Culture, Competence, and the Corporation*

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Abstract

We provide an economic treatment of two central ideas from management studies: corporate culture, and corporate competence. We follow Weber and Camerer’s (2003) experimental work, which identifies both the importance of cultural norms in communication, and the efficiency costs of moving to an unfamiliar culture. We argue that communication through tacit, cultural, channels can mitigate principal agent problems within organizations. The cultural displacement associated with job changes therefore has an agency cost. This reduces employee mobility and binds the employee’s competences to the employer. Hence, the employer will finance training in general skills. If new information systems reduce the cultural specificity of communication channels then employees will become more mobile, and the burden of training will shift from employers to professional schools.

KEY WORDS: Corporate culture, training, information technology.

JEL CLASSIFICATION: J24, L20, M14, M53, O33
In this paper we provide an economic framework in which we analyze two ideas which are common in the management literature, but which have received little attention from economists: firstly, the idea that corporations derive value from the possession of core competencies; and secondly, the notion of corporate culture. We present a theory in which the firm serves as a repository of cultural norms, and we show that the existence of these norms is sufficient to tie skilled personnel to the firm. Our analysis suggests a positive relationship between an organization’s cultural differentiation and its incentive to train its staff. We discuss the impact of innovations in communication technologies upon this relationship.

Culture has been studied by social scientists since the word was introduced to the English language by the anthropologist Edward B. Tylor (1871). It is notoriously hard to define: culture is experienced in different ways according to the perspective of the observer (Martin, 1992). In this paper we adopt Bower’s (1966) early characterization of culture as “the way we do things around here.” Specifically, we assume that cultural norms determine the way that information is communicated within the firm.

The importance of shared cultural norms and experience in organizational communication is illustrated by experimental work performed by Weber and Camerer (2003). They gave one subject 8 from 16 pictures and they recorded the time he took to give another subject sufficient information to enable her to select the same pictures in the same order. Over 20 rounds of the same game, the time taken to perform this task dropped an average of 249 seconds to 48, as the subjects found verbal shortcuts for describing the pictures. Merging teams so that one member was unfamiliar with the private language developed by the other two raised task completion times to 130 seconds, after which convergence to pre-merger competence levels was extremely slow.

In line with Weber and Camerer, we consider a model of the firm in which employees acquire cultural skills in the early part of their careers without which they cannot operate efficiently. As discussed above, culture is a set of learned tacit skills which facilitate communication within the organization. For example, the sales person may learn how to use a set of organization-specific artifacts to communicate her activities to her supervisor. Some types of cultural skill are associated with the employee’s profession: for example, she may learn to write reports in a certain way, or to use common software to generate management accounting data. Other cultural skills are associated with the firm: these may include an understanding of the formal and informal lines of reporting in the organization, and an understanding of the most effective ways to attract attention and
support when needed. An employee who leaves the firm after acquiring cultural skills will retain her professional cultural knowledge, but will lose her firm culture. This corresponds to the cultural dissonance which Weber and Camerer identify.

In our model, learned cultural skills help to overcome agency problems between the manager and the employees. As in Alchian and Demsetz (1972), the role of the manager is to monitor the actions of the firm’s employees and hence to ensure that they act in a value-maximizing way. Monitoring is performed using both formal and informal reporting and communication lines which rely upon cultural skills. An employee who is not versed in the organization’s culture is unable properly to communicate with his managers and so cannot be monitored as effectively. In this case, contracting between the firm and the employee is impaired and the value to the firm of his skills is reduced.

Hence, in our model cultural norms substitute for explicit contracting when information is hard to gather and to verify. The employee uses his general skills to invest in a firm-specific project which will go ahead only if his tacit communication within the firm generates adequate monitoring systems: the likelihood of this is increasing in the cultural match between employee and firm. As a result, employee effort and social welfare is increasing in the strength of the cultural match.

We analyze in this setting the interaction between cultural skills and functional skills. Functional skill is part of an agent’s general human capital and it is perfectly transferable, in the sense that it will be equally as productive if deployed in an organization which can provide the necessary infrastructural support. For example, a stockbroker could take her client knowledge to any of a number of firms and be equally productive in any of them.

Functional skill can be acquired either through professional training paid for by the employee, for example on an MBA programme, or it can be paid for by the employer through mentoring in the early stages of the career. Since an employee’s productivity is adversely affected by the cultural displacement which he experiences upon leaving the firm, the employer will earn quasi-rents from his functional skill. If these quasi-rents are sufficiently high then the employer will be prepared to pay for training: if not the employee will purchase it himself. Corporate investment in general skills and the accumulation of firm-specific competencies are therefore susceptible to a cultural explanation.

Our analysis suggests the existence of a cultural channel which links communications technology to employee training and mobility. Advances in information processing which codify previously tacit
communications systems will serve to raise the importance of professional culture at the expense of organization-specific cultural norms in monitoring employees. This will reduce the efficiency cost of changing firms and so will increase the return which an employee earns on his functional skills. This will undermine the employer’s incentives to train its staff and so will result in an increased demand for professional qualifications.

For an example of industrial changes arising when formal language substitutes for firm-level cultural norms, we turn to the investment banking business. The introduction of risk management techniques based around Value at Risk (VaR) reports has recently revolutionized trading activities. VaR reports provide simple and standardized statistics about risk-taking which allow managers easily to quantify the risks which their employees are taking. Since its introduction in the early 1990s in J. P. Morgan, VaR reporting has become ubiquitous. It has provided standard codes for communicating risk information within the firm and has replaced earlier approaches which relied to a large extent upon informal communication and trust. Hence, VaR represents an advance in the power of risk managers’ professional language. In line with our model, it has reduced the rent which corporations earn from their trading activities. As a result, one would expect employees to bear a greater proportion of their training costs. Casual empiricism confirms that this is the case.

Our work is related to several strands of literature. The labour economics literature also focusses upon the relative importance of general and specific human capital. The theory of human capital was first developed by Becker (1964). Becker argued that, because employees will in a competitive labour market capture all of the returns to their general human capital, they will pay for it themselves. In two important papers, Acemoglu and Pischke (1998, 1999) show that firms may pay for general training in the presence of labour market frictions which compress wage differentials for skilled workers. They suggest that wage differentials may be compressed because of labour market matching costs of one type or another, or because of labour market intervention which raises minimum wages. Our argument is similar, but focusses upon the role of firm-specific language and norms in resolving commitment problems, and the concomitant problems associated with moving to a new cultural environment.

Murphy and Zábonjník (2004a, 2004b) argue that advances in management and computer science have caused an increased emphasis upon general, as opposed to firm-specific, skills. They use this to explain increased CEO mobility and wages, and they provide evidence in support of their theory that CEOs are more likely than ever before to have an MBA. While we emphasize the
The importance of changed language and culture in a world with a fixed body of general skill, Murphy and Zábonjník’s data is also supportive of our theory.

The switch to employee-financed training arises in our model when technological advances increase the importance of professional relative to firm culture. One could interpret this as an improved ability to write objective, as opposed to subjective, employment contracts. We show (proposition 4) that this raises welfare because the employee, unlike the firm, internalizes the benefits which functional skills bring in the labour market. This result contrasts with Baker, Gibbons and Murphy (1994), who argue in a model without labour markets that better objective contracts may lower welfare by undermining implicit contracting based in a repeated game upon subjective performance measures.

Since in our model cultural links serve to bind employees to their employer, it is meaningful in our set-up to talk about “corporate competencies.” Corporate competencies represent the organization’s collective expertise: they were discussed at length by Prahalad and Hamel (1990), who argued that corporations should design strategies in order to extend and to exploit their competencies. Crucial to the notion of a competence is the difficulty of replicating it in other corporations. Prahlad and Hamel’s seminal paper spawned a literature examining the implications of their ideas: see for example Collis and Montgomery (1995), and Conner (1991). But people, not companies, have skills, and the idea of the corporate competence is hard to reconcile with contractual views of the corporation. In providing a cultural language-based explanation for the competence, our paper therefore bridges a gap between the strategic management and economics literatures.

A substantial sociological literature examines the consequences of shared culture. Schein (1984) supports our assertion that culture generates shared assumptions and behaviour patterns. He argues that culture is manifested in organizational artifacts such as modes of access to senior management and the location of the coffee machine. At a deeper level, culture represents the deeply-held shared values of the corporation’s employees.

Our identification of cultural norms with the organization’s informal communication channels is consistent with much of the small economics literature on culture. Arrow (1974) suggests that organizational culture is constituted of codes developed within organizations to coordinate activities. Crémer (1993) thinks of culture as a store of common knowledge and language upon which the employees can draw to save costly time when responding to external stimuli. Further discussion of optimal organizational codes is provided by Crémer, Garicano and Prat (2004). Lazear (1999)
notes that cultural assimilation simplifies communication and hence facilitates trade between individuals. A related literature relates organizational form to the processing and communication of information (see for example Bolton and Dewatripont, 1994). However, in none of these papers does informal communication have a role in overcoming agency problems.

Although we believe that our model captures an essential quality of corporate culture, it leaves some aspects unaddressed. Much of the management literature addresses the behavioural characteristics of organizational culture. Kreps (1990) argues that culture can enable corporate actors to decide between multiple equilibria, and shows how reputational incentives can sustain cultural norms. Hermalin’s (1991) survey expands upon this theme. A related question is the extent to which a productive culture can be nurtured: this is addressed by Rob and Zemsky (2002) in a model in which workers reciprocate the behaviour which they experience from their peers.

Cultural assimilation is left unmodelled in our paper. Carillo and Gromb (1999, 2002) address this question by examining worker incentives to make culture-specific investments. Such investments generate positive externalities for other workers as they increase the firm’s costs of cultural change. The interesting problem of managerial myopia when assessing local culture and the obstacles it places in the way of cultural change is currently unexamined in the economics literature.

We formalize our discussion in sections 1 and 2 with a simple model of a two firm economy. Section 3 examines the relationship between culture and investment in training. Section 4 contains a discussion of our results and suggests some extensions to the model. Section 5 concludes.

1. Model

We consider the interaction between two firms and an employee. The action unfolds over two periods: period 1 runs from time 0 to time 1 and period 2 runs from time 1 to time 2.

We are concerned in this model with the incentives for general human capital production and their interaction with the operation of the labour market. We refer to general human capital as functional skill. The cost of acquiring functional skill $\lambda$ is $C(\lambda)$, where $C'(\cdot) > 0$ and $C''(\cdot) > 0$. At time 0 the employee has no functional skills but he can if he wishes purchase training before starting work. We write $\lambda^e$ for the level of functional skill which he purchases.

Employees cannot be productive without both functional skills and product knowledge. We think of product knowledge as essentially tacit: it can be acquired only through on-the-job experience and
not in a classroom. Product knowledge acquired in one firm is transferable to the other one. An example from investment banking is detailed knowledge of a specific client’s needs and preferences. Sales people who learn about a client frequently take the client with them if they move from one investment bank to another.

At time 0 the employee enters the labour market. He acquires product knowledge by working in period 1, but while he does so he is unproductive.\(^1\) During period 1 the employer can elect if it chooses to pay for additional training which will raise the employee’s functional skill level to \(\lambda \equiv \lambda^c + \lambda^f\). At the end of the period the employee re-enters the labour market and in period 2 he uses his functional skills productively.

In period 2 the employee can use his functional skill and product knowledge to work at a productive task. The employee exerts set-up effort \(e \in \mathbb{R}_{\geq 0}\) at cost \(e^2/2\). After he has done so, the task cannot be transferred to another firm. The employer can invest \(K > 0\) in the project, which will then return \(R\) or 0. The probability that the task returns \(R\) is \(\pi(e, \lambda)\). We assume that \(\pi\) is twice continuously differentiable, increasing in \(e\) and \(\lambda\), and concave in \((e, \lambda)\).

We assume that there is an agency problem between the employer and the employee. The employee will appropriate the returns from the project if he can, and such appropriation is not provable in court. Appropriation can be prevented only by active employer monitoring. Monitoring systems are created after the employee has exerted effort \(e\) to set up the project. They depend upon the employee’s ability to use the firm’s formal and informal lines of reporting to generate the information which the employer needs to oversee his actions. In other words, they rely upon the cultural fit between the employee and the employer. We formalize this dependency by defining the employee’s cultural fit with the firm to be the probability \(\kappa\) that the employee establishes a satisfactory monitoring system capable of preventing appropriation. In the absence of such a system, the employer will anticipate appropriation and will refuse to go ahead with the project.

Employees have no cultural skills at time 0, but they acquire them during period 1. At time 1, the employee’s fit with the period 1 employer is equal to \(\kappa_h\); his fit with the competitor is \(\kappa_p < \kappa_h\). The reduction \(\Delta \kappa \equiv \kappa_h - \kappa_p\) in cultural fit experienced by employees who switch employers at time 1 corresponds to the cultural displacement observed in Weber and Camerer’s (2003) experimental work. Although cultural fit reflects the employee’s ability to communicate with his manager, it

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\(^1\)This assumption is intended to ensure that an inexperienced employee is not valuable and is without significant loss of generality. It would be a simple matter to modify the model to allow the employee to perform unskilled clerical work in the first period.
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does not affect his productivity.

We interpret the cultural parameters \( \kappa_p \) and \( \kappa_h \) as follows. The employee’s ability to communicate with any employer in the market is captured by \( \kappa_p \), which is therefore a measure of professional culture, or of the power of the professional languages which carry professional cultures. Professional languages are product-specific: management accountants learn to communicate using one language, and commercial bank loan officers use another. Professional languages may become more powerful, and hence \( \kappa_p \) may increase, in response to technical factors such as improved computers or better mathematical models, or to social ones, such as the creation of standard-setting professional bodies. The employee’s cultural fit \( \kappa_h \) is greater at his current employer because he is able to use methodologies, reporting methods, and language which are peculiar to the firm. In other words, the parameter \( \Delta \kappa \) reflects the employee’s degree of firm-specific culture, or the degree of cultural differentiation between the firm and its competitors.

In our formal model, we treat the employee’s professional culture \( \kappa_p \) and his total culture \( \kappa_h \) at his current employer as independent variables. An alternative approach would be to regard professional and firm-specific (\( \Delta \kappa \)) cultures as independent. One approach can of course be turned into the other through a change of variables and much of the time, our presentation is unaffected by the choice. The only material difference is the effect under the two approaches of an independent change in professional culture, \( \kappa_p \). In our set-up, \( \kappa_h \) stays constant when this happens and so professional culture substitutes for firm-specific culture. When we discuss changes to professional culture, we have in mind technical advances which serve to codify previously tacit forms of communication. Such codification seems likely to replace informal firm-specific modes of communication and so we argue that our choice of independent variables is the more natural one.

The time line for the second period project is illustrated in figure 1. As discussed in the introduction, cultural fit serves to resolve a contracting problem: the greater the cultural fit, the more likely the firm is to be able effectively to contract with its employees. Conditional upon a successful monitoring system, the firm will write a contract with the employee under which he is (optimally) paid only when the task succeeds. We write \( w \) for the wage payment in the event that the task returns \( R \).

It follows from this discussion that the expected second period income of an employee with functional skill \( \lambda \) working in a firm with cultural fit \( \kappa \) and exerting effort \( e \) in exchange for incentive
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| Employee exerts set-up effort $e$ | Adequate monitoring systems established with probability $\kappa$ | Employer decides whether to invest | Returns realise: Employee paid Only after success |

![Figure 1: Second period project time line.](image)

The wage $w$ is

$$I(\kappa, w, \lambda, e) \equiv \kappa w \pi(e, \lambda) - e^2/2.$$  \hfill (1)

The expected profit which a firm generates from this employee is as follows:

$$P(\kappa, w, \lambda, e) \equiv \kappa \{ (R - w) \pi(e, \lambda) - K \}. \hfill (2)$$

In the following sections we solve this model by backwards induction to determine the personal and firm-level investments $\lambda^e$ and $\lambda^f$ in human capital.

### 2. Second Period Decisions

Given cultural fit $\kappa$, employment contract $w$, and functional skill $\lambda$, the optimal period 2 effort level $\bar{e}(\kappa, w, \lambda)$ satisfies the following first order condition:\footnote{To avoid notational clutter, throughout the paper we use subscripts to denote partial derivatives: hence $\pi_1(e, \lambda) = \frac{\partial \pi}{\partial e}(e, \lambda)$, $\pi_{11}(e, \lambda) = \frac{\partial^2 \pi}{\partial e^2}(e, \lambda)$ and so on.}

$$\bar{e} = \kappa w \pi_1(\bar{e}, \lambda). \hfill (3)$$

Let $\bar{I}(\kappa, w, \lambda) \equiv I(\kappa, w, \lambda, \bar{e}(\kappa, w, \lambda))$ be the income level corresponding to $\bar{e}(\kappa, w, \lambda)$. Define $w^h(\kappa_h, \kappa_p, w, \lambda)$ to be the wage which the existing employer has to pay to give an employee with functional skill $\lambda$ the expected income which he would derive from a wage $w$ paid by the competitor:

$$\bar{I}(\kappa_h, w^h(\kappa_h, \kappa_p, w, \lambda), \lambda) = \bar{I}(\kappa_p, w, \lambda). \hfill (4)$$

**Lemma 1** $w^h(\kappa_h, \kappa_p, w, \lambda) = \kappa_p w / \kappa_h < w$.

Lemma 1, whose proof appears in the appendix, is intuitively obvious. An employee who exerts a high level of functional effort will be rewarded for doing so only if his employer is able to understand his project. So a poor cultural fit reduces the responsiveness of income to effort and
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hence undermines the effectiveness of incentive pay. As a result, employers with a strong cultural fit with their employees can pay them less.

Define

\[ w^* (\kappa_h, \kappa_p, \lambda) \equiv w^h (\kappa_h, \kappa_p, R, \lambda); \]  
\[ e^* (\kappa_h, \kappa_p, \lambda) \equiv e (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda). \]  

**Proposition 1** The employee does not change firms in the second period. If the incumbent firm cannot profit from employing him, he is unemployed. Otherwise, he is paid \( w^* (\kappa_h, \kappa_p, \lambda) \) provided condition (7) is true:

\[ \kappa_h (R - w^* (\kappa_h, \kappa_p, \lambda)) \left[ \frac{\pi_1 (e^* (\kappa_h, \kappa_p, \lambda), \lambda)}{\pi (e^* (\kappa_h, \kappa_p, \lambda), \lambda)} \right]^2 < 1. \]  

When condition (7) is false the employee is paid the wage \( \hat{w} > w^* (\kappa_h, \kappa_p, \lambda) \) which makes the condition hold with equality.

The proof of proposition 1 appears in the appendix. It follows intuitively because the highest payment which the competitor can afford to make is \( R \) and an offer of \( w^* (\kappa_h, \kappa_p, \lambda) \) from the existing employer is equally attractive to the employee. The employer experiences a direct cost when wages are increased, and an indirect gain from the higher effort which they induce. Condition (7) is satisfied when the former effect outweighs the latter. This occurs when professional languages are sufficiently powerful (\( \kappa_p \) is sufficiently high) for competitive pressures to allow the employee to extract a significant rent from his functional skills. We assume for the remainder of the paper that this is the case. Then the employer makes positive profits provided

\[ (R - w^* (\kappa_h, \kappa_p, \lambda)) \pi (e^* (\kappa_h, \kappa_p, \lambda), \lambda) - K > 0. \]  

Again, we assume that this is the case.

Proposition 1 states that the employee will not change firms after the first period of his career. This is true because his existing employer’s superior cultural skills give it an advantage in resolving the principal/agent problem which exists between them. The existing employer can therefore match any outside option, and still retain some of the profits from the employee’s actions. In other words, functional skills which reside in the corporation after one period will remain there.

Our model therefore provides an explanation for the creation of “corporate competencies”: although these are general skills which reside within the employees of the firm, they are tied to
the firm by cultural norms which are valuable in aligning the employee’s incentives with those of
the employer. This remark applies equally to the retention within the firm of product knowledge.
A unique corporate culture allows the firm to establish property rights over key assets such as
client relationships. It may help to explain the existence of organizations such as investment banks,
which exist essentially to invest in and to maintain informational assets whose ownership cannot
be established in court.

We make the following definitions:

\[ I^* (\kappa_h, \kappa_p, \lambda) \equiv \bar{I} (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda); \]  

\[ P^* (\kappa_h, \kappa_p, \lambda) \equiv P (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda, e^* (\kappa_h, \kappa_p, \lambda)); \]  

\[ \pi^* (\kappa_h, \kappa_p, \lambda) \equiv \pi (e^* (\kappa_h, \kappa_p, \lambda), \lambda). \]  

With assumption (7), \( w^* \), \( e^* \), \( I^* \), \( P^* \), and \( \pi^* \) are respectively the equilibrium period 2 wage,
effort level, employee income, employer profits and probability of task success. Define \( W^* (\kappa_h, \kappa_p) \) as follows:

\[ W^* (\kappa_h, \kappa_p, \lambda) \equiv I^* (\kappa_h, \kappa_p, \lambda) + P^* (\kappa_h, \kappa_p, \lambda). \]  

\( W^* \) is the total expected second period surplus after investments in the employee’s functional skill
\( \lambda \) have been performed and hence is a measure of \textit{ex post} welfare.

The following proposition examines comparative statics with respect to \( \kappa_h \) and \( \kappa_p \).

**Proposition 2**

1. \( e^* \), \( \pi^* \) and \( I^* \) are unaffected by changes in \( \kappa_h \) and are increasing in \( \kappa_p \);

2. \( P^* \) is increasing in \( \Delta \kappa \);

3. \( W^* \) is increasing in \( \kappa_p \) and in \( \kappa_h \).

The intuition for these results is as follows. Firstly, recall that the employee’s period 2 income
is equal to his outside option. This is unaffected by \( \kappa_h \), and so nor is his effort or income. Since
the probability of project success depends upon employee effort this is also unaffected by \( \kappa_h \) and is
increasing in \( \kappa_p \).

Part 2 of the proposition is best illustrated by setting \( \kappa_h w^* (\kappa_h, \kappa_p, \lambda) = \kappa_p R \) in the expression
for \( P^* \) to yield the following expression:

\[ P^* (\kappa_h, \kappa_p, \lambda) = R \Delta \kappa \pi^* (\kappa_h, \kappa_p, \lambda) - K \kappa_h. \]  

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Cultural differentiation ($\Delta \kappa$) imposes a productivity cost upon employees who switch firms at time 1 and this cost is extracted as period 2 rent by the initial employer. Intuitively therefore, we expect greater cultural differentiation to raise employer profits. Note though that changes in $\Delta \kappa$ have an indirect effect upon $\pi^*$ so that we need to check that this effect does not outweigh their direct impact upon $P^*$. This is accomplished in the appendix.

Finally, by part 1 of the proposition, increases in $\kappa_h$ do not affect the probability of project success, but they do affect the probability $\pi^*$ of project execution and so raise ex post welfare. This increase accrues entirely to the period 1 employer without affecting the employee’s wealth. Increases in the power $\kappa_p$ of professional languages raise the probability $\pi^*$ of project success without changing the probability of project execution and hence raise ex post welfare. This increase flows entirely to the employee, who by part 2 of the proposition will in this case also receive a wealth transfer from the employer.

We conclude this section by examining the comparative statics with respect to functional skill ($\lambda$).

**Proposition 3**

1. $w^*$ is unaffected by $\lambda$, while $I^*$ is increasing in $\lambda$;
2. $e^* (\kappa_h, \kappa_p, \lambda)$ is increasing in $\lambda$ if and only if $\pi_{12} (e^*, \lambda) > 0$;
3. $\pi^*$ and $P^*$ are all increasing in $\lambda$ if and only if
   \[
   e_3^* (\kappa_h, \kappa_p, \lambda) > -\frac{\pi_2 (e^*, \lambda)}{\pi_1 (e^*, \lambda)},
   \]
   which is true if and only if
   \[
   \pi_{12} (e^*, \lambda) > -\frac{1 - \kappa_h w^* (\kappa_h, \kappa_p, \lambda) \pi_{11} (e^*, \lambda) \pi_2 (e^*, \lambda)}{\kappa_h w^* (\kappa_h, \kappa_p, \lambda) \pi_1 (e^*, \lambda)},
   \]
4. $W^*$ is increasing in $\lambda$ if and only if
   \[
   e_3^* (\kappa_h, \kappa_p, \lambda) > -\frac{\pi_2 (e^*, \lambda)}{\pi_1 (e^*, \lambda)} \left( 1 + \frac{\kappa_h w^* (\kappa_h, \kappa_p, \lambda)}{R \Delta \kappa} \right),
   \]
   which is true if and only if
   \[
   \pi_{12} (e^*, \lambda) > -\frac{1 - \kappa_h w^* (\kappa_h, \kappa_p, \lambda) \pi_{11} (e^*, \lambda) \pi_2 (e^*, \lambda)}{\kappa_h w^* (\kappa_h, \kappa_p, \lambda) \pi_1 (e^*, \lambda)} \left( 1 + \frac{\kappa_h w^* (\kappa_h, \kappa_p, \lambda)}{R \Delta \kappa} \right).
   \]

At first blush, it may appear surprising that $\partial w^*/\partial \lambda = 0$ so that an increase in functional skill does not earn the employee a higher incentive wage. The reason for this is that increasing
functional skill raises the value of the outside option, but it also raises the value of staying at the existing employer. The employer need only compensate the employee to the extent that the former effect outweighs the latter. In our set-up, these effects cancel one another out. Note though that increased functional skill raises the value of projects which receive the go-ahead from the existing employer and hence that it increases the employee’s expected income: $\partial I^*/\partial \lambda > 0$.

Intuitively, increased functional skill will cause the employee to work harder only when skill and effort are complements: i.e., when $\pi_{12} > 0$. When $\pi_{12} < 0$ so that effort substitutes for skill, increased functional skill will serve to lower the employee’s effort.

Condition (14) states that increased skill will raise welfare (and profits and the success probability) when it serves to increase effort further than would be required to remain on the same iso-$\pi$ curve. This is equivalent (equation 15) to the statement that effort and skill are not too easily substituted: when they are, the employee’s gains are partially at the expense of the employer. The final part of the proposition shows that for sufficiently low $e_3^* (\kappa_h, \kappa_p, \lambda)$, the employee gains from higher $\lambda$ are entirely at the employer’s expense, and that \textit{ex post} welfare is reduced.

3. Investment in Functional Skills

In this section we analyze the employee’s time 0 and the employer’s period 1 equilibrium investments in functional skill, which we denote by $\lambda^e (\kappa_h, \kappa_p)$ and $\lambda^f (\kappa_h, \kappa_p)$ respectively. We write

$$\lambda (\kappa_h, \kappa_p) \equiv \lambda^e (\kappa_h, \kappa_p) + \lambda^f (\kappa_h, \kappa_p)$$

for the total investment in human capital. Firstly, we establish the following result for the marginal returns to functional skill for the employee and the employer:

\textbf{Lemma 2}

1. $I_{33}^* (\kappa_h, \kappa_p, \lambda) < 0$;

2. $P_{33}^* (\kappa_h, \kappa_p, \lambda) < 0$ if and only if condition (16) is satisfied:

$$e_{33}^* (\kappa_h, \kappa_p, \lambda) < -\frac{1}{\pi_1 (e^*, \lambda)} \left\{ \pi_{11} (e^*, \lambda) (e_3^*)^2 + 2 \pi_{12} (e^*, \lambda) e_3^* + \pi_{22} (e^*, \lambda) \right\},$$

where $e^*$ and its derivatives are evaluated at $(\kappa_h, \kappa_p, \lambda)$.

Note that the curly-bracketed term in condition (16) is negative by virtue of the concavity of $\pi$. The condition therefore requires $e_{33}^*$ to be small and positive, or negative. This is ultimately a
statement about the third derivatives of \( \pi \): we adopt it as an assumption.\(^3\)

We define \( L^e (\kappa_h, \kappa_p) \) and \( L^f (\kappa_h, \kappa_p) \) as follows:

\[
I^*_3 (\kappa_h, \kappa_p, L^e (\kappa_h, \kappa_p)) = C' (L^e (\kappa_h, \kappa_p));
\]

\[
P^*_3 (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p)) = C' (L^f (\kappa_h, \kappa_p)),
\]

and we assume that

\[
I^* (\kappa_p, \kappa_p, L^e (\kappa_p, \kappa_p)) - C (L^e (\kappa_p, \kappa_p)) > 0.
\]

Conditional upon no investment by the employer, \( L^e (\kappa_h, \kappa_p) \) is the investment which the employee makes in functional skill (\( \lambda \)); assumption (19) states that this level of investment is individually rational for the employee when \( \kappa_h = \kappa_p \). Since \( I^* \) is \( \kappa_h \)-invariant, the employee’s participation constraint in the absence of employer investment is satisfied for all \( \kappa_h \).

If \( P^*_3 (\kappa_h, \kappa_p, \lambda) \) was negative then the employer would never invest in functional skill. To rule out this uninteresting case, we assume that condition (14) is satisfied. Then, subject to satisfying his participation constraint, the employer would be prepared to invest in functional skill precisely until the total level of skill was \( L^f (\kappa_h, \kappa_p) \).

**Lemma 3** In equilibrium, either:

1. \( \lambda^e (\kappa_h, \kappa_p) = L^e (\kappa_h, \kappa_p) \) and \( \lambda^f (\kappa_h, \kappa_p) = 0; \)
2. or \( \lambda^e (\kappa_h, \kappa_p) = 0 \) and \( \lambda^f (\kappa_h, \kappa_p) = L^f (\kappa_h, \kappa_p). \)

**Proof.** If \( \lambda^f (\kappa_h, \kappa_p) = 0 \) then by definition \( \lambda^e (\kappa_h, \kappa_p) = L^e (\kappa_h, \kappa_p). \) If \( \lambda^f (\kappa_h, \kappa_p) > 0 \) then again by definition, \( \lambda^f (\kappa_h, \kappa_p) = L^f (\kappa_h, \kappa_p). \) In this case an employee who invested in functional skills would therefore bear some of the costs of training without changing period 2 functional skills: employee investment is valuable only insofar as it generates a period 1 return. But investment in the employee’s skill does not bear fruit until after period 1 on-the-job learning. Hence \( L^e (\kappa_h, \kappa_p) = 0. \)

\(^3\)For example, it is easy to see that the assumption will be true whenever \( \pi (e, \lambda) \) takes the form \( f (e) + g (\lambda) + k (e) l (\lambda) \) for concave \( f, g, k, \) and \( l, \) with \( f'' \) and \( k'' \) both negative, or small and positive.

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We write $E^e$ and $E^f$ for the employee’s expected income in cases (1) and (2) of lemma 3, and $F^e$, $F^f$ for the firm’s income:

\[
E^e(\kappa_h, \kappa_p) \equiv I^*(\kappa_h, \kappa_p, L^e(\kappa_h, \kappa_p)) - C(L^e(\kappa_h, \kappa_p));
\]

\[
E^f(\kappa_h, \kappa_p) \equiv I^*(\kappa_h, \kappa_p, L^f(\kappa_h, \kappa_p));
\]

\[
F^e(\kappa_h, \kappa_p) \equiv P^*(\kappa_h, \kappa_p, L^e(\kappa_h, \kappa_p));
\]

\[
F^f(\kappa_h, \kappa_p) \equiv P^*(\kappa_h, \kappa_p, L^f(\kappa_h, \kappa_p)) - C(L^f(\kappa_h, \kappa_p)).
\]

The IR constraint (19) therefore reduces to $E^e(\kappa_p, \kappa_p) > 0$ and lemma 3 shows that the training IR constraint for the employer is $F^f(\kappa_h, \kappa_p) > 0$. In the case where this does not hold, it is convenient to define $L^f(\kappa_h, \kappa_p) = 0$. With this definition it is obvious that $\lambda^e(\kappa_h, \kappa_p) = L^e(\kappa_h, \kappa_p)$ precisely when $E^e(\kappa_h, \kappa_p) \geq E^f(\kappa_h, \kappa_p)$. We now examine the effect of the cultural parameters $\kappa_h$, $\kappa_p$ upon incentives to invest in functional skills:

**Lemma 4**

1. $L^e$ is unaffected by $\kappa_h$ and is increasing in $\kappa_p$;
2. $L^f$ is increasing in $\kappa_h$. It is decreasing in $\kappa_p$ precisely when condition (20) is satisfied:

\[
\Delta \kappa \pi_{23}^* \left( \kappa_h, \kappa_p, L^f(\kappa_h, \kappa_p) \right) < \pi_{3}^* (\kappa_h, \kappa_p). \tag{20}
\]

The intuition behind part 1 of lemma 4 is straightforward. Since changes in $\kappa_h$ do not affect the employee’s outside option they do not affect his incentives to invest in functional skills; increases in $\kappa_p$ raise the return which he can extract from his human capital and so incentivize its augmentation.

The first statement in part 2 is intuitively obvious: since the employer captures all of the efficiency gains from higher $\kappa_h$, we must have $\partial L^f / \partial \kappa_h > 0$. The second statement highlights two effects which a higher $\kappa_p$ has upon the employer’s incentives. Firstly, there is a direct effect represented by the term on the right hand side of equation (20): *ceteris paribus*, advances in professional languages reduce the share of the returns to functional skill which the employer can capture and hence reduce his incentive to invest in functional skill. Secondly, there is an indirect effect represented by the term on the left hand side of the equation. The marginal effect of $\lambda$ may be altered by changes in $\kappa_p$ and this may affect investment incentives either positively or negatively, according to the sign of $\pi_{23}^*$.

Further expansion of expression (20) is not illuminating. For the remainder of the paper we will assume that the direct effects of increases in $\kappa_p$ outweigh the indirect effects and hence we
adopt equation (20) as an assumption. Like equation (16), it is satisfied for a number of plausible functional forms for $\pi$.

Let
\[
W(\kappa_h, \kappa_p) \equiv W^*(\kappa_h, \kappa_p, \lambda(\kappa_h, \kappa_p)) - C(\lambda(\kappa_h, \kappa_p))
\]
be the *ex ante* expected welfare: in case (1) of lemma 3 we have $W = E^e + F^e$, and in case 2, $W = E^f + F^f$.

**Proposition 4** There exists $\bar{\kappa}_h(\kappa_p) \leq \infty$, with $\bar{\kappa}_h(\kappa_p) > 0$, such that $\lambda^e(\kappa_h, \kappa_p) = L^e(\kappa_h, \kappa_p)$ precisely when $\kappa_h \leq \bar{\kappa}_h(\kappa_p)$. Expected welfare $W(\kappa_h, \kappa_p)$ is continuously increasing in $\kappa_h$ for $\kappa_h \neq \bar{\kappa}_h(\kappa_p)$, and exhibits a discontinuous drop at $\kappa_h = \bar{\kappa}_h(\kappa_p)$.

![Figure 2: Payment for general human capital.](image)

Proposition 4 is illustrated in figure 2. It is the natural consequence of our formalization of corporate culture. When cultural differences serve to reduce labour mobility, functional skills are valuable to the employee only insofar as they would affect his productivity at competitor firms. When the cultural gulf between firms is so great that the employee would find it hard to employ his functional skills anywhere else, the employer will have a greater incentive to invest in training than the employee, and the employee will no longer pay for his own education.

Note better professional languages (higher $\kappa_p$) serve to increase employee’s ability to communicate with other employers and hence raise the marginal value which he places upon functional skill. The range of $\kappa_h$ values for which the employee educates himself is therefore increasing in $\kappa_p$. 

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We show in the proof of proposition 4 that $E^e$ is unaffected by $\kappa_h$ and that $E^f$, $F^e$ and $F^f$ are all increasing in $\kappa_h$ away from $\bar{\kappa}_h$. Note that the employee decides whether or not to invest in functional skills. At the critical value $\bar{\kappa}_h(\kappa_p)$, $E^e(\kappa_h, \kappa_p) = E^f(\kappa_h, \kappa_p)$ and he is indifferent between training himself and relying upon training provided by his employer. We therefore have:

**Corollary 1** The employee’s welfare is a continuous weakly increasing function of $\kappa_h$. The employer’s welfare is a continuous increasing function of $\kappa_h$ away from $\bar{\kappa}_h(\kappa_p)$, and the employer experiences the whole of the welfare drop at $\bar{\kappa}_h(\kappa_p)$.

We have argued that a drop in cultural differentiation which takes the firm from slightly above to slightly below $\bar{\kappa}_h(\kappa_p) - \kappa_p$ arises when improved communications reduce the importance of tacit cultural communications. In our model, this shift from subjective to objective contracting raises welfare. The reason is that when the employee takes responsibility for his training, he internalizes the benefit which it brings him in the time 1 labour market. In contrast, the employer does not account for this effect when making training decisions. As we note in the introduction, our result is in contrast with those obtained in a different context by Baker et al. (1994).

4. Discussion

4.1. Technological Shocks in Professional Services Firms

We argue that our theory is of particular relevance in the context of professional services firms such as law partnerships and investment banks. Professional services firms sell skills which are embedded in their employees. Employees have traditionally received much of their functional training on-the-job. Junior employees are typically cash-constrained and hence, while they may to some extent pay for their training as they receive it through lower wages or longer working hours, some payment must occur after the training has occurred. In a world of imperfect contracts and inalienable human capital, post-education payment is limited by the size of the employee’s outside option.

Our model provides a positive role in this context for cultural differences: when skills are acquired on the job, the difficulty of adapting to a new corporate culture restricts the employee’s mobility and hence guarantees the employer a return on his investment in training. It is clear from our model that, in the case where the employee is unable to acquire functional skills without on the job training, cultural differences are essential to their dissemination.
Notwithstanding these observations, there has been in recent years a significant drop in workplace training in professional services. For example, Hillman (2001) documents a sharp rise in lawyer mobility in the United States. Coincident with this rise has been a sharp reduction in training activities: Hillman states that

“Mentoring is haphazard, if it exists at all. Firms point to the new economics of law practice and ask law schools to do more.” (ibid, p. 1078).

This observation would be consistent in our model with an increase in the power $\kappa_p$ of professional languages: according to proposition 4, this will increase the range of cultural parameters $\kappa_h$ for which employees will purchase their own training. Similarly, the last two decades have seen a sharp increase in the demand for M.B.A. graduates in investment banking. The increased reliance upon self-training in investment banking is again explicable in terms of an increase in the professional culture parameter, $\kappa_p$.

One possible explanation for increased $\kappa_p$ values in professional services firms is recent advances in information technology. The advent of the personal computer and the development of intelligent software has increased the codifiability of functional skill. At the same time, it has standardized much of the reporting which occurs within professional services firms. For example, the advent in the investment banking world of standard risk management systems such as J.P. Morgan’s RiskMetrics has given investment banks the tools they need to monitor new hires and to ensure that they are immediately productive. In short, we contend that technological advantages have raised the base quality of within-firm communication and hence have reduced the importance of firm-specific cultural artifacts in facilitating new business. As a result human capital is more mobile and more training is purchased outside the firm.

### 4.2. Employee Mobility

The increased emphasis upon professional training identified above in the law and investment banking industries has occurred at the same time as a marked increase in employee mobility. In our simple model, employees never switch firms: this feature clearly does not fit the real world. Introducing such a feature would however be a simple matter, although it would significantly increase the complexity of the model without adding any further insights.

One natural way to incorporate labour mobility would be to endow each company with a
different production technology, with the stochastic return $R$ on successful projects drawn from a common distribution before the opening of the time 1 labour market. Employees would continue to work in period 2 for the firm at which their functional skills would be most useful. As the two companies’ returns would be i.i.d., the most likely period two employer would be the one with the highest cultural fit, but with non-zero probability the competitor’s returns would exceed those of the incumbent employer by a sufficient amount to compensate for its lower cultural fit. Naturally, as $\Delta \kappa$ decreased, the probability of this event would increase and greater employee mobility would occur simultaneously with an increased emphasis upon self-training. Moreover, in this extension anticipated labour mobility would serve further to reduce the incumbent employer’s incentives to invest in employee skills.

4.3. Team Transfers

The investment bank Credit Suisse First Boston attempted in July 2004 simultaneously to lure twenty five equity and fixed income origination and syndication staff from Dresdner Kleinwort Wasserstein, offering in the process to more than double their compensation.4 “Team transfers” of this type, under which an entire division moves from one firm to another, are increasingly common in investment banking. We believe that our model goes some way towards explaining this phenomenon.

We have argued that employees are most effective when operating in a familiar culture. The discussion so far suggests that this culture exists at the level of the firm. In practice, however, it is probably concentrated in the people with whom the employee has the most frequent work contact. We call a subset of the firm whose important contact is with one another a “team”. Such a team was the target of the CSFB raid in the preceding paragraph. Our model shows that the general skills possessed by a team are most valuable when they operate as a group. Rival employers would therefore prefer to employ all, rather than a part of, the team.

Team transfers are not restricted to the investment banking industry. For example, Fee, Hadlock and Pierce (2004) show that American football coaches are often hired and fired as a group. The shared language of the team is suggestive of team, as opposed to firm, human capital. Team human capital is examined in the context of contract renegotiation by Chillemi and Gui (1997). Chillemi

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4“CSFB launches raid to lure 25 DrKW staff,” Patrick Jenkins and Claudia Wanner, Financial Times, 1 July 2004.
and Gui show that workers can obtain wage premia when renegotiating in the presence of team human capital by threatening to walk out, and thus imposing a negative externality upon the team. To the extent that team members could be replaced by other employees of the firm with similar cultural skills, this effect will be attenuated. Such replacement is however impossible in the context of the team transfer, which may explain the high salary rises associated with team transfers.

4.4. Optimal Cultural Differentiation

In our model cultural differences tie employees to the corporation and hence allow the company to acquire competencies. We therefore provide an explanation for the assertion, common in the management studies literature, that a unique organizational culture can give organizations a competitive edge (Wilson and Rosenfeld, 1990). This observation has generated a related literature which discusses the extent to which corporations can “manage” their cultures (Peters and Waterman, 1982).

Most academic writers have concluded that active cultural management is extremely difficult: cultural norms are so deeply embedded in the groups which manifest them that they are hard to measure or even to quantify (Barney, 1986). Our model suggests that, far from representing a problem to the corporation, this feature of culture is central to its value. To the extent that culture can be codified, it can be replicated. Replication of a successful culture will allow employees to move between corporations without experiencing the costs of cultural dissonance: in the context of our model, it will raise the power $\kappa_p$ of professional languages. Cultural differences are valuable to the corporation precisely because they cannot be codified.

Nevertheless, we can use our model to think about cultural management. Active cultural management will in our set-up result in an increase in $\kappa_h$. We know from corollary 1 that this will raise employer welfare everywhere except at $\bar{\kappa}_h (\kappa_p)$, where it will cause a discontinuous drop in welfare. For fixed $\kappa_p$ an employer with control over cultural norms will therefore elect to raise $\kappa_h$ either to $\bar{\kappa}_h - \varepsilon$, or will raise them sufficiently far above $\bar{\kappa}_h$ to cover the welfare loss which he experiences as he starts to train. Provided the costs of cultural differentiation are not too severe, for given $\kappa_p$ one would therefore expect corporations to differentiate themselves as far as possible along cultural lines.

In practice, the extent to which this form of differentiation is possible depends upon the number
of companies and the topology of the $\kappa$ space. For example, suppose that cultural values are located on a circle of radius $\rho$, and that cultural differentiation is measured along its circumference. Then with $n$ firms the maximum degree of differentiation will be $2\pi \rho / n$: if this is just in excess of $\bar{\kappa}_h$ then firms will optimally cluster. A detailed model of this type of effect would allow us to examine the effects of cultural differences upon entry incentives, and is outside the scope of this paper.

The effect of an increase in $\kappa_p$ upon the employer’s preferred $\Delta \kappa$ is in our model ambiguous. Since $\bar{\kappa}_h'(\kappa_p) > 0$, firms with $\Delta \kappa = \bar{\kappa}_h - \varepsilon$ might be expected to react by attempting to raise $\kappa_h$. On the other hand, firms with $\kappa_h$ slightly above $\bar{\kappa}_h$ might find it cheaper to reduce cultural specificity to below the new $\bar{\kappa}_h$ than to raise it still further. To the extent that increases in $\kappa_p$ presage increased costs of cultural management, one might expect the latter effect to obtain. In any case, we have argued above that cultural fine-tuning of the type discussed in this paragraph is probably impossible.

5. Conclusion

In this paper we examine the consequences for general human capital production and mobility of corporate culture. We view corporate culture as comprising tacit firm-level communication skills which are essential to managerial monitoring. An employee’s general human capital is productive only insofar as he can be monitored, and the cultural dissonance which he will experience upon leaving his firm therefore impairs his ability to sell his skills elsewhere. This observation explains firm investment in general human capital.

Cultural norms are important in resolving agency problems only insofar as communication technologies are tacit. We argue that advances in information processing and reporting have in recent years facilitated the codification of many formerly tacit channels of reporting. This has increased employee mobility and has disincentivized on-the-job transfer of general skills. As a result we have in recent years witnessed an increasing demand for professional degrees such as MBAs.

We argue that corporate competencies arise when employees and their general human capital are culturally bound to their organizations. This generates a new insight into the competence-based view of corporate strategy. When cultural differentiation enables corporations to generate quasi-rents from competencies, it seems likely that corporations should enter businesses which reinforce
and which rely upon their cultural differences. Hence, corporate strategy is to a large extent associated with cultural management. We leave this observation for later research.

References


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### Appendix

**Proof of Lemma 1**

Applying the Envelope Theorem to equation (1) yields the following

\[
\bar{I}_1 (\kappa, w, \lambda) = w \pi (\bar{e} (\kappa, w, \lambda), \lambda) ;
\]

\[
\bar{I}_2 (\kappa, w, \lambda) = \kappa \pi (\bar{e} (\kappa, w, \lambda), \lambda) .
\]  

Differentiate equation (4) with respect to \(\kappa_h\) to obtain

\[
w_h^h (\kappa_h, \kappa_p, w, \lambda) = - \frac{\bar{I}_1 (\kappa_h, w^h, \lambda)}{\bar{I}_2 (\kappa_h, w^h, \lambda)} = - \frac{w^h (\kappa_h, \kappa_p, w, \lambda)}{\kappa_h} .
\]

Now define \(G (\kappa_h, \kappa_p, w, \lambda) \equiv \kappa_h w^h (\kappa_h, \kappa_p, w, \lambda)\). Then \(G (\kappa_p, \kappa_p, w, \lambda) = \kappa_p w\) and \(G_1 (\kappa_h, \kappa_p, w, \lambda) = w^h (\kappa_h, \kappa_p, w, \lambda) + \kappa_h w^h (\kappa_h, \kappa_p, w, \lambda) = 0\), so that \(\kappa_h w^h (\kappa_h, \kappa_p, w, \lambda) = \kappa_p w\) as required.

Note that the right hand side of equation (23) is the slope of iso-income curves in \(\kappa_h - w^h\) space: the equation simply states that \(w^h\) will be altered in response to changes in \(\kappa_h\) in order to maintain the income of an employee with cultural fit \(\kappa_h\) at \(I (\kappa_p, w, \lambda, \bar{e} (\kappa_p, w, \lambda))\). Such an employee has effort level \(\bar{e} (\kappa_h, w^h (\kappa_h, \kappa_p, w, \lambda), \lambda)\) : examination of the expression for \(I\) therefore suggests that we must have \(\bar{e} (\kappa_h, w^h (\kappa_h, \kappa_p, w, \lambda), \lambda) = e (\kappa_p, w, \lambda)\). This is indeed the case; it is demonstrated in the special case where \(w = R\) in the proof of proposition 2 below.
Proof of Proposition 1

Write \( \tilde{P}(\kappa, w, \lambda) \equiv P(\kappa, w, \lambda, \bar{e}(\kappa, w, \lambda)) \) for the profits earned by an employer with cultural fit \( \kappa \) paying incentive wage \( w \) to an employee with functional skill \( \lambda \).

Trivially, no wage \( w \) below \( w^*(\kappa_h, \kappa_p, \lambda) \) can be an equilibrium since whichever employer pays it, the competitor can afford a wage at which the employee earns a higher income. The incumbent can always attract the employee by paying \( w^*(\kappa_h, \kappa_p, \lambda) \). This generates the highest return for the incumbent precisely when \( \tilde{P}_2(\kappa, w, \lambda) < 0 \); he will otherwise select the wage \( \hat{w} > w^* \) at which \( \tilde{P}_2(\kappa_h, \hat{w}, \lambda) = 0 \). To differentiate \( \tilde{P} \), we require the following, obtained from equation (3):

\[
\bar{e}_1(\kappa, w, \lambda) = \frac{w \pi_1(\bar{e}(\kappa, w, \lambda), \lambda)}{1 - \kappa w \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)}; \tag{24}
\]

\[
\bar{e}_2(\kappa, w, \lambda) = \frac{\kappa \pi_1(\bar{e}(\kappa, w, \lambda), \lambda)}{1 - \kappa w \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)} = \frac{\kappa}{w} \bar{e}_1(\kappa, w, \lambda). \tag{25}
\]

Then we have

\[
\tilde{P}_2(\kappa, w, \lambda) = P_4(\kappa, w, \lambda, \bar{e}(\kappa, w, \lambda)) \bar{e}_2(\kappa, w, \lambda) + P_2(\kappa, w, \lambda, \bar{e}(\kappa, w, \lambda))
\]

\[
= \kappa \pi(\bar{e}(\kappa, w, \lambda), \lambda) \left\{ \frac{\kappa (R - w)}{1 - \kappa w \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)} \left[ \pi_1(\bar{e}(\kappa, w, \lambda), \lambda) \right]^2 \left( \frac{\pi(\bar{e}(\kappa, w, \lambda), \lambda)}{\pi(\bar{e}(\kappa, w, \lambda), \lambda)} - 1 \right) \right\},
\]

from which condition (7) follows. Provided he makes a profit from doing so, the incumbent will therefore employ the employee in the second period.

Proof of Proposition 2

Set \( w = R \) in equation (23) to obtain \( w_1^*(\kappa_h, \kappa_p, \lambda) = -w^*(\kappa_h, \kappa_h, \lambda) / \kappa_h \). Then from equations (6), (24) and (25),

\[
e_1^*(\kappa_h, \kappa_p, \lambda) = \bar{e}_1(\kappa_h, w^*(\kappa_h, \kappa_p, \lambda), \lambda) + \bar{e}_2(\kappa_h, w^*(\kappa_h, \kappa_p, \lambda), \lambda) w_1^*(\kappa_h, \kappa_p, \lambda)
\]

\[
= \bar{e}_1(\kappa_h, w^*(\kappa_h, \kappa_p, \lambda), \lambda) \left\{ 1 - \frac{\kappa_h}{w^*(\kappa_h, \kappa_p, \lambda)} \right\} = 0.
\]

Similarly, differentiating equation (5) yields

\[
w_2^*(\kappa_h, \kappa_p, \lambda) = R \pi(\bar{e}(\kappa_p, R, \lambda), \lambda) / \{ \kappa_h \pi(\bar{e}(\kappa_h, w^*(\kappa_h, \lambda, \lambda), \lambda), \lambda) \}.
\]
Since $e_1^* = 0$ we have $e^* (\kappa_p, \kappa_p, \lambda) = e^* (\kappa_h, \kappa_p, \lambda)$ and hence $w_2^* (\kappa_h, \kappa_p, \lambda) = R / \kappa_h$. Hence, again using equations (24) and (25),

$$e_2^* (\kappa_h, \kappa_p, \lambda) = \frac{R \pi_1 (\bar{e} (\kappa, w, \lambda), \lambda)}{1 - \kappa_h w^* (\kappa_h, \kappa_p, \lambda) \pi_{11} (\bar{e} (\kappa, w, \lambda), \lambda)} > 0.$$  

Now differentiate equation (11) to obtain

$$\pi_1^* (\kappa_h, \kappa_p, \lambda) = \pi_1 (e^* (\kappa_h, \kappa_p, \lambda), \lambda) e_1^* (\kappa_h, \kappa_p, \lambda) = 0;$$

$$\pi_2^* (\kappa_h, \kappa_p, \lambda) = \pi_1 (e^* (\kappa_h, \kappa_p, \lambda), \lambda) e_2^* (e^* (\kappa_h, \kappa_p, \lambda), \lambda)$$

$$= \frac{R \pi_1 (e^* (\kappa, w, \lambda), \lambda)^2}{1 - \kappa_h w^* (\kappa_h, \kappa_p, \lambda) \pi_{11} (\bar{e} (\kappa, w, \lambda), \lambda)} > 0.$$  

Differentiation of equation (9) and using equations (21) and (22) yields

$$I_1^* (\kappa_h, \kappa_p, \lambda) = \bar{I}_1 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda) + \bar{I}_2 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda) w_1^* (\kappa_h, \kappa_p, \lambda)$$

$$= w^* (\kappa_h, \kappa_p, \lambda) \pi (e^* (\kappa_h, \kappa_p, \lambda), \lambda) + \kappa_h \pi (e^* (\kappa_h, \kappa_p, \lambda), \lambda) (-w^* (\kappa_h, \kappa_p, \lambda) / \kappa_h) = 0;$$

$$I_2^* (\kappa_h, \kappa_p, \lambda) = \bar{I}_2 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda) w_2^* (\kappa_h, \kappa_p, \lambda)$$

$$= R \pi^* (\kappa_h, \kappa_p, \lambda) > 0.$$  

For the second part of the proposition, it is simplest to express $P^*$ as in equation (13). Differentiate to obtain:

$$P_1^* (\kappa_h, \kappa_p, \lambda) = R \pi^* (\kappa_h, \kappa_p, \lambda) + R \Delta \kappa \pi_1^* (\kappa_h, \kappa_p, \lambda) - K$$

$$= R \pi^* (\kappa_h, \kappa_p, \lambda) - K > 0 \text{ by assumption (8);}$$

$$P_2^* (\kappa_h, \kappa_p, \lambda) = R [\Delta \kappa \pi_2^* (\kappa_h, \kappa_p, \lambda) - \pi^* (\kappa_h, \kappa_p, \lambda)]$$

$$= R \left[ \frac{R \Delta \kappa [\pi_1 (e^* (\kappa_h, \kappa_p, \lambda), \lambda)]^2}{1 - \kappa_p R \pi_{11} (e^* (\kappa_h, \kappa_p, \lambda), \lambda)} - \pi^* (\kappa_h, \kappa_p, \lambda) \right]$$

$$< 0 \text{ by assumption (7).}$$

Equation (12), immediately yields

$$\mathcal{W}_1^* (\kappa_h, \kappa_p, \lambda) = I_1^* (\kappa_h, \kappa_p, \lambda) + P_1^* (\kappa_h, \kappa_p, \lambda) > 0;$$

$$\mathcal{W}_2^* (\kappa_h, \kappa_p, \lambda) = \frac{\Delta \kappa [R \pi_1 (e^* (\kappa_h, \kappa_p, \lambda), \lambda)]^2}{1 - \kappa_p R \pi_{11} (e^* (\kappa_h, \kappa_p, \lambda), \lambda)} > 0.$$  

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Proof of Proposition 3

Firstly, note from lemma 1 that \( w^* (\kappa_h, \kappa_p, \lambda) = \kappa_p R / \kappa_h \) and hence that \( w_3^* = 0 \). From equation (9),

\[
I_3^* (\kappa_h, \kappa_p, \lambda) = I_2 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda) w_3^* (\kappa_h, \kappa_p, \lambda) + I_3 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda)
\]

\[
= \kappa_h w^* (\kappa_h, \kappa_p, \lambda) \pi_2 (e^* (\kappa_h, \kappa_p, \lambda), \lambda) > 0 .
\]

(26)

Now differentiate equation (3) to obtain

\[
\bar{e}_3 (\kappa, w, \lambda) = \frac{\kappa w \bar{\pi}_{12} (\bar{e} (\kappa, w, \lambda), \lambda)}{1 - \kappa w \bar{\pi}_{11} (\bar{e} (\kappa, w, \lambda), \lambda)}.
\]

(27)

From equation (6) we have

\[
e_3^* (\kappa_h, \kappa_p, \lambda) = \bar{e}_2 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda) w_3^* (\kappa_h, \kappa_p, \lambda) + \bar{e}_3 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda)
\]

\[
= \bar{e}_3 (\kappa_h, w^* (\kappa_h, \kappa_p, \lambda), \lambda),
\]

(28)

from which part 2 follows immediately.

From equation (13) \( P_3^* (\kappa_h, \kappa_p, \lambda) = R \Delta \kappa_3 (\kappa_h, \kappa_p, \lambda) \), so that \( P_3^* (\kappa_h, \kappa_p, \lambda) \) has the same sign as \( \pi_3^* (\kappa_h, \kappa_p, \lambda) \). Equation (11) has the following \( \lambda \) derivative:

\[
\pi_3^* (\kappa_h, \kappa_p, \lambda) = \pi_1 (e^* (\kappa_h, \kappa_p, \lambda), \lambda) \pi_3 (\kappa_h, \kappa_p, \lambda) + \pi_2 (e^* (\kappa_h, \kappa_p, \lambda), \lambda),
\]

(29)

from which condition (14) is immediate. Condition (15) is obtained by substituting for \( e_3^* (\kappa_h, \kappa_p, \lambda) \) using equations (27) and (28).

The proof of part 4 is entirely analogous to that of part 3.

Proof of Lemma 2

Recall from equation (26) that \( I_3^* (\kappa_h, \kappa_p, \lambda) = \kappa_p R \pi_2 (e^* (\kappa_h, \kappa_p, \lambda), \lambda) \). Hence

\[
I_{33}^* (\kappa_h, \kappa_p, \lambda) = \kappa_p R \left[ \pi_{12} (e^* (\kappa_h, \kappa_p, \lambda), \lambda) \pi_3^* (\kappa_h, \kappa_p, \lambda) + \pi_2 (e^* (\kappa_h, \kappa_p, \lambda), \lambda) \right] .
\]

Substituting for \( e_3^* (\kappa_h, \kappa_p, \lambda) \) using equations (27) and (28) and rearranging, gives us \( I_{33}^* (\kappa_h, \kappa_p, \lambda) < 0 \) if and only if

\[
\kappa_h w^* (\kappa_h, \kappa_p, \lambda) \left[ \pi_{11} \pi_{12} - (\pi_{12})^2 \right] - \pi_{22} > 0,
\]

(30)

where every \( \pi \) derivative is evaluated at \( (e^* (\kappa_h, \kappa_p, \lambda), \lambda) \). The square-bracketed term in condition (30) is positive by concavity of \( \pi \) and so the condition is always satisfied.

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Equation (13) implies that \( P_{33}^* (\kappa_h, \kappa_p, \lambda) = R \Delta \kappa \pi_{33} (\kappa_h, \kappa_p, \lambda) \). Differentiation of equation (29) yields

\[
\pi_{33}^* = \pi_{11} (e_3^*)^2 + 2\pi_{12} e_{33}^* + \pi_{22} + \pi_1 e_{33}^*,
\]

where as usual every term involving \( \pi \) is evaluated at \((e^* (\kappa_h, \kappa_p, \lambda), \lambda)\) and every term in \( e^* \) is evaluated at \((\kappa_h, \kappa_p, \lambda)\). Rearranging this expression yields condition (16).

**Proof of Lemma 4**

From equation (17), \( L_1^e (\kappa_h, \kappa_p) = \frac{I_{33}^e (\kappa_h, \kappa_p, L^e (\kappa_h, \kappa_p))}{C_{\infty}^e (L^e (\kappa_h, \kappa_p))} \), which is zero since \( I_1^* \equiv 0 \). Similarly, \( L_2^* (\kappa_h, \kappa_p) = \frac{P_{33}^* (\kappa_h, \kappa_p, L^e (\kappa_h, \kappa_p))}{C_{\infty}^e (L^e (\kappa_h, \kappa_p))} \). \( I_{32}^* = R \{ \pi_2 (e^*, \lambda) + \kappa_p \pi_{12} (e^*, \lambda) e_3^* \} \), where all starred functions are evaluated at \((\kappa_h, \kappa_p, L^e (\kappa_h, \kappa_p))\). Substituting \( e_3^* = e_3^* \pi_{12} (e^*, \lambda) \) we obtain \( I_{32}^* = R \{ \pi_2 (e^*, \lambda) + \pi_1 (e^*, \lambda) e_3^* \} \) and hence that \( I_{32}^* > 0 \) if and only if condition (14) is satisfied: this is assumed to be the case. Hence \( L_2^e (\kappa_h, \kappa_p) > 0 \).

Differentiate equation (18) to obtain \( L_1^f (\kappa_h, \kappa_p) = \frac{P_{31}^e (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p))}{C_{\infty}^e (L^f (\kappa_h, \kappa_p))} \), and observe that \( P_{31}^e (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p)) = R \{ \pi_3^e + \Delta \kappa \pi_{31}^e \} = R \pi_3^e \), where we use the fact that \( \pi_1^e \equiv 0 \). Hence, invoking assumption (16), \( L_1^f (\kappa_h, \kappa_p) > 0 \).

Finally, \( L_1^f (\kappa_h, \kappa_p) = \frac{P_{31}^e (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p))}{C_{\infty}^e (L^f (\kappa_h, \kappa_p))} \). Condition (20) follows by differentiating equation (13).

**Proof of Proposition 4**

Using the fact that \( I_1^* \equiv 0 \) and the Envelope Theorem, \( E_1^e (\kappa_h, \kappa_p) = 0 \). Also,

\[
E_1^f (\kappa_h, \kappa_p) = I_3^e (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p)) L_1^f (\kappa_h, \kappa_p) > 0.
\]

Since \( L^f (\kappa_h, \kappa_p) = 0 \) and \( \lambda^e (\kappa_h, \kappa_p) = L^e (\kappa_h, \kappa_p) \) if and only if \( E^e (\kappa_h, \kappa_p) \geq E^f (\kappa_h, \kappa_p) \) the existence of \( \kappa_h (\kappa_p) \) follows immediately.

To show that \( \kappa_h^* (\kappa_p) > 0 \), it is sufficient to show that \( E^e (\kappa_h, \kappa_p) - E^f (\kappa_h, \kappa_p) \) is increasing in \( \kappa_p \) when \( \kappa_h = \kappa_h (\kappa_p) \). We have \( E_2^e (\kappa_h, \kappa_p) = I_2^e (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p)) \) and \( E_2^f (\kappa_h, \kappa_p) = I_2^f (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p)) + I_3^e (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p)) L_2^f (\kappa_h, \kappa_p) \), since \( I_3^e > 0 \) and by assumption (20), \( L_2^f (\kappa_h, \kappa_p) < 0 \). We prove in lemma 4 that \( I_{23}^* > 0 \) and we know that \( L^e (\kappa_h, \kappa_p) > L^f (\kappa_h, \kappa_p) \) at \( \kappa_h \), so finally \( E_2^f (\kappa_h, \kappa_p) < E_2^e (\kappa_h, \kappa_p) \), as required.
For $\kappa_h < \bar{\kappa}_h (\kappa_p)$, $W(\kappa_h, \kappa_p) = E^e (\kappa_h, \kappa_p) + F^e (\kappa_h, \kappa_p)$; since $E^e_1 (\kappa_h, \kappa_p) = 0$ we have

$$W_1 (\kappa_h, \kappa_p) = F^e_1 (\kappa_h, \kappa_p)$$

$$= P^*_1 (\kappa_h, \kappa_p, E^e (\kappa_h, \kappa_p)) + P^*_3 (\kappa_h, \kappa_p, L^e (\kappa_h, \kappa_p)) L^e_1 (\kappa_h, \kappa_p)$$

$$= P^*_1 (\kappa_h, \kappa_p, L^e (\kappa_h, \kappa_p)) > 0,$$ by lemma 4.

For $\kappa_h > \bar{\kappa}_h (\kappa_p)$, $W(\kappa_h, \kappa_p) = E^f (\kappa_h, \kappa_p) + F^f (\kappa_h, \kappa_p)$. We have shown that $E^f_1 (\kappa_h, \kappa_p) > 0$. Using the Envelope Theorem, $F^f_1 (\kappa_h, \kappa_p) = P^*_1 (\kappa_h, \kappa_p, L^f (\kappa_h, \kappa_p)) > 0$ also.

Finally, note that $E^e (\kappa_h, \kappa_p) = E^f (\kappa_h, \kappa_p)$ when $\kappa_h = \bar{\kappa}_h (\kappa_p)$ and hence that $L^e (\kappa_h, \kappa_p) > L^f (\kappa_h, \kappa_p)$. It follows that $F^f (\kappa_h, \kappa_p) < F^e (\kappa_h, \kappa_p)$ and hence that $W$ drops by $F^f (\kappa_h, \kappa_p) - F^e (\kappa_h, \kappa_p)$ at $\bar{\kappa}_h (\kappa_p)$. 

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