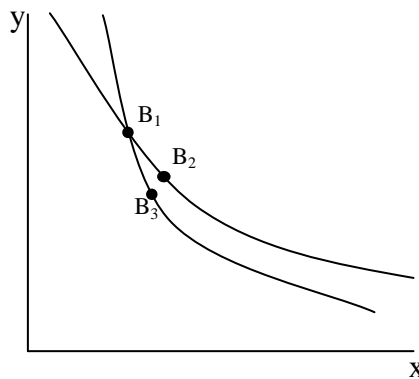


Week 1 - Individual Demand Theory Notes

1. Explain what properties of consumer preference orderings imply that indifference curves never cross, and are always downward sloping and convex. What is the significance of the curvature of the indifference curves and how does this relate to the marginal rate of substitution?

The possibility of indifference curves crossing is ruled out by the assumptions of transitivity and monotonicity. In the diagram below, since B_1 is weakly preferred to B_2 (because they lie on the same indifference curve), and B_3 is weakly preferred to B_1 (for the same reason), transitivity implies that B_3 is weakly preferred to B_2 . However, since B_2 is strictly preferred to B_3 (because monotonicity requires that more of both goods is always better), B_3 cannot be weakly preferred to B_2 .



The curvature of the indifference curves shows the willingness of an individual to exchange one good for the other. The tighter the curvature (the limiting case being perfect complements, where the curve has become so tight it is a square corner), the less willing is an individual to exchange because a larger amount of one good is required to compensate for the loss of the other good. The shallower the curvature (the limiting case being the case of perfect substitutes, where indifference curves are straight lines), the more willing is the individual to exchange because a smaller amount of one good is required to compensate for a given loss of the other. It should also be clear from the above discussion that the curvature of the indifference curves is closely connected with the degree to which goods are substitutes or complements. However, it is important to remember that whether goods are substitutes or complements depends upon the size of both the substitution effect and the income effect, whereas the curvature of the indifference curves only determines the size of the substitution effect. However, it is definitely the case that the shallower the curvature of the indifference curves, the larger the substitution effect, and the larger the substitution effect the more likely goods are to be gross substitutes *ceteris paribus* (because the substitution effect always takes demand away from the good which has become more expensive and towards the one which has become cheaper).

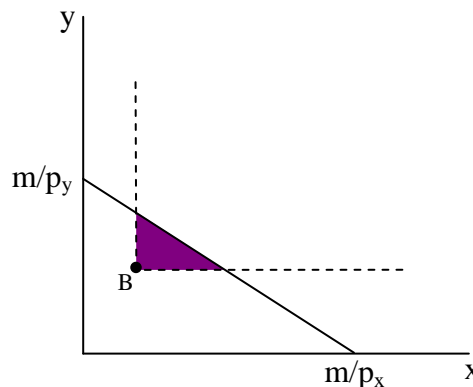
The marginal rate of substitution (MRS) is the slope of the indifference curve. It is derived mathematically for a non-linear indifference curve by taking the constant slope of the straight-line tangent to the curve at the particular point of interest. Intuitively, the absolute value of the MRS is the ratio between the marginal amount of good y that must be given to compensate for a marginal loss of good x , and the marginal loss of good x . The higher is the marginal utility of good x , the more utility is lost when good x is taken away. The lower the marginal utility of good y , the more

of it must be given to compensate for a given utility loss. Hence it is also intuitive that the MRS is the ratio $-MU_x/MU_y$. The MRS changes along a non-linear indifference curve. For the downward-sloping convex indifference curves which result from well-behaved preferences, the MRS is always negative, and always decreases (becomes greater in absolute value) as the amount of good x decreases. This is because a consumer with well-behaved preferences is less willing to exchange good x for more of good y the less of good x they have, because the convexity of their preferences implies that they prefer averages to extremes.

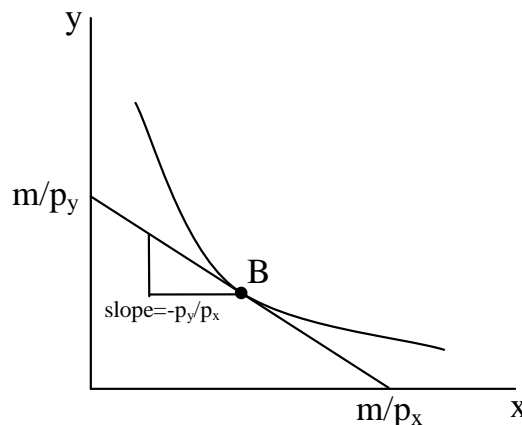
2. Explain under what conditions a consumer will maximize utility by setting their MRS equal to the price ratio between the two goods.

(i) The parameters m , p_x and p_y define a budget set, which is the set of possible points that the consumer could choose to consume (they must choose one point from this set). We use the theoretical framework based on the assumption of consistent, well-behaved preferences (including non-satiation and strict convexity) to predict the optimal bundle that the consumer will choose. This optimal choice must possess a number of properties:-

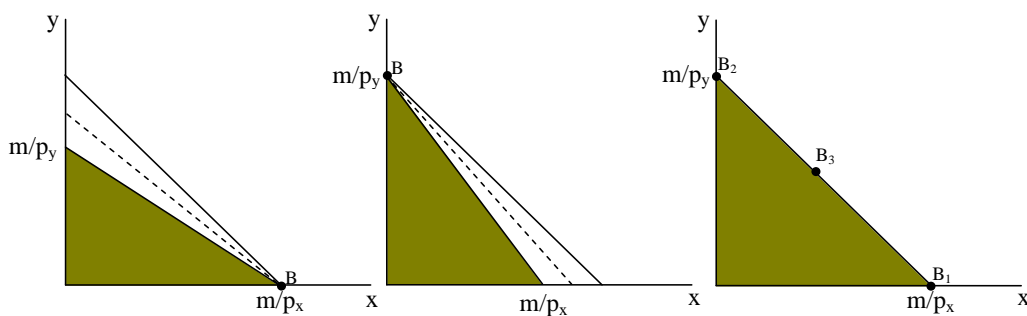
- It must lie on the frontier of the budget set, the **budget constraint**. This results from the assumption of monotonicity. If a point B below the budget set were chosen, there would be points north-east of B which are strictly preferred to B (shaded in the diagram below):-



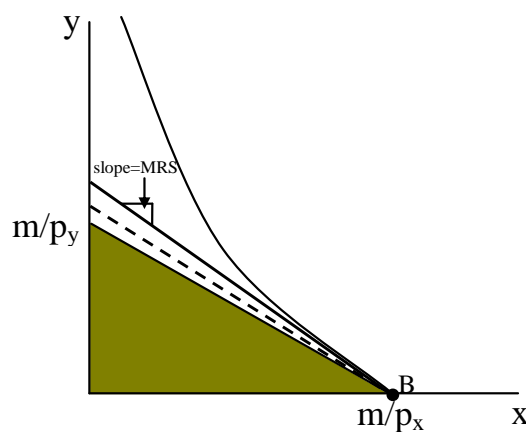
- At the optimally chosen bundle, the indifference curve will usually be tangential to the budget line. In other words, the MRS (the slope of the indifference curve) must be equal to the price ratio (the slope of the budget line). The reason is that otherwise the consumer could reach a higher indifference curve within the same budget set by altering the chosen bundle.



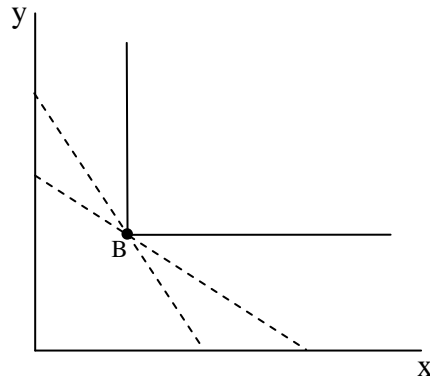
This argument does not hold in the same neat form in cases where strict convexity and non-satiation do not hold. For example, when goods are perfect substitutes, we can generally no longer say that the MRS is the only exchange rate that would lead B_1 to be chosen, and sometimes B_1 will not be the strictly preferred bundle along the budget line. The diagram below illustrates the three possibilities for the optimal bundle when the goods are perfect substitutes. Either the consumer's entire income is spent on one of the two goods (these are called 'boundary solutions' or 'corner solutions'), or the budget line is the same as the indifference curve, in which case there are an infinite number of equally preferred bundles (e.g. B_1 , B_2 and B_3 in the diagram furthest to the right). At a corner solution, the MRS is not generally equal to the price ratio, and there are other exchange rates represented for example by the dotted line, which still lead to B being the optimal bundle.



Boundary solutions **may** occur in the case of strictly convex preferences. However, they are a special case whereas with perfect substitutes they are the usual outcome. So, with strictly convex preferences the MRS is **usually** equal to the price ratio, but the chosen bundle is always strictly preferred to all others available along both the budget constraint and the line with slope equal to the MRS going through the chosen bundle B:



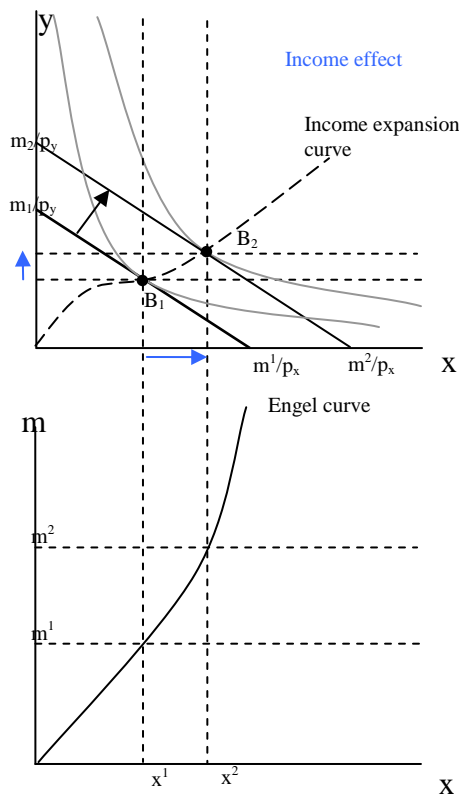
“Kinky” solutions occur in the case of perfect complements (where strict convexity does not hold) because there is no proper tangency condition (the absolute value of the MRS is infinite at the corner of the indifference curve):



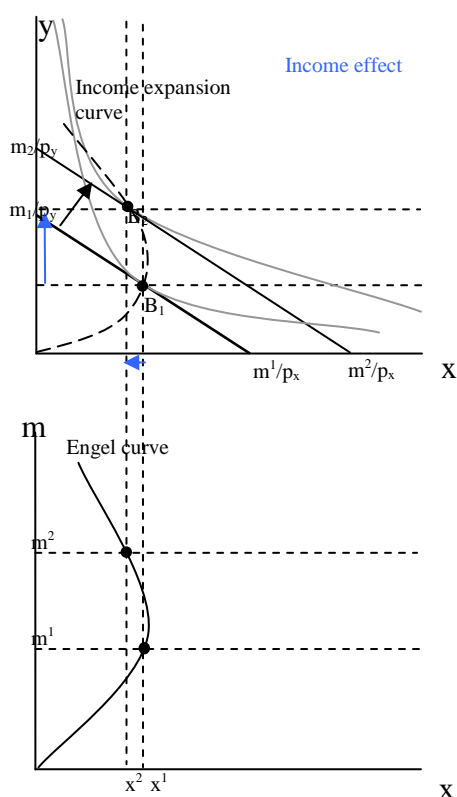
A similar effect occurs at any kinked indifference curve. These “kinky” solutions have the important feature in common with corner solutions that there is no longer a unique price ratio that will lead to utility maximization at the optimally chosen bundle. Therefore the observed price ratio does not fully reveal the subjective MRS of the consumer in the usual way. As will become relevant later on in the term when we look at welfare economic theory, the assumption that price ratios fully reflect and efficiently utilize information about consumer preferences depends upon the assumption of well-behaved preferences.

3. Derive the income offer curve and the Engel curve for (1) a normal good and (2) an inferior good. What difference does it make if the consumer’s preferences are homothetic?

The diagram below shows the derivation of the Engel curve for a normal good:



Below is illustrated the derivation of the Engel curve for an inferior good.

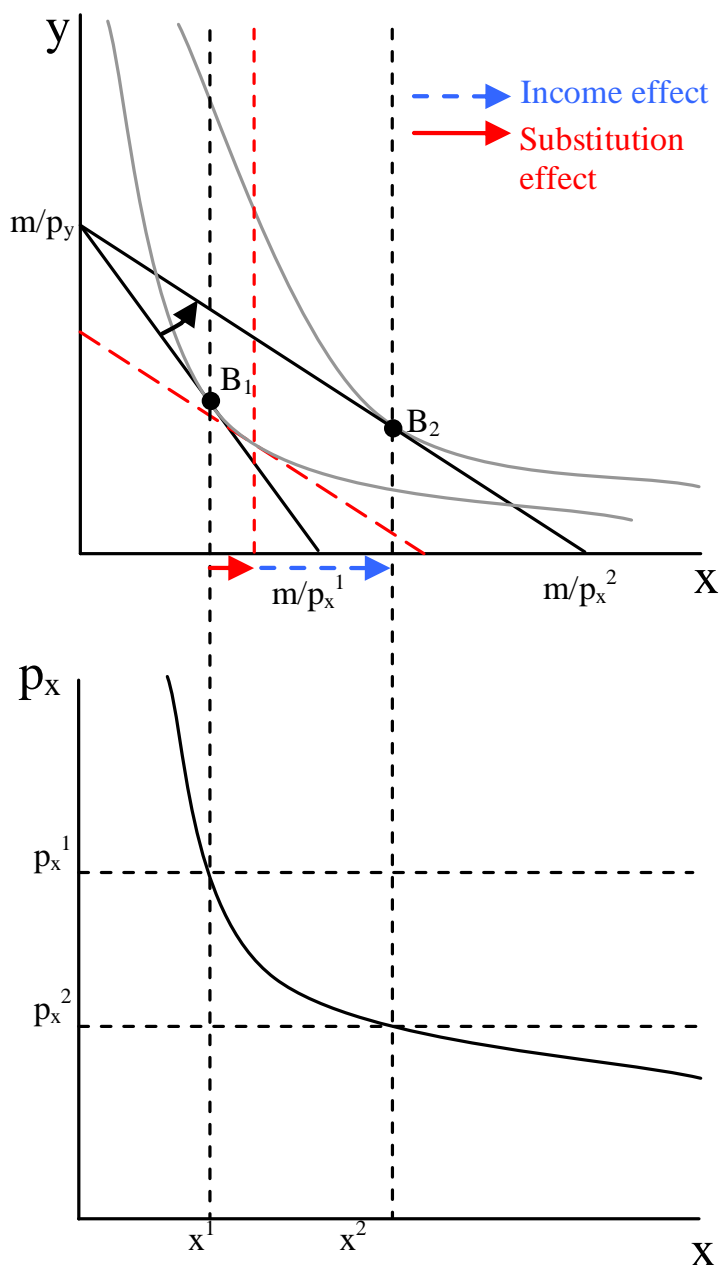


If the good was homothetic, the price offer curve and Engel curve would both be straight lines through the origin. Both goods would therefore be normal (and would have an income elasticity of 1, which means the consumption of both goods always increases by X% whenever income increases by X%).

4. Use indifference curve analysis to derive the Marshallian demand curve for (a) a normal good, (b) an inferior good which obeys the law of demand and (c) a Giffen good. Why must a normal good always obey the law of demand. Hence why must a Giffen good always be inferior.

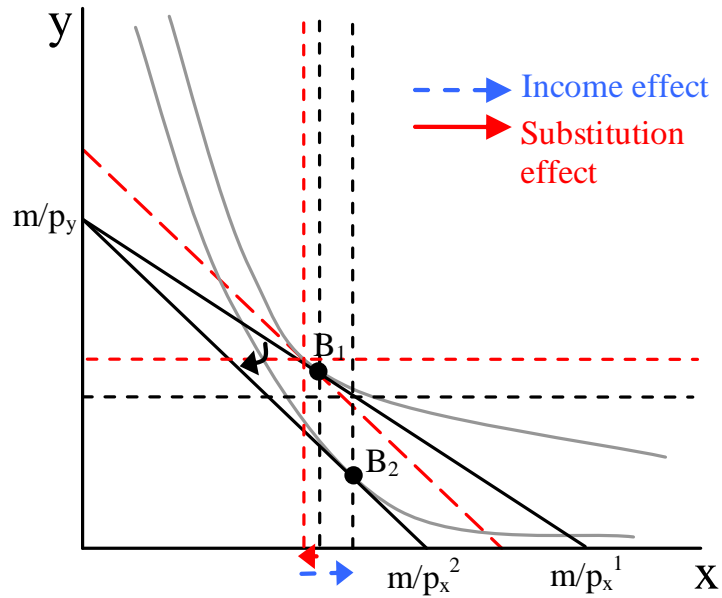
The diagram overleaf illustrates the derivation of the Marshallian demand curve for a normal good. It is downward sloping. This is because a normal good, as we will show by indifference curve analysis, must by definition obey the Law of Demand. For a Giffen good, the Marshallian demand curve is upward sloping. Thus a Giffen good does not obey the Law of Demand, and so cannot be normal, and therefore must be inferior. It is, however, possible for a good to be inferior, but not a Giffen good, if the income effect is never sufficiently large to outweigh the substitution effect. Such a good has a downwards sloping demand curve but it will be less price responsive/elastic than a normal good, ceteris paribus, because the income effect works against the substitution effect. These three cases are illustrated.

When the price of good x drops from p_x^1 to p_x^2 the budget line pivots outwards. The overall effect of the drop in price is that the optimal bundle changes from B_1 to B_2 . The income and substitution effects on the amount of good x consumed operate in the same direction: consumer B consumes more of good x both because it is relatively cheaper and because they have more 'purchasing power' at the lower price (the budget set expands).



A Giffen good **must** be inferior because when the price of a good increases, the substitution effect always unambiguously decreases demand for that good. The income effect must therefore increase the demand for that good in order for the overall effect to be positive. Since the budget set is shrinking, for the income effect to

be positive, the good must be inferior. The diagram below illustrates a Giffen good. When the price of Giffen good x increases from p_x^1 to p_x^2 , the amount of x demanded increases in the move of the optimal chosen bundle from B_1 to B_2 .



For most inferior goods however, the income effect, although positive, will not outweigh the substitution effect, and so a rise in the price of the good will still cause the amount of the good demanded to drop:-

