Robust control of heterogeneous vehicular platoon with uncertain dynamics and communication delay

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Abstract: Platoon formation of highway vehicles has the potential to significantly enhance road safety, improve highway utility, and increase traffic efficiency. However, various uncertainties and disturbances that are present in real-world driving conditions make the implementation of vehicular platoon a challenging problem. This study presents an H-infinity control method for a platoon of heterogeneous vehicles with uncertain vehicle dynamics and uniform communication delay. The requirements of string stability, robustness and tracking performance are systematically measured by the H-infinity norm, and explicitly satisfied by casting into the linear fractional transformation format. A delay-dependent linear matrix inequality is derived to numerically solve the distributed controllers for each vehicle. The performances of the controlled platoon are theoretically analysed by using a delay-dependent Lyapunov function which includes a linear quadratic function of states during the delay period. Simulations with a platoon of heterogeneous vehicles are conducted to demonstrate the effectiveness of the proposed method under random parameters and external disturbances.

1 Introduction

The platooning of autonomous vehicles for highway offers many benefits, e.g. enhanced road safety, improved highway utility and increased fuel economy [1]. These benefits are provided by ensuring that all the vehicles move at a harmonised speed with desired spatial distributions [1]. The pioneering work of platoon control dates back to the 1990s. An earlier work was done by PATH in California, which introduced many well-known topics in terms of configuration, stability, platoon performances, and so on [1,2].

As pointed out by Hedrick et al. [1] the control topics of a platoon are divided into two tasks: (i) to implement safe and efficient control of platoon formation, stabilisation and dissolution; (ii) to carry out controls for throttle/brake actuators of each vehicle via feedback control law. The first task coordinates the movement of a group of neighbouring vehicles, while the second task concerns the motion of each individual vehicle. A platoon system can be considered as a combination of node dynamics, information flow topology, distributed controllers and formation geometry [3–7]. Many advanced control methods have been introduced into platoon control since 1990s, for instance, Sheikhholeslam and Desoer [8], Rajamani et al. [9], Zhou and Peng [10], Barooah et al. (2009) [11], Desjardins and Chaib-draa [12], Dunbar and Caveney [13] and Zheng et al. [14]. Some real-world applications of platoon control have been demonstrated recently, including the GCDC in the Netherlands [15], SARTRE in Europe [16] and Energy-ITS in Japan [17]. A recent review on platoon control can be found in [5].

Two practical challenges for current development of platoon control are how to properly address the uncertain node dynamics and how to handle the emerging vehicle-to-vehicle (V2V) communications. Short safety gaps and string stable operations can be guaranteed when linear platoon models without disturbances are available [5]. Unfortunately, the accurate models of engine, clutch, gearbox, wheel/tyre and braking systems are often non-linear. Moreover, these models also depend on changing conditions of the ageing vehicle and varying environment/roads. Many existing research has been focused on radar-based information flow topologies, which are not suitable since large deployment of V2V communications such as dedicated short range communications (DSRC), vehicular ad-hoc network (VANET), and mobile ad-hoc network (MANET) [18,19]. The V2V communications generate more abundances on topology types, including two-predecessor following type, two-predecessor-leader following type and the bidirectional-leader type [7,19]. In addition, communication networks, however, generally introduce time delays and drop packets. For example, Liu and Goldsmith [20] examined the robustness of longitudinal controller designs and found out that string stability is seriously compromised by communication delays. Guo and Yue [21] investigated the controller design for automated vehicles whose sensors have limited sensing capability. Hakan and Paolo [22–24] gave out a multi-objective H-infinity synthesis method for a homogeneous platoon with the leader and predecessor following communication topology.

The platoon control is more challenging when simultaneously considering the uncertain dynamics and communication delays which are both present in current real-world driving conditions. To the best of our knowledge, few studies systematically take into account the heterogeneity, dynamic uncertainties and uniform communication delay. There have been attempts to optimise a string of vehicles with respect to quadratic criterion including several performance requirements, e.g. Stankovic et al. [4]; however, the resulting control strategies are not in a way of directly robust design. The purpose of this paper is to develop an H-infinity control method for heterogeneous platoon, which can guarantee both robust stability to parameter uncertainties and uniform communication delay, while achieving the desired tracking performance and string stability requirement.

The remainder of this paper is organised as follows: Section 2 introduces the modelling of a heterogeneous platoon; Section 3 describes the control objectives for the platoon; Section 4 synthesises the H-infinity controller for each vehicle. The analyses of closed-loop control performance are given in Section 5. Section 6 demonstrates its effectiveness with a heterogeneous platoon, and Section 7 concludes this paper.
2 Uncertain model for vehicle dynamics
2.1 Uncertainty and disturbance analysis

A platoon is said to be homogeneous if all vehicles have identical dynamics; otherwise it is heterogeneous. In reality, a vehicular platoon would be more likely to be heterogeneous since different types of vehicles might be involved. Here, we consider a platoon composed of a series of non-identical passenger cars, shown in Fig. 1.

The powertrain structures of vehicle longitudinal dynamics are assumed to be similar and all of them include engine, driveline, brake system, aerodynamic drag, tire friction, rolling resistance and gravitational force, and so on [25, 26]. The full non-linear model of vehicle i is expressed as

\[
\begin{align*}
\tau_{e,i} \dot{\omega}_{e,i} &= T_{e,i} - T_{e,i}, & T_{e,i} &= \text{MAP}_i\left(\omega_{e,i}, \alpha_i\right), \\
J_{e,i} \dot{\omega}_{e,i} &= T_{e,i} - T_{b,i}, & T_{b,i} &= C_{TC,i}\left(\omega_{e,i}\right)\omega_{e,i}, \\
M_i \ddot{v}_i &= T_{f,i} - F_{a,i} - F_{f,i} - F_{s,i}, \\
\tau_{b,i} \dot{\omega}_{b,i} &= K_{b,i} P_{b,i} - T_{b,i}, & T_{b,i} &= \eta_{b,i} J_{b,i} \omega_{b,i}, \\
T_{f,i} &= K_{TC,i}\left(\omega_{b,i}\right)\omega_{b,i}, & \omega_{b,i} &= i_{g,i} J_{b,i} \frac{v_i}{r_i}, \\
F_{a,i} &= \frac{1}{2} \rho C_{w,i} A_i \left(v_i + v_w\right)^2, & F_{f,i} &= M_i g \cos(\theta) f_r, \\
F_{s,i} &= M_i g \sin(\theta),
\end{align*}
\]

where the subscript \(i\) represents the \(i\)th vehicle and definitions of the parameters and variables in (1) are listed in Table 1. This full non-linear model will be used to validate the proposed robust controller in Section 6.

For the sake of controller design, it is further assumed that: (i) the tire longitudinal slip is negligible, and the powertrain dynamics are lumped into a first-order inertial transfer function; (ii) the vehicle body is considered to be rigid and symmetric; and (iii) the influence of pitch and yaw motions is neglected. Then, the model of vehicle i is simplified to be

\[
\begin{align*}
T_{d,i} &= \frac{1}{\tau_d + 1} T_{\text{const},i}, \\
\dot{v}_i &= \frac{1}{M_i} \left[ \frac{T_{d,i} R_{el,i} \eta_{i}}{\tau_d} - \frac{1}{2} \rho C_{w,i} A_i \left(v_i + v_w\right)^2 \right] - g \sin(\theta), \\
\dot{\theta}_i &= f_r \cos(\theta) + \sin(\theta),
\end{align*}
\]

where \(T_{d,i}\) is the lumped torque generated by engine or braking system, \(T_{\text{const},i}\) is the driving torque command, \(R_{el}\) is the gear ratio of the drive-train, \(\theta_i\) is the equivalent road resistance coefficient and \(\tau_i\) represents the time constant of powertrain dynamics. As a heterogeneous platoon, all involved cars have different parameters, and moreover these parameters vary with operating conditions, listed in Table 2 [27, 28]. In addition, external disturbances such as wind gust and road slope also influence the stability and car-following performance of the platoon. The typical probability density of wind speed and road slope is demonstrated in Fig. 2 [29, 30]. Here, instead of directly using probability function, we assume that both these disturbances are bounded in a specific threshold, which corresponds to the values of 95% confidence level.

As an evidence to show how large the uncertainties can be, the uncertain range of frequency responses from torque command \(T_{\text{com}}\) to speed \(v\) is drawn in Fig. 3 by using linearised model at different equilibrium point. It is found that vehicle dynamics are greatly affected by the heterogeneity and model uncertainty. It is well-known that even small model perturbation can lead to the instability of closed-loop control system. Therefore, it is necessary to consider the robustness of closed-loop stability, string stability and tracking performance when designing the platoon controller.

Table 1 Definition of parameters and variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>throttle position</td>
</tr>
<tr>
<td>(T_{\text{com}})</td>
<td>steady-state engine torque</td>
</tr>
<tr>
<td>(T_e)</td>
<td>transient engine torque</td>
</tr>
<tr>
<td>(T_f)</td>
<td>pump torque</td>
</tr>
<tr>
<td>(T_t)</td>
<td>turbine torque</td>
</tr>
<tr>
<td>(K_{TC})</td>
<td>torque ratio</td>
</tr>
<tr>
<td>(\eta)</td>
<td>final gear ratio</td>
</tr>
<tr>
<td>(r)</td>
<td>tyre radius</td>
</tr>
<tr>
<td>(T_s)</td>
<td>driving force</td>
</tr>
<tr>
<td>(v)</td>
<td>vehicle speed</td>
</tr>
<tr>
<td>(C_a)</td>
<td>coefficient of aerodynamic drag</td>
</tr>
<tr>
<td>(\eta_C)</td>
<td>time constant of braking system</td>
</tr>
<tr>
<td>(\rho)</td>
<td>air density</td>
</tr>
<tr>
<td>(g)</td>
<td>gravity constant</td>
</tr>
<tr>
<td>(v_w)</td>
<td>wind speed</td>
</tr>
<tr>
<td>(a_g)</td>
<td>engine speed</td>
</tr>
<tr>
<td>(\tau_e)</td>
<td>time constant of engine dynamics</td>
</tr>
<tr>
<td>(M)</td>
<td>engine torque characteristics</td>
</tr>
<tr>
<td>(M_p)</td>
<td>inertia of flywheel</td>
</tr>
<tr>
<td>(\eta_C)</td>
<td>pump capacity coefficient</td>
</tr>
<tr>
<td>(\eta_g)</td>
<td>gear ratio</td>
</tr>
<tr>
<td>(\eta_{\text{mech}})</td>
<td>mechanical efficiency of driveline</td>
</tr>
<tr>
<td>(M_i)</td>
<td>vehicle mass</td>
</tr>
<tr>
<td>(T_b)</td>
<td>braking torque</td>
</tr>
<tr>
<td>(F_s)</td>
<td>aerodynamic drag</td>
</tr>
<tr>
<td>(K_s)</td>
<td>total braking gain</td>
</tr>
<tr>
<td>(F_r)</td>
<td>rolling resistance</td>
</tr>
<tr>
<td>(A_i)</td>
<td>rolling resistance</td>
</tr>
<tr>
<td>(A_i)</td>
<td>front cross-area</td>
</tr>
<tr>
<td>(f_r)</td>
<td>coefficient of rolling resistance</td>
</tr>
<tr>
<td>(\theta)</td>
<td>road slope</td>
</tr>
</tbody>
</table>

Table 2 Range of model parameters for typical passenger cars [27, 28]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Nominal</th>
<th>Minimum (%)</th>
<th>Maximum (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_i)</td>
<td>kg</td>
<td>1400</td>
<td>800</td>
<td>2000</td>
</tr>
<tr>
<td>(\eta_{\text{e}})</td>
<td>–</td>
<td>0.92</td>
<td>0.86</td>
<td>0.98</td>
</tr>
<tr>
<td>(\tau_{\text{e}})</td>
<td>s</td>
<td>0.5</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>(\eta_{\text{f}})</td>
<td>–</td>
<td>0.34</td>
<td>0.29</td>
<td>0.39</td>
</tr>
<tr>
<td>(A_i)</td>
<td>m²</td>
<td>2.24</td>
<td>1.58</td>
<td>2.9</td>
</tr>
<tr>
<td>(l_{f})</td>
<td>m</td>
<td>0.012</td>
<td>0.01</td>
<td>0.014</td>
</tr>
<tr>
<td>(\theta)</td>
<td>deg.</td>
<td>0</td>
<td>–17.0°</td>
<td>17.0°</td>
</tr>
<tr>
<td>(v_w)</td>
<td>m/s</td>
<td>0</td>
<td>–5.9°</td>
<td>5.9°</td>
</tr>
</tbody>
</table>

– a: Hilly road
– b: Highway and expressway
2.2 Vehicle uncertain model

The heterogeneity of node dynamics is considered as model uncertainties on the average model. The inverse model compensation technique is used to eliminate the non-linearities in longitudinal dynamics for the purpose of controller design [4, 7, 15, 20]

$$T_{com} = \left( M_{i0} u_i + \frac{1}{2} p C_w A_i v_i^2 + M_{i0} g f_i \right) \frac{R_i}{g_i},$$

where $M_{i0}$ and $f_{i0}$ are the nominal parameters for node $i$, $R_i$ and $v_i$ are measured by onboard sensors, $u_i$ is the new control input after linearisation. Note that this inverse model compensation technique cannot be accurately implemented, because many parameters cannot be accurately measured, such as the friction force and efficiency coefficient. The linearised longitudinal dynamics of node $i$ becomes

$$a_i = k_i u_i + \epsilon_i,$$

where $k_i$ is the system gain and $\epsilon_i$ is the external disturbance arising from the uncertainties of vehicle parameters and environmental factors

$$k_i = \frac{M_{i0}}{M_i},$$

$$\epsilon_i = \frac{p C_w A_i}{2M_i} \left( 2v_i v_0 + v_0^2 - \frac{\tau_i}{\tau_i + 1} v_i^2 \right) + g \left( \frac{k_i}{\tau_i + 1} f_0 - s_i \right).$$

On the basis of (4), the state space model of node $i$ in the platoon is

$$\delta_{\xi} = D_i - D_0,$$

$$\delta u_i = v_{i-1} - v_i,$$

$$a_i = k_i u_i / \tau_i - a_i / \tau_i + d_i,$$

where $D_i$ is the distance between any two consecutive vehicles, $D_0$ is the desired distance, $\delta_{\xi}$ is the distance error, $\delta_{\xi}$ is the relative speed and $d_i = (s + 1/\tau_i) e_i$. Define a state vector $X_{Vi} = [\delta_{\xi} \delta_{\xi} a_i]$ for node $i$. Moreover, its dynamics can be rewritten into a compact form

$$X_{Vi} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} X_{Vi} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_i + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} X_{Vi(-1)} + \begin{bmatrix} 0 \\ 0 \\ d_i \end{bmatrix},$$

where $b_i \in \mathbb{R}$ represents the system gain at steady-state condition of node $i$, $a_i \in \mathbb{R}$ represents the time lag in the powertrain dynamics of node $i$, defined as

$$b_i = \frac{k_i}{\tau_i}, \quad a_i = \frac{1}{\tau_i}.$$

The two parameters $a_i$ and $b_i$ are bounded by the following equation

$$a_i \in \left[ \frac{1}{\tau_i}, \frac{1}{\tau_i} \right], \quad b_i \in \left[ \frac{k_i}{\tau_i}, \frac{k_i}{\tau_i} \right].$$

Fig. 2 Distribution of external disturbances [29, 30]

- a Probability density of wind speed of four sites in South Dakota: Alfred, Green River, Olga and Ray/wheelock
- b Probability density of road slope of typical hilly road and highway

Fig. 3 Uncertainty of frequency response of linearised model

- a Amplitude
- b Phase

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where \# symbols are the upper and lower bounds of parameters, respectively, shown in Table 2. Note that \( d_i \) is considered as external disturbance, which will be attenuated by multiplied with a weighting function. Equations (7)–(9) are actually taken as the uncertain model for robust controller design.

### 3 Problem description of platoon control

A central task of platoon control is to improve the traffic flow capacity while ensuring safety. Therefore, string stability becomes the primary performance requirement, which requires the spacing errors decrease as they propagate along the vehicle stream. For a string unstable platoon, so-called ‘slinky’ effect will happen and thus cause potential traffic jam and even rear-end collision [3, 6, 7, 9]. It is well-known that string stability is guaranteed when the frequency characteristic of external disturbance \( d \) in this application, \( d_i \) is in general distributed in the low-frequency range, and so the parameter \( a \) and \( b \) are selected to attenuate the low-frequency energy. A bigger cutting frequency of \( w_p(s) \) implies a higher requirement of tracking performance. The values of \( k_d \), \( k_a \), and \( k_i \) are determined according to the optimal requirement of \( \delta_i \), \( \delta_x \), and \( a_i \), respectively. A bigger weighting coefficient implies more stringent optimal requirement of the corresponding state. Its state-space transfer function is

\[
X_{i+1} = A_{i}X_{i} + B_{i}X_{i},
\]

\[
q_{i} = C_{i}X_{i} + D_{i}X_{i}.
\]

Then, the disturbance attenuation ability (Req. 2) is equivalent to

\[
\frac{\|q_{i}\|}{\|d_{i}\|} < 1.
\]

A practical platoon control approach needs to ensure both internal stability and string stability with a sufficient level of performance [31]. The string stability in the sense of \( \ell_{2} \) norm was mathematically formalised by Swaroop and Hedrick [3]. If \( \ell_{2} \) string stability holds for a platoon, there exists a bounded set of initial states such that the maximum spacing error remains uniformly bounded. A weaker condition is \( \ell_{2} \) string stability. It guarantees that the energy of the spacing errors does not amplify along the platoon. The two conditions are equivalent for a homogeneous platoon [31]. Here according to the \( \ell_{2} \) string stability definition, Req. 3 is designed to be [21]

\[
\frac{\|\delta_{i}\|}{\|\delta_{i}\|_{12}} < 1.
\]

Referring to the \( \mu \)-synthesis method, Req. 1–Req. 3 described by (12), (15) and (16), respectively, is equivalent to the robust stabilisation problem by introducing a virtual perturbation as shown by Fig. 4 [32, 33]. Then, the objective is to find a state feedback controller, such that the \( H_{\infty} \) norm of the closed-loop system is smaller than one. The model of on-vehicle platoon and the state feedback controller are described by (19) and (21), respectively.

To design the \( H_{\infty} \) controller, the uncertain model of node \( i \) is converted to the linear fractional transformation (LFT) format

\[
X_{i} = A_{i}X_{i} + A_{i}X_{i+1} + B_{i}u_{i} + B_{i}w_{i},
\]

\[
z_{i} = C_{i}X_{i} + D_{i}u_{i},
\]

where (see (18))

\[
A_{i} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & \frac{(\omega + \omega)}{2} \end{bmatrix}, \quad A_{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_{i} = \begin{bmatrix} 0 \\ 0 \\ \frac{b + \beta}{2} \end{bmatrix},
\]

\[
B_{i+1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad B_{i+2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C_{i} = \begin{bmatrix} 0 & 0 & \frac{(\omega - \omega)}{2} \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{i} = \begin{bmatrix} 0 \\ \frac{(\alpha + \beta)}{2} \end{bmatrix},
\]
The vehicular platoon is controlled by a distributed state feedback

\[ u_i(t) = k_{ix}X_i(t) + \sum_{j=1}^{N} k_{ij}X_j(t - t_{ij}), \quad i = 1, \ldots, N, \]  

which subjects to \( T_{\text{TH}}(s) < 1 \), where \( T_{\text{TH}}(s) \) is the transfer function from \( W \) to \( Z \). The symbol \( t_{ij} \) is the uniform communication delay, satisfying \( 0 < t_{ij} \leq h \). The communication delay incurred in platoons is highly dependent on the network architecture and underlying wireless channel. It also depends on how the control law is executed. If the controller is designed to update its output at the receipt of needed information, the communication delay among vehicles can be arbitrary. This is because the data sources are distributed and the capacity of the wireless channel is interference-limited. Thus, we cannot have all the vehicles transmit data packets simultaneously in a reliable way. Another possible approach is to install a universal clock in each vehicle of the platoon so that vehicles can be synchronised to update their controllers at the same time [20].

4 Synthesis of H-infinity controllers

The Lyapunov theorem and integral inequality are used to derive the delay-dependent condition for H-infinity performance of the vehicular platoon [34, 35]. Before giving the main theorem, we first introduce the following lemmas:

Lemma 1: If we define

\[ V[X(t)] = \int_{-h}^{0} X^T(\alpha)QX(\alpha)\,d\alpha \]

and \( Q^T = Q > 0 \), then the derivative of \( V[X(t)] \) is

\[ \dot{V}[X(t)] = hX^T(t)QX(t) - \int_{-h}^{0} X^T(\alpha)QX(\alpha)d\alpha \]

Proof:

\[ V[X(t)] = \int_{-h}^{0} \left[ \int_{0}^{t} X^T(\tau)QX(\tau)\,d\tau \right] \,d\beta - \int_{-h}^{0} \left[ \int_{0}^{t} X^T(\alpha)QX(\alpha)d\alpha \right] \,d\beta, \]

\[ = \int_{-h}^{0} \int_{0}^{t} X^T(\tau)QX(\tau)d\beta \,d\tau - \int_{-h}^{0} \int_{0}^{t} X^T(\alpha)QX(\alpha)\,d\alpha \,d\beta, \]

\[ = h \int_{0}^{t} X^T(\alpha)QX(\alpha)\,d\alpha - \int_{-h}^{0} \int_{0}^{t} X^T(\alpha)QX(\alpha)\,d\alpha \,d\beta. \]
Lemma 2: If \( Q^T Q > 0 \), for any signal vector \( X(t) \), we have

\[
- \int_{t-h}^{t} \dot{X}(\alpha) Q X(\alpha) d\alpha \leq \left[ X(t) X(t-h) \right]^T \left[ \begin{array}{cc} -Q & Q \\ Q & -Q \end{array} \right] \left[ \begin{array}{c} X(t) X(t-h) \end{array} \right].
\]

Proof: The state feedback control logic is rewritten as

\[
U(t) = K_1 X(t) + K_2 X(t - t_d).
\]

The closed-loop system is

\[
\dot{X}(t) = A_d X(t) + A_d X(t - t_d) + B_d W, \quad Z = C_d X(t) + D_d X(t - t_d),
\]

where

\[
A_d = A + B_1 K_1, \quad C_d = C + D K_1, \quad A_d = B_2 K_2, \quad D_d = D K_2.
\]

Define a Lyapunov function

\[
V[X(t)] = V_1[X(t)] + V_2[X(t)] + V_3[X(t)],
\]

\[
V_1[X(t)] = X^T(t) M_1^{-1} X(t), \quad V_2[X(t)] = \int_{t-t_d}^{t} \int_{\alpha}^{t} X^T(\tau)(M_1^{-1} M_2^* + e I) X(\tau) d\tau d\alpha,
\]

\[
V_3[X(t)] = \int_{t-t_d}^{t} X^T(\tau)(M_1^{-1} M_2^* + e I) X(\tau) d\tau.
\]

The derivative of \( V_1[X(t)] \) is

\[
\dot{V}_1[X(t)] = X^T(t) M_1^{-1} \dot{X}(t) + \dot{X}^T(t) M_1^{-1} X(t).
\]

Substituting (28) into (30) yields (see (31))

\[
\dot{V}_1[X(t)] \leq e \dot{h} X^T(t) X(t) - e \int_{t-t_d}^{t} X^T(\tau)(e I) X(\tau) d\tau.
\]

By Lemma 1, the derivative of \( V_2[X(t)] \) is

\[
\dot{V}_2[X(t)] = \dot{e} \dot{h} X^T(t) X(t) - \dot{e} \int_{t-t_d}^{t} X^T(\tau)(e I) X(\tau) d\tau.
\]

By Lemma 2, inequality (33) is obtained (see (33))

The derivative of \( V_3[X(t)] \) is

\[
\dot{V}_3[X(t)] = X^T(t) (M_1^{-1} M_2^* + e I) X(t).
\]

Combining (31), (33) and (34), \( \dot{V}[X(t)] \) satisfies (see equation (35) at the bottom of the next page)
Substituting (35) into (36), \( T_{zw}(s) < \gamma \) is held only if
\[
\begin{bmatrix}
\si A_{cl}^T A_d + \si A_{cl}^T M_1^{-1} A_d + A_{cl}^T M_1^{-1} + M_1^{-2} M_2^2 + C_{cl}^T C_{cl} \\
\si e I + A_{cl}^T M_1^{-1} + D_{cl}^T C_{cl} \\
B_{cl}^T M_1^{-1} + \si e h B_{cl}^T A_d \\
\end{bmatrix} \\
\times \begin{bmatrix}
\si A_{cl}^T A_d - e I + D_{cl}^T D_d \\
\si e h A_{cl}^T B_d \\
\end{bmatrix} < 0.
\]  
(37)

By using the Schur supplement theorem, (37) is converted to (38).

Multiplying \( T = \text{diag}(M_1 \ I \ I \ I \ I \ I) \), inequality (38) is equivalent to linear matrix inequality (LMI) (39). (see (36))

\( \dot{V}[X(t)] \leq \begin{bmatrix} X(t) & X(t) \end{bmatrix}^T \begin{bmatrix}
\si A_{cl}^T A_d + \si A_{cl}^T M_1^{-1} A_d + A_{cl}^T M_1^{-1} + M_1^{-2} M_2^2 + C_{cl}^T C_{cl} \\
\si e I + A_{cl}^T M_1^{-1} + D_{cl}^T C_{cl} \\
B_{cl}^T M_1^{-1} + \si e h B_{cl}^T A_d \\
\end{bmatrix} \begin{bmatrix}
X(t) & X(t) \\
X(t) & X(t) \end{bmatrix} + \begin{bmatrix}
\si A_{cl}^T A_d - e I + D_{cl}^T D_d \\
\si e h A_{cl}^T B_d \\
\end{bmatrix} < 0. \)
(35)

\[
\int_0^1 [Z(\tau)]^2 - \gamma^2 |W(\tau)|^2 \, d\tau = \int_0^1 \left[ \frac{1}{2} \dot{V}[X(\tau)] + V[X(0)] - V[X(\tau)] \right] \, d\tau \\
= \int_0^1 \begin{bmatrix}
M_1^{-1} A_d + A_{cl}^T M_1^{-1} + D_{cl}^T C_{cl} & 0 \\
0 & C_{cl}^T C_{cl} \\
\end{bmatrix} \begin{bmatrix}
X(t) & X(t) \\
X(t) & X(t) \end{bmatrix} + \begin{bmatrix}
A_{cl}^T A_d & A_{cl}^T M_1^{-1} A_d + C_{cl}^T D_{cl}^T & M_1^{-1} M_2 \\
0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
X(t) & X(t) \\
X(t) & X(t) \end{bmatrix} \, d\tau + V[X(0)] - V[X(\tau)]. 
\]  
(36)

Substituting (26) into (39), it is found that LMI (39) is identical to LMI (25). LMI (39) means that the H-infinity norm from \( W \) to \( Z \) is smaller than \( \gamma \), which measures the platoon performances of robust stability, disturbance attenuation ability and string stability simultaneously.

Remark 1: When constructing LMI (25), the constraint of communication topology is realised by properly defining the structure of matrix \( N_2 \). If vehicle \( i \) communicates with vehicle \( j \), \( N_2(i, j) = [0 \ 0 \ 0] \); otherwise \( N_2(i, j) = [0 \ 0 \ 0] \).

Remark 2: The main theorem is a sufficient condition for the robust performance state feedback controller and the conversation can be reduced by using the structure information of the virtual perturbation in Fig. 4. Since the virtual perturbation has a diagonal structure, it is invariant if multiplied with another diagonal matrix and the inverse on both sides. Then, the state feedback controller

\[
\begin{bmatrix}
A_{cl}^T A_d + A_{cl}^T M_1^{-1} + D_{cl}^T C_{cl} & 0 \\
0 & C_{cl}^T C_{cl} \\
\end{bmatrix} - \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\end{bmatrix} < 0. 
\]  
(38)

\[
\begin{bmatrix}
A_{cl}^T A_d + A_{cl}^T M_1^{-1} + D_{cl}^T C_{cl} & 0 \\
0 & C_{cl}^T C_{cl} \\
\end{bmatrix} - \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\end{bmatrix} < 0. 
\]  
(39)
can be further optimised by using the $D$-$K$ procedure of $\mu$-synthesis and the detail can be found in [32].

**Remark 3:** One shortcoming of the proposed method is that the synthesis procedure depends on the platoon length, which means that the re-calculation of robust controllers might be needed as vehicles enter or depart from the platoon. In practical use, one remedy for look-ahead topologies is to define the maximum allowable length $N_{\text{max}}$. The state feedback coefficient is solved numerically by using LMI (25) for the platoon with maximum length. When the platoon length $N$ is smaller than $N_{\text{max}}$, only the control input calculated from the state of vehicle whose index is not greater than $N$ is used.

**Remark 4:** Since $T_{\text{Zw}}(s)\omega_{\infty} < 1$ establishes for any $W$, if $w_j = 0, j = i + 1, \ldots, N$, we have $\delta d_{i2}^2 < \delta d_{i1}^2 + \sum_{j=1}^{N} \left( W_{ij}^2 + Z_{ij}^2 + d_{j1}^2 \right)$. This inequality means that $\delta d_{i2}$ is uniformly bounded by $\delta d_{i1}$ and the perturbations arising from $\Delta_{ij}$ and external disturbances.

5 Simulations and discussions

To validate the performance of the proposed control method, a series of simulations were performed for a heterogeneous platoon. Besides a leading vehicle, there are ten followers. Each is selected to be a passenger car with a 1.6 l gasoline engine, a torque converter, a six-speed automatic transmission, two driving and two driven wheels, as well as a hydraulic braking system [36]. The follower is assumed to be able to communicate with the leader and two vehicles in front of it, which is so-called the two-predecessor-leader following typed communication topology [7]. The heterogeneity is reflected by different vehicle mass and time lag of powertrain dynamics. Constant spacing strategy is used to formulate the platoon, with desired distance as 20 m. As shown in Fig. 5, the leading vehicle runs with a naturalistic acceleration profile from driver experiment data, which lasts for around 20 min with speed varying between 9 and 21.5 m/s. The maximum and minimum accelerations are 0.9 and $-2.5 \text{ m/s}^2$, respectively.
5.1 Simulation validation

The first simulation is to validate the closed-loop stability and string stability of the platoon. Considering the practical driving environment, the simulation condition is defined as: (i) vehicle mass $M_i$ and time constant of powertrain are time invariant and each vehicle has different mass and time constant as shown by Figs. 6a and 6b, respectively; (ii) the communication delay is time varying and changes every 0.1 s randomly. As representative, the distribution of the communication delay between the leader and vehicle 1 is depicted by Fig. 7a; and (iii) external disturbances arising from periodic variation road slope and wind are added. One periodic profile of road slope and wind speed is shown by Figs. 7b and 7c, respectively.

As representative, the responses of vehicles 1, 5 and 10 are shown in Fig. 8. As shown in Fig. 8a, the maximum distance tracking error occurs at about 14 min and is about 4 m. The maximum tracking error is smaller than the desired distance, which equals 10 m. The vehicle platoon runs safely without collisions. As in Fig. 8b, the maximum velocity tracking error is about 1 m/s. Fig. 8c is the control input of vehicle. The platoon control system is robust stable and has satisfied tracking performance, even though each vehicle has different dynamics and there also exist model uncertainties and random communication delay.

Furthermore, as shown in Fig. 9a, the root mean square (RMS) of distance error of vehicle 1 is about 0.8 m, which is the biggest among all the vehicles. The RMSs of other vehicles are $<0.05$ m. The RMS of distance error does not increase along vehicle stream. Moreover, simulations have been conducted with different platoon length. The maximum distance error under different platoon length is shown in Fig. 9b, from which it is found that the maximum distance error stays uniformly bounded with increasing platoon length.

5.2 Robustness limit analysis

The main theorem in Section 4 is a sufficient condition, not necessary, for the robust performance of vehicular platoon with
communication delay. It implies that a state feedback controller may still acceptable, even if it cannot satisfy LMI (25). To find out the real robustness limit, more numerical simulations were conducted to analyse the influence of parameter uncertainties and communication delays on the platoon performances. The RMS of distance error is used to reflect the tracking performance of the whole platoon

\[
\sigma_{\text{d}} = \frac{\sum_{i=1}^{N} \left( \int_{0}^{T} \delta_{\text{d}}(t) \, dt \right)}{N \cdot T},
\]  

(40)

where \( N \) is the number of vehicles in the platoon and \( T \) is the simulation time. Fig. 10a shows the RMS of distance error \( \sigma_{\text{d}} \) with different maximum possible communication delays. If the communication delay is smaller than 0.9 s, \( \sigma_{\text{d}} \) is smaller than 0.1 m. It increases slowly with the communication delay. The vehicle platoon remains stable. When the communication delay becomes longer than 1.1 s, \( \sigma_{\text{d}} \) increases quickly with uncertain level. The influence of parameter uncertainties and communication delay on the platoon performances. If a larger communication delay exists, the allowed model uncertainty becomes smaller.

Some open questions are worthy to be further investigated: (i) only identical communication delay is considered, which is a very strict requirement in practical. If the communication delay is not identical and satisfies some random distribution, the robust optimisation problem of the state feedback H-infinity controller needs to be further studied; and (ii) to convert the vehicular platoon control problem to an H-infinity stabilisation problem, a virtual perturbation is introduced. Though the D-K procedure of \( \mu \)-synthesis can be used to reduce this conversation numerically, the necessity condition of the existence of required H-infinity controller is not given.

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8 References