SUPERVALUATION RECONSTRUCTED

John Broome University of St Andrews

This is a very preliminary version written in 1995. But even this version has benefited from useful advice I received from Adam Morton. I should be very grateful for any more useful advice you may be able to offer.

1. Supervaluation theory

The most popular account of the logic of vague terms is, perhaps, supervaluation theory.¹ Here is a outline of the theory, as it is normally presented. Every vague term could be made precise, or sharpened, in a number of different ways. That is to say, it could be given a number of different sharp interpretations. For instance, 'bald' could be interpreted as having fewer than a thousand hairs on the head, or as having fewer than two thousand hairs on the head, and so on. None of these is the actual meaning of 'bald', which is vague, but they are all ways 'bald' might be acceptably sharpened. By sharpening its terms, any sentence containing vague terms can also be sharpened in a number of different ways. Let us say a sentence is 'supertrue' if and only if it is true in all of its sharpenings. Supervaluation theory says a sentence is true if and only if it is supertrue. For instance, 'Serge is bald' is true if and only if Serge has fewer than a thousand hairs on his head, and fewer than two thousand, and fewer than . . ., and so on.

Supervaluationists deny bivalence: they claim that some sentences are neither true nor false. Suppose Miles has fifteen hundred hairs on his head, and suppose 'bald' can be made sharp in the ways I described. Then 'Miles is bald' is true on some sharpenings of 'bald' and false on others. (I shall call this sentence and others like it 'borderline cases'.) Since it is not the case that 'Miles is bald' is true on all ways of making it precise, according to supervaluation theory this sentence is not true. It is also not the case that 'Miles is not bald' is true on all ways of making it precise, so this sentence is not true either. A sentence is false if and only if its negation is true. Consequently, 'Miles is bald' is not false. It is neither true nor false.

On the other hand, supervaluationism does accept the law of excluded middle: for any sentence 'S', 'S or not S' is true. This is because 'S or not S', being valid in classical logic, must be true in all its sharpenings. Suppose 'S' is 'Miles is bald'. 'Miles is bald or Miles is not bald' is true, even though neither 'Miles is bald' nor 'Miles is not bald' is true. It is characteristic of supervaluationism that a disjunction may be true without either of its disjuncts being true. Excluded middle, applied to borderline cases, is an example.

2. The objection

In his book *Vagueness*, Timothy Williamson argues powerfully against supervaluationism. His main objection is that supervaluationists do not honour the disquotational principle that 'S' is true if and only if R, where 'R' is the translation of 'S' into the metalanguage. They cannot retain this principle whilst rejecting bivalence,

¹ The fullest account of the supervaluation theory of vagueness is Kit Fine's in 'Vagueness, truth and logic'.

he says. Here is a simplified version of his argument.² Assume the disquotational principle:

(1) 'S' is true if and only if R.

In particular, 'not S' is true if and only if not R. But 'not S' is true if and only if 'S' is false. So

(2) 'S' is false if and only if not R.

Suppose now that 'S' is a borderline case, so that according to supervaluationism:

'S' is not true and 'S' is not false.

In this, replace "S" is not true' and "S" is not false' by their equivalents from (1) and (2):

Not *R* and not not *R*.

This is a contradiction (even in logical systems that do not permit elimination of double negation). So supervaluationism and the disquotational principle are incompatible. Since Williamson persuasively argues that the disquotational principle is incontrovertible, this is a powerful objection to supervaluationism.³

However, Williamson himself provides the material for reconstructing supervaluationism in a form that is not subject to his objection. In this paper I shall take the disquotational principle for granted, and reconstruct the supervaluationist theory on that basis, taking my guidance from Williamson.

3. The language

Take a language that has a standard interpretation. I shall take English (with a few terms excluded, as I shall explain), with the standard meanings for its words. I assume the standard interpretation assigns vague meanings to some words. Other interpretations of the language assign different meanings to its words. Some of these interpretations will be sharp, in that they assign precise meanings to all the words of the language. Some sharp interpretations will be sharpenings of the standard interpretation. 'Sharpening' is a primitive notion, but it is easy to understand. For example, having fewer than one thousand hairs on the head is a sharpening of the

² See *Vagueness*, pp. 162, 188–9.

³ Fine is willing to reject the disquotation principle. See 'Vagueness, truth and logic', p. 297.

standard interpretation of 'bald', but having fewer than half a million hairs on the head is not. The standard interpretation determines which interpretations are sharpenings of it; it determines a range of sharpenings. Other vague interpretations besides the standard one determine different ranges of sharpenings. Unless I say otherwise, when I mention a sharpening I mean a sharpening of the standard interpretation. Remember that a sharpening assigns sharp meanings to *all* the words of the language.⁴

The metalanguage in which this paper is written is English, like the object language. The difference is simply that a few technical terms including 'sharpening' are excluded from the object language but included in the metalanguage. I shall treat the object language as literally a part of the metalanguage.⁵ This means that object-language terms and sentences can be *used* as well as mentioned within the metalanguage; I can write in material as well as formal mode. For instance, I shall say '**Miles is bald** in some sharpenings', as well as ''**Miles is bald**' is true in some sharpenings'. More often I shall use schematic letters standing in for object-language sentences: I shall say '**S** in some sharpenings', as well as ''**S**' is true in some sharpenings'. Since the metalanguage has the standard interpretation, truth in the metalanguage is truth in the standard interpretation. I shall print object-language sentences and terms (and schematic letters representing them) in bold, when they are either used or mentioned in the metalanguage. This is merely a typographic device that I use for the sake of clarity. It has no formal significance.

Including the object language within the metalanguage carries a risk. Metalinguistic reasoning might lead to conclusions that contradict truths of the object language. But it will turn out that this does not happen.

The contradictions I am thinking of are not the semantic paradoxes; I assume those have already been satisfactorily dealt with. The predicates 'true' and 'false' are not amongst the terms that are excluded from the object language. Saul Kripke has shown how a language can contain its own truth predicate without contradiction.⁶ Kripke's method requires certain paradoxical sentences to be neither true nor false. I shall simply ignore those sentences.

I do things this way for two reasons. One is that writing in material mode simplifies the statements I need to make. The other is that it helps separate two tasks; one to delimit truth, and the other to analyse vagueness. The first is not the subject of this paper; the second is. The two tasks cannot be separated completely, since in analysing vagueness we are concerned with the truth conditions of vague sentences. But since truth is not my subject, I shall try to make it as transparent as possible. Writing in material mode helps, and so does embedding truth in the object language; both techniques help to concentrate the metalanguage on vagueness rather than truth.

⁴ Different words cannot always be sharpened independently of each other. See Williamson, *Vagueness*, pp. 145–6.

⁵ This is Alfred Tarski's advice in 'The semantic conception of truth', p. 50.

⁶ 'Outline of a theory of truth'.

Setting paradoxical sentences aside, I shall assume the object language conforms to classical logic under any sharp interpretation. In particular, for any sentence 'S', in any sharp interpretation either 'S' will be true or else '**not** S' will be true. Also, in any sharpening of the standard interpretation, I assume the disquotational principle that 'S' is true if and only if S.

I next need to define a special sort of conditional in the metalanguage. I shall call it an 'assertive conditional' and write it 'if_a'. Its purpose is to make it possible to state precise conditions under which a vague sentence can be asserted, without implicitly denying its vagueness. It leaves open the possibility that some sentences are neither asserted nor denied. 'S if_a T' is defined to mean that S is asserted if T is. An assertive conditional does not support contraposition: 'not T if_a not S' does not follow from 'S if_a T'. The ordinary material conditional 'S if T' is equivalent to 'S if_a T and not T if_a not S'. The biconditional 'S iff_a T' means that if either of S or T is asserted, the other is too.

4. Supervaluation and truth

I shall formulate the basic principle of supervaluation theory as:

(3) Supervaluation: $S \text{ iff}_{a} S$ in all sharpenings.

For instance: **Miles is bald** iff_a **Miles is bald** in all sharpenings. If it is not the case that S in all sharpenings, supervaluation says 'S' cannot be asserted. But it does not say 'S' can be denied. Indeed, it would be inconsistent for it to do so. Substituting '**not** S' for 'S', supervaluation implies:

Not *S* iff_a not *S* in all sharpenings.

Therefore, when 'S' is a borderline case, so that S in some sharpenings and **not** S in others, we cannot assert **not** S. That is to say, we cannot deny S.

Supervaluation applies to all sentences in the object language including sentences containing the truth predicate. So:

(4) 'S' is true iff_a 'S' is true in all sharpenings.

Since 'S' is true in a sharpening if and only if S in that sharpening (because the disquotation principle applies in each sharpening), 'S' is true in all sharpenings if and only if S in all sharpenings. Consequently, from (3) and (4),

(5) **'S' is true** iff_a **S**.

It is also the case that 'S' is not true in a sharpening if and only if not S in that sharpening (because each sharpening is classical and the disquotation principle applies

in each). So 'S' is not true in all sharpenings if and only if not S in all sharpenings. Consequently, by supervaluation,

(6) 'S' is not true iff_a not S.

(5) and (6) are together equivalent to: 'S' is true if and only if S. That is to say, we assert 'S' is true' whenever we assert 'S' and only then, and we deny 'S' is true' whenever we deny 'S' and only then. The object-language truth is disquotational, therefore. This conclusion of metalinguistic reasoning is confirmed in the object language:

'S' is true if and only if S,

because 'S' is true if and only if S in every sharpening. There is no conflict between object language and metalanguage here, then. Furthermore, the disquotational nature of truth means that 'true' attaches to just the sentences that are indeed true. For consistency this had to be so: since the object language is part of the metalanguage, 'true' simply means true. Fortunately, no inconsistency has emerged.

5. Excluded middle and bivalence

In every sharpening, *S* or not *S*, so

Excluded middle: *S* or not *S*.

This raises the possibility of another inconsistency between the object language and metalinguistic reasoning. In borderline cases, we can assert neither that S nor that **not** S. So we seem able to say 'neither S nor **not** S', which contradicts excluded middle.

However, 'neither S nor **not** S' not only contradicts excluded middle, it contradicts itself. The first clause, 'neither S', asserts that not S, which the second clause, 'nor **not** S', denies. So in borderline cases it cannot be the case that neither S nor **not** S. Nor does supervaluation imply it is. It implies simply that we can neither assert S nor deny it, and we can neither assert **not** S nor deny it. This does not contradict excluded middle. Perhaps it lies uncomfortably with excluded middle, but this is a discomfort supervaluation theory that disjunctions may be asserted without asserting either of their disjuncts. Excluded middle for borderline cases is just an example.

Up to now, supervaluationists have had a way of easing the discomfort. In borderline cases, they would say that 'S' is neither true nor false. But this relief has been cut off by Williamson's argument that bivalence cannot be denied like this. Indeed, we can demonstrate bivalence in the object language. In every sharpening (since I am ignoring paradoxical sentences), 'S' is true or 'S' is false, so

Bivalence: 'S' is true or 'S' is false.

This itself raises a possible conflict with the metalanguage. In borderline cases, supervaluation implies we cannot assert 'S' is true, and we cannot assert 'S' is false either. But actually this does not contradict bivalence. We cannot deny 'S' is true any more than we can assert it, because we cannot assert 'S' is not true. Bivalence is another disjunction that may be asserted even when neither of its disjuncts may be asserted.⁷ This may be uncomfortable, but we can live with it, just as supervaluationists have always lived with excluded middle.

6. Assertion

But here is another possible contradiction. Within each sharpening, it is surely the case that 'S' can be asserted or it can be denied. Consequently, 'S' can be asserted or it can be denied. Yet I have said that if 'S' is a borderline case, it can neither be asserted nor denied. Surely this is a contradiction.

I see two possible responses. One is to refuse 'assert' and 'deny' a place in the object language. Then the object-language sentence "S' can be asserted or it can be denied' would not exist, so it could not contradict my metalinguistic remark. But there is a difficulty with this response. If 'assert' and 'deny' exist within the metalanguage, we need to worry about the logic of this metalanguage. We can treat it as itself a new object language and discuss it in a higher-order metalanguage. Then the only way of preventing the same contradiction from arising will be to reject the supervaluation principle applied to this new extended object language. In effect, we will have to reject supervaluation for 'assert' and 'deny'. But now I have in this paper applied supervaluation to 'true' and 'false', that seems to be an unprincipled arbitrary exclusion. Worse, 'assert' and 'deny' would have got themselves into the unsatisfactory position that 'true' and 'false' occupy in existing versions of supervaluationism. In existing versions, the supervaluationist principle is not applied to 'true' and 'false', because they are held within the metalanguage. This is the reason why these versions are inconsistent with the disquotation principle. For a borderline 'S', they deny 'S' is true, whilst not denying S. Supervaluation applied to 'true' would prevent them from denying 'S' is true. Similarly, if 'assert' and 'deny' are held within the metalanguage, and supervaluation is not applied to them, we shall have to reject this disquotation principle:

'S' can be asserted if and only if S.

Indeed, I have already contradicted this principle: I have denied that borderline

⁷ Compare Kripke, 'Outline of a theory of truth', p. 77. Kripke there accepts a version of bivalence as a consequence of supervaluationism in a different context.

sentences can be asserted, whilst not denying those sentences. This is surely unsatisfactory.

There is a more radical response that would avoid this unsatisfactory conclusion. I offer it only as a suggestion, because I do not have it worked out in detail. A language needs a truth predicate for well-known reasons,⁸ but it may not need 'assert' and 'deny' as well. My suggestion is to exclude these terms from the language altogether. I used them in explaining my account of truth for vague sentences, but in principle I could have done without them. When 'S' is a borderline case, I said it can neither be asserted nor denied, but instead I could simply have refused either to assert or deny it. Because neither asserting nor denying something is not very expressive in written text, I found myself actually using the terms 'assert' and 'deny', but in principle they were not necessary. I also used these terms in defining the assertive conditional if_a. But if we regularly withheld assertion or denial from some sentences, our regular conditional would be the assertive one; it would not support contraposition. The assertive conditional would not need defining, but we could use it to define the stronger conditional 'if' that does support contraposition: we could define 'S if T' as 'S if_a T and not T if_a not S'. The terms 'assert' and 'deny' are useful for explaining the supervaluationist theory to people who assume that every sentence is assertible or deniable. But if we abandon that assumption, they are redundant. Without them, we would then have a consistent metalanguage.

7. Degrees and definiteness

Still, it useful to have something that *can* be said about borderline cases. Let us define an operator '**to a degree**' in the object language by:

S to a degree if and only if S in some sharpenings and not S in some sharpenings.

Now, when 'S' can neither be asserted not denied, we can say 'S to a degree'. Notice that S to a degree if and only if 'S' is true to a degree.

I make the operator 'to a degree' rather than, say, 'is indefinite' because it opens up one way of expanding the theory. We might develop a detailed theory of degrees. I would expect it to consist of a set of objects called 'degrees', with a general method for assigning a degree to each statement. 'S is true' will have the same degree as 'S'. For well-known reasons,⁹ degrees will certainly not generate a degree-functional logic: the degree of a compound such as a conjunction will not be determined by the degrees of its components. Moreover, degrees will certainly not be linearly ordered: many

⁸ For instance, see Stephen Read, *Thinking About Logic*, pp. 18–31.

⁹ See Fine, 'Vagueness, truth and logic', p. 269–71, or Williamson, *Vagueness*, pp. 135–8.

statements are bound to have incomparable degrees. For instance, I would not expect the degrees of 'Alamein was the turning-point of the war' and 'Belgium is a small country' to be comparable. However, there will be some ordering structure amongst degrees. For instance, Cats are soft to a greater degree than dogs are soft. I would expect this to express a sufficient condition for comparability of degrees:

S to a greater degree than T if S in all sharpenings where T, and S in some sharpenings where not T.

But an account of degrees is not a job for this paper.

Balancing the operator 'to a degree' we may add another operator 'definitely', defined by:

Definitely *S* if and only if *S* in all sharpenings.

'Definitely' and 'to a degree' are precise predicates, unlike 'true', which is vague.

8. Validity

Since validity is truth in all interpretations, before I can discuss it I need to generalize the supervaluation principle to other interpretations besides the standard one:

Supervaluation. S in an interpretation iff_a S in all sharpenings of that interpretation.

'S' is valid if and only if S in all interpretations, and a derivation of 'T' from premises ' S_1 ', ' S_2 '... ' S_n ' is valid if and only if T in all interpretations where S_1 and S_2 and ... S_n . Take first a sentence 'S' that does not contain the special operators '**definitely**' and '**to a degree**'. Suppose 'S' is classically valid. Then S in all sharp interpretations, because classical logic applies in those interpretations. Consequently, by supervaluation, S in all vague interpretations too, because S in every sharpening of every vague interpretation. So S in all interpretations, and therefore 'S' is valid. Next take a derivation of 'T' from ' S_1 ', ' S_2 '... ' S_n ' where none of these sentences contain the special operators. If this derivation is classically valid, then T in any sharp interpretation, then S_1, S_2, \ldots and S_n . If S_1, S_2, \ldots and S_n in some vague interpretation, then S_1, S_2, \ldots and S_n in every sharpening of that interpretation. Consequently T in every sharpening of that interpretation, so T in the vague interpretation itself. In sum, T in every interpretation in which S_1, S_2, \ldots and S_n . So the derivation is valid. We can conclude that classically valid sentences and derivations remain valid in supervaluation theory.

For sentences and derivations that involve the special operators, I shall employ a

formal device that comes from Kit Fine.¹⁰ Let a 'refinement' be a pair consisting of a sharp interpretation and an interpretation of which it is a sharpening. In effect, this construction simply refines the individuation of sharpenings. A sharp interpretation is inevitably a sharpening of many interpretations (including itself). The construction splits it into many refinements, one for each interpretation of which it is a sharpening. Like an interpretation, a refinement is defined to assign values to sentences. It retains all the same truths as its parent sharp interpretation: if S in the sharp interpretation, then S in the refinement. Consequently, classical logic applies in refinements. From now on, we can take the sharpenings of an interpretation to be refinements rather than their parent sharp interpretations. The supervaluation principle continues to apply.

A refinement is also allotted some new truths besides the ones it inherits from its parent sharp interpretation. First I must generalize the definitions of '**to a degree**' and '**definitely**' to other interpretations besides the standard one.

S to a degree in an interpretation if and only if *S* in some sharpenings of that interpretation and **not** *S* in some sharpenings of it.

Definitely S in an interpretation if and only if S in all sharpenings of that interpretation.

These definitions define the operators only in interpretations and not refinements (which are technically pairs of interpretations). This is intuitively appropriate: the concepts '**definitely**' and '**to a degree**' belong intuitively to vague interpretations rather than their sharpenings. But now comes Fine's useful formal move: if a '**definitely**' sentence or a '**to a degree**' sentence is true in an interpretation, we assign it to the interpretation's sharpenings (which are refinements) too. If **definitely** *S* in an interpretation, then **definitely** *S* in all its sharpenings, and if *S* **to a degree** in an interpretation, then *S* **to a degree** in all its sharpenings.

A sentence 'S' is valid if and only if S in every interpretation. We know already that classically valid sentences are valid. We can add as a valid sentence:

S if definitely S.

The reason is this. If **definitely** S in any refinement of an interpretation, then **definitely** S in the interpretation. This implies (by the definition of '**definitely**') that S in all refinements of that interpretation, including the one where **definitely** S. So S in any refinement where **definitely** S. Therefore (by classical logic), in every refinement S if **definitely** S. Consequently (by supervaluation) S if **definitely** S in every interpretation.

However, the converse 'if S then definitely S' is not valid because there are some

¹⁰ It is a simplified version of the account on p. 293 of 'Vagueness, truth and logic'.

refinements of some interpretations (specifically those interpretations where S to a degree) in which S but not definitely S. In those refinements it is not the case that if S then definitely S, and this is therefore not the case in those interpretations either.

A derivation of 'T' from ' S_1 ', ' S_2 '... ' S_n ' is valid if and only if T in all interpretations where S_1 and S_2 and ... S_n . The derivation of 'S' from 'definitely S' is valid, though I shall not spell out the argument. More surprisingly, 'definitely S' can be validly derived from 'S'. The reason is that every interpretation where S is one where S in all of its sharpenings (by supervaluation). Consequently definitely S in that interpretation. Note that for validity we do not require the conclusion of a derivation to be true in every *refinement* in which the premises are true, because a refinement is not an interpretation.¹¹

At this point classical logic has broken down. In classical logic, if '*T*' can be validly derived from ' S_1 ', ' S_2 '... ' S_n ', then 'if S_1 and S_2 and ... S_n then *T*' is valid. We have discovered that 'definitely *S*' can be validly derived from '*S*', but we know 'If *S* then definitely *S*' is not valid.¹² The trouble is that classical logic does not allow for sentences that are neither asserted nor denied. Because the derivation is valid, we know the assertion of '*S*' permits the assertion of 'definitely *S*'. But the denial of 'definitely *S*' does not permit the denial of '*S*' because '*S*' may be neither asserted nor denied. 'If *S* then definitely *S*', however, implies the contrapositive 'If not definitely *S* then not *S*'. That is why it does not follow from the validity of the derivation of 'definitely *S*' from '*S*'. What will follow is the assertive conditional 'If_a *S* then definitely *S*'.

But it does not follow yet, because I have not yet added the assertive conditional to the object language. As I did with the special operators, I shall define it initially for an interpretation rather than a refinement:

S if_a T in an interpretation if and only if either S in all sharpenings of that interpretation or it is not the case that T in all sharpenings of it.

Then we add that if S if_a T in an interpretation, then S if_a T in all sharpenings of that interpretation. It is now easy to check that 'If_a S then definitely S' is valid. The outcome is that supervaluationist logic is a modification of classical logic, using this additional connective. The usefulness of the assertive conditional is that it accurately links precise and vague sentences.

9. Williamson

In most of this reconstruction of supervaluation theory – a response to Timothy Williamson's objection – I have been following Williamson's own advice. Developing

¹¹ See Williamson, Vagueness, pp. 147–8, for a discussion of this point.

¹² See Williamson, Vagueness, p. 151.

a definition of Kit Fine's, Williamson defines an object-language predicate ' $true_{T}$ ' of sentences by the condition that:

'S' is true_T in a refinement if and only if 'S' is true in that refinement.

Williamson says supervaluationists are wrong to identify truth with supertruth; instead they should identify it with $truth_{T}$. That is to say, they should take the true sentences to be just the ones that are $true_{T}$. But the definition he gives of $truth_{T}$ does not immediately put us in a position to follow this advice. More needs to be done first, because the definition does not specify the conditions under which sentences *are* $true_{T}$ (in the standard interpretation); it only gives us the conditions under which a sentence is $true_{T}$ in a refinement. In his discussion, Williamson implicitly assumes that the $true_{T}$ sentences are the ones that are $true_{T}$ in every sharpening. This is implicitly to adopt supervaluationism. For instance, Williamson claims that $truth_{T}$ is disquotational, but that follows only from the supervaluation principle.

Truth as I have treated it is nothing other than $truth_T$, and I have followed Williamson's advice in identifying it with ordinary truth. But I have explicitly adopted supervaluation too, and my entire account of vagueness depends on it. Williamson claims that, once truth is identified with $truth_T$, nothing significant is left of supervaluation theory. I think this is incorrect. To be sure, supervaluationists up to now have denied bivalence and the disquotation principle, and they ought not to have done that. But the core of their treatment of vague terms remains intact.

Some supervaluationists have hoped to analyse vagueness within a precise metalanguage. This is why Fine rejected the disquotation principle.¹³ Fine wanted truth to be precise, and "S" is true' cannot be precise if it is equivalent to a vague sentence 'S'. Williamson says the quest for a precise metalanguage should be abandoned, and I have taken his advice. In my account, truth is just as vague as the sentences it is predicated of. As I have defined them so far, the operators 'definitely' and 'to a degree' are precise, but that is only because I have ignored second-order vagueness. First-order vagueness is the vagueness of the borderline between men who and bald and those who are not bald. Second-order vagueness is the vagueness of the borderline between being bald and not being bald. There will be third-order and higher-order vagueness too. Second-order vagueness occurs because it is not fully determinate which sharp interpretations count as sharpenings of a vague interpretation. The notion of 'sharpening' is itself vague, that is to say. It follows that the operators are vague too.

I shall not include an account of second-order and higher-order vagueness in this paper, because I have nothing to add to Fine's existing account. Williamson has an objection to Fine's account too. It implies there is a sharp boundary between bald men and men who are borderline cases of some sort or other – men on some second-order or third-order or higher-order borderline. If this boundary exists, its position is plainly

¹³ Fine, 'Vagueness, truth and logic', p. 297.

unknown to us. But then, says Williamson, if there can be this sharp unknown boundary, why should there not be a sharp unknown boundary between bald men and men who are not bald? The whole idea that there are borderline cases may be a mistake.

Perhaps this is a cogent objection to supervaluationism; I do not know. This paper is addressed to the different objection that supervaluationist logic is defective because it denies the principle of disquotation. I do not think that objection is fatal.

References

Fine, Kit, 'Vagueness, truth and logic', Synthese, 30 (1975), pp. 265–300.

- Kripke, Saul, 'Outline of a theory of truth', *Journal of Philosophy*, 72 (1975), pp. 690–716. Reprinted in Robert L. Martin (ed.), *Recent Essays on Truth and the Liar Paradox*, Oxford University Press, 1984, pp. 53–82. Page references to the reprinted version.
- Read, Stephen, *Thinking About Logic: An Introduction to the Philosophy of Logic*, Oxford University Press, 1995.
- Tarski, Alfred, 'The semantic conception of truth', *Philosophy and Phenomenological Research*, 4 (1944), pp. 341–76.

Williamson, Timothy, Vagueness, Routledge, 1994.