SRAFFA'S STANDARD COMMODITY

JOHN BROOME

Birkbeck College, University of London

In Sraffa's *Production of Commodities by Means of Commodities* [3] quite a lot of space is taken up by "the standard commodity", but it is, perhaps, not obvious why it is given so much attention. It is not required in the development of most of what are thought of as Sraffa's significant innovations, including the theory of price determination, the possibility of reswitching, and the "reduction to dated labour". Although a newcomer to Sraffa's work has available several helpful expositions, they often leave the role of the standard commodity rather obscure. I hope that he or she may find some further explanation useful.

What is at issue is the distribution of the economy's net product between workers and capitalists (I shall ignore other classes). If one of the two classes gets a bigger share of the product, the other must get a smaller one. Since the sharing out of income is channelled through the wage and the profit rate, it seems obvious at first sight that the wage and the profit rate must have some inverse relationship to one another. If one goes up, the other must go down. A corollary is that, given the wage, the profit rate is determined.

What is obvious at first sight is not so obvious when it comes to be worked out in detail, as we shall see. But this simple insight is nevertheless a valuable one to preserve: the distribution of income must, in some sense or other, be a matter of dividing up a definite total product. A theory of distribution does well to take account of this from the start. Some, however, do not, and instead determine the incomes of different classes in ways that are apparently independent of one another. Neoclassical theory, for instance, fixes the income of each factor separately by its marginal product. But having done so it turns out that these separately determined incomes may not add up to the total product which is actually to be shared out, so the theory then has to make special arrangements of one sort or another to get around this "adding up" problem. For another example, Adam Smith also offers separate theories determining the incomes of the different classes. Simplifying greatly, the income of workers is given by the supply of and demand for labour, and that of capitalists by the supply of and demand for capital. There is nothing really wrong with this, since the interaction of supply and demand will indeed normally be the proximate determinant of income distribution. But the markets in labour and capital are actually very closely linked: the supply of capital is what creates the demand for labour since it is capital that hires the workers and provides them with means of production. This is simply the way in which the relationship between the incomes of workers and capitalists, competing for a given total, shows itself in the context. Smith certainly knew how capital creates the demand for labour, but he did not appreciate the link this makes between the profit rate and wages. As a result he committed certain errors which Ricardo was keen to point out. [See 2, pp. xxxiii-xxxv and p. 46.]

Ricardo kept always in mind that the distribution of income constitutes a *division* of "the produce of the earth". He says, for instance:

> There can be no rise in the value of labour without a fall of profits. If the corn is to be divided between the farmer and the labourer, the larger the proportion that is given to the latter, the less will remain for the former. So if cloth or cotton goods be divided between the workman and his employer, the larger the proportion given to the former, the less remains for the latter. [2, p. 35.]

Sraffa sets out from the same point, and one of his aims is to make the notion more precise: the product has to be divided up, so what exactly does this tell us about the relation between the wage and the profit rate?

If the world contained only one product, the answer would be straightforward. Suppose only corn were produced, as follows:

$$a \text{ units of corn } \& \ \ell \text{ units of labour} \rightarrow 1 \text{ unit of corn.}$$

(I assume constant returns to scale. More will be said about this below.) If the annual gross product is N units, the annual net product is $N(1-a)$. Let the wage rate in corn be w. Then the wage bill is $Nw\ell$. Capitalists receive the rest, $N(1-a) - Nw\ell$. (This, the capitalists' share, I shall henceforth call "the surplus".) Suppose, like Sraffa, that labour is paid at the end of the year (nothing except convenience hangs on this). Then the capitalists' capital is only seed corn, Na. So the profit rate is:

$$r = \frac{N(1-a) - Nw\ell}{Na}$$

$$\rightarrow \quad r = \frac{1 - a - w\ell}{a} \tag{1}$$

Here, then, is the expected inverse relationship between the wage rate and the profit rate. Notice two things. Firstly, notice that we worked in physical units of corn; no question of valuation arose. The profit rate was calculated as a physical ratio because both capital and the surplus were quantities of corn. Notice, secondly, that the calculation was done in terms of aggregates for the economy, aggregate surplus was divided by aggregate capital. We could equally easily have asked "what is the profit *per* unit of corn grown?", and the algebra would have been trivially different. The difference becomes important later.

In an economy with several products, however, there are complications. First of all, a decision has to be made about a standard for measuring the wage; some product must be selected as numeraire. The relation between the wage and the profit rate will be different according to which is chosen. It is conceivable (though actually it turns out that this cannot happen, as is shown in the last paragraph of this paper) that a fall in the profit rate might bring about a rise in the wage relative to one product but a fall relative to another. As we shall see, a suitable choice of numeraire can be very important.

The chief problem that arises when there are several products to deal with is how to aggregate them. The economy's net product, its capital, and the shares of workers and capitalists will be collections of different sorts of things. To compare them as aggregates, the collections will have to be weighted and added up. The profit rate, for instance, will be the *value* of the surplus divided by the *value* of aggregate capital. Here, by the "value" of a collection of products I do not mean the labour embodied in it, but what is ordinarily meant by the word: the constituents of the collection multiplied by their respective prices and added up. In other words, the correct weights to use for calculating the profit rate are prices. But this leads to a problem. Competition between capitalists will ensure that each industry earns the same profit rate. Hence the price of each product will be given by its cost of production marked up appropriately to earn the going rate of profit. Normally, a different rate of profit will make prices different. Yet, as I have just said, in order to work out the profit rate we apparently need to know prices beforehand, so as to know the value of capital and of the surplus. It looks as though calculating the profit rate will involve some circularity: prices have to be known first, but prices depend on the profit rate.

Marx attempted to avoid the circularity as follows [1, Part II]. The profit rate he calculated by dividing the labour embodied ("value" in the Marxist sense) in the surplus by the labour embodied in capital. That is, he used as weights the labour embodied in products, rather than their prices. The labour embodied in a product is a technical matter, independent of prices and of distribution. So, this way, the profit rate is discovered in advance of prices, and the price of each product can next be worked out by applying the appropriate mark-up, now known, to its cost of production. Unfortunately, as Marx realised [1, p. 161], the answer got this way is wrong. To find the profit rate, you really do need to weight the constituents of surplus and of capital by prices, not by labour embodied, and any other method is wrong. Marx's calculation leads to something Marxists call "the value rate of profit", as opposed to "the price rate of profit". The latter is what we have been talking about, the profit rate in the common or garden sense.

According to Sraffa [2, p. xlviii], Ricardo was troubled by the problem I have mentioned. It appeared to him like this. Ricardo was interested in how the total product was divided up. But every time the division changes, if, say, wages increase at the expense of profits, then the prices of commodities will change, and hence the value of the total product will change. It is hard to think about the division of a total if the total alters every time the division alters. There is another difficulty, too. When the distribution of income changes, the value of the total product will change for *two* reasons. Firstly, as I have said, prices will change. But secondly the actual physical composition of the product will be affected by its distribution. What is produced will depend on what is demanded. Suppose wages are increased. Then the demand for wage goods will presumably expand and the demand for capitalists' goods will contract. Wage goods industries will find themselves more profitable than others, causing a migration of capital into those industries and an increase in their output. In the end, after enough capital has moved, a constant profit rate will be restored throughout the economy, but with the composition of output different from what it was originally.

Because of these two things, and especially because of the second one, it is no longer possible, if there is more than one product, to think of the distribution of income as a matter of dividing up a previously fixed total. The primitive insight we set out from seems to have come to nothing. But we shall see that it can be rescued, though in a rather rarefied form, and this is what the standard commodity does. Another approach is needed. So far we have spoken of aggregates, aggregate capital and aggregate surplus. But what we have is evidently a simultaneous determination problem. Sraffa drops Marx's aggregate approach and works instead with the general equilibrium equations of the economy. The equilibriating mechanism is the competition which equalises the profit rate in each industry. Through it, prices and profit rate are simultaneously determined. Let there be n industries, and let the technique in the j'th industry be:

a_{1j} units of product 1 & a_{2j} units of product 2 & ....

& a_{nj} units of product n & ℓ_j units of labour $\rightarrow$ 1 unit of product j.

The technology may be represented more briefly by an $(n \times n)$ input-output matrix A, and a $(1 \times n)$ vector of labour coefficients ℓ. Implicitly, I am assuming constant returns to scale. Sraffa claims [3, Preface] that the assumption of constant returns is not necessary to his theory. He says that he is treating the output of each industry as given, never changing, so there is no relevance in what would happen if the scale were altered. Yet he does consider different distributions of the product; in fact, that is what his whole theory is about. And I have just explained that a different distribution normally implies a different composition of the product. So, we *must* take account of different scales of output in various industries, which means that, if the technical coefficients are to stay the same, we have to assume constant returns to scale. (This does not mean that Sraffa's results will only be true if there are constant returns to scale, but it does mean that it will be a more complicated business to establish them if there are not.) Sraffa never takes note of the mechanism which equalises the profit rate in different industries, but as Marx knew [1, pp. 195–6] this mechanism works by alterations in the scale of output.

Let the wage be w, and the price vector p. The net income to a capitalist from the production of a unit of product j is

$$p_j - (p_1a_{1j}+p_2a_{2j}+\ldots+p_na_{nj}+w\ell_j),$$

what he sells it for less its cost of production. The capital required in this operation is $(p_1a_{1j}+p_2a_{2j}+\ldots+p_na_{nj})$, so the profit rate is

$$r = \frac{p_j - (p_1a_{1j}+p_2a_{2j}+\ldots+p_na_{nj}+w\ell_j)}{(p_1a_{1j}+p_2a_{2j}+\ldots+p_na_{nj})}$$

$$\rightarrow p_j = (p_1a_{1j}+p_2a_{2j}+\ldots+p_na_{nj})(1+r) + w\ell_j.$$

The corresponding equations for all the industries may be written in matrix form:

$$p = pA(1+r) + w\ell. \tag{2}$$

These equations are enough, given the wage rate, to fix all price ratios and the profit rate (provided the matrix A is productive and indecomposable).

Let us attend to the solution of these equations for the profit rate in terms of the wage rate. This is where the standard commodity comes in. It is defined as a non-negative eigen vector of A, corresponding to A's largest eigen value (which can be proved to be positive and, provided A is productive, less than one). There is, of course, an infinite range of such eigen vectors, each a scalar multiple of the others. Although Sraffa has a way of fixing the scale of his standard commodity, only its proportions are important, so we may pick any of these eigen vectors arbitrarily. Write the standard commodity s. Since s is defined as an eigen vector of A:

$$As = \gamma s \tag{3}$$

where γ is the corresponding eigen value. It is still open to us to choose a numeraire, and we choose the standard commodity. The "price" of the standard commodity, what it costs to buy, is ps. So we make

$$ps = 1. \tag{4}$$

The wage, like every other price, is thus measured in terms of the standard commodity. Postmultiply (2) by s:

$$ps = pAs(1+r) + w\ell s$$

$$\rightarrow ps = ps\gamma(1+r) + w\ell s \qquad \text{(from (3))}$$

$$\rightarrow 1 = \gamma(1+r) + w\ell s \qquad \text{(from (4))}$$

$$\rightarrow r = \frac{1 - \gamma - w\ell s}{\gamma} \tag{5}$$

Compare (5) with (1). We have again the fixed inverse relation between the wage and the profit rate, a relation just like the one in the one-product world. By using the standard commodity as numeraire we have neatly cut through that circle where prices depend on the profit rate and the profit rate on prices. Prices have been eliminated from the equations. The settling of the distribution of income has been separated from the determination of relative prices.

How does this trick work? Imagine an economy whose gross output was actually the standard commodity, s. This is (apart from the question of scale) what Sraffa calls "the standard system". Such an economy would be in a way quite like the economy which produces only corn. The eigen equation (3) says that, to produce output s, the inputs to production must be γs. That is, the inputs will be proportional to the outputs. The labour employed will be ℓs. We could write the technology of the standard system:

γ units of standard commodity & ℓs units of labour

$\rightarrow$ 1 unit of standard commodity.

This accounts for the similarity between equations (1) and (5). In (1), wages are expressed in corn, which is why in (5) wages had to be expressed in standard commodity, corn's analogue. Hence the standard commodity was used as numeraire.

If, now, in addition, workers actually consume goods in the proportions of the standard commodity, then the surplus capitalists have left after paying wages will be in the proportions of the standard commodity also. In that case the surplus is physically comparable to their capital, just as in the corn economy. Each is a quantity of standard commodity. The profit rate is the ratio of the one to the other. The division can be done in physical terms, and relative prices do not come into it.

But what of the economies which happen not to produce in the proportions of the standard system? Here we find an advantage of the general equilibrium approach above the aggregate approach. Sraffa is able to say "The actual system consists of the same basic equations as the standard system, only in different proportions" [3, Section 31]. It sounds a rather lame remark, but no one can doubt its correctness. There are the same processes of production, and a change in the scale of any of them does not alter the equations relating the price of output to its cost. In whatever proportions the different products are produced, the profit rate and the relative prices must be the same. That, though rather lame, does seem to be all there is to say about it.

So what, then, is the achievement of the standard commodity? We set out from the insight that, because workers and capitalists are competing for a share of the economy's net product, the wage can only increase at the expense of a fall in the profit rate, and *vice versa*. This insight, however, ran into trouble because the economy's net product actually consists of many different goods. To make a single total they must be aggregated, but their aggregate value is liable to change whenever its distribution changes, since distribution affects prices. Thus there no longer seems to be a fixed total for sharing out. The standard commodity is a device for restoring the insight, but not in the way that Marx, for one, had in mind (and perhaps Ricardo too, but I have avoided the thorny question of what Ricardo wanted to do with *his* "invariable standard of value"). Marx hoped to aggregate the net product differently, by some measure, other than its value (in the ordinary sense), which was not subject to re-evaluation. The standard commodity, on the other hand, picks out just that special pattern of production for the economy, the standard system, where there would be no need to aggregate different products to make a single total, by any method, because the division can be seen in physical terms without it.

This *is* a survival of the original insight, but in a very much attenuated form. It does point to a sort of fixed total that can be divided in different proportions between the classes. But the fixed total is not the net product of the economy as it actually is, but of the standard system which can be made out of it. The standard commodity is undoubtedly a mathematical convenience, as is shown by the simplicity with which we calculated the wage-profit relationship, thanks to its help as numeraire. But, more than that, I think it offers an interpretation or clarification of the simple wage-profit relationship by linking it, doubtless rather indirectly, with Ricardo's simple intuition about the division of the produce of the earth.

I think, then, that the standard commodity is valuable for exposition and elucidation, but I would not like to claim more for it. After all, the inverse relationship between the wage and profit rate may be demonstrated quite adequately without its assistance. It only requires more brute force and less finesse in the mathematics. From equation (2) we can get:

$$p(I-A(1+r)) = w\ell$$

$$\rightarrow \frac{p}{w} = \ell(I-A(1+r))^{-1}$$

I have assumed $(I-A(1+r))$ to be non-singular, and I also assume $(I-A(1+r))^{-1}$ to be non-negative. These assumptions merely require r to be small enough to make w positive, the only economically significant possibility. Differentiating the last equation:

$$\frac{d}{dr}\left(\frac{p}{w}\right) = \ell(I-A(1+r))^{-1}A(I-A(1+r))^{-1}.$$

Since ℓ, A and $(I-A(1+r))^{-1}$ are all non-negative, so is every component of the right hand side of this equation. For all j, then, $\frac{d}{dr}(p_j/w)$ is non-negative. That is, an increase in r causes an increase, or at least no change, in the price of every product relative to the wage. To put it the other way round, if the profit rate goes up, the wage goes down, or at least stays the same, relative to *every* product. We established before that the wage goes down relative to the standard commodity, but we now have this more general result (which Sraffa proved too, incidentally [3, Section 49]), without mentioning the standard commodity.

References

1. Karl Marx, *Capital*, Volume 3 (London, Lawrence and Wishart, 1974).
2. Piero Sraffa (ed.), *The Works and Correspondence of David Ricardo*, Volume 1 (*On the Principles of Political Economy and Taxation*, with an Introduction by Piero Sraffa) (Cambridge University Press, 1951).
3. Piero Sraffa, *Production of Commodities by Means of Commodities* (Cambridge University Press, 1960).