DEDEKIND'S CATEGORICITY THEOREM, INDUCTION, AND MATHEMATICAL COMMUNICATION

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Abstract: Dedekind's Categoricity Theorem states that any structures satisfying the axioms of second-order Peano arithmetic are isomorphic. Philosophers of mathematics (e.g. Shapiro, Isaacson) have repeatedly claimed that this result has significant implications with respect to the determinacy of our understanding of the natural numbers. At the same time, it is also widely acknowledged that the significance accorded to Dedekind's Theorem depends on whether we can justifiably assume that second-order quantifiers are interpreted relative to the standard (i.e. full) semantics for second-order logic. This in turn suggests that the common claim – i.e. that the theorem helps to secure the determinacy of arithmetical reference – is predicated on substantial set theoretic assumptions. While acknowledging such a dependence, Parsons (1990, 2008) and Lavine (1999) have both proposed a means by which the proof of Dedekind's Theorem can be reconstrued as a demonstration in so-called full schematic arithmetic (i.e. a variant of first-order Peano arithmetic in which the induction schema may be extended to predicates defined over arbitrary first-order languages extending that of first-order arithmetic). On this basis, they have suggested that not only does the theorem lack significant set theoretic presuppositions, but it may also be viewed as demonstrating that any two mathematical agents who are able to communicate must conclude that structures which satisfy their axioms will be isomorphic. I will argue that when properly understood, the arguments Parsons and Lavine have offered for the latter conclusion are either question begging or rest on faulty assumptions about the range of predicates to which an agent might justifiably extend his induction schema. I will also discuss their former claim in light of recent work by Simpson and Yokoyama on the reverse mathematics of Dedekind's Theorem and its relation to the existence of non-standard models.