Competition and Information Production in Market Maker Models

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Abstract: The microstructure literature models the mechanisms through which fundamental information is incorporated into market prices. This paper extends previous models by endogenising information production and analysing incentives for costly information production. In contrast to the existing literature, increasing the number of informed traders can result in reduced price informativeness. When prices have an allocative role this has welfare consequences: the regulatory implications of a dichotomy between private and public incentives for information gathering are discussed.

Keywords: market maker model, information acquisition, competition, liquidity, price informativeness

1. INTRODUCTION

Starting from the seminal work of Kyle (1985), a substantial literature has arisen which examines market maker intermediated markets. These models analyse in a noisy rational expectations framework the strategic behaviour of informed traders who wish to maximise the return which they earn on their information in anonymous trading with uninformed ‘liquidity’

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traders. This literature provides an information-theoretic foundation for price formation in traded markets and has deepened our understanding of the mechanisms by which information is reflected in prices, and of the relationship between adverse selection and market liquidity. In this paper I extend the existing literature by modelling the costly production of information in the Kyle framework. Conclusions in the existing literature about price informativeness are shown not to be robust. The model of this paper has implications for the design of exchanges and it suggests avenues for further research into the welfare economics of financial markets.

In standard microstructure models a risk neutral and competitive market-maker interacts with informed and uninformed traders. The informed traders strategically anticipate the effects of their trading upon the market and so are not competitive price-takers. The trading motives of the uninformed traders and the source and quality of the informed traders’ information are typically unmodelled. Much of the literature in this field examines the consequences of relaxing some of these assumptions. For example, Spiegel and Subrahmanyan (1992) and Dow and Rahi (2000, 2001) examine models in which the uninformed traders are strategic agents with a hedging motive for trade. Admanti and Pfleiderer (1988) use the Kyle formalism to examine intraday trading patterns. Chowdhry and Nanda (1991), Pagano (1989) and Bhushan (1991) examine the distribution of trade between different marketplaces. See O’Hara (1995) for a review of the applied theoretical literature in this field.

One area which has been underinvestigated is the relationship between the cost of information and competition levels between informed traders. The related incentives determine the level of information which is generated and therefore the informativeness of prices in financial markets. Some work does consider the acquisition of information. Verrecchia (1982) examines information acquisition by risk averse competitive (price-taking) traders and derives some comparative statics for price informativeness. A classic paper by Grossman and Stiglitz (1980) examines the equilibrium level of informed trade and its effect upon price informativeness when information has a fixed cost, access to information production technologies is
unrestricted, and traders are risk-averse competitive price-takers. Their paper demonstrates that perfectly informative financial markets are impossible and shows that the informativeness of prices is inversely related to the cost of information. This work is extended to the imperfectly competitive case by Kyle (1989), who considers free entry by risk averse strategic traders with access to a costly signal of given quality. Incentives for information production and revelation have been examined in a different context in models of Initial Public Offerings.¹ Several papers examine the relation² between public revelation and private foreknowledge of information, but none examines the effect of competition upon the intensity of information production and hence upon the informativeness of prices.

In this paper I approach the problem of information generation from a different angle. Rather than fixing the cost and the quality of information and endogenising informed trader entry as the above authors do, I follow the market microstructure literature and fix the number of informed traders exogenously and I endow them with a technology which allows them to decide the amount which they will spend upon information production and hence the quality of the information which is available to them. This allows me to quantify incentives for information gathering.

The incentives which informed agents have to gather information are of interest for two reasons. Firstly, they will affect the profitability of their trades and hence the expected costs which liquidity traders must meet. These issues are the primary focus of the existing literature and this paper does not address them in detail. Instead it concentrates upon a second concern: the informativeness, or efficiency, of prices. When stock prices have an allocative role in guiding productive investment, price informativeness has welfare consequences (Grossman, 1995; and Subrahmanyam and Titman, 2001). The relationship between the social and private incentives for information production by speculators was examined in Hirshleifer’s (1971) paper. More

¹ See Benveniste and Spindt (1989), Benveniste and Wilhelm (1990), Sherman and Titman (2002) and references therein.

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recently, Dow and Rahi (2001) provide a welfare analysis of the effects documented by Hirshleifer, but without endogenising the information acquisition process.

In providing an explicit model for information production, this paper identifies an effect which has been ignored in the existing literature. When several informed traders compete, each will earn a lower rent upon his informational assets. When information is costly, there will be a reduced incentive for information production and prices will tend to be less informative. I show that when a plausible concave relationship is assumed between investment in information and its precision, this effect will dominate the usual positive efficiency effects of increased competition. In other words, and contrary to previous work in this field, increasing the number of informed traders in a market place can reduce information-gathering incentives and with them the informativeness of prices.

Similarly, when information is given exogenously an increase in fundamental uncertainty reduces the informativeness of ex ante prices and hence increases the rents which informed traders earn from their information. In this paper, increased informational rents provide incentives for increased information production. I show that this effect can outweigh the effect identified in the existing literature: in other words, an increase in fundamental uncertainty can, by increasing information-gathering incentives, increase the informativeness of prices.

Finally, I examine the role of uninformed traders. In the existing literature, uninformed traders hide the activities of speculators so that an increase in the number of liquidity traders increases speculator trading volume and with it information rents. The informativeness of prices is unaffected. In contrast, I show that with endogenous information production an increase in the number of liquidity traders increases price informativeness by raising information rents and hence information-gathering incentives. As I discuss in the conclusion to this paper, my results on price informativeness have implications for the optimal design of financial markets.

Section 2 presents the model and defines the equilibrium concept which is used in the remainder of the paper. Section 3 shows that a unique equilibrium exists. Section 4 discusses the comparative statics of the equilibrium and examines the
informativeness of prices. The final section concludes the paper, notes some of the policy implications of the work and suggests avenues for further investigation. One of the proofs appears in the Appendix.

2. THE MODEL

I consider a market for a single risky security, whose value $S$ is a random variable:

$$\tilde{S} = \bar{S} + \tilde{\alpha},$$

where the expected value $\bar{S}$ of the security is common knowledge and $\tilde{\alpha}$ is a randomly distributed noise term with mean 0 and variance $V_\alpha$. There are three types of agents in the model: a risk neutral competitive market maker, $m$ liquidity traders, and $n$ informed traders.

As Spiegel and Subrahmanyam (1992) note, the trading motives of the liquidity traders can be endogenised by assuming that they wish to hedge endowment shocks. In this paper, I instead follow the standard microstructure line and assume that the trade sizes $\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_m$ of the liquidity traders are determined exogenously as independent draws from a normal distribution with mean 0 and variance $V_L V_\alpha$. For every $i, \tilde{y}_i$ is independent of $\tilde{\alpha}$.

The informed traders have access to a costly information-gathering technology which gives them an imperfect signal of $\tilde{\alpha}$. I assume that each of the informed traders can access at a cost the same information set. Each trader can observe a signal

$$\tilde{\alpha} + \tilde{\varepsilon} \sqrt{\frac{V_\alpha}{\pi(c)}},$$

where $\tilde{\varepsilon}$ is a draw from $N(0,1)$ which is the same for every trader, and $c$ is the trader’s expenditure on information. $\tilde{\varepsilon}$ is independent of $\tilde{\alpha}$ and of every $\tilde{y}_i$. The precision $\frac{\pi(c)}{V_\alpha}$ of the signal is an increasing concave function of its cost $c$: in other words, $\pi' (.) > 0$ and $\pi'' (.) < 0$. Conditional upon their signals, the informed traders will select trade sizes $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$.

3 Throughout the paper, random variables are indicated by a tilde ($\tilde{X}$): realisations of the variables are shown unadorned ($X$).
Denote by

\[ \tilde{\omega} \equiv \sum_{i=1}^{n} \tilde{x}_i + \sum_{j=1}^{m} \tilde{y}_j \]

the total order flow which the market maker observes. She cannot distinguish between informed and uninformed traders and this therefore comprises her entire information set.

The market functions in the following fashion. Firstly, the informed traders decide how much to spend upon information production. Then informed and uninformed traders pass their orders to the market maker as limit orders. The market maker is competitive and sets a price at which her expected profit is equal to zero.

Amongst the multiplicity of possible equilibria for this model, I will examine those which are linear and symmetric.

**Definition** A symmetric linear Nash equilibrium for this game is one in which:

1. The market maker’s pricing strategy is:

   (a) Competitive: \( E\{\tilde{P}|\tilde{\omega}\} = 0; \)
   (b) Linear: \( \tilde{P} = \tilde{S} + \lambda \tilde{\omega}. \)

2. The informed traders’ strategies are:

   (a) Symmetric: all spend \( c^*; \)
   (b) Linear: informed trader order size has a linear dependence upon the received signal:

   \[ x = \beta (\tilde{\alpha} + \tilde{\epsilon} \sqrt{\frac{V_{\alpha}}{\pi(c)}}). \]

3. Informed traders’ actions maximise their expected income, given the actions of other informed traders.

In searching for linear Nash equilibria I am following the existing literature. There are a multiplicity of possible Nash
strategies for information production. Restriction to symmetric equilibria is necessary to ensure model tractability and seems plausible when all traders have the same information production technology. Throughout the paper, I will further follow the existing literature by assuming that the respective numbers $m$ and $n$ of uninformed and informed traders are common knowledge.

3. EXISTENCE AND UNIQUENESS OF EQUILIBRIA

Proposition 1 proves the existence of equilibria for the model, and gives the equilibrium parameter values. Its proof appears in the Appendix.

**Proposition 1:** A unique symmetric linear Nash equilibrium always exists. The equilibrium parameter values $\lambda^*$ and $c^*$ solve the following simultaneous equations:

$$
\lambda = \frac{1}{n+1} \sqrt{\frac{n\pi(c)}{m(1 + \pi(c))V_L}}; 
$$

(1)

$$
1 = \frac{V_\alpha \pi'(c)(2\pi(c) + n + 1)}{2\lambda(1 + \pi(c))^2(n + 1)^2}. 
$$

(2)

The equilibrium value for the informed order size parameter $\beta$ is as follows:

$$
\beta = \sqrt{\frac{m}{n} \frac{\pi(c^*)}{1 + \pi(c^*)} V_L}. 
$$

(3)

Equation (2) is the first order condition which each informed trader solves when selecting the information production level $c$ in a symmetric equilibrium. It states that the marginal cost $1$ of information production must be equal to the marginal income on the right hand side of the equation which new information generates. Note that:

$$
\frac{d}{dc} \left( \frac{\pi'(c)(2\pi(c) + n + 1)}{(1 + \pi(c))^2} \right) = \pi''(c) \left( \frac{2\pi(c) + n + 1}{(1 + \pi(c))^2} \right)
$$

$$
- 2\pi'(c)^2 \left( \frac{1}{(1 + \pi(c))^2} + \frac{n - 1}{(1 + \pi(c))^3} \right) < 0,
$$
since the precision $\pi$ (.) is assumed concave and $n \geq 1$. Equation (2) therefore yields a unique optimal level of information production. The cost and income curves are illustrated in Figure 1.

Note that the income curve is shifted upwards by a higher $V_\alpha$ and by a higher market depth $\frac{1}{\lambda}$, so that $c^*$ is increasing in these quantities. Increasing the number $n$ of informed traders lowers the income curve and hence results in a reduced level of equilibrium information production. This is because an increase in $n$ intensifies the competition between the market makers and so diminishes their expected information rents. It therefore reduces their expenditure on information production.

Equation (1) is derived in the Appendix using standard arguments which rest upon the projection theorem. It shows that the market liquidity $\frac{1}{\lambda}$ is increasing in both informed and uninformed trader numbers, in the expenditure $c$ upon information, and in the variance of the liquidity traders’ order size. The two reaction curves are plotted together on Figure 2. They have a unique crossing point, from which the existence and uniqueness of the symmetric linear equilibrium follows.

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**Figure 1**

Optimal Expenditure on Information Production is at $c^*$: Changing $\lambda$, $n$, or $V_\alpha$ will Shift the Income Curve and Hence Change $c^*$
4. PROPERTIES OF THE EQUILIBRIUM

(i) Comparative Statics

**Proposition 2:** The comparative statics of the equilibrium liquidity and information production parameters $\lambda^*$ and $c^*$ are as follows:

1. $\frac{\partial c^*}{\partial V_\alpha} > 0; \frac{\partial \lambda^*}{\partial V_\alpha} > 0$.

2. $\frac{\partial c^*}{\partial V_L} > 0; \frac{\partial \lambda^*}{\partial V_L} < 0$.

3. $\frac{\partial c^*}{\partial m} > 0; \frac{\partial \lambda^*}{\partial m} < 0$.

4. $\frac{\partial c^*}{\partial n} < 0; \frac{\partial \lambda^*}{\partial n} < 0$.

**Proof:** The bold lines in Figure 3 are the reaction curves which are discussed in Section 3 and illustrated in Figure 2. We can determine the comparative statics of the equilibrium by considering the curve movements which are induced by changes in the exogenous parameters of the model.

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Figure 2: The Unique Symmetric Linear Nash Equilibrium Occurs at $(\lambda^*, c^*)$, where the Reaction Curves of the Market Maker and the Informed Traders Cross.
Recall from the discussion in Section 3 that increases in $m$ and in $V_L$ will shift the $\lambda$ reaction curve to the left without altering the $c$ reaction curve. The equilibrium point will therefore be shifted to a point $A$, as illustrated in the figure. This proves parts 2 and 3 of the proposition. As discussed in Section 3, increases in $V_\alpha$ will shift the $c$ reaction curve upwards without altering the $\lambda$ curve, so that the equilibrium moves to a point $B$ as in the figure: this proves part 1 of the proposition.

Finally, recall from Section 3 that increases in $n$ will lower the $\lambda$ and the $c$ reaction curves so that the equilibrium point becomes $C$, as indicated in the figure. The new value of $c^*$ is certainly lower. Equation (1) implies that:

$$
\frac{\partial \lambda^*}{\partial n} = \frac{\partial \lambda}{\partial n} \Bigg|_{\lambda=c^*} + \frac{\partial \lambda}{\partial c^*} \frac{\partial c^*}{\partial n} 
$$

$$
= \sqrt{\frac{\pi^*}{(1 + \pi^*)V_L}} \left\{ \frac{1}{2\sqrt{n(n+1)}} - \frac{\sqrt{n}}{(n+1)^2} \right\} + \frac{\partial c^*}{\partial n} \frac{1}{2(n+1)} \sqrt{n} \frac{mV_L}{\sqrt{\pi^*(1 + \pi^*)}} \pi'(c^*)
$$

$$
= \frac{1 - n}{2(n+1)^2} \sqrt{\frac{\pi^*}{n(1 + \pi^*)V_L}} + \frac{\partial c^*}{\partial n} \frac{1}{2(n+1)} \sqrt{n} \frac{mV_L}{\sqrt{\pi^*(1 + \pi^*)}} \pi'(c^*)
$$

$$
< 0.
$$

This concludes the proof.

The intuition for these results is as follows.

When the *ex ante* variability $V_\alpha$ of the security’s value increases, the value of the informed traders’ information increases and
hence expenditure on information production will also increase. When there is more fundamental uncertainty trading is riskier and trading volumes will be lower. Informed trading will therefore be more visible and as a result market liquidity will drop: equivalently, \( \lambda^* \) will increase.

An increase in the noise level \( V_L \) will enable informed traders to hide their activities more effectively and so will raise the equilibrium return on information production, which will therefore increase. A greater degree of noise will render informed trading less visible and market liquidity will therefore increase. An identical argument explains part 3 of the proposition.

As discussed in Section 3, an increase in the number of informed traders results in higher competition levels, reduced informational rent and hence a reduced level of information production, so that \( \frac{\partial \lambda^*}{\partial n} < 0 \). The increase in \( n \) has two effects upon \( \lambda \). Firstly, an increased \( n \) with no change in information production renders the order flow observed by the market maker more informative and hence decreases market liquidity (increases \( \lambda \)). However, the increased competition between informed traders and the reduction in information production has an offsetting effect: although more informed traders are passing orders to the market maker, the information content of each order is reduced. This will tend to increase market liquidity (reduce \( \lambda \)). It transpires that the second effect dominates.

(ii) Price Informativeness

In this section I examine the consequences of endogenous information acquisition for price informativeness. Price informativeness is inversely related to the following expression:

\[
\text{var}\{V|P\} = \text{var}\{\tilde{\alpha}|\tilde{S} + \lambda \tilde{\omega}\} = V_\alpha(1 - \rho^2),
\]

where \( \rho = \frac{\lambda \text{cov}(\tilde{\alpha}, \tilde{\omega})}{\lambda V_\alpha^2 V_L^2} \) is the correlation coefficient between \( \tilde{\alpha} \) and \( \tilde{S} + \lambda \tilde{\omega} \).

Proposition 3 shows how price informativeness is related to \( n \), \( V_\alpha \), \( V_L \) and \( m \).

**Proposition 3:**

1. Price informativeness is decreasing in \( n \) if and only if:
2. Price informativeness is increasing in $V_\alpha$ if and only if:

$$-\frac{\partial c^*}{\partial n} \pi'(c^*) > \frac{\pi(c^*) (1 + \pi(c^*))}{n(n + 1)};$$

3. Price informativeness is increasing in $V_L$ and $m$.

Increases in $n$ have two opposing effects upon the informativeness of prices. Firstly, a higher $n$ results in more competition between market makers and so increases the information content of prices. Secondly, the competition which comes from a higher $n$ reduces the rent which informed traders earn on information and so reduces information production and with it the informativeness of prices. Proposition 3 indicates that when

$$-\frac{\partial c^*}{\partial n} \pi'(c^*) > \frac{\pi(c^*) (1 + \pi(c^*))}{n(n + 1)},$$

the second effect is stronger than the first. This will be the case when either $n$ is sufficiently high, so that the effects of competition upon information rents are particularly severe, or when the precision of information is sufficiently cost-sensitive ($\pi'$ ($c^*$) is sufficiently high), so that a slight reduction in expenditure in response to increased competition has a large effect upon information production.

Similar intuition applies to increases in fundamental uncertainty, $V_\alpha$. For a given precision $\pi(c)$, a higher $V_\alpha$ reduces the informativeness of prices and hence increases the information rent which accrues to informed traders. As shown in Proposition 2, this in turn increases information gathering incentives and hence the informativeness of prices. Part 2 of Proposition 3 indicates that the second effect will outweigh the first precisely when

$$\frac{\partial c^*}{\partial V_\alpha} \pi'(c^*) > \frac{(n + 1 + \pi(c^*)) (1 + \pi(c^*))}{n V_\alpha}.$$
information rents, and price informativeness is unaffected. However, as demonstrated in Section 4(i), with endogeneous information production increased rents will lead to greater investment in information. Proposition 3 demonstrates that the increased level of information production will increase price informativeness.

The results of this proposition are at variance with microstructure models in which information is acquired exogenously. Since speculators cannot in those models reduce their expenditure on information production increased competition between them results unambiguously in increased price informativeness, and increased fundamental uncertainty in reduced price informativeness. Proposition 3 shows that endogenising information production can in some circumstances reverse these conclusions. The results for the number of informed traders and the variance of their order size are even more striking: endogenising information production renders price informativeness an increasing function of these parameters and hence always alters the conclusions of previous models.

These results may have implications for market efficiency and hence for social welfare. I return to this theme in the conclusion.

5. CONCLUSION

The literature on market microstructure examines the strategic interplay between informed and uninformed traders who meet in an anonymous marketplace, typically inter-mediated by a risk neutral competitive market maker. It models the process by which information is impounded in prices and shows how the informativeness of prices is affected by exogenous parameters, such as the number of informed and uninformed traders. The relevant models have however, typically ignored the process by which information is acquired. This is important, as price informativeness depends not only upon the numbers of traders and the uncertainties to which they are exposed, but also upon the *ex ante* incentives for gathering information. This paper extends the existing literature by endogenising information production.
It is thus able to comment upon the relationship between the private incentives for information acquisition and the public good which price informativeness provides.

When prices have an allocative role, price informativeness has important welfare consequences. These effects are typically ignored by the agents who generate information and in consequence there may be either under- or over-production of information (Hirshleifer, 1971). I show in this paper that information production incentives depend critically upon competition levels between market makers and upon the nature of the liquidity traders with whom they trade. When competition levels increase information rents are reduced and incentives for information production are diminished. I show in Proposition 3 that this may reduce the informativeness of prices and thus reduce the allocative efficiency of market prices. Conversely, an increase in the number of uninformed traders will increase information production and with it the informativeness of prices.

These effects are ignored by the existing literature, in which increased competition between informed traders leads inevitably to more informative prices and hence to welfare gains. They suggest that access to the information gathering technology should be limited. This might be achieved by charging traders a Pigouvian tax for access to information technology. Although trader entry is unmodelled in this paper, it seems likely that such a tax would in equilibrium reduce the number of informed traders to the level where their expected profits just equalled the tax. One could think of the high cost of an exchange seat as such a tax. Restrictions upon participation in the initial stage of an Initial Public Offering could be explained in a similar way (see Sherman and Titman, 2001).

The relevance of uninformed traders to information production is also critical. The profits which informed traders make come at the expense of uninformed traders, who ignore the social benefits which price informativeness brings. If uninformed traders can trade without participating in the price formation process, they will do so. Prices will be less informative, but the expected losses sustained by uninformed traders will also be reduced. Trading off-exchange at exchange prices is one way in which uninformed traders can accomplish
this. This type of trade will reduce the rent earned by informed traders on their rent and so will diminish the informativeness of prices and possibly also social welfare. This argument provides a strong rationale for limiting access to exchange prices by delaying price reporting to non-exchange members. It also suggests that the recent explosion in trading via electronic crossing networks may be socially undesirable.

The observations in the above paragraphs suggest a possible research programme. Categorical policy statements can be made only in the context of a general equilibrium model in which all of the players are acting strategically and where the social benefits of improved allocative efficiency are captured through the utility functions of shareholders. This work would bridge the finance and welfare economics literatures and might generate insights into the importance of market institutions. This paper takes a first step towards an understanding of the relevant issues.

APPENDIX

Proof of Proposition 1
Suppose that every informed trader except the $k$th follows strategies given by parts 2(a) and 2(b) of Definition 1. To determine the $k$th trader’s optimal information expenditure $c_k$, I determine for a given $c_k$ the optimal $x_k$ conditional upon a given signal $\alpha + \varepsilon \sqrt{\frac{V}{\sigma(c_k)}}$. This yields a maximum expected profit conditional upon $\alpha + \varepsilon \sqrt{\frac{V}{\sigma(c_k)}}$. Taking expectations of this with respect to $\alpha$ and $\varepsilon$ yields the expected profit from an information expenditure of $c_k$. $c_k$ is selected to maximise this expression. To ensure a symmetric equilibrium, $c^*$ and $\beta$ are finally selected so that the optimal $c_k$ is equal to $c^*$ and so that the $k$th trader’s decision rule is given by part 2(b) of Definition 1.

Write $\pi_k$ for $\pi(c_k)$ and $\pi^*$ for $\pi(c^*)$. Then given expenditure $c_k$ and a signal $\alpha + \varepsilon \sqrt{\frac{V}{\sigma_k}}$, the expected profit which the $k$th trader earns from a position size $x_k$ is
The $k$th informed trader will select $x_k$ so as to maximise this expression. Differentiation immediately yields

\[
x_k = \frac{\alpha + \varepsilon \sqrt{\frac{V_\alpha}{\pi_k}}}{1 + \pi_k} \left\{ \frac{n}{2\lambda} - \beta \left( \frac{n - 1}{2} \right) \left( \pi_k + \sqrt{\pi^*} \right) \right\}.
\] (5)

Inserting equation (5) into the objective function equation (4) yields the following:

\[
\frac{\lambda}{(1 + \pi_k)^2} \left( \alpha + \varepsilon \sqrt{\frac{V_\alpha}{\pi_k}} \right)^2 \left\{ \frac{\pi_k}{2\lambda} - \beta \left( \frac{n - 1}{2} \right) \left( \pi_k + \sqrt{\pi^*} \right) \right\}^2 - c_k.
\] (6)

To determine the optimal information production for the $k$th informed trader, I firstly take expectations of equation (6) to determine the trader’s expected profit from information expenditure of $c_k$:

\[
\frac{V_\alpha \lambda}{(1 + \pi_k)^2} \left( \frac{1}{\pi_k} \right) \left\{ \frac{\pi_k}{2\lambda} - \beta \left( \frac{n - 1}{2} \right) \left( \pi_k + \sqrt{\pi^*} \right) \right\}^2 - c_k
\]

\[
= \frac{V_\alpha \lambda \pi_k}{1 + \pi_k} \left\{ \frac{1}{2\lambda} - \beta \left( \frac{n - 1}{2} \right) \left( 1 + \frac{1}{\sqrt{\pi_k \pi^*}} \right) \right\}^2 - c_k.
\]

The first order condition at the optimal level $c_k$ of information production is as follows:
\[ V_{\alpha} \lambda \pi'(c_k) B \left\{ B + \frac{\pi_k \beta (n-1)}{2 \pi_k \sqrt{\pi^*} \pi_k} \right\} = 1, \]  \hspace{1cm} (7)\]

where

\[ B \equiv \frac{1}{\lambda} \left( \frac{1}{2 \lambda} - \beta \left( \frac{n-1}{2} \right) \left( 1 + \frac{1}{\sqrt{\pi^*} \pi_k} \right) \right). \]

For any \( \beta \) and \( c^* \), the equilibrium information expenditure \( c_k \) of the \( k \)th speculator must satisfy equation (7) and in response to a subsequent signal \( \left( \alpha + \varepsilon \sqrt{\frac{V_{\alpha}}{\pi_k}} \right) \) he will select trade size \( x_k \) given by equation (5). A symmetric equilibrium will occur whenever equations (7) and (5) are satisfied with \( c_k = c^* \) and in addition the optimal \( x_k \) satisfies \( x_k = \beta \left( \alpha + \varepsilon \sqrt{\frac{V_{\alpha}}{\pi_k}} \right) \). The condition on \( x_k \) is equivalent to:

\[ \frac{1}{1 + \pi_k} \left\{ \frac{\pi_k}{2 \lambda} - \beta \left( \frac{n-1}{2} \right) \left( \frac{1}{\sqrt{\pi^*} \pi_k} \right) \right\} = \beta. \]  \hspace{1cm} (8)\]

Setting \( \pi_k = \pi^* \) and solving for \( \beta \) we obtain

\[ \beta = \frac{\pi^*}{\lambda (n+1)(1+\pi^*)}. \]  \hspace{1cm} (9)\]

Substituting equation (9) into equation (7) and setting \( \pi_k = \pi^* \) we obtain the following condition:

\[ V_{\alpha} \lambda \pi'(c^*) \left( \frac{1}{2 \lambda} - \frac{n-1}{2 \lambda (n+1)} \right) \left\{ \frac{1}{1 + \pi^*} \left( \frac{1}{2 \lambda} - \frac{n-1}{2 \lambda (n+1)} \right) + \frac{n-1}{2 \lambda (n+1)(1+\pi^*)^2} \right\} = 1, \]

which reduces to equation (10), as required:

\[ \frac{V_{\alpha} \pi'(c^*) (2\pi^* + n + 1)}{2 \lambda (n+1)^2 (1+\pi^*)^2} = 1. \]  \hspace{1cm} (10)\]

Part 1(a) of Definition 1 and the projection theorem imply that

\[ P(\tilde{\omega}) = \tilde{S} + \lambda \tilde{\omega} \]

\[ = E \{ \tilde{S} | \tilde{\omega} \} = \tilde{S} + \frac{\text{cov}(\tilde{S}, \tilde{\omega})}{\text{var}(\tilde{\omega})} \tilde{\omega}, \]
whence
\[
\lambda = \frac{\text{cov}(\tilde{S}, \tilde{\omega})}{\text{var}(\tilde{\omega})}. \tag{11}
\]

The equilibrium value for \(\tilde{\omega}\) is
\[
\text{cov}(\tilde{S}, \tilde{\omega}) = \text{cov}(\tilde{\alpha}, \tilde{\omega}) = \beta n V_{\alpha}, \tag{12}
\]
\[
\text{var}(\tilde{\omega}) = n^2 \beta^2 (V_{\alpha} + V_{/\pi^*}) + m V_{L} V_{\alpha}
= \frac{V_{\alpha}}{\pi^*} (n^2 \beta^2 (1 + \pi^*) + m \pi^* V_{L}). \tag{13}
\]

Substituting equations (12) and (13) into equation (11) and solving for \(\lambda\) finally yields equation (14), which was to be proved:
\[
\lambda = \frac{1}{n + 1} \sqrt{\frac{n \pi^*}{m(1 + \pi^*) V_{L}}}. \tag{14}
\]

The existence and uniqueness of the simultaneous solution to equations (10) and (14) is proved in the main body of the paper, using Figures 1 and 2. The expression for \(\beta\) is derived by substituting for \(\lambda\) in equation (9).

**Proof of Proposition 3**

The correlation coefficient between \(\tilde{\alpha}\) and \(\tilde{S} + \lambda \tilde{\omega}\) is
\[
\rho = \lambda \frac{\text{cov}(\tilde{\alpha}, \tilde{\omega})}{\lambda V_{\alpha} \frac{1}{2} V_{\tilde{\omega}}^2}.
\]

Using equation (11), we obtain
\[
\rho^2 = \frac{\text{cov}(\tilde{\alpha}, \tilde{\omega}) \text{cov}(\tilde{\alpha}, \tilde{\omega})}{V_{\omega} V_{/\alpha}} = \lambda \beta n = \frac{n}{n + 1} \frac{\pi^*}{1 + \pi^*},
\]
so that \(\text{var}\{V|P\} = V_{\alpha} \frac{(1 + n + \pi^*)}{(n + 1)(1 + \pi^*)}.\) It follows after some simple manipulation that
\[
\frac{d}{dn} \text{var}\{V|P\} = \left[ \frac{\partial}{\partial n} + \frac{\partial \pi^*}{\partial n} \frac{\partial}{\partial \pi^*} \right] \text{var}\{V|P\}
= \frac{V_{\alpha}}{(1 + n)^2 (1 + \pi^*)^2} \left\{ - \frac{\partial \pi^*}{\partial n} \pi'(\pi^*) n (1 + n) - \pi^* (1 + \pi^*) \right\}, \tag{15}
\]

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from which part 1 of the proposition follows immediately.

For the second part, note that

$$\frac{d}{dV} \text{var}\{V|P\} = \frac{1}{(n+1)(1+\pi^*)^2} \left\{ (n+1+\pi^*)(1+\pi^*) - nV \frac{\partial \pi^*}{\partial V} \right\}.$$ 

Finally, note that

$$\frac{d}{dV_L} \text{var}\{V|P\} = -V \frac{n}{(n+1)(1+\pi^*)^2} \frac{\partial \pi^*}{\partial V_L} < 0;$$

$$\frac{d}{dm} \text{var}\{V|P\} = -V \frac{n}{(n+1)(1+\pi^*)^2} \frac{\partial \pi^*}{\partial m} < 0.$$ 

REFERENCES


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