What's Observable in Special and General Relativity?

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MY MAIN CLAIM: what's observable in special relativity

is the same as

what's observable in general relativity
(whatever that is)

Quantum Gravity

> . . . one of the basic difficulties in quantum gravity is to understand which physical quantities should be predicted by the theory.
>
> And the answer seems simple: the observable quantities in the quantum theory should be the same as in the classical theory, or at least a subset of these. Rather remarkably, however, the problem of what precisely is observable is far from being trivial even in *classical* general relativity.

(Rovelli, *What is observable in classical and quantum gravity?*)

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  2. Observables must be gauge-invariant
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  (A supposed contrast to the observables of SR)

The hole argument

- Given a model $(\mathcal{M}, g, T)$ of a GR theory, general covariance implies that $(\mathcal{M}, d * g, d * T)$ is also a model.
- If these models represent physically distinct solutions, the theory is indeterministic.
- So, the models are simply distinct mathematical representations of the same physical solution (d is a gauge transformation).

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1. either substantivalism or determinism is false?
2. the only quantities that are observable are Diff$(\mathcal{M})$-invariant?
3. $\mathcal{M}$ is a 'gauge artifact'?
4. 'a point is not a diffeomorphism invariant entity' . . . 'there are no points in a physical spacetime'?

‘the physicists’ view’ *vs.* ‘the philosophers’ view’

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> Philosophical responses to [Earman and Norton’s] version of the [hole] argument divide quite strikingly into two camps. On the one hand, there are those who criticize the argument on the grounds that it relies upon a naive approach to modality [Bartels, Brighouse, Butterfield, Maudlin, Hoefer, Maidens, Stachel]. . .
>
> The second sort of response to the hole argument is more radical, and its popularity more telling as a measure of the insularity of contemporary philosophy of space and time. [It denies] . . . that the hole argument has *anything* at all to teach us about the nature of space-time [Leeds, Mundy, Liu, Rynasiewicz]
>
> (Belot and Earman, *From Metaphysics to Physics*)

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- the ‘identities’ of the points of $\mathcal{M}$ are (allegedly) determined independently of g and T
- primitive intra-model numerical distinctness $\Rightarrow$(?) primitive trans-model identities
- physics is obviously modally committed, but not via trans-model identities

Spacetime: $\mathcal{M}$, or $(\mathcal{M}, g)$?

Does g represent:

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Spacetime: $\mathcal{M}$, or $(\mathcal{M}, g)$?

Does g represent:

1. spacetime structure?
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There are reasons to answer ‘yes’ to (1):

- stress-energy and ‘gravitation’
- non-vanishing of its components, signature
- its indispensability
- continuity with pre-GR theories
    - ESPECIALLY CONCERNING OBSERVABILITY/ EMPIRICAL CONFIRMATION (when tidal effects are ignorable)

The hole argument and GR *vs.* SR

- HA *does* tell us something about nature
  1. the models $(\mathcal{M}, g, T)$ and $(\mathcal{M}, d * g, d * T)$ of a generally covariant theory do not represent physically distinct solutions
  2. $R(p)$ is not observable
- But why claim these lessons are not applicable to pre-GR theories, which can be formulated generally covariantly?
- Is $R(x)$ an observable of non-generally covariant formulations of these theories?
- Trivial versus substantive general covariance (Stachel, Earman)

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By means of a purely mathematical reformulation of the equations representing the theory, and with, at most, purely mathematical complications connected with that reformulation, any physical theory can be brought into agreement with any, arbitrary relativity postulate, even the most general one

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If one . . . recalls that, in the final analysis, all physical observations consist in the determination of purely topological relations (‘coincidences’) between objects of spatio-temporal perception, from which it follows that no coordinate system is privileged by these observations, then one is forced to the following conclusion: By means of a purely mathematical reformulation of the equations representing the theory, and with, at most, purely mathematical complications connected with that reformulation, any physical theory can be brought into agreement with any, arbitrary relativity postulate, even the most general one **and this without modifying any of its content that can be tested by observations**

Stachel: definitions

chronogeometric structures “characterize the behavior of (ideal) clocks and measuring rods” (1993, 134)

affine structures “characterize the behavior of freely falling . . . test particles” (*ibid.*)

dynamical structures characterize “the behavior of physical fields and/or particles in spacetime. . . usually specified by requiring that the dynamical variables be subject to a set of differential equations” (1993, 135)

individuating field *any* structure on a differentiable manifold that individuates the points of the manifold by some property or properties that characterize(s) each of the points (1993, 139)

Stachel: GR versus SR

> . . . we can now identify the missing element in the resolution of the hole argument . . . To justify the identification of a whole equivalence class of drag-along metrics with *one* gravitational field, we must stipulate that, in general relativity, there is no structure on the differentiable manifold that is *both* independent of the metric tensor *and* able to serve as an individuating field (1993, 140)

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> [In 1905, Einstein gave a] careful definition of a physical frame of reference. He defined it in terms of a network of measuring rods and a set of suitable-synchronized clocks, all at rest in an inertial system. He used such a system of rods and clocks to give physical meaning to a preferred coordinate system associated with the inertial frame of reference. In my language (admittedly anachronistic), the rods and clocks serve to establish a nondynamical individuating field for the points of Minkowski spacetime. (1993, 141)

Stachel’s principle of general covariance

The Principle of General Covariance (1) There are no individuating fields in spacetime that are independent of the metric tensor field; (2) the metric field determines both chronogeometrical and inertio-gravitational structures of spacetime; and (3) The metric field obeys a set of generally covariant field equations. (Stachel 1993)

QUESTION: How does a generally covariant formulation of a **SR** theory fare?

pre-GR theories as cosmological theories? I

- In any non-gravitational theory, it is always possible to choose a reference system which is not affected by the system under consideration (Rovelli 1991)
- in non-generally relativistic physics, considering the reference systems as external non-dynamical objects is *not* an approximation (Rovelli 1991)
- There is no problem of time in theories of isolated systems, embedded inside the universe. . . The reason is that if the system modeled is understood to include only part of the universe one has the possibility of referring to a clock in the part of the universe outside the system which is modeled by the theory. This is generally what the t in the equations of classical and quantum mechanics refers to. (Smolin 2000)

pre-GR theories as cosmological theories? II

Diffeomorphism invariance is the technical implementation of a physical idea, due to Einstein. The idea is a deep modification of the pre-general-relativistic (pre-GR) notions of space and time. In pre-GR physics, we assume that physical objects can be localized in space and time with respect to a fixed non-dynamical background structure. Operationally, this background spacetime can be defined by means of physical reference-system objects, but these objects are considered as dynamically decoupled from the physical system that one studies. This conceptual structure fails in a relativistic gravitational regime. (Rovelli 1998)

- Generally covariant versions of pre-GR theories?
- Does the metric structure in these represent an "individuating field"?

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- The observables in the non-generally covariant versions of a theory should outstrip diffeomorphic-invariants because the coordinate system *encodes* metrical information

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- A moral of the dynamical, constructive approach to SR: the reference objects should not be thought of as non-dynamical, nor need they be excluded from the system under study
- Barbour on ephemeris time: the system under study can define it's own coordinate system
- The observables in the non-generally covariant versions of a theory should outstrip diffeomorphic-invariants because the coordinate system *encodes* metrical information
- It is absurd to suppose that classical pre-GR theories should be understood as incapable of treating the act of measurement itself

When is a symmetry a gauge symmetry?

Gauge transformations $:=$ transformations that connect solutions that represent the same physical state or history

- A major motivation for interpreting a tfmn as a gauge tfmn is often that doing so is a way to avoid indeterminism

Now the obvious danger here is that determinism will be trivialized if, whenever it is threatened by non-uniqueness, we are willing to sop up the non-uniqueness in temporal evolution with what we regard as gauge freedom to describe the evolution in different ways. **Is there then some non-question begging and systematic way to identify gauge freedom and to characterize the genuine observables?**

(Earman, *An Ode to the constrained Hamiltonian formalism*)

The constrained Hamiltonian formalism and Gauge

Restrict to eqns derivable from an action that is (quasi-)invariant under a group of transformations that depend on arbitrary functions of all the independent variables
Noether's 2nd theorem tells us the Euler-Lagrange equations are not independent of one another, leading to:

1. (apparent) 'underdetermination'
2. a constrained Hamiltonian theory

Dirac: transformations of the phase space that are generated by the first class constraints are to be interpreted as gauge transformations.
Observables are taken to be phase functions that are constant along gauge orbits

More lines from the Ode

Suppose, for example, that you want to be a relationist about space and time, and also that you want to acknowledge the striking success in the use of Minkowski spacetime for formulating theories of modern physics. You could reconcile these two desires by saying that the relational spatiotemporal structure of physical events conforms to those prescribed by Minkowski spacetime while at the same time denying that physical events are in any literal sense located in a spacetime container. But to make such a stance consistent requires treating a Poincaré boost of matter fields on Minkowski spacetime as a gauge transformation. . . By contrast, an application of the constraint apparatus to Maxwell electromagnetic theory and other standard special relativistic theories does not produce the verdict that the Poincaré group is a gauge group.

The moral for SR?

Verdict in the case of GR is that the observables are Diff$(\mathcal{M})$-invariant fns on the space of (H_{ij}, K_{ij}) pairs that are embeddable in GR spacetimes. . .
These are not what's observable in GR.
In the case of *generally covariant* formulations of SR, the apparatus is *silent*.
(It is not silent on generally covariant versions of SR theories derived from a generally covariant action. Are these the same theory? Distinguish:

- parametrized theories
- theories with lagrange multiplier fields to enforce field equations for background structure.)

Conclusion

- What’s special about GR is not to be found in what’s observable in GR
- The question is not “why is GR so hard to quantize?”, but “why does the quantization of pre-GR theories give us empirically successful theories when in reality there is no *non-quantum* (dynamical) metric of GR to provide a background with respect to which such theories can be defined?”