor have the hope, nor even the desire. It is enough to be given answers to the things that should be asked, given that sometimes one cannot be given an answer to the questions commonly asked.

I think that to be a philosopher is to give explanations of the phenomena. But those who seek something in things beyond the phenomena, or beyond those things that follow from the phenomena, cannot even explain what they seek, so it is not surprising if they cannot be satisfied by the explanations.

The clever French Jesuit Tournemine recently provided me with an elegant example.[10] After he had offered some praise of my pre-established harmony – which seemed to provide an explanation of the agreement that we perceive between body and soul, i.e., the agreement between our own and others besides our own –, he said that he still desired one thing, namely, an explanation of the union as well, agreement being one thing union another. I responded that, whatever that metaphysical union is that the schools add over and above agreement, it is not a phenomenon and there is no notion of, or acquaintance with, it. Thus, I could not have intended to explain it.[11]

It is the same with the things at issue between us. I hold that the primitive or derivative force that is conceived of in extension and bulk as outside perceiving things is not

a thing but a phenomenon, just like extension itself, bulk, and motion, which are no more things than an image in a mirror or a rainbow in a cloud.[12] And, in fact, to look for something beyond the phenomena here seems to me just as if someone were to deny that he was satisfied by the explanation provided for the phenomena of an image, as if there were some, I know not what, essence of the image left to be explained.

Arguments, in my opinion, cannot prove the existence of anything besides perceivers and perceptions (if you subtract their common cause), and the things that should be admitted in them. In a perceiver, these are the transitions from perception to perception, with the same subject remaining; in perceptions, the harmony of perceivers. For the rest, we invent natures of things and wrestle with the chimeras of our minds as if with ghosts. There is an active force and a passive force in every perceiver;[13] the active in the transition to the more perfect, the passive in the opposite. And there is an infinity of perceivers. Indeed, there are as many as there are simple substances, i.e., monads. The order of these among themselves, expressed by our phenomena, constitutes the notions of time and space. That which results from the passions of the perceivers and limits the phenomena themselves, taken as a whole, gives rise to the apparition of bulk, i.e., of the passive force of bodies.

As for the rest, even if these things can be demonstrated once a subject whose perceptions change has been admitted, nevertheless, it is enough for me to assume that which is usually conceded, i.e., that there is a certain force in the perceiver for forming new perceptions for itself from prior ones – which is the same as if you said that a new perception sometimes follows from some prior perception. Ancient and modern philosophers commonly recognize this in one instance, namely the voluntary operations of the soul. But I think that it holds always and everywhere and is sufficient for all the phenomena, with both great uniformity and great simplicity in things.

Besides, you will easily see from this that material substances are not destroyed but conserved, provided that they are sought in that which is dynamic, which reveals itself through the phenomena, i.e., in the active and passive force of perceivers, not outside of them. But extension, like time and bulk and the motion that consists of their variations, disappears into the phenomena no less than real qualities do, and exists, as Democritus said, more by convention than in reality.[14] This is also sufficiently proven by the transitory and relative nature of motion, and that famous labyrinth of the continuum, in which we entangle ourselves because of a false conception of time, space, and bulk.

In real things there is nothing but discrete quantity, i.e., a multitude resulting from[15] true unities. Continuous quantity, which is not apparent but exact, pertains to ideal things and possibilities, since it involves something indefinite or indeterminate, which is not allowed by the actual nature of things.