Lecture 8: Parity Violation

8.1 Parity

Parity is spatial inversion: $(\vec{x}, t) \mapsto (-\vec{x}, t)$. In odd-dimensional spaces this maps objects of one handedness onto their incongruent counterparts.

8.1.1 If parity is not a symmetry...

One might think that:

1. For some handed processes, the probability of one handedness differs from the probability of the other handedness ($\mathrm{Co}^{60} \rightarrow \mathrm{Ni}^{60} + e^- + \overline{\nu}_e$).

2. The laws do not take the same form in coordinates systems related by parity (*cf.* Einstein's statement of the relativity principle).

3. If M is a model of T then "$P(M)$" is not a model of T and does not represent a physically possible world.

8.1.2 Experimental verification

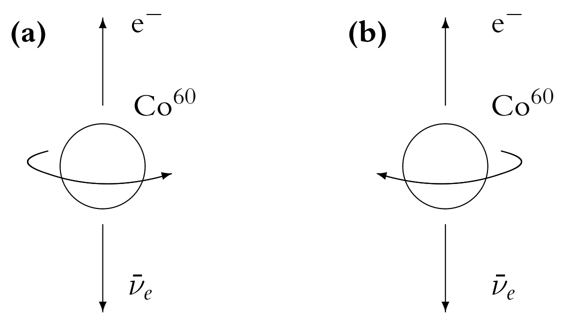

Figure 1: $\mathrm{Co}^{60} \rightarrow \mathrm{Ni}^{60} + e^- + \overline{\nu}_e$

Scenario (a) is much more probable that scenario (b).

8.2 Substantivalist arguments

8.2.1 Earman's 1989 argument

The failure of mirror image reflection to be a symmetry of laws of nature is an embarrassment for the relationist account sketched...for as it stands that account does not have the analytical resources for expressing the law-like asymmetry for the analogue of Kant's hand standing alone. Putting some 20th century words into Kant's mouth, let it be imagined that the first created process is a [$\mathrm{Co}^{60} \rightarrow \mathrm{Ni}^{60} + e^- + \overline{\nu}_e$] decay. The absolutist has no problem in writing laws in which [(a)] is more probable than [(b)], but the relationist...certainly does since for him [(a)] and [(b)] are supposed to be merely different modes of presentation of the same relational model. Evidently, to accommodate the new physics, relational models must be more variegated than initially thought. (Earman 1989, 148)

8.2.2 Comments

1. Is it true that: "The absolutist has no problem in writing laws in which [(a)] is more probable than [(b)]"? To ground a distinction between [(a)] and [(b)] via substantivalism one needs to be a haecceitist.

   > It seems wrong for a law of nature to contain reference to a particular, contingent physical object. But it seems (to me) at least as wrong for a law of nature to contain reference to particular bits of space... (Hoefer 2000, 253).

2. Might whether the decay is 'probable' depend on its congruence or otherwise to typical *subsequent* decays? *Cf.* Weyl (1952, 21) and consider David Lewis's account of probabilistic laws (Lewis 1994, 233–6).

3. If handedness is not an intrinsic matter, so that (a) and (b) are intrinsically identical, what could *ground* their differential likelihoods?

   > God could no doubt *see to it* that certain kinds of particles always decay into configurations of the same handedness. But we need to be able to suppose that the result in question comes about through law rather than divine supervision. How can it be law that particles always... display decay modes of one orientation rather than another, if orientation is not intrinsic? If one particle has decayed in left-handed fashion, how does the next particle 'know' that it should do likewise? It's instruction cannot be to trace a pattern of a certain intrinsic description; it can only be to do what the first particle did.
   >
   > The problem here is not 'action at a distance', though perhaps that will trouble some. It is rather that the required laws would make ineliminable reference to particular things, whereas it is generally supposed to be of the essence of laws that they state relations of kind to kind. (Van Cleve 1991, 21–2)

We have parity violating laws. Does physics, after all, characterize left and right in intrinsic terms?

8.2.3 Symmetries and spacetime structure: inertial structure

- that the laws of classical mechanics are invariant under boosts (in addition to time-independent translations and rotations) means that there is no absolute standard of rest—Newton's absolute space is undermined—*but...*

- that the laws are *not* invariant under time-dependent rotations and arbitrary time-dependent translations, indicates that there are dynamically preferred ways of matching-up the points of space at different times that are independent of matter and its relative motion. We have:

- preferred coordinate systems (the inertial frames)
- inertial structure (an affine connection) that is naturally given a realistic (substantival) interpretation (or so the argument goes).

8.2.4 Symmetries and spacetime structure: an orientation field

Is the failure of parity to be a symmetry analogous to the failure of boosts to rotating frames to be symmetries?

In particular:

1. are there preferred coordinates? (R.H. vs. L.H. coordinates?)
2. is there an associated spacetime structure field? (An orientation field?)

> . . . the 'absolutist' asserts that "the scientific treatment of motion . . . requires some absolute quantities . . . such as handedness. To make these quantities meaningful requires the use of an orientation, and this structure must be a property of or inhere in something distinct from bodies. *The only plausible candidate for the role of supporting the nonrelational structures is the spacetime manifold*." (Huggett 2000, 236; *cf.* Earman 1989, 125)

8.3 Relationalist responses

8.3.1 *PCT* symmetry

Were [parity] a symmetry, then applied to the world as a whole it would follow that only quantities and relations invariant under the transformation would be real: two spacetime models, the one the spatial inversion of the other, would describe the same world, and the same handed objects within it. The hand considered in itself, in an otherwise empty space, would be neither left nor right handed. . . Of course it has turned out that spatial inversion is not a symmetry of the standard model, but an argument to a similar effect remains: the combination of space, time, and matterantimatter inversion (TCP symmetry) is demonstrably a symmetry of any relativistic quantum theory. From Leibniz Equivalence, it follows that the world does not have one TCP orientation rather than the other. Its mirror image, on inverting matter and antimatter and the arrow of time, is identically the same. (Saunders 2003)

8.3.2 Two toy models

Model 1

- Everything is either a red or a green hand.
- Red hands are never created, but can "decay" into green hands.
- Only red 'left' hands can decay into green 'left' hands.

- Red 'right' hands never decay and no green 'right' hands exist at all.

Model 2

- Red hands are 'charged'; they attract or repel one another.
- Both left and right red hands can be 'negatively' and 'positively' charged.
- Only 'negatively' charged red left hands decay into green (left) hands.
- Only 'positively' charged right hands decay into green right hands.

8.3.3 The relational content of the laws

1. All red hands which decay into green ones are *handed in the same way*.
2. Red hands that *repel each other* and which can decay into green hands are handed in the same way; red hands that attract each other and which can decay into green hands are handed in the opposite way.

- There are two types of electrons (leptons of a certain mass that couple to photons), distinguished in terms of the sets of interactions (sets "A" and "B") in which they can participate (these can be specified relationally).
- All electrons which are involved in A-type interactions are *handed in the same way*; similarly for B-type electrons. A-type and B-type electrons are oppositely handed.

8.4 Parity violation and Dirac spinors

The massless Dirac field obeys:

$$i\gamma^\mu \partial_\mu \psi(x) = 0$$

derivable from the Lagrangian density:

$$\mathcal{L}(x) = \overline{\psi}(x) i\gamma.\partial\psi(x)$$

These are invariant under parity:

$$\psi(\vec{x}, t) \mapsto U(P)\psi(\vec{x}, t)U(P) = \eta_P \gamma^0 \psi(-\vec{x}, t), \eta_P^2 = 1$$

8.4.1 The 'left' and 'right' chiral components

Define:

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi \qquad \psi_R = \frac{1}{2}(1 + \gamma_5)\psi$$

where:

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \Rightarrow \{\gamma^\mu, \gamma_5\} = 0.$$

The chiral subscripts are appropriate since:

$$\gamma^0(1 \pm \gamma_5) = (1 \mp \gamma_5)\gamma^0$$

so:

$$U(P)P_L\psi(x)U(P) \equiv \eta_P\gamma^0 P_L\psi(x_P) = P_R\eta_P\gamma^0\psi(x_P) = P_R U(P)\psi(x)U(P)$$

Also, consider:

$$\psi(x) = \sum_{p,\lambda}[a(p,\lambda)u(p,\lambda)e^{-ip.x} + b(p,\lambda)^\dagger v(p,\lambda)e^{ip.x}]$$

where:

$$\gamma.p\, u(p,\lambda) = 0 \;\Rightarrow\; (1 - 2\mathbf{S}.\hat{\mathbf{p}}\gamma_5)u(p,\lambda) = 0.$$

One has:

$$(1 + 2\mathbf{S}.\hat{\mathbf{p}})u_L(p,\lambda) = 0, \quad (1 - 2\mathbf{S}.\hat{\mathbf{p}})u_R(p,\lambda) = 0.$$

The chiral components separately obey the massless Dirac equation:

$$i\gamma^\mu\partial_\mu\psi_L = 0, \quad \text{and} \quad i\gamma^\mu\partial_\mu\psi_R = 0.$$

The *helicity* of a particle is the projection of its spin in its direction of motion. Helicity eigenstates of a spin-$\frac{1}{2}$ particle involve the spin being either aligned or anti-aligned with the particle's direction of motion. These constitute two incongruent, 'handed' objects. By definition, *left-handed* massless particles are particles of negative helicity (their spin is opposite to their direction of motion), while *right-handed* particles are particles of positive helicity.

The difference between negative and positive helicity is very much like the difference between left and right. One cannot *describe* which is which, one must ultimately demonstrate the difference.

8.4.2 An explicit representation

In the chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}.$$

We have:

$$i(\partial_0 - \sigma.\nabla)\psi_L = 0; \quad i(\partial_0 + \sigma.\nabla)\psi_R = 0$$

and under infinitesimal Lorentz transformations:

$$\psi_L \mapsto (1 - i\theta.\frac{\sigma}{2} - \beta.\frac{\sigma}{2})\psi_L; \quad \psi_R \mapsto (1 - i\theta.\frac{\sigma}{2} + \beta.\frac{\sigma}{2})\psi_R.$$

NB: this is all consistent with there being no intrinsic difference between ψ_L and ψ_R.

8.5 Electro-weak theory

Consider just the electron and its neutrino:

$$\mathcal{L}_{\text{kin}}(x) = \overline{\psi_e} i\gamma.\partial\psi_e + \overline{\psi_\nu} i\gamma.\partial\psi_\nu \ .$$

Rewrite as:

$$\mathcal{L}(x) = \overline{L}(x) i\gamma.\partial L(x) + \overline{R}(x) i\gamma.\partial R(x) \ .$$

where L is a *doublet* of the left-handed components of the neutrino and electron fields: $L(x) = \binom{\nu_L(x)}{e_L(x)}$, and R is a singlet involving the right-handed component of the electron field: $R(x) = e_R(x)$.

$\mathcal{L}$ is invariant under the $SU(2)$ transformations:

$$L(x) \to e^{\frac{1}{2}i\alpha.\tau} L(x), \quad \overline{L}(x) \to e^{-\frac{1}{2}i\alpha.\tau} \overline{L}(x), \quad R(x) \to R(x) \ ,$$

It is also invariant under independent $U(1)$ phase transformations of L and R separately. In particular, under $L \to e^{-i\frac{1}{2}\chi} L$ and $R \to e^{-i\chi} R$ which may be written $\psi \to e^{i\chi Y}\psi$ where $Y = -\frac{1}{2}$ for L and $Y = -1$ for R.

A *local* $SU(2) \times U(1)$ transformation is written:

$$\psi \to e^{i\alpha(x).\mathbf{T} + i\chi(x)Y} \psi(x)$$

where $\mathbf{T} = \frac{1}{2}\tau$ acting on L and $\mathbf{T} = \mathbf{0}$ acting on R.

For a gauge invariant Lagrangian, replace the derivatives with the covariant derivatives:

$$D_\mu = \partial_\mu - ig\mathbf{A}_\mu(x).\mathbf{T} - ig' B_\mu(x) Y \ .$$

The result is:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + g\overline{L}\gamma^\mu \frac{1}{2}\tau L.\mathbf{A}_\mu + g'(\frac{1}{2}\overline{L}\gamma^\mu L + \overline{R}\gamma^\mu R) B_\mu \ .$$

Since R is a singlet under the chosen $SU(2)$ transformations, it does not couple to the three component fields of $\mathbf{A}_\mu$ at all. That the left and right fields are assigned different values of "weak hypercharge" Y means that the strengths of their coupling to the $U(1)$ gauge field B_μ is different.

Coupling to the Higgs field leaves just one massless gauge boson. The 'physical' gauge fields corresponding to gauge bosons of definite mass are given by

$$W_\mu = \frac{1}{\sqrt{2}}(A_{1\mu} - iA_{2\mu}); \quad Z_\mu = \cos\theta_W A_{3\mu} - \sin\theta_W B_\mu; \quad A_\mu = \sin\theta_W A_{3\mu} - \cos\theta_W B_\mu \ ,$$

where $\tan\theta_W = g'/g$. With $e \equiv g\sin\theta_W$, the Lagrangian is rewritten:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \frac{g}{2\sqrt{2}}(J^\mu W_\mu + J^{\mu\dagger} W_\mu^\dagger) + e j^\mu_{e.m.} A_\mu + \frac{g}{2\cos\theta_W} J^\mu_n Z_\mu \ ,$$

$$\begin{aligned} \text{where} \quad J^\mu(x) &= \overline{L}(x)\gamma^\mu(\tau_1 + i\tau_2)L(x) = \overline{\nu}(x)\gamma^\mu(1-\gamma_5)e(x) \\ j^\mu_{e.m.} &= \overline{L}\gamma^\mu\frac{1}{2}(\tau_3 - 1)L - \overline{R}\gamma^\mu R = -\overline{e}\gamma^\mu e \\ J^\mu_n &= \overline{L}\gamma^\mu\left(\cos^2\theta_W \tau_3 + \sin^2\theta_W 1\right)L - 2\sin^2\theta_W\theta_W \overline{R}\gamma^\mu R \\ &= \frac{1}{2}\left[\overline{\nu}\gamma^\mu(1-\gamma_5)\nu - \overline{e}\gamma^\mu(1-\gamma_5 - 4\sin^2\theta_W)e\right] \ . \end{aligned}$$

8.6 Open questions

1. Does the Standard Model suggest that left- and right-handed particles differ intrinsically? One might view, e.g., hypercharge as an intrinsic property but that does not suggest that "being left-handed" is an intrinsic matter.

2. Does the theory indicate the existence of extra spacetime structure? As Huggett stresses, the coordinate-free expression of the physics introduces an orientation field. But one does not need to reify such a field in order to make sense of the theory (recall the toy models). However, without such a field, there is a *type* of 'non-locality' (recall the Newtonian mechanics analogy).

3. Are there problems with regarding such a field as physical?

   - Is it explanatory?
   - Could it be made dynamical. . . could it be quantized!?
   - It introduces *ineliminable* "differences without differences"; do we really want to say that worlds just like ours except that in them *right-handed* electrons couple to *W*s are genuinely distinct possibilities, obeying different laws?

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