Aid and the Supply Side:
Public Investment, Export Performance and
Dutch Disease in Low Income Countries

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Supplementary Materials

Model equations
Appendix I: Model equations

This appendix sketches the equations of the basic simulation model used in this paper. GAMS programmes corresponding to the variants of the model can be obtained at:

http://www.economics.ox.ac.uk/Members/christopher.adam/Adam_Bevan_GAMS_programmes.htm

The model

Sectors \((i)\)
- Private \((ip)\)
  - Food
  - Cash Crops
  - Manufacturing
  - Services
- Public
  - Public Services \((pub)\)

Rural \((ir)\) \{Food, Cash Crops\}
Urban \((iu)\) \{Manufacturing, Services, Public\}

Labour categories \((lc)\)
- Unskilled labour \((u)\)
- Skilled labour \((s)\)

Households \((hh)\)
- Rural \((rur)\)
- Urban unskilled \((urbu)\)
- Urban skilled \((urbs)\)

Prices

Import prices \(pm_i = E.pm_i^w(1 + \tau_i^m)\)
(1)

Export prices \(pe_i = E.pe_i^w(1 - \tau_i^p)\)
(2)

Composite consumption prices \(pc_i = \frac{pd_i.xd_i + pm_i.m_i}{q_i}\)
(3)

Composite output prices \(px_i = \frac{pd_i.xd_i + pe_i.e_i}{x_i}\)
(4)

Value added prices \(pva_i = px_i - \sum_j a_{i,j}.pc_j\)
(5)

Capital goods prices \(pk_i = \sum_j b_{i,j}.pc_j\)
(6)
Composite labour price

\[ pL_i = \frac{\Phi_{i,s} w_s L_{i,s} + \Phi_{i,u} w_u L_{i,u}}{LC_i} \]  

(7)

Output and Factor Demands

Production Function

\[ x_i = A_i H_i^{\alpha_h} LC_i^{\alpha_l} K_i^{\alpha_k} KG^{\alpha_{kg}} \]  

(8)

Labour market FOC

\[ LC_i = \frac{\alpha_l L_{i,s} p \nu a_i x_i}{pL_i} \]  

(9)

Labour aggregation

\[ LC_i = B \left[ \omega \frac{L_{i,s}^{(\alpha_l-1)}}{\alpha_l} + (1-\omega) \frac{L_{i,u}^{(\alpha_l-1)}}{\alpha_l} \right] \]  

(10)

Skill composition

\[ \left( \frac{L_u}{L_s} \right) = \left[ \left( \frac{\omega}{1-\omega} \right) \left( \frac{\Phi_{i,s} w_s}{\Phi_{i,u} w_u} \right) \right]^{\sigma_l} \]  

(11)

Labour market equilibrium

\[ \sum_i L_{i,lc} + L_{pub,lc} = \bar{L}_{lc} \]  

(12)

Composite output

\[ x_i = C_i \left[ \beta_i e^{\frac{1+\sigma}{\alpha}} + (1-\beta_i) x d_i \right] \]  

(13)

Relative supplies

\[ \left( \frac{e}{xd} \right) = \left[ \left( \frac{1-\beta}{\beta} \right) \left( \frac{pe}{pd} \right) \right]^{\sigma_l} \]  

(14)

Composite consumption

\[ q_i = D_i \left[ \delta m^{\frac{\sigma_q-1}{\sigma_q}} + (1-\delta) xd_i^{\frac{\sigma_q-1}{\sigma_q}} \right] \]  

(15)

Relative demands

\[ \left( \frac{m}{xd} \right) = \left[ \left( \frac{\delta}{1-\delta} \right) \left( \frac{pd}{pm} \right) \right]^{\sigma_c} \]  

(16)

Demand

Intermediate goods demand

\[ nd_j = \sum_j a_{i,j} x_j \]  

(17)

Final consumption

\[ cd_{i,hb} = y_{i,hb} + \sum_j p c_j (1+\tau_j y_{j,hb}) \]  

(18)

Consumption share

\[ \Psi_{i,hb} = \frac{pc_j (1+\tau_j)^{(1-\sigma_c)} \theta_{i,cb}}{\sum_j pc_j (1+\tau_j)^{(1-\sigma_c)} \theta_{j,cb}} \]  

(19)
Capital formation 

\[ pk_i dk_i = \kappa_i (1 + (r^*_i - \bar{r})) \left[ s - E \Delta Z - pk_{pub}(dk_{pub} + dk_{g}) \right] \]  

(20)

Investment 

\[ id_i = \sum_j b_{i,j}[dk_i + dk_{g}] \]  

(21)

Government expenditure 

\[ g = (p_{v,a} + \sum_j a_{pub,j} p_{c,j}) \bar{g} + \sum_k \Phi_{pub,k} w_k L_{pub,k} + \sum_i \eta_i (KG - KG_0) \]  

(22)

Income and Saving 

Rural Income 

\[ y_{rar} = \sum_{\nu} p_{v,a}, x_{\nu} \]  

(23)

Urban unskilled 

\[ y_{arua} = \sum_{lu} w_{a,u} \Phi_{lu,u} L_{lu,u} \]  

(24)

Urban skilled 

\[ y_{arbs} = \sum_{lu} p_{v,a} x_{lu} - y_{arua} + i^{d} \bar{B} \]  

(25)

Disposable income 

\[ y_{dhh} = y_{hh} (1 - \tau_{hh}^D) + E(rmit_{hh} + trns_{hh}) \]  

(26)

Government revenue 

\[ tr = \sum \left[ E \left( \tau_{m}^i p_{m,i} m_i + \tau_{e}^i p_{e,i} e_i \right) + \tau_{c}^i p_{c} \sum c_{d,i,hh} \right] + \sum \tau_{hh}^D y_{hh} \]  

(27)

Household saving 

\[ s_{hh} = \varphi_{hh} y_{dhh} \]  

(28)

Government saving 

\[ s_{G} = tr - g - \sum_{hh} trns_{hh} - i^{d} \bar{B} - E i^{f} \bar{F} \]  

(29)

Saving 

\[ s = \sum_{hh} s_{hh} + s_{G} + E aid \]  

(30)

Equilibrium Conditions 

Balance of payments 

\[ \sum \left( p_{m,i} m_i - p_{e,i} e_i \right) + i^{f} \bar{F} = aid + \sum_{hh} rmit_{hh} + \Delta Z \]  

(31)

Goods 

\[ q_i = \sum_{hh} c_{d,i,hh} + nd_i + \bar{g}_i + \bar{G}_i + \sum \eta_i (KG - KG_0) \]  

(32)

Variables 

\[ E \] exchange rate 

\[ pm^w \] world price of imports 

\[ pm \] domestic price of imports 

\[ pe^w \] world price of exports 

\[ pe \] domestic price of exports 

\[ pd \] price of domestic good 

\[ pc \] price of composite consumption good 

\[ px \] output price 

\[ pva \] value added price 

\[ pk \] capital price (by destination) 

\[ pl \] price of composite labour 

\[ w \] nominal wage rate
Φ  wage distribution factor
τ^m  tariff rate
τ^e  export duty rate
τ^c  indirect tax rate
τ^d  direct tax rate
x  domestic output
q  composite supply
xd  domestic sales
e  exports
m  imports
H  land
LC  composite labour
L  skill-specific employment
L  skill-specific labour supply
K  private capital stock
KG  public capital stock
a  input-output matrix
b  capital composition matrix
nd  intermediate demand
cd  final consumption demand
θ  consumption shares
γ  subsistence consumption
dk  private investment demand (by destination)
dkg  public investment demand (by destination)
id  investment demand (by origin)
r  sectoral profit rate
g  total government expenditure
bar  government consumption
om  marginal O&M rates
rmit  remittances
trns  budget transfers
y  gross factor income
yd  disposable income
s  saving
ϕ  propensity to save
tr  government revenue
i^d  domestic interest rate
i^f  foreign interest rate
B  domestic debt stock
F  external debt stock
Z  official reserves
aid  net aid flows
σ_l  elasticity of substitution (labour)
σ_t  elasticity of transformation(output)
σ_q  elasticity of substitution (demand)
σ_c  elasticity of substitution (consumption)