Lecture 6: Individuality and Indiscernibility

> Identity is utterly simple and unproblematic. Everything is identical to itself; nothing is ever identical to anything else except itself. There is never any problem about what makes something identical to itself; nothing can ever fail to be. And there is never any problem about what makes two things identical; two things never can be identical. (Lewis 1986, 192–3)

6.1 Seemingly legitimate questions

- Is the nonidentity of numerically distinct individuals grounded, or can one take numerical diversity as primitive?
- What is the relation between intra-world numerical diversity (and its ground) on the one hand and trans-world identity, essentialism and haecceitism on the other?

If one holds that the distinctness of individuals requires a *qualitative* ground one is naturally led to...

6.2 The Principle of the Identity of Indiscernibles

> There are never in nature two beings which are perfectly alike and in which it would not be possible to find a difference that is internal or founded upon an intrinsic denomination. (Leibniz 1991, 62)

Formally:

$$\forall x \forall y [\forall F [Fx \leftrightarrow Fy] \rightarrow x = y]$$

Restrictions on the properties that F is to be understood as ranging over yield principles of varying strength:

Strong PII F ranges over only *pure intrinsic* properties

Weak PII F ranges over only *pure* (intrinsic or relational) properties

Trivial PII F ranges over *impure* as well as pure properties

6.2.1 Leibniz's PII: which and why?

- Leibniz advocated strong PII, at least applied to Monads.
- 1. PSR $\Rightarrow$ PII (in particular in the correspondence with Clarke, suggesting that the PII is a contingent truth);
  2. predicate-in-subject doctrine of truth $\Rightarrow$ PII (in *First Truths*, suggesting that the PII is a necessary truth) (See Cover & O'Leary-Hawthorne 1999, Ch. 5.)

6.2.2 Counterexamples and responses

Symmetric worlds Surely a world containing nothing but two intrinsically identical iron spheres, each a mile in diameter and two miles apart is a genuinely possible world? (Black 1952)

Hacking suggests that we can always reinterpret any such description as of a world containing no indiscernibles, e.g. as containing just a single sphere (Hacking 1975). Note that glib talk of "non-Euclidean spaces" can be misleading; this strategy is more radical than it might at first appear (Belot 2001).

6.3 Quantum Mechanics and the PII

Does the exclusion principle mean that a relatively strong form of the PII is at least physically necessary when restricted to *fermions*? (Weyl suggested that the exclusion principle should be called the "Pauli-Leibniz principle".) For consider the example of bound electrons in an atom: no two can share their values of energy, total angular momentum, spin etc.

Actually, it seems we should draw the reverse conclusion: when applied to 'identical' fermions (i.e., fermions with the same state-independent properties: mass, charge and total spin), it seems that weak PII is false of physical necessity! The problem is that the state of any composite system containing 'identical' fermions is required to be antisymmetric under even permutations of the particles:

$$\Psi(\ldots, x_i, \ldots, x_j, \ldots) = -\Psi(\ldots, x_j, \ldots, x_i, \ldots)$$

where x_i and x_j are generalized coordinates of the ith and jth 'identical' fermions respectively. More abstractly, $P_{ij}|\Psi\rangle = -|\Psi\rangle$. It follows that the reduced density matrices of each 'identical' fermion are the same. As an example consider the singlet state, $1/\sqrt{2}(|\uparrow_{z1}\rangle\,|\downarrow_{z2}\rangle - |\downarrow_{z1}\rangle\,|\uparrow_{z2}\rangle)$; the density matrix representing the state of both particle 1 and particle 2 is: $\frac{1}{2}\,|\uparrow_z\rangle\,\langle\uparrow_z| + \frac{1}{2}\,|\downarrow_z\rangle\,\langle\downarrow_z|$.

Where a and b are 'identical' fermions which are components of a composite system in state Ψ, and c is some other, arbitrary component of the composite system, it follows that, for arbitrary observables Q, Q':

1. $\mathrm{pr}_\Psi(Q_a$ has value $q) = \mathrm{pr}_\Psi(Q_b$ has value $q)$
2. $\mathrm{pr}_\Psi(Q_a$ has value q/Q'_b has value $q') = \mathrm{pr}_\Psi(Q_b$ has value q/Q'_a has value $q')$
3. $\mathrm{pr}_\Psi(Q_a$ has value q/Q'_c has value $q') = \mathrm{pr}_\Psi(Q_b$ has value q/Q'_c has value $q')$.

(See Butterfield 1993, 459–66; *cf.* Margenau 1944, French & Redhead 1988.) One can resist the conclusion by denying that quantum mechanics is complete (see van Fraassen 1991, 423–33), or by denying that it even makes sense to talk of an entangled fermion as having its own state (which determines its intrinsic properties), so that the question of whether two fermions have the same such state does not even arise (see Massimi 2001).

6.4 A Quinean Principle of the Identity of Indiscernibles

Recall Robert Adam's question:

> Is the world—and are all possible worlds—constituted by purely qualitative facts, or does thisness hold a place beside suchness as a fundamental feature of reality? (Adams 1979, 5)

It is a mistake to think that a commitment to the adequacy of purely qualitative facts (a denial of differences *solo numero*) entails either weak PII or strong PII as characterized above. Simon Saunders has recently championed a proposal of Quine's (Saunders 2003; Quine 1986, 63–4).

Suppose we have a first-order language equipped with a stock of finitely many basic predicates. There is an essentially unique analysis of identity in predicative terms. '$x = y$' is true on some assignment of values to the variables *just if* every sentence of the following form is true on the same valuation:

$$Fx \leftrightarrow Fy \tag{1}$$

$$\forall z((Rxz \leftrightarrow Ryz) \wedge (Rzx \leftrightarrow Rzy)) \tag{2}$$

$$\forall z \forall w((Pxzw \leftrightarrow Pyzw) \wedge (Pwxz \leftrightarrow Pwyz) \wedge \ldots) \tag{3}$$

$$\vdots$$

for all basic 1-place predicates Fx, 2-place predicates Rxy and so on.

On this definition, Max Black's spheres do not count as identical. Let Rxy stand for "x is two miles from y." Consider a valuation that assigns one sphere to x and the other to y.[1] On this valuation, the sentence (2) comes out false. Each sphere is not two miles from exactly those things that the other sphere is two miles from, for each sphere is two miles from the other and not two miles from itself. In general, it is enough that there are (purely qualitative) relations which are *symmetric and irreflexive* for the existence of indiscernible (in the traditional sense) yet distinct relata to be possible. Should we think of the sphere's being two miles apart as the *ground* of their numerical diversity? (For further discussion of the proposal that relations can individuate in this way, and of other proposals, see Cover & O'Leary-Hawthorne 1999, 270–89.)

6.4.1 Back to quantum mechanics

The proposal works well for fermions: the exclusion principle does guarantee that there is always some irreflexive relation true of them. However, 'identical' bosons violate even the Quinean PII (see Cortes 1976). Rather than seeing them as individuals which violate the Quinean PII, one can give up viewing them as individuals at all (see Redhead & Teller 1992).

6.4.2 Possible Worlds

Does the Quinean view entail anti-haecceitism? Worlds that differ purely haecceitistically violate traditional forms of the PII, and the Quinean PII sanctions such violations

[1] How is this assignment to be made?!

only if there are real relations between such objects that can ground their diversity. What could these be in the case of possible worlds?

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