SOME KEY THEOREMS IN SOCIAL CHOICE THEORY

[Comment: STRATEGY-PROOF in all of the theorems listed below refers to "weak dominant strategy implementability" - that is, agents must always be incentivised to truthfully reveal their private information, REGARDLESS of whether or not they expect others do so.
In game theoretic terms, this means that we only impose the assumption of INDIVIDUAL RATIONALITY, but not PERFECT COMMON KNOWLEDGE OF RATIONALITY, in seeking for a viable social choice mechanism.]

{A} Arrow (1951) - Under UNRESTRICTED DOMAIN with at least 3 options: any social choice rule satisfying the WEAK PARETO PRINCIPLE and INDEPENDENCE OF IRRELEVANT ALTERNATIVES (imposing ORDINAL NON-COMPARABLE UTILITY) must be DICTATORIAL.

{B} Sen (1970) - Under UNRESTRICTED DOMAIN with at least 3 options: any social choice rule satisfying the WEAK PARETO PRINCIPLE must violate MINIMAL LIBERALISM.

{C} Gibbard-Satterthwaite (1973, 1975) - Under UNRESTRICTED DOMAIN with at least 3 options: any STRATEGY-PROOF mechanism must be DICTATORIAL.

[Comment: VCG mechanisms are examples of non-dictatorial strategy-proof mechanisms, but only work under the restricted domain of quasi-linear preferences. They also do NOT achieve PARETO EFFICIENCY due to budget imbalance. For instance, if BUDGET FEASIBLE (a VCG mechanism set up to never run a deficit) then there is a deadweight welfare loss due to the budget surplus which occurs if some agents are pivotal and which must be "burned" in order to achieve strategy-proofness. Simple majority rule is also a strategy-proof mechanism if there are only 2 options or if the domain is restricted to single-peaked preferences.]

{D} Hurwicz (1960, 1972), generalised by Ledyard and Roberts (1974): In an economy consisting of private goods (Hurwicz) or private and public goods (Ledyard and Roberts) (RESTRICTED DOMAIN):
No INDIVIDUALLY RATIONAL mechanism can be STRATEGY-PROOF, hence no INDIVIDUALLY RATIONAL mechanism can guarantee PARETO EFFICIENCY.

[Comment: One important implication of the Hurwicz Theorem is that neither Walrasian price adjustment in a private goods economy nor the Lindahl mechanism in a public goods economy can be strategy proof if the number of agents is finite, nor can any other conceivable individually rational mechanism. (If there is an infinite continuum of agents, then Walrasian/Lindahl price adjustment leads to efficient revelation of private information only for EXCLUDABLE goods, so Lindahl still fails to work efficiently in the case of PURE PUBLIC GOODS due to the "free-rider problem".)]

{E} Green and Laffont (1979) - Under quasi-linear preferences (RESTRICTED DOMAIN): (a) any STRATEGY-PROOF mechanism must be VCG, and (b) no VCG mechanism can guarantee BUDGET BALANCE, hence: (c) no mechanism can guarantee PARETO EFFICIENCY.

{F} Mitsui (1983) - Under quasi-linear preferences (RESTRICTED DOMAIN): the budget imbalance from VCG mechanisms disappears asymptotically (i.e. as the number of agents becomes large), hence VCG mechanisms are ASYMPTOTICALLY PARETO EFFICIENT.

[Comment: Such asymptotically Pareto-efficient VCG mechanisms are NOT individually rational, as implied by Theorem {D}.]