Dynamical Modeling and Distributed Control of Vehicle Platoons

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Outline

1. Introduction
2. Modeling: the four-component framework
3. Analysis: Stability and robustness
4. Synthesis: Design of DMPC
5. Conclusion
1. Introduction

- **Vehicle Platoon**

- **Control Objectives**
  a) to ensure all the vehicles in the same group to move at the same speed with the leader
  b) to maintain the desired spaces between adjacent vehicles

- **Potential Benefits**
  - Improve traffic efficiency, enhance road safety, and reduce fuel consumption, etc.
  - The earliest implementation can date back to the PATH program during the last eighties

- **Real-world experiments**
  - USA - PATH
  - Europe - SARTRE
  - Japan - Energy ITS
1. Introduction

- View platoons from a networked control perspective

Typical Communication Topology

Different Communication Topologies

Connected Vehicle by V2V

- New challenges: Variety of topologies

1. Dynamical Modeling
2. Performance Analysis
3. Controller Synthesis

1. The four-component framework
2. Stability and robustness analysis
3. Design of Distributed Model Predictive Control
Outline

1. Introduction

2. Modeling: the four-component framework

3. Analysis: Stability and robustness

4. Synthesis: Design of DMPC

5. Conclusion
2. Modeling: the four-component framework

- Networks of Dynamical Systems
  
  ➢ From Control Perspective
  1. Dynamics + Communication
  2. Control Theory + Graph Theory
  
  ➢ Research topics
  1. Dynamic: single integrator, double integrator, linear dynamic, nonlinear dynamic
  2. Communication: data rate, switching topology, time-delay

- Applications


Vehicle platoons can be viewed as a special one-dimensional network of dynamical system
2. Modeling: the four-component framework

- Modeling of Platoons under the four-component framework

- **Vehicle Dynamics:** linear dynamics, nonlinear dynamics;
- **Formation Geometry:** constant spacing, time headway policy.
- **Distributed Controller:** linear controller, MPC, robust control;
- **Information Flow Topology:** PF, PFL, BD, etc;

Explicitly highlight the influence of different components!
2. Modeling: the four-component framework

- **Categorization of existing works** [Li and Zheng *et al.*, 2015]

### Table I. Categorization of Platoon Control

<table>
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<th>Node Dynamics (ND)</th>
<th>Third-order model</th>
<th>SISO model</th>
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<td>Stability Margin</td>
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<td>Coherence Behavior</td>
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Note: The table contains a comprehensive list of references and categories related to platoon control, including node dynamics, information flow topology, distributed controller, and formation geometry. Each category is associated with specific references for more detailed information.
2. Modeling: the four-component framework

- **Typical case**

1. Linear dynamics

   \[ \dot{x}_i(t) = Ax_i(t) + B_1 u_i(t) + B_2 w_i(t) \]

   \[ x_i(t) = \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1/\tau \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1/\tau \end{bmatrix} \]

2. Constant spacing policy

   \[
   \begin{align*}
   \lim_{t \to \infty} \|v_i(t) - v_0(t)\| &= 0, \quad i = 1,2,\ldots N \\
   \lim_{t \to \infty} \|p_{i-1}(t) - p_i(t) - d_{i-1,i}\| &= 0
   \end{align*}
   \]

3. Communication topology

   Pinning matrix \( \mathcal{P} \), Adjacency matrix \( \mathcal{A} \), Laplacian matrix \( \mathcal{L} \)

4. Linear controller

   \[ u_i(t) = -\sum_{j \in \mathcal{I}_i} \left[ k_p (p_i - p_j - d_{i,j}) + k_v (v_i - v_j) + k_a (a_i - a_j) \right] \]

**Closed-loop dynamics**

\[ \dot{X} = \left\{ I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B_1 k^T \right\} \cdot X + I_N \otimes B_2 \cdot W \]
Outline

1. Introduction
2. Modeling: the four-component framework
3. Analysis: Stability and robustness
4. Synthesis: Design of DMPC
5. Conclusion
3. Analysis: Stability and Robustness

- **Performance Index: linear platoon**

\[
\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B_1 k^T\} \cdot X + I_N \otimes B_2 \cdot W
\]

- **Closed-loop Stability:**

- **Stability Margin:**

- **Robustness index:**

\[
\|w_i(t)\|_{L^2} = \int_0^{+\infty} (w_i(t))^2 \, dt < \infty
\]

\[
AF_{f2l} = \sup \frac{\|\tilde{P}_N\|_{L^2}}{\|w_1\|_{L^2}} = \|G_{f2l}(s)\|_{\mathcal{H}_\infty}
\]

\[
AF_{a2a} = \sup \frac{\|Y\|_{L^2}}{\|W\|_{L^2}} = \|G_{a2a}(s)\|_{\mathcal{H}_\infty}
\]

\[
\mathbb{R} \quad \mathbb{Im} \quad \text{Stability Margin}
\]

\[
d_{\text{min}}
\]
3. Analysis: Stability and Robustness

**Stability Region Analysis** [Zheng et al. 2014, ITSC]

Consider a homogeneous platoon with linear controllers given by

\[
\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes Bk^T\}X
\]

If graph $G$ satisfies certain conditions (all the eigenvalues of $\mathcal{L} + \mathcal{P}$ are positive real numbers), the platoon is asymptotically stable if and only if

\[
\begin{cases}
  k_p > 0 \\
  k_v > k_p \tau / \min(\lambda_i k_a + 1) \\
  k_a > -1 / \max(\lambda_i)
\end{cases}
\]

**Proof sketch:**

Similarity transformation + Routh–Hurwitz stability criterion

\[
S(I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes Bk^T) = \bigcup_{i=1}^{N} \{S(A - \lambda_i Bk^T)\}
\]

\[
|sI - (A - \lambda_i Bk^T)| = s^3 + \frac{\lambda_i k_a + 1}{\tau} s^2 + \frac{\lambda_i k_v}{\tau} s + \frac{\lambda_i k_p}{\tau}.
\]
3. Analysis: Stability and Robustness


Consider a homogeneous platoon with linear controllers given by

$$\dot{X} = \{I_N \otimes A - (L + P) \otimes B k^T\}X$$

(2.1) if the graph $G$ is in Bidirectional topology, then the stability margin decays to zero as $O(1/N^2)$

(2.2) if the graph $G$ is in BDL topology, then the stability margin is always bounded away from zero.
### 3. Analysis: Stability and Robustness

**Stability Margin Improvement : Asymmetric control** [Zheng *et al.* 2016, IEEE CST]

Consider a homogeneous cooperative ACC under the BD topology with the asymmetric controller architecture given by

\[
\dot{X} = \{I_N \otimes A - (\mathcal{L}_{BD} + \mathcal{P}_{BD}) \epsilon \otimes Bk^T\}X
\]

(3.2) For any fixed \( \epsilon \in (0,1) \), the stability margin is bounded away from zero and independent of the platoon size \( N \) (\( N \) can be any finite integer).

**Asymmetric control**

The controller is called asymmetric, if

\[
\begin{align*}
    k_i^f &= (1 + \epsilon)k, k_i^b = (1 - \epsilon)k & i = 1, \ldots, N - 1 \\
    k_N^f &= (1 + \epsilon)k,
\end{align*}
\]

where \( \epsilon \in (0,1) \) is called the asymmetric degree. Note that if \( \epsilon = 0 \), then it is reduced to the symmetric case.
3. Analysis: Stability and Robustness

- **Stability Margin Improvement : Asymmetric control** [Zheng et al. 2016, IEEE CST]

  - **Tradeoff:** Convergence Speed and Transient Performance
    - Benefit: bounded stability margin → good for convergence speed
    - Cost: overshooting phenomena in transient process.

- Space errors for homogeneous platoons under BD topology with different asymmetric degree $\epsilon$. (a) $\epsilon=0$ (symmetric); (b) $\epsilon=0.2$; (c) $\epsilon=0.4$; (d) $\epsilon=0.6$
3. Analysis: Stability and Robustness

- **Stability Margin Improvement : Topological Selection** [Zheng et al. 2016, IEEE CST]

Consider a homogeneous platoon with linear controllers given by

\[ \dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\}X \]

(3.1) if the graph \( G \) is undirected, to maintain bounded stability margin, it needs at least lots of followers (i.e. \( \Omega(N) = O(N) \)) to obtain the leader’s information.

To maintain bounded stability margin, the tree depth of graph \( G \) should be a constant number and independent of the platoon size \( N \).
3. Analysis: Stability and Robustness

- **Stability Margin Improvement : Topological Selection** [Zheng et al. 2016, IEEE CST]

- Extending information flow to reduce the tree depth is one major way to guarantee a bounded stability margin.
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4. Synthesis: Design of DMPC

Nonlinear Heterogeneous platoons

Design a distributed controller for a heterogeneous platoon considering **nonlinear dynamics, input constraints** and **variety of communication topologies**

### Nonlinear Heterogeneous model

\[
\begin{align*}
\dot{p}_i(t) &= v_i(t) \\
\frac{\eta_{T,i}}{r_{w,i}} T_i(t) &= m_i \ddot{v}_i(t) + C_{A,i} v_i^2(t) + m_i g f_i \\
\tau_i \dot{T}_i(t) + T_i(t) &= u_i(t)
\end{align*}
\]

### Design of DMPC

Discretization

\[
x_i(t + 1) = \phi_i(x_i) + \psi_i \cdot u_i(t)
\]

### Stability analysis

### Simulation

### Linear homogeneous platoon

### Performance analysis

### Benchmark
4. Synthesis: Design of DMPC

- **Local open-loop optimal control problem**

**Problem $F_i$:** For $i \in \{1,2, \ldots, N\}$ at time $t$

\[
\min_{U_i} J_i \left( y_i^p (:|t), u_i^p (:|t), y_i^a (:|t), y_{-i}^a (:|t) \right) \\
= \sum_{k=0}^{N_p-1} l_i \left( y_i^p (k|t), u_i^p (k|t), y_i^a (k|t), y_{-i}^a (k|t) \right)
\]

s.t.

\[
\begin{align*}
\dot{x}^p_i (k+1|t) &= \phi_i \left( x^p_i (k|t) \right) + \psi_i \cdot u_i^p (k|t) \\
y_i^p (k|t) &= y x_i^p (k|t) \\
k &= 0, \ldots, N_p - 1 \\
x_i^p (0|t) &= x_i(t) \\
u_i^p (k|t) &\in \mathcal{U} \\
y_i^p (N_p|t) &= \frac{1}{|\Pi_i|} \sum_{j \in \Pi_i} (y_j^a (N_p|t) - \tilde{d}_{j,i}) \\
T_i^p (N_p|t) &= h_i \left( v_i^p (N_p|t) \right)
\end{align*}
\]

Move at constant speed at the end of predictive horizon

This is based on the local average of neighboring outputs. Thus, any node does not need to \textit{a priori} know the desired set point,
4. Synthesis: Design of DMPC

- **Local open-loop optimal control problem**

  - **Construction of local cost function**

    \[
    l_i \left( y_i^p(k|t), u_i^p(k|t), y_i^a(k|t), y_{-i}^a(k|t) \right) \\
    = \left\| Q_i \left( y_i^p(k|t) - y_{\text{des},i}(k|t) \right) \right\|_2 \\
    + \left\| R_i \left( u_i^p(k|t) - h_i \left( v_i^p(k|t) \right) \right) \right\|_2 \\
    + \left\| F_i \left( y_i^p(k|t) - y_i^a(k|t) \right) \right\|_2 \\
    + \sum_{j \in \mathbb{N}_i} \left\| G_i \left( y_i^p(k|t) - y_j^a(k|t) - \tilde{d}_{i,j} \right) \right\|_2
    \]

    - Tracking leader: \( p_i = 0, Q_i = 0 \)
    - Penalize the input: \( R_i \geq 0 \)
    - Maintain its assumed output: \( F_i \geq 0 \)
    - Maintain the assumed output of its neighbors: \( G_i \geq 0 \)

    This output is sent to the nodes in set \( \emptyset_i \).

    \( \emptyset_j = \{ j_1, j_2, j_3, j_4 \} \).

    \( \mathbb{N}_i = \{ i_1, i_2, i_3, i_4 \} \).

    Stability
4. Synthesis: Design of DMPC

- Local open-loop optimal control problem
- Algorithm of distributed model predictive control

(I) Initialization: At time \( t = 0 \), assume that all followers are moving at a constant speed

\[
\begin{align*}
  u_i^a(k|0) &= h_i(v_i(0)), & k = 0,1,\ldots,N_p - 1, \\
  y_i^a(k|0) &= y_i(0), & \text{for transmitting.}
\end{align*}
\]

(II) Iteration of DMPC: At \( t > 0 \), for all node \( i = 1,\ldots,N \)

1. Optimize Problem \( F_i \), yielding optimal control sequence \( u_i^*(k|t) \), \( k = 0,1,\ldots,N_p - 1 \)
2. Compute the assumed control input \( i.e. \), \( u_i^a(k|t+1) \) for next step by disposing first term and adding one additional term

\[
u_i^a(k|t+1) = \begin{cases} 
  u_i^*(k+1|t), & k = 0,1,\ldots,N_p - 2 \\
  h_i(v_i^*(N_p|t)), & k = N_p - 1
\end{cases}
\]

3. Transmit \( y_i^a(k|t+1) \) to the nodes in set \( \mathbb{O}_i \).
Receive \( y_{-i}^a(k|t+1) \) from the nodes in set \( \mathbb{N}_i \);
Compute \( y_{\text{des},i}(k|t+1) \) if \( \mathbb{P}_i \neq \emptyset \)
4. Implement the control effort, \( i.e. \),
\( u_i(t) = u_i^*(0|t) \)
5. Increment \( t \) and go to step (1).

What’s the requirement for stability?
4. Synthesis: Design of DMPC

**Assumption 1 (Unidirectional topology):** The graph $G$ contains a spanning tree rooting at the leader, and the communications are unidirectional from preceding vehicles to downstream ones.

- **Sufficient conditions [Zheng et al. 2016, IEEE CST]**

  If $G$ satisfies **Assumption 1**, a platoon under proposed DMPC is asymptotically stable if satisfying

  \[
  F_i \geq \sum_{j \in \Omega_i} G_j, \quad i \in \mathcal{N}
  \]

  The main strategy is to construct a proper Lyapunov function for the platoon and prove its decreasing property.

  sum of local cost functions
4. Synthesis: Design of DMPC

The desired trajectory \( v_0 = \begin{cases} 20 \text{ m/s} & t \leq 1 \text{ s} \\ 20 + 2t \text{ m/s} & 1 \text{ s} < t \leq 2 \text{ s} \\ 22 \text{ m/s} & t > 2 \text{ s} \end{cases} \)

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<tr>
<th>Weights</th>
<th>PF</th>
<th>PLF</th>
<th>TPF</th>
<th>TPLF</th>
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(a) PF  
(b) PLF  
(c) TPF  
(d) TPLF
5. Conclusion

- **Vehicle Platoon**

  Vehicular platoon can bring many potential benefits, e.g., Improving traffic capacity; Enhancing highway safety; Reducing road congestion

- **Four-component framework**

- **Platoons under “homogeneity + linear feedback”**
  1) Stability Region Analysis
  2) Scaling of Stability Margin
  3) Improvement of Stability Margin: a) Asymmetric control and b) topological selection

- **Platoons under “heterogeneous + nonlinear”**
  1) We propose a novel DMPC algorithm for vehicle platoons with nonlinear dynamics and unidirectional topologies
  2) A sufficient condition is derived to guarantee asymptotic stability.
References


**Four-component Framework**


**Stability Region and Stability Margin**


**Design of DMPC and robust controller**

2. F. Gao, S. E. Li, Y. Zheng and D. Kum, Robust Control of Heterogeneous Vehicular Platoon with Uncertain Dynamics and Communication Delay, *IET Intelligent Transport Systems*, 2016; accepted.