Lecture 5: Leibniz equivalence and Putnam's paradox

5.1 Recap: Responses to the hole argument

Let $\mathcal{M}_1 = \langle M, g, T \rangle$ and $\mathcal{M}_2 = \langle M, d^*g, d^*T \rangle$ be two models of GR where d is a non-trivial diffeomorphism, possibly a hole diffeomorphism.

Haec $\mathcal{M}_1$ and $\mathcal{M}_2$ represent different physical situations, i.e., they represent different possible worlds.

LE $\mathcal{M}_1$ and $\mathcal{M}_2$ represent the same possible world.

One If $\mathcal{M}_1$ is taken to represent a possible world, then $\mathcal{M}_2$ does not represent a possible state of affairs at all.

5.2 Denying (LE): the "acid test"

> Fortunately we do not need to settle this reformulation problem. What ever reformulation a substantivalist may adopt, they all must agree concerning the acid test of substantivalism, drawn from Leibniz. If everything in the world were reflected East to West (or better, translated 3 feet East), retaining all the relations between bodies, would we have a different world? The substantivalist must answer yes since all the bodies of the world are now in different spatial locations, even though the relations between them are unchanged...
>
> ...The diffeomorphism is the counterpart of Leibniz' replacement of all bodies in space in such a way that their relations are preserved. (Earman & Norton 1987, 521)

5.3 Maudlin's metrical essentialism

Maudlin considers and rejects a substantivalist interpretation of models that grounds (LE): treat the points of M as akin to bound variables. The reason he rejects it is that such a substantivalist cannot endorse 'Leibniz–Clarke counterfactuals'.

> Take as a paradigm Leibniz shift the displacement of all physical objects in a Neo-Newtonian or Minkowski space-time 3 meters to the North. According to the substantivalist such a shift is metaphysically possible and would result in an ontologically distinct state of affairs...
>
> The problem is that in specifying the Leibniz shift we must refer to physical event locations not via bound variables but by name. We want to say that in the shifted situation objects that are *here*... would be *there*. If one really believes in event locations, believes that there is a deep ontological fact about at which space-time point a particular event occurred, then one ought to be able to discuss the possibility of that event... occurring somewhere else.
>
> ...The restriction to bound variables simply has no reasonable justification within the substantivalist program. So we should allow the mathematical

> points in our solutions of the field equations to act as *names* of physical spacetime points. (Maudlin 1989, 315)

> the parts of space derive their character from their positions, so that if any two could change their position, they would change their character at the same time and each would be converted numerically into the other. The parts of duration and space are understood to be the same as they really are because of their mutual order and position; nor do they have any hint of individuality apart from that order and position which consequently cannot be altered. (Newton 1962, 132)

But consider:

1. (A subset of) the various properties that an object actually has, and the various relations that it stands in, are essential to its being numerically distinct from other objects.

2. (A subset of) the various properties that an object actually has, and the various relations that it stands in, are essential to it in the sense that it is metaphysically impossible for that very object to have existed with *different determinations* of the same *types* of properties and relations.

Other problems (*cf.* Earman 1989, 201):

- GTR as a guide to which properties are essential
- Non-isomorphic models

5.4 Relationalism

One way to embrace (LE), and thereby to avoid the hole problem, is to opt for relationalism. One adopts a realist attitude to the fields; only M in $\langle M, g, T \rangle$ is to be understood in a instrumentalist fashion.

With regard to the attractiveness of this option, note:

1. Much might seem to hang on the status of the metric field. Is it naturally interpreted as representing spacetime structure, or as a (material?) field (the "gravitational" field) much like, e.g., the electromagnetic field. On this question, see Maudlin (1990), Rynasiewicz (1996), Rovelli (1997), Hoefer (1998). Key points to consider include: then indispensability of the metric, its fixed signature, whether the role it plays in GR is different from the role of spacetime structure fields in other theories and whether it carries energy and momentum.

2. Must the relationalist give "a direct characterization of the reality underlying a Leibniz-equivalence class [of substantivalist models]" (Earman 1989, 171)?

5.5 'Sophisticated' substantivalism

Mundy (1992), Rynasiewicz (1994), Brighouse (1994), Hoefer (1996) have all argued that the substantivalist is not committed to (Haec) and that he is (more or less) entitled to (LE).

The tenability of this position is related to a number of issues. In particular:

1. Does treating spacetime points as real entities commit one to haecceitistic differences? (Recall the "acid test".)

2. Does a straightforward reading of the mathematical formalism sanction haecceitistic differences?

3. How well-motivated is anti-haecceitism as a philosophical position?

> [Manifold substantivalism] can be characterized in terms of political economies as an exercise in the division of labor. The differential geometer constructs the Ms and then passes them on to the physicist who proceeds to test them for suitability as a basis for a general-relativistic model. . . it is assumed that questions of identity and individuation of points of M have been settled prior to the introduction of the g-field and the T-field; indeed, the very characterization of fields [given above] takes for granted the identity of the elements in the point set, the topology on the set, and the differential structure. (Earman 1989, 180)

> Is the world—and are all possible worlds—constituted by purely qualitative facts, or does thisness hold a place beside suchness as a fundamental feature of reality? (Adams 1979, 5)

5.6 Putnam's Paradox

> It is shown that the hole argument is really Putnam's argument restricted to spacetime theories. This is because the gauge theorem is about the inscrutability of reference, not about the indeterminacy (sic) of possible worlds. Thus, the gauge freedom in spacetime theories expresses a semantic fact rather than one about ontology; and the hole argument is really against the metaphysical determinacy of reference. (Liu 1996, 243)

Earman and Norton's gauge theorem If $\langle M, O_1, \ldots, O_n \rangle$ is a model of a local spacetime theory and h is a diffeomorphism from M onto M, then the carried along tuple $\langle M, h^* O_1, \ldots, h^* O_n \rangle$ is also a model of the theory. *Proof* The vanishing of tensor quantities is preserved under diffeomorphism. (*Cf.* Earman & Norton 1987, 520)

The "Putnam Theorem" Suppose that we have a language, L, which consists of, inter alia, a set of n-place predicates, $P_1, \ldots, P_s$. For any theory, T, of L, if a first-order structure is a model of T under a (non-trivial) interpretation, I, there is always another first-order structure under another (non-trivial) interpretation

> K (different from I), which is also a model of T that makes all statements in T true in exactly those possible worlds in which they are true under I. (Liu 1996, 245)

Let A be a first-order structure that makes true some "complete description" T. (T will be a set of sentences of some language L and A is said to be a *model* for the set of sentences Γ.) A will consist of a set of elements D (the *domain of discourse*), an assignment of elements of D to the names of L, and assignments of n-tuples of elements of D to the n-place predicates of L. The theorem states that the structure A' will also be a model of T where A' is obtained from A by the following procedure. Let $|\phi|_A$ stand for the assignment given to the L expression ϕ by the assignment A. Considers a non-trivial permutation π of the elements of D and let $\pi*$ represent the obvious action on sets of n-tuples of D generated by π (let the action of $\pi*$ on elements of D be just that of π). One can then generate A' by assigning to all expressions ϕ the set-theoretic object $\pi * |\phi|_A$. That is $|\phi|_{A'} = \pi * |\phi|_A$ for all expressions ϕ of L.

Now suppose T is a description in our "spacetime theory language" that is intended to be a description of the actual world (it could be a maximally specific description, with respect to a specific set of coordinate charts, of every field). Suppose A is a model for T. The domain of A will be some differential manifold M, and some map from M to actual spacetime points will represent the intended interpretation of our spacetime vocabulary. Let A' be obtained from A by a "hole diffeomorphism" which is just a special case of a permutation of the domain of A. By "Putnam's theorem," A' will also be a model for T. But now, under the same map from M onto spacetime, A' represents an unintended interpretation. Putnam points out that holding true the sentences of T does not determine which of the reference relations represented by A or A' was intended. In fact, the conclusion is worse than this (because of the Löwenheim-Skolem theorem): *any* consistent set of sentences that does not assert that our world is finite has some model of the same cardinality as the set of actual spacetime points. Therefore there are infinitely many model–world isomorphisms according to which the actual world makes *every possible* description in spacetime language true.

> . . . a mathematical model can be converted into a scheme of abbreviation [interpretation?] by a bijection of the domain of the model onto a set of actual individuals. Thus, the class of cases in which Model Literalism [a.k.a. (Haec)] leads to an objectionable postulation of distinct but indiscernible physical situations coincides with the class of cases in which there exists underdetermined schemes of references according to the inscrutability arguments. Put slightly differently, in those cases in which we want to deny that a given pair of models represent distinct situations, each model has the status of an unintended interpretation relative to the other. If we take the pair to be *co-intended*, then we are committed to regarding the difference between them as indicative of a real difference in possibilities. (Rynasiewicz 1994, 420–1)

References

Adams, R. M. (1979), 'Primitive thisness and primitive identity', *Journal of Philosophy* **76**, 5–26.

Brighouse, C. (1994), Spacetime and holes, *in* D. Hull, M. Forbes & R. Burian, eds, 'Proceedings of the 1994 Biennial Meeting of the Philosophy of Science Association', Vol. 1, Philosophy of Science Association, East Lansing, MI, pp. 117–25.

Earman, J. (1989), *World Enough and Space-Time: Absolute versus Relational Theories of Space and Time*, MIT Press, Cambridge MA.

Earman, J. & Norton, J. (1987), 'What price substantivalism? the hole story', *British Journal for the Philosophy of Science* **38**, 515–25.

Hoefer, C. (1996), 'The metaphysics of space-time substantivalism', *Journal of Philosophy* **93**, 5–27.

Hoefer, C. (1998), 'Absolute versus relational spacetime: For better or worse, the debate goes on', *British Journal for the Philosophy of Science* **49**, 451–67.

Liu, C. (1996), 'Realism and spacetime: Of arguments against metaphysical realism and manifold realism', *Philosophia Naturalis* **33**, 243–63.

Maudlin, T. (1989), The essence of space-time, *in* A. Fine & J. Leplin, eds, 'Proceedings of the 1988 Biennial Meeting of the Philosophy of Science Association, Vol. 2', Philosophy of Science Association, East Lansing, Michigan, pp. 82–91.

Maudlin, T. (1990), 'Substances and space-time: What Aristotle would have said to Einstein', *Studies in History and Philosophy of Science* **21**, 531–61.

Mundy, B. (1992), Space-time and isomorphism, *in* D. Hull, M. Forbes & K. Okruhlik, eds, 'Proceedings of the 1992 Biennial Meeting of the Philosophy of Science Association', Vol. 1, Philosophy of Science Association, East Lansing, Michigan, pp. 515–27.

Newton, I. (1962), De Gravitatione, *in* A. Hall & M. Hall, eds, 'Unpublished Scientific Papers of Isaac Newton', Cambridge University Press, Cambridge, pp. 123–46.

Rovelli, C. (1997), Halfway through the woods: Contemporary research on space and time, *in* J. Earman & J. Norton, eds, 'The Cosmos of Science', University of Pittsburgh Press, Pittsburgh, pp. 180–223.

Rynasiewicz, R. A. (1994), 'The lessons of the hole argument', *British Journal for the Philosophy of Science* **45**, 407–36.

Rynasiewicz, R. A. (1996), 'Absolute versus relational space-time: An outmoded debate?', *Journal of Philosophy* **93**, 279–306.