Background Independence: what's special about GR?

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Motivation: another decade of dispute

Traditional 'textbook' wisdom

-ve part: ***General covariance*** restricts a theory's formulation, not its content.

- it does not implement a generalized relativity principle
- or abolish 'factitious' causes.

+ve part: GR differs from SR (solely?) in having a ***dynamical*** metric field:

- the '**absolute objects**' programme (Anderson, Friedman, Hiskes, Pitts)

Quantum gravity wisdom

GR, unlike SR, is a ***background independent*** theory. Background independence is:

1. linked to the satisfaction of a *substantive* general covariance requirement
2. meant to have radical implications for GR's observables

Conclusions

1. 'Substantive' principles of general covariance either
   (a) fail to distinguish GR and SR (*pace* Stachel, Earman), or
   (b) fail to have differential implications for the observables of GR and SR (e.g., Anderson's and Rovelli's principles)
2. Background (in)dependence has no direct implications for what's observable in *classical* (i.e. non-quantum) theories
3. The results of Anderson's absolute object programme fail to match the intuitions that motivate it:
   - background independent $\neq$ lacking absolute objects
4. Given (1) and (2), the interesting question is:
   - how does background-dependent quantum theory emerge from a background-independent quantum cosmology?

Familiar varieties of general covariance

T is 'generally covariant' iff

GC1 the equations of motion/field equations of T transform in a generally covariant manner under an arbitrary coordinate transformation

GC2 the equations of motion/field equations of T relate 'intrinsic, coordinate-free' objects; they are true independently of coordinate systems.

GC3 if $(\mathcal{M}, O_1, O_2, \ldots, O_N)$ is a model of T, then so is $(\mathcal{M}, d^*O_1, d^*O_2, \ldots, d^*O_N)$ for any $d \in \text{diff}(\mathcal{M})$

Varieties of substantive 'general covariance'

GC4 (GC3) + this diffeomorphism invariance is a *gauge symmetry*

GC5 the theory only counts 'diffeomorphic invariants' as genuine physical magnitudes

'Einstein' The theory takes its simplest form when expressed generally covariantly.

'Anderson' T is generally covariant just if diff$(\mathcal{M})$ is (a subgp of) the *symmetry group* of T. (The symmetry gp of T is the largest subgp of the covariance gp of T that is also a symmetry group of the absolute objects.)

Rovelli T is generally covariant iff a smooth displacement of ***only the dynamical fields*** over the manifold, sends solutions of the equations of motion into solutions of the equations of motion.

Earman (GC4) where: if T has a Lagrangian(/Hamiltonian) formulation, diff$(\mathcal{M})$ is a gauge group if **[AND ONLY IF]** diff$(\mathcal{M})$ is a variational symmetry(/the generators of diffeomorphisms commute with the first class constraints)

A qualification concerning 'Einstein' general covariance

"Of two theoretical systems compatible with experience, the one is to be preferred that is the simpler and more transparent from the standpoint of the absolute differential calculus" (Einstein 1918)

"The hypothesis that the geometry of physical space is represented best by a formalism which is covariant with respect to general coordinate transformations, and that a restriction to a less general group would not simplify that formalism, is called *the principle of general covariance*" (Bergmann 1942, 159; NB Bondi's 1959 objection)

Why general covariance is vacuous: the history

Einstein's hopes

- SR's 'epistemological defect': involves causes that are
  (i) unobservable and; (ii) affect but are unaffected
- A fully general relativity principle would remove it
- General covariance as a way to implement a fully general RP.

The irony

- The 'point-coincidence' resolution of the hole dilemma seems to imply:
- Any theory can be formulated generally covariantly

Einstein post Kretschmann

1. GR removes the 'epistemological defect' by addressing (ii)
2. The simplicity defence of general covariance, but contrast:
   - simplest theory versus
   - simplest form of a theory

Rehabilitating substantive general covariance?

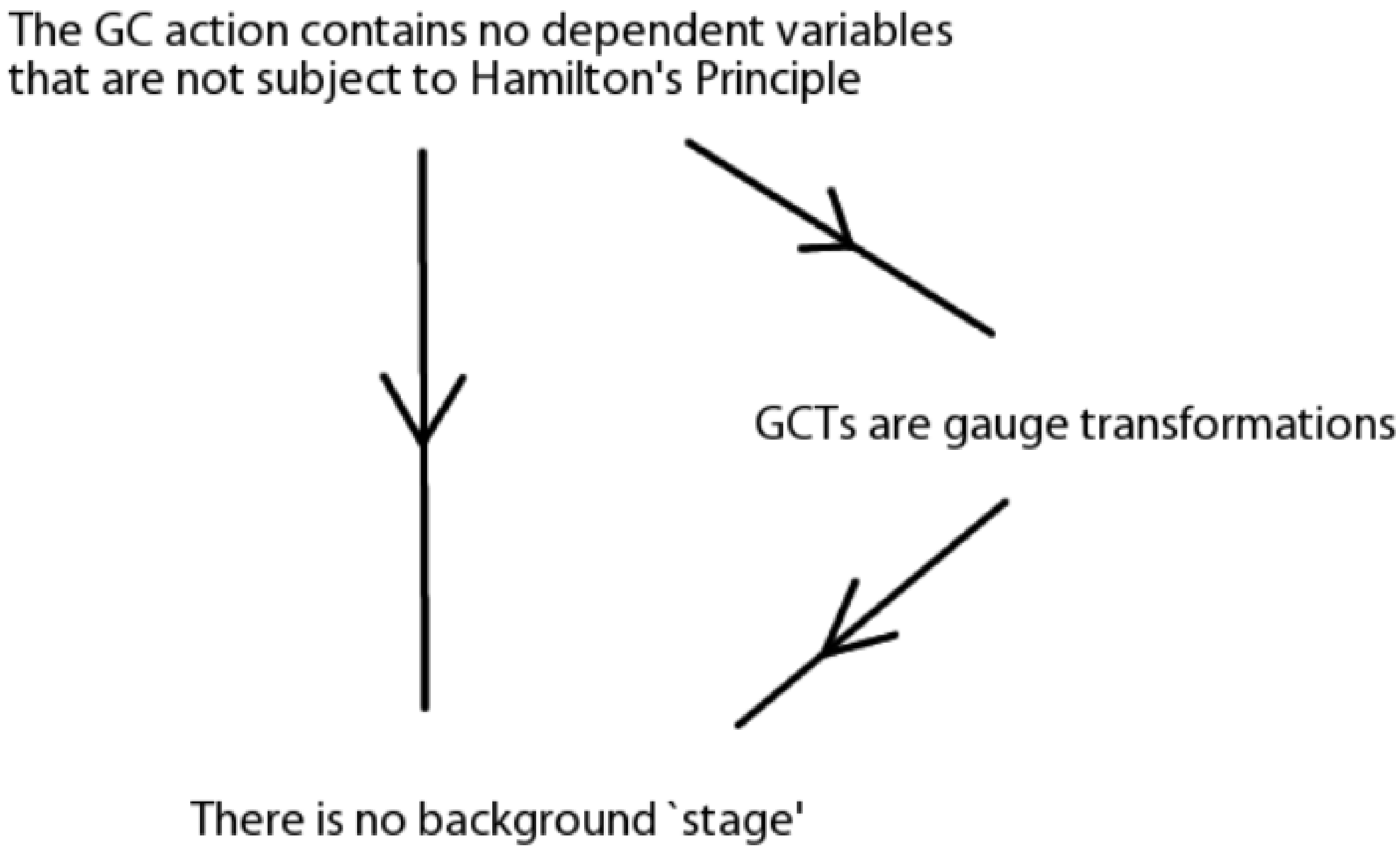
The key conceptual difficulty of quantum gravity is therefore to accept the idea that we can do physics in the absence of the familiar stage of space and time…

This absence of the familiar spacetime "stage" is called the *background independence* of the classical theory. Technically, it is realized by the gauge invariance of the action under (active) diffeomorphisms…

In turn, gauge invariance under diffeomorphism[s] (or *diffeomorphism invariance*) is the consequence of the combination of two properties of the action: its invariance under arbitrary changes of coordinates and the fact that there is no nondynamical "background" field. (Rovelli 2004, 10)

- **'gauge invariance'** := the generally covariant action contains no dependent variables that are not subject to Hamilton's principle?
- Only when the generally covariant action contains no dependent variables that are not subject to Hamilton's principle are diffeomorphisms gauge?

Rovelli's logic?



Earman on substantive general covariance

the substantive requirement of general covariance lies in the demand that diffeomorphism invariance is a gauge symmetry of the theory at issue. This requirement is termed “substantive” because it is *not* automatically satisfied by a theory that is formally generally covariant, i.e. a theory whose equations of motion/field equations are written in generally covariant coordinate notation or, even better, in coordinate-free notation…

the physics literature contains a generally accepted apparatus that applies to a very broad range of spacetime theories and that serves to identify the gauge freedom of any theory *in the class*. The verdict of the apparatus is, for example, that there is something special about Einstein’s GTR as regards general covariance: GTR satisfies substantive general covariance ***whereas formally generally covariant forms of special relativistic theories*** *(e.g. the equations for the source-free Maxwell field written in generally covariant notation)* ***need not satisfy substantive general covariance.*** (Earman 2006, 444, 445)

When to see gauge freedom

Suppose T's r equations of motion/field equations are derivable from an action $S = \int d^p x L(\mathbf{x}, \mathbf{u}, \mathbf{u}^{(n)})$; $\mathbf{x} = (x^1, \ldots x^p)$, $\mathbf{u} = (u^1, \ldots u^r)$. $\mathcal{G} \ni (\mathbf{x}, \mathbf{u}) \to (\mathbf{x}', \mathbf{u}')$ is a ***variational symmetry group*** just in case the infinitesimal generators of $\mathcal{G}$ leave L form invariant up to a divergence term.

Noether's 2nd theorem If the parameters of $\mathcal{G}$ are s arbitrary functions of the independent variables, then there are s independent equations relating the r 'Euler expressions'.

Earman's proposal "variational symmetries containing arbitrary functions of the independent variables connect equivalent descriptions of the same physical situation, i.e. are gauge transformations." (450)

Example: the SR Klein-Gordon field

Non-generally covariant form:

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} - \frac{\partial^2\Phi}{\partial t^2} - m^2\Phi = 0$$

Generally covariant, coordinate-independent form:

$$\eta^{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0$$

Derivable from:

$$\mathfrak{U}(\Phi, \eta) = \int \frac{1}{2}(\eta^{ab}\nabla_a\Phi\nabla_b\Phi + m^2\Phi^2)\sqrt{-\eta}d^4x$$

in which Φ is varied but η_{ab} is not.

"The action admits the Poincaré group as a variational symmetries…The apparatus sketched above renders the verdict that there is no non-trivial gauge freedom in the offing." (2006, 452)

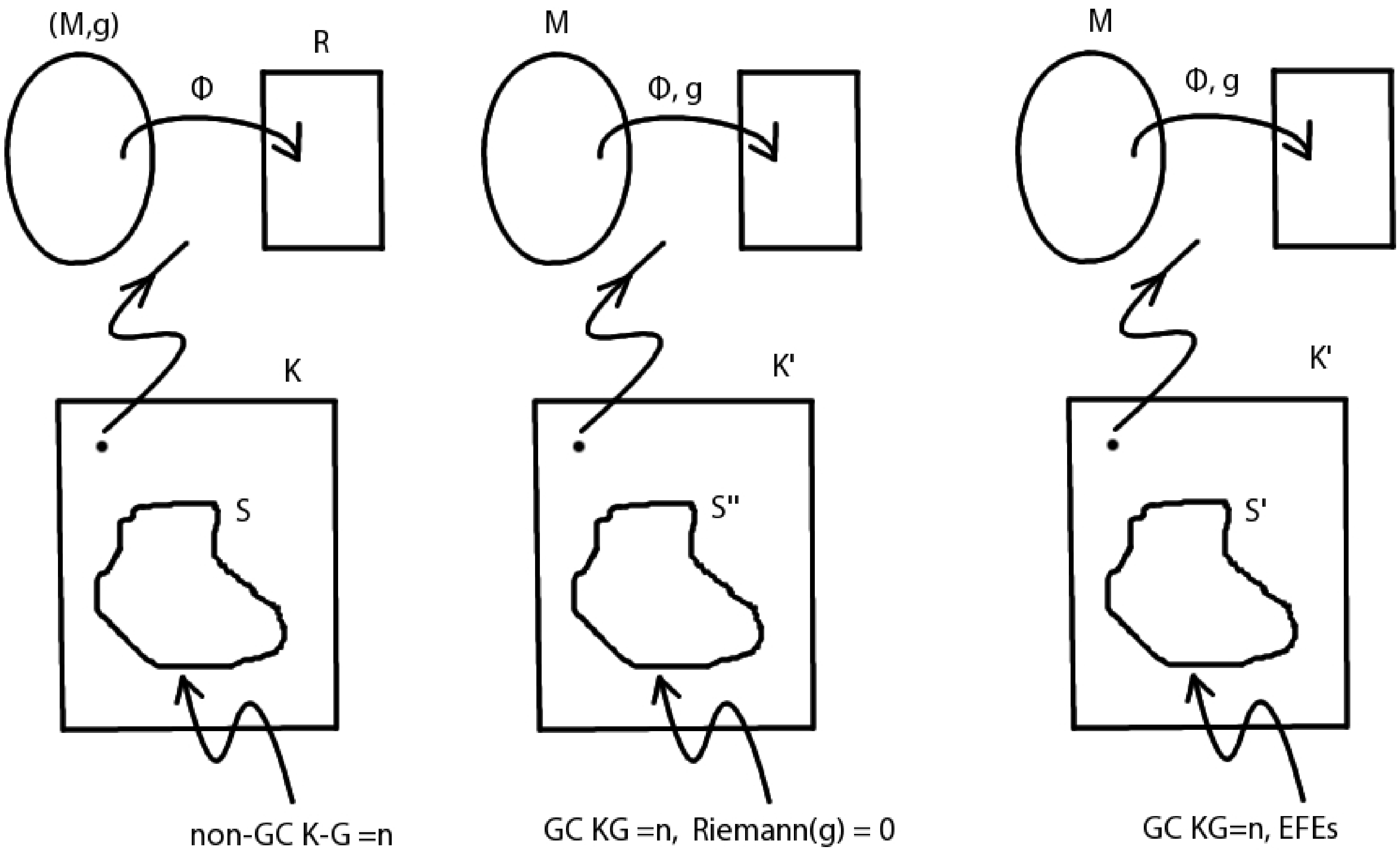
Why this interpretation is problematic

1. Diff(M) *is* a variational symmetry of $\mathfrak{U}$!

2. Criterion seems to say "*if* $\mathcal{G}$ is an N2 variational symmetry, then $\mathcal{G}$ is a gauge group" (WTSG). Earman needs "$\mathcal{G}$ is a gauge group *only if* $\mathcal{G}$ is an N2 variational symmetry" (WNTSG). NB he already admits that the apparatus is silent on non-Lagrangian theories.

3. Physicists see (WTSG) as motivated by a desire to avoid indeterminism. This provides just as good grounds for regarding Diff(M) as a gauge group of the (generally covariant) SR Klein-Gordon theory.

Three theories



A digression on observables

Does (G4) imply (G5)?

"the only gauge invariant quantities constructible…from Φ and g_{ab} will be *completely* non-local quantities such as integrals over all spacetime of functionals of Φ. Coincidence quantities constructible from Φ and g_{ab}—such as the value of Φ being such-and-such where-and-when the value of g_{ab} is such-and-such—won't be diffeomorphic invariants since… g_{ab} is homogeneous and isotropic" (2006, 456; my emphasis).

This is not a problem! One still has diffeomorphic invariants like:
'the value of Φ being such-and-such ***at such-and-such a distance from*** the value of Φ being such-and-such.'

It is in terms of this sort of observable that one should understand the content of ***non****-generally covariant* formulations of the theory. So…

Rovelli on Newtonian observables

For Newton, the coordinates $\vec{x}$ that enter his main equation $\vec{F} = m\frac{d^2\vec{x}(t)}{dt^2}$ are the coordinates of absolute space. However, since we cannot directly observe space, the only way we can coordinatize space points is by using physical objects. The coordinates $\vec{x}$ of the object moving along the trajectory $\vec{x}(t)$ are therefore defined as the distances from a chosen system O of objects, which we call a "reference frame"...

In other words, the physical content of [Newton's 2nd law] is actually quite subtle: there exist reference objects O with respect to which the motion of any other object A is correctly described by [$\vec{F} = m\frac{d^2\vec{x}(t)}{dt^2}$]...

Notice also that for this construction to work it is important that the objects O forming the reference frame are not affected by the motion of the object A. There shouldn't be any dynamical interaction between A and O.

(Rovelli 2004, 87–8)

A preferable point of view?

- Reference objects should not be thought of as non-dynamical, nor need they be excluded from the system under study (Morals of the dynamical, constructive approach to SR)

- Astonomy and ephemeris time: the system under study can itself be used to pick out the preferred set of coordinate systems

- It is absurd to suppose that classical pre-GR theories should be understood as incapable of treating the act of measurement itself

- The observables in a non-generally covariant versions of a theory outstrip diffeomorphic-invariants only because the coordinate system *encodes* information about fields that don't figure in the equations explicitly (e.g., the metric)

Back to Rovelli: ‘active diffeomorphism invariance’

Active diff invariance should not be confused with passive diff invariance, or invariance under change of coordinates. ... A field theory is formulated in manner invariant under passive diffs (or change of coordinates), if we can change the coordinates of the manifold, re-express all the geometric quantities (dynamical **and non-dynamical**) in the new coordinates, and the form of the equations of motion does not change. A theory is invariant under active diffs, when a smooth displacement of the dynamical fields (**the dynamical fields alone**) over the manifold, sends solutions of the equations of motion into solutions of the equations of motion. Distinguishing a truly dynamical field, namely a field with independent degrees of freedom, from a nondynamical field disguised as dynamical (such as a metric field g with the equations of motion Riemann$[g] = 0$) might require a detailed analysis (for instance, Hamiltonian) of the theory. (gr-qc/9903045)

What are non-dynamical fields?

1. Absolute objects (Anderson, Friedman, Pitts)
   (a) fields that are the same in every model of the theory
   (b) counterexamples (dust velocity field, Torretti's constant curvature theory, $\sqrt{-g}$ in (certain formulations of) GR)
2. Non-variational fields (Hiskes, Rovelli, Baez?)
   (a) Rosen/Sorkin actions

The original idea was that "the dynamical quantities depend on the absolute elements but not vice versa" (Anderson 1967, 73); **an absolute object "affects the behavior of other objects but is not affected by these objects in turn."** (Anderson and Gautreau 1969, 1657)

A **background field** is a field that violates **the action–reaction principle**. Non-variationality and absoluteness do not capture this notion.

“In an absolute framework the properties of any entity are defined with respect to a single entity-which is presumed to be unchanging. An example is the absolute space and time of Newton, according to which positions and motions are defined with respect to this unchanging entity.” (Smolin, *The Case for Background Independence*)

GR vs SR versions of the Klein Gordon field

	GR version	SR version
models	$\langle M, g_{ab}, \Phi \rangle$	$\langle M, g_{ab}, \Phi \rangle$
field eqns	$g^{ab}\nabla_a \nabla_b \Phi - m^2\Phi = 0$ $R_{ab} - \frac{1}{2}R g_{ab} = T_{ab}(\Phi)$	$g^{ab}\nabla_a \nabla_b \Phi - m^2\Phi = 0$ $R_{abcd}(g) = 0$
action	$S = \int \frac{1}{2}\sqrt{-g}\,(\mathcal{L}_M + \mathcal{L}_G)\, d^4x$ $\mathcal{L}_M = g^{ab}\Phi_{;a}\Phi_{;b} + m^2\Phi^2$ $\mathcal{L}_G = R(g)$	$S = \int \frac{1}{2}\sqrt{-g}\mathcal{L}_M d^4x$ $\mathcal{L}_M = g^{ab}\Phi_{;a}\Phi_{;b} + m^2\Phi^2$
$\delta S = 0$	for arbitrary variations of Φ and g	for arbitrary variations of Φ, *but not* g

Sorkin's example, discussed by Earman

models	$\langle M, g_{ab}, \Phi, \theta \rangle$
field eqns	$g^{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0$ $R_{abcd}(g) = 0 \ldots$
action	$S = \int \frac{1}{2}\sqrt{-g}(\mathcal{L}_M + \mathcal{L}_G)d^4x$ $\mathcal{L}_M = g^{ab}\Phi_{;a}\Phi_{;b} + m^2\Phi^2$ $\mathcal{L}_G = \theta^{abcd}R_{abcd}$
$\delta S = 0$	for arbitrary variations of *all* dependent variables

A background-dependent Klein-Gordon field theory?

models	$\langle M, g_{ab}, \Phi \rangle$
field eqns	$g^{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0$ $R_{ab} - \frac{1}{2}Rg_{ab} = 0$
actions	$S_1 = \int \frac{1}{2}\sqrt{-g}(\mathcal{L}_M + \mathcal{L}_G)d^4x$ $S_2 = \int \frac{1}{2}\sqrt{-g}\mathcal{L}_G d^4x$ $\mathcal{L}_M = g^{ab}\Phi_{;a}\Phi_{;b} + m^2\Phi^2$ $\mathcal{L}_G = R(g)$
$\delta S_1 = 0$ $\delta S_2 = 0$	for arbitrary variations of Φ, *but not* g for arbitrary variations of g
OR $\delta S = 0$	$S = \int \frac{1}{2}\sqrt{-g}(\mathcal{L}_M + \mathcal{L}_G)d^4x$, $\langle M, g_{ab}, \Phi, \Theta^{ab} \rangle$ $\mathcal{L}_G = \Theta^{ab}G_{ab}(g)$ for arbitrary variations of *all* dependent variables

g is not an absolute object in this theory. Intuitively it *is* dynamical, although it acts but is not acted upon.

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