

Fiscal Deficits and Growth in Developing Countries

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Abstract

This paper examines the relation between fiscal deficits and growth for a panel of 45 developing countries. Based on a consistent treatment of the government budget constraint, it finds evidence of a threshold effect at a level of the deficit around 1.5% of GDP. While there appears to be a growth payoff to reducing deficits to this level, this effect disappears or reverses itself for further fiscal contraction. The magnitude of this payoff, but not its general character, necessarily depends on how changes in the deficit are financed (through changes in borrowing or seigniorage) and on how the change in the deficit is accommodated elsewhere in the budget. We also find evidence of interaction effects between deficits and debt stocks, with high debt stocks exacerbating the adverse consequences of high deficits.

Keywords: Fiscal deficits, growth, threshold effects, developing countries.

JEL Codes: H3, H6, O4

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1. Introduction

A great deal of attention has been devoted in both theoretical and empirical literatures to the possible impact of various fiscal magnitudes on growth. In general, the theoretical literature has been careful to respect the government budget constraint, which imposes the requirement that a change in one magnitude has to be matched by offsetting changes elsewhere. This has not usually been true of the empirical literature, which has frequently examined the consequence of variations in a subset of budget items, implicitly assuming that the (hidden) offsets elsewhere are growth neutral¹. Since the offsetting changes are unspecified, and could take a variety of different forms, this assumption is strong – all possible offsetting combinations are being treated as neutral. For example, consider the common case where a study includes the share of government consumption expenditure in GDP as one of the regressors, and interprets the coefficient on this variable as a measure of the impact on the growth rate of a small increase in this share. This interpretation assumes that the coefficient would be invariant to whether the increase was financed by a one-for-one reduction in capital spending, or one-for-one increases in grant aid, tax revenue, or deficit financing. Furthermore, it assumes that this neutrality would hold within each of these categories, for example between monetizing the deficit and increased public borrowing.

Even in the theoretical literature, much of the effort has been devoted to consideration of revenue neutral shifts in (more or less) distortionary taxes, and compositional shifts in (more or less) productive expenditures in isolation, or to combinations of the two in a balanced budget configuration. While there is also an extensive literature devoted to the consequences of budget deficits, much of this has been in a context of lump sum taxation and expenditure which is unproductive (as modelled, for example, by lump sum transfers)². Very little attention has been accorded to the fiscal issue of most

¹ For an extensive survey of the literature from this perspective, see Gemmell (2001).

² We here follow the common terminology for distinguishing between expenditure which enhances output - is 'productive' - and that which does not. 'Unproductive' expenditure may however be of high

practical interest, where variations in distortionary taxes and/or productive expenditures may be partly offset by changes in deficit financing³.

This paper examines this question in the context of developing countries. Most of the existing empirical analyses in this area assume that the relation between deficits and growth is linear⁴. We show that while a linear representation tends to fit the data reasonably well for our sample of developing countries, it nonetheless masks important and policy-relevant non-linearities, especially at low levels of the fiscal deficit. In particular we show that for a sample of low- and middle-income countries, the relationship is not linear: the gains to growth of fiscal contraction are most marked as the deficit falls from a high level, but these taper out well before the economy reaches a balanced budget position. Our empirical analysis suggests a statistically significant non-linearity in the impact of the budget deficit on growth at around 1.5% of GDP. However, this non-linearity reflects the underlying composition of deficit financing; a corresponding threshold effect characterises the effect of seigniorage financing on growth, but there is no evidence from our data of a similar non-linearity arising from debt-financing.

This type of non-linear relationship is quite plausible *a priori*. The distortionary impact of taxation is increasing in the tax rate, while that of a very small deficit may be low. Hence, for a given level of government spending, a shift from a balanced budget to a (small) deficit may temporarily reduce distortions, depending on the composition of deficit financing⁵. If these distortions impact on growth rather than simply on output levels, it is therefore feasible that growth will be maximized when there is some recourse to deficit financing.

The rest of the paper is organized as follows. Section 2 considers how deficit financing might be considered in the context of distortionary taxes and productive

social value, for example by entering utility directly; hence it should not be confused with waste, though it might include that.

³ For example, in one of the leading textbooks on growth, Barro and Sala-I-Martin (1995), the discussion of government and growth (pages 152-161) presupposes a continuously balanced budget, so consideration of public debt is excluded. In another, Aghion and Howitt (1998), government as a fiscal institution barely makes an appearance; the only consideration of public debt is relegated to one of the problem sets. (Problem 7 of Chapter 1).

⁴ For example Easterly *et al* (1994), Miller and Russek (1997) and Kneller *et al* (2000). One notable exception is Giavazzi *et al* (2000).

expenditure. We set up a simple overlapping generation model in the tradition of Diamond (1965) which incorporates high-powered money in addition to debt and taxes. Section 3 describes the empirical model and the estimation strategy, which allows for both threshold effects in the deficit and its financing, and interactions between the deficit and debt stocks. Section 4 presents our econometric results based on data from the IMF's Government Finance Statistics for 45 developing countries over the period 1970-99, and Section 5 concludes.

2. Deficit financing when government operations are not neutral

The model

In our empirical application, we will be concerned with the impact on growth of both deficit flows and debt stocks, both independently and interactively. Most of the data are drawn from economies which are far from balanced growth or "steady state" configurations. To motivate this empirical analysis we therefore set up a simple Overlapping Generations (OLG) model of savings behaviour which is then embedded in an endogenous growth model along the lines of the model of government and growth due to Barro (1990) and Barro and Sala-i-Martin (1992). We restrict our attention to a general flat-rate tax on output, so that the possibility of shifts in the composition of taxes is ignored. The government spending activity simply involves purchases of current output; there is no separate production process. Government spending may either create a congestible productive public good or it may enter (some) consumers' utility directly, but have no consequences for output⁵. Our model differs from theirs in one crucial respect; the government budget need not be balanced, and a deficit may be financed either by printing money and/or issuing debt. Debt may be domestically or externally held; domestic debt carries an interest rate equal to the net private rate of return. For the most part we assume that external debt is available on exogenous terms, including the case where a rationed amount of external debt is available at an exogenously determined concessionary interest rate⁷.

⁵ The longer run effects may be quite different, since the deficit will also raise the debt stock or inflation rate over time, and that will in general also impact on growth.

⁶ This 'unproductive' expenditure could be expenditure by those controlling the government in their own interests, with no benefit to the general population, or expenditure which enters consumers' utility functions directly, but in a way that is neutral to their inter-temporal decisions. In both cases, the only impact it has on growth is via the financing requirements it imposes.

⁷ The model abstracts from real exchange rate issues, so there is no "secondary burden" of external debt.

Individuals

Individuals live for two periods, inelastically supplying a unit of labour in the first, and consuming in both. Population, and, equivalently, the labour force, grows at the constant rate n . Given the structure of the model, this has no implications for the dynamics, but means that the growth rate needs to be deflated by n to obtain per capita income growth. There are no intergenerational transfers, and utility takes the additive logarithmic form:

$$U = b \ln c_1 + (1-b) \ln c_2 \quad (1)$$

where c_1, c_2 are first and second period consumption and b is a preference parameter⁸.

Production

Since the empirical application is to countries with very different rates of population growth, it is unappealing to model growth effects subject to a stationary population. However, many endogenous growth models suffer from scale effects, where average and marginal products of capital are proportional to the size of the workforce. The model utilized here avoids that shortcoming, but this complicates the growth relations somewhat. The productive public expenditure, denoted G_p , is here supposed to have benefits in proportion to the aggregate labour force; i.e. it is congestible in the number of workers over whom it is spread. There is also a positive externality associated with the overall capital intensity of production in the economy. The representative firm's production function takes the Cobb Douglas form⁹:

$$y_i = A^{a+b} l_i^{1-a} k_i^a (K/D)^b (G_p/L)^{1-a-b} \quad (2)$$

where the firm is indexed by i , and has constant returns in the factors it hires. l_i is firm i 's labour force, and k_i its capital stock. L is the aggregate labour force, and K the aggregate capital stock. A is a productivity index, common across firms and

⁸ This functional form has the convenient and, in the context of developing countries, not wholly unrealistic consequence that saving is inelastic with respect to the interest rate. Specifically, if the net of tax wage is w , then first period consumption is bw and saving is $(1-b)w$, and this holds whatever the combination of assets, structure of returns, and anticipated tax rate.

constant over time. Abstracting from depreciation, competitive markets then ensure that:

$$r = (1-t) \partial y_i / \partial k_i = (1-t) a A^{a+b} (k_i / l_i)^{a-1} (K / L)^b (G_p / L)^{1-a-b} \quad (3)$$

where t denotes the flat-rate tax on output. Hence for the economy in aggregate, and letting $g_p = G_p / Y$,

$$Y = A^{a+b} K^{a+b} G_p^{1-a-b} = AK g_p^{(1-a-b)/(a+b)} \quad (4)$$

$$r = (1-t) a A g_p^{(1-a-b)/(a+b)} \quad (5)$$

and the net-of-tax wage bill is:

$$W = (1-t)(1-a)Y. \quad (6)$$

Government

All government activities are scaled relative to GDP (Y). The government makes two kinds of expenditures, a productive type, as noted above, in relative amount

$g_p = G_p / Y$ and an unproductive type in amount g_u . It receives grant aid in ratio to

income of a_e , and sets a flat-rate output tax at rate t . Domestic debt is all short-term.

The stock outstanding at the start of period t is D_{dt} which is retired in the period and

fresh one-period debt in the amount D_{dt+1} is floated. The interest rate on domestic

debt in period t , r_{dt} , is equal to the net of tax return to capital, possibly intermediated

by an inflation effect (see below). The end-of-period debt-income ratio is

$D_{dt+1} / Y_t = \Delta_{dt+1}$, and the beginning of period ratio is $\frac{D_{dt}}{Y_t} = \frac{D_{dt}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} = \frac{\Delta_{dt}}{(1+g_t)}$. Similar

relations hold for external debt, indexed by e , except that r_e is exogenous.

⁹ Time subscripts are omitted to simplify the notation where that does not risk confusion.

The government also obtains financing from seigniorage, partly from real growth in the economy, partly (perhaps) from inflation. This is in the amount $\Sigma_t / Y_t = s_t$. The conventional deficit after grants and interest payments is therefore

$$d_t Y_t = \Sigma_t + (D_{dt+1} - D_{dt}) + (D_{et+1} - D_{et}). \quad (7)$$

where d_t is the ratio of this deficit to GDP in period t .

The government budget constraint, relative to GDP, is:

$$t_t = g_{pt} + g_{ut} - a_{et} + r_d \Delta_{dt} + r_e \Delta_{et} - d_t \quad (8)$$

or, by substituting for the components of the deficit:

$$t_t = g_{pt} + g_{ut} - a_{et} - s_t + \left(\frac{(1+r_d)}{(1+g_t)} \Delta_{dt} - \Delta_{dt+1} \right) + \left(\frac{(1+r_e)}{(1+g_t)} \Delta_{et} - \Delta_{et+1} \right). \quad (9)$$

Money

Money enters the model as an input into production, specifically the process of capital accumulation, and into consumption, as part of the transactions technology. In both cases firms and households can economize on money so that their demand for real balances is a function of the rate of inflation, which is assumed to be known with perfect foresight. The full monetary mechanism is sketched in an annex, “Money in the OLG Model”, which is available from the authors on request; here we present only its main features.

The representative firm’s capital formation in period t requires capital goods and working capital (real balances). Capital formation is financed from the savings of the cohort who were young in $t-1$. Part of total savings are absorbed by the domestic government bond issue, D_{dt} , with the balance represented by goods and real balances from $t-1$. Net savings are then transformed into capital goods and working capital, the real value of the latter being determined by the government’s choice of inflation (equivalently its issue of high powered money). This generates the firm’s demand for money as:

$$m_{pt} = m^p(I_t, p_t) \quad (10)$$

where I_t denotes the investible resources available to the firm, $m^p_I > 0$, and $m^p_p < 0$.

Inflation therefore reduces the demand for real money balances and also drives a wedge between investible funds and installed capital. High inflation means that a given quantum of investible resources yields a lower amount of useable capital.

Hence:

$$K_t/I_t = j(p_t) = j_t \quad (11)$$

which is monotonically decreasing in p_t ¹⁰.

The representative consumer's demand for money is somewhat simpler consisting of a standard cash in advance constraint of the form:

$$m_{ct} = m^c(W_t, p_t) \quad (12)$$

where W_t is the net of tax wage bill, $m^c_w > 0$, and $m^c_p < 0$. Combining these demands, aggregate real balances are $m_t = m_{ct} + m_{pt}$. Government receives total seigniorage from these combined balances of $\Sigma_t = (1 + p_t)m_t - m_{t-1}$. Expressed as a share of GDP, this is denoted $s = \Sigma/Y$.

The monetary component of the model therefore incorporates a real seigniorage mechanism where the government sets inflation (the growth of nominal high-powered money) as a tax instrument, taking into account the impact on the tax base (real balances) which is determined, in the light of inflation, by firms and consumers. Thus, on the one hand, inflation generates additional seigniorage for government, lowering the distortionary impact of tax financing. On the other hand, it itself induces a distortion, reducing the efficiency with which investible resources are transformed

¹⁰ Also, $j_t \leq 1$ as $p_t \geq 0$. This specification means that the gross rate of return to saving only coincides with the marginal product of capital when inflation is zero; the ratio between the two is also given by j_t

into productive capital¹¹. The monetary model is then spliced onto the real model previously outlined to derive an expression for the growth rate of the economy in terms of the fiscal factors.

The growth rate

From (1) aggregate saving is simply:

$$S_t = (1-b)W_t = (1-b)(1-a)(1-t_t)Y_t,$$

and from (11):

$$K_{t+1} = j_{t+1}I_{t+1} = j_{t+1}(S_t - D_{dt+1}).$$

Now the growth rate of output between periods t and $t+1$ is:

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} - 1 = \frac{AK_{t+1}g_{pt+1}^{(1-a-b)/(a+b)}}{Y_t} - 1.$$

Hence by substitution we obtain the growth equation:

$$g_{t+1} = Aj_{t+1}[(1-b)(1-a)(1-t_t) - \Delta_{dt+1}]g_{pt+1}^{(1-a-b)/(a+b)} - 1 \quad (13)$$

which shows that growth between t and $t+1$ depends on the tax rate in t , the domestic debt stock at the end of t , and the inflation rate and government productive spending in $t+1$ ¹².

Equations (13) and (9) provide the basis for a partial analysis of the one-period growth effects of various government policy changes to which we now turn.¹³

¹¹ It also reduces the efficiency with which produced goods are turned into consumption, and hence lowers welfare. But, in the present context, this mechanism does not affect the growth rate. Notice also that inflation lowers the rate of return to saving. Given the specification of utility, this does not alter the savings ratio.

¹² Following Barro (1990), the provision of public services involves a call on resources at the same time as it enhances output.

Trade-offs involving the tax rate

To illustrate the relevant properties of this model we proceed by analysing the growth consequences of a change in the tax rate offset by each of the other six budget components in turn. Any other pair wise combination can be inferred from this. Specifically, (13) is differentiated with respect to the instrumental changes under consideration, and (9) is differentiated to ensure that these changes respect budget balance.

- (i). First, and most obviously, a tax-financed rise in unproductive expenditure is always growth reducing, though it might still be welfare enhancing if it entered individual utility functions.
- (ii). By the same token, increased grant aid is growth enhancing if it is used to reduce taxes, to lower domestic debt, or to increase productive spending. Its effect on growth is neutral if it is used to finance non-productive expenditure.
- (iii). A (sustained) rise in the tax rate to finance a (sustained) increase in productive expenditure will be growth enhancing if:

$$(1-b)(1-a) \left[(1-t_t) - g_{pt} \frac{a+b}{1-a-b} \right] > \Delta_{dt+1} \quad (14)$$

This trade-off is intermediated by the level of domestic debt. The higher domestic debt the lower the growth-enhancing levels of both taxation and productive expenditure.

- (iv). The impact of reducing the output tax and financing the deficit by increased seigniorage is less straightforward. Since the system is complex, we do not offer a full discussion of its properties here. However the outlines are clear enough. Raising the

¹³ The present model is a rich one, but a fuller exploration of its properties would take us too far afield in the present context. To keep the discussion manageable we will assume that if the expenditure ratio (g_p) is changed in the current period, it will be maintained at its new value in the following period. Hence there may be offsetting effects on growth of financing increased spending.

inflation rate from a sufficiently low level does create additional seigniorage, permitting a cut in taxes which enhances growth, but it is also directly inimical to growth. It is unclear, *a priori*, which effect predominates. If the demand for money is essentially consumption driven (so that j is close to one and insensitive to inflation), then there is little growth impact from inflation, and growth will be maximised if inflation is set close to the seigniorage maximising level. On the other hand, if the demand for money is essentially production driven, there is likely to be a serious growth penalty associated with inflation.¹⁴

The effect on this trade-off of domestic debt is somewhat surprising. Total differentiation of (13) with respect to the inflation rate yields:

$$dg = Ag_p^{(1-a-b)/(a+b)} \left[(1-b)(1-a)j(1-t) \left\{ \frac{-dt}{(1-t)} + \frac{dj}{j} \right\} - \Delta_d dj \right] \quad (15)$$

The trade-off so far discussed is the term in the curly brackets. The inclusion of domestic debt has two consequences. First, while, as already noted, its existence is inimical to growth, it tends to make inflation financing more likely to be attractive (since $dj < 0$). Second, inflation has the effect of altering the interest cost of domestic public debt. Specifically, $r_d = Aag_p^{(1-a-b)/(a+b)} j(1-t)$, so the impact of inflation on this rate also has the sign of $\left\{ \frac{-dt}{(1-t)} + \frac{dj}{j} \right\}$. This second effect will therefore tend to offset the first, but will be much weaker.

(v). The impact on growth of financing a tax cut by increasing domestic debt has the sign of $-(1-b)(1-a)dt_t - d\Delta_{dt+1} = -[1 - (1-b)(1-a)]d\Delta_{dt+1}$ which is always negative. This effect is linear in the deficit and invariant to the level of the debt stock, though the latter does help determine the tax rate itself. Notice that domestic debt is damaging even if $r_d < g$. Whereas in the exogenous growth model it can be beneficial to crowd out capital if the interest rate is very low, because the economy is dynamically

¹⁴ Simulations based on reasonable parameter values for the model presented in the annex referred to earlier suggest that this cost may dominate any gains from the tax reduction permitted by the increased seigniorage.

inefficient, this is never so in the endogenous growth case. Crowding out capital through domestic debt is always costly, even if the higher debt ratio is more than self-financing.

(vi). External debt differs in two ways. First it does not have the direct crowding out effect of domestic debt¹⁵. Second, the interest rate will in general depend on some supply curve of foreign savings¹⁶. Here we assume that this rate is exogenous. There are two cases to consider. In the first, $r_e > g$, and increased external debt sooner or later imposes a net budgetary cost. Thus the impact effect on growth of using increased external finance to permit a temporary tax cut is positive ($dt_t = -\Delta_{et+1}$); however, when the external debt income ratio is stabilised at its new higher level, the tax rate must rise above its original level ($dt = \frac{r_e - g}{1 + g} d\Delta$), so the growth rate falls below its original level. In this case growth is positively associated with the deficit and negatively associated with the debt stock.

The second case is where external finance is available on concessionary terms, and specifically $r_e < g$. Leaving aside the practically important issue of the likely finite horizon for this concessionary window, we stress the other feature, that the supply is rationed at any one time. Increased external debt now assists growth in the stock-income ratio as well as the flow, and is analytically equivalent to changes in the aid flow, a_e .

Summary

This discussion has posited two types of government spending, and five ways of financing it, taxes, grants and three forms of deficit finance. It suggests that while the impacts on growth of taxes and grants are reasonably straightforward, the impact of the deficit is likely to be complex, depending on the financing mix and the outstanding debt stock. In particular, deficits may be growth enhancing if financed by limited seigniorage; they are likely to be growth inhibiting if financed by domestic

¹⁵ Of course, it may have an indirect effect if it discourages investment because of reduced confidence.

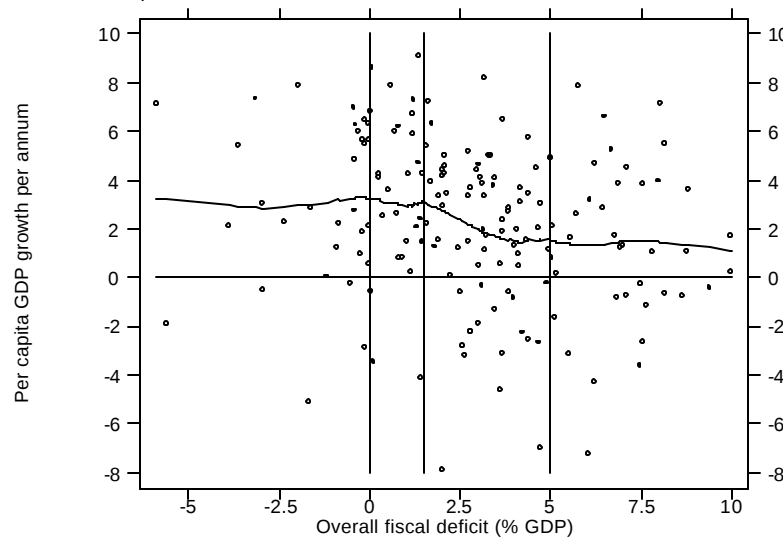
¹⁶ If, instead, we followed Diamond in assuming that the interest rate on external debt equalled that on domestic debt, it would always pay (in the endogenous growth model without real exchange rate or confidence effects) to use additional external finance to retire domestic debt.

debt¹⁷; and to have opposite flow and stock effects if financed by external loans at market rates.

3. Empirical model and estimation strategy

Our empirical results are based on the growth equation (13) and the representation of the government budget constraint given by (8) and (9). Before presenting our econometric evidence, it is instructive to consider a simple scatter plot of our data. Figure 1 plots average annual per capita income growth against the fiscal deficit after grants and interest payments for 45 non-OECD countries for the period 1970-1999¹⁸. This scatter is overlaid with a semi-parametric estimate of the sample relationship between the two (see Robinson, 1988). For convenience we have also plotted vertical lines corresponding to a balanced budget, and deficits of 1.5 percent and 5 percent of GDP.

Figure 1: Fiscal Deficit and Growth in Non-OECD Countries 1970-1999



The plotted line in Figure 1 is derived from a semi-parametric model of the form $y = Zb + f(x) + e$ where Z denotes a vector of control variables, x is the fiscal deficit after grants and interest on debt and $f(x)$ is a potentially non-linear function. The model is estimated non-parametrically on the pooled sample of countries. The control variables are defined and described in Table 1 below. Asymptotically efficient

¹⁷ The starkness of this result depends on the 100% crowding out property of the present model. In practice, the role of domestic debt as a component in the liquidity of the financial system also suggests caution in relying on the sharp difference between money and debt adopted here.

¹⁸ Details of country coverage and data definitions are provided in Table 1 and the Appendix.

estimates of $\hat{b}$ are computed by sorting the data on x and differencing the sorted data, allowing $f(x)$ to be derived residually as $f(x) = y - Z\hat{b}$ (Yatchew, 1998). The non-parametric function $f(x)$ is then plotted against average annual per capita GDP growth as a locally weighted smoothing kernel estimator with a bandwidth of 0.5.

Given the high variance in the data, this evidence is only tentative but it does suggest that while the dominant feature of the data is the negative correlation between fiscal deficits and growth, there is also a hint that this effect may not be linear¹⁹. To examine this possibility in a more robust fashion, we proceed in three steps. First we examine the role of the deficit itself on growth, based on the budget constraint defined in (8) above. Next, in line with equation (9), we substitute the deficit for its financing. In both cases we test for and find important non-linearities in these relationships. Finally we examine the stock-flow interaction between the deficit and net public indebtedness. Our first growth regression model is therefore of the form:

$$gy_{it} = b'X_{it} + w'W_{it} + qh_{it}[W_1 - \tilde{W}_1]_{it} + u_{it}$$

$$h_{it} = \begin{cases} 1 & \text{if } W_1 > \tilde{W}_1 \\ 0 & \text{if } W_1 \leq \tilde{W}_1 \end{cases} \quad (16)$$

$$\text{and } u_{it} = m_i + l_t + e_{it}.$$

Equation (16) is a standard fixed-effects panel data model. Countries are indexed by i , time, defined in terms of five-year periods, by t , and u_{it} is a two-way error term with m_i denoting time-invariant country-specific effects, l_t common time-varying effects, and e_{it} the idiosyncratic error component. h is an indicator variable and b, w and q parameters to be determined by the data. The dependent variable gy is average *per capita* income growth, and X is a vector of control variables (consisting of the rate of growth of population to proxy for the growth of the labour supply, the level of investment, and the start of period income). The vector W consists of the elements of the government budget (tax revenue, non-tax revenue, grants, expenditure, net lending, interest on debt and the deficit), where W_1 is the fiscal deficit, conventionally defined after grants and interest on debt.

¹⁹The bivariate linear regression of growth on the fiscal deficit using these pooled data has the form $y = 2.919 - 0.233x$ where the standard errors on the intercept and slope coefficients are 0.346 and 0.082

This specification allows the marginal effect of the fiscal deficit on growth to vary around a threshold value of the deficit, represented by $\tilde{W}_1$ ²⁰. Since the fiscal variables are bound together by the government budget identity, estimation of the parameters of equation (16) requires we eliminate one fiscal variable to avoid perfect co-linearity amongst the regressors. Defining the budget identity as $\sum_{j=0}^l W_j = 0$ and substituting out one of the fiscal factors, denoted W_0 , equation (16) is re-written as:

$$gy_{it} = b'X_{it} + \sum_{j=1}^m y_j' W_{jit} + qh_{it}[W_1 - \tilde{W}_1]_{it} + u_{it} \quad (17)$$

where the coefficient $y_j = (w_j - w_0)$ now measures the marginal impact of fiscal factor W_j on growth, net of the marginal impact of the excluded factor W_0 . As Kneller *et al* (2000) note, much of the empirical work in this area falls into the trap of implicitly assuming that the eliminated category is growth-neutral. Though widely prevalent in the literature, this interpretation is sustained only by assumption: the government budget identity means that it is neither possible to identify directly from the data the gross effect of any fiscal factor on growth – i.e. the w_j parameters -- nor to subject the assumption of neutrality to empirical testing.

Researchers in this area tend to disaggregate the fiscal accounts so as to select a category of revenue or expenditure (or the deficit) which may plausibly be assumed to be non-distortionary²¹. Though desirable, this assumption of neutrality is neither necessary nor is it likely to hold in general. Given the limitations on the coverage and quality of fiscal data in developing countries, as well as the heterogeneity across countries, it is difficult to identify any revenue or expenditure category that is plausibly growth-neutral across all countries, without disaggregating the data to a point where sample coverage is severely compromised. Since we cannot choose an

respectively. The equation has an overall R^2 of 0.047. Not surprisingly given the variance in the data it is not possible to statistically discriminate between these simple linear and non-parametric regressions.

²⁰ In principle, of course, there may be non-linearities elsewhere in the budget, particularly in revenue and expenditure items, but we do not explore this possibility in this paper.

²¹ For example, Kneller *et al*, (2000) and Miller and Russek (1997). Kneller *et al* (2000) find two fiscal variables (one expenditure and one revenue aggregate) which have equal net (and hence equal gross) impacts on growth. Despite their claim to the contrary (p 178) though, this equality does not constitute a test for neutrality but only of whether they are equally distortionary.

obvious ‘growth-neutral’ fiscal category we partition total non-interest expenditure into two groups, ‘productive expenditure’ defined as expenditure on health, education, infrastructure, public order and safety (including defence) and public administration, all of which have been identified to have some growth enhancing element, *ceteris paribus*, and a ‘residual expenditure’ category consisting of economic services, recreation and culture, plus other miscellaneous expenditure. Although tempting to do so, we do not assume that ‘residual expenditure’ is the direct counterpart of the ‘unproductive expenditure’ defined in Section 2. We cannot assume its effect on growth is zero and hence the coefficient estimates reported below must be read as measuring the effect of a particular fiscal factor on growth net of the effect of this residual expenditure category.

Clearly, the impact of the fiscal deficit on growth cannot be considered independently of its financing. Our second model therefore substitutes the deficit by its sources of financing. Incomplete and poor quality data means we cannot accurately distinguish between domestic and external borrowing across our sample of countries. We are able to use data on reserve money combined with the budget identity to distinguish between seigniorage (denoted s and defined as the sum of the inflation tax and the growth in real balances and expressed as a share of GDP: $s_t = p_t m_t + \dot{m}_t$) and total debt financing of the deficit (defined as the residual and represented by b), again allowing for potential threshold effects in both financing components. Our second growth regression takes the form:

$$gy_{it} = b'X_{it} + \sum_{j=2}^m y_j W_{jt} + q_1 s_t + q_2 b_t + q_3 h_{it}^s [s - \tilde{s}]_{it} + q_4 h_{it}^b [b - \tilde{b}]_{it} + u_{it} \quad (18)$$

where $\tilde{s}$ and $\tilde{b}$ denote estimated seigniorage and debt-financing thresholds, if they exist.

Finally, we re-estimate (17) and (18) allowing for interaction effects between a country’s deficit and its stocks of debt (both external and domestic) and real money

balances²². This allows us to examine the extent to which the flow effects of the deficit on growth are moderated by the degree of net indebtedness.

Data, Estimation and Econometric Issues

Our data consist of a panel of 45 non-OECD countries covering the period from 1970-99²³. We follow standard practice and compute 5-year averages so as to smooth over some of the cyclical features of the data. This gives us a potential sample size of 270 but because of missing data this is reduced to a usable sample (before taking lags) of 184 observations²⁴.

*** Table 1 here ***

Table 1 provides a summary of the characteristics of the data and defines the variable mnemonics used in the paper. The dominant feature of the data is the growth slowdown from the mid-1970s. This affected all countries worldwide but the recovery amongst the developing countries in our sample has been weak, with average per capita growth in the late 1990s still lower than that enjoyed 20 years earlier. The growth slowdown was accompanied initially by a steady rise in external government indebtedness and only in the 1990s by a fiscal adjustment. Against this background both investment and the level and composition of revenue and expenditure remained comparatively stable. It follows, therefore, that these two factors will not in themselves explain the slowdown and recovery in growth, although they may well explain variations around this trend movement. As a consequence, all the models presented below include a set of time dummy variables to capture common time-varying factors not otherwise included in our model. We do not report the

²² As we note later, limitations on the quality of fiscal data mean we are unable to extend our empirical analysis to examine the interaction between the way the deficit is financed and the level of indebtedness; we are restricted to a consideration of the interaction between the overall deficit and the level of indebtedness.

²³ See the Appendix for the country coverage of our data. Our sample is broadly comparable to that used by Devarajan *et al* (1996).

²⁴ The limiting constraint on the sample is the fiscal data which have been compiled from the IMF's Government Finance Statistics. Accurate fiscal and deficit financing data are notoriously difficult to collect in all countries and this is especially true for lower income countries. Using period averages helps a bit but we were nonetheless obliged to eliminate a large number of countries for want of data. Unfortunately, exclusion from the data sample is a far from random process: in many countries an early victim of fiscal distress is the timely and accurate reporting of statistics to the GFS. As with other work in this area, our results are therefore likely to embody potential biases arising from this endogenous self-selection process.

coefficients on these time-dummy variables but they are generally strongly significant and pick up much of the common growth slowdown²⁵.

Estimation of our growth regressions forces us to confront two important econometric issues. The first concerns the characteristics of the fiscal data. For our sample as a whole, the principal components of the budget have a much lower variation over time and across countries than per capita income growth. The cross-country coefficient of variation for per capita growth is around 2.5 for the pooled data sample but only 0.8 for tax revenue and 0.7 for total non-interest expenditure. The principal exception is the budget deficit itself which has a sample coefficient of variation of approximately 1.3. The low degree of variability in fiscal aggregates relative to growth (both across countries and over time) stacks the deck against finding statistically strong effects from regression analysis of the kind carried out here, especially for conventional taxation and expenditure aggregates²⁶.

The second problem is that fiscal performance is highly likely to be endogenous to economic growth, at least in the short-run. We tackle this in two ways. First, we work with (fixed) five-year averages of the data which eliminates some of the short-run cyclical simultaneity between growth and fiscal performance. Second we specify our empirical model so that fiscal factors impact on growth with a lag²⁷. An obvious alternative strategy is to assume that fiscal factors and growth are jointly determined and instrument the contemporaneous effect of fiscal factors on growth. We report the results of estimating our core model in this manner in Appendix Table II. These suggest that the alternative specification does not change radically the point estimates,

²⁵ The median value for the coefficients on the time dummies in the models reported in Table 3, measured relative to the average growth from 1975-79 are as follows (where the t-values are in parenthesis): 1980-84, -1.9% (2.29); 1985-89, -1.1% (1.26); 1990-94, -1.7% (1.99); and 1995-99, -0.9% (1.06).

²⁶ Easterly (1995) suggests two reasons why fiscal aggregates, especially revenue shares, have such low variance. The first is that 'natural experiments', in which countries have moved to very high or very low (average) tax rates, which would help identify the impact of revenue and expenditure on growth, are hard to find. This contrasts to the evidence on inflation and growth where there are numerous extreme inflation episodes allowing the relationship between inflation and growth to be identified with some precision. The second is that cross-country data consist only of revenue shares and not marginal rates so that variations in tax rates (which might be substantial) may not be manifest in tax revenue shares, especially if the informal economy is large and scope for tax evasion exists.

²⁷ Devarajan *et al* (1996) employ a similar strategy. They use annual data and use a five-year moving average of the annual growth rate as their dependent variable. Since their sampling interval (annual data) is less than their moving-average lag, error serial correlation is introduced requiring a correction to the error covariance matrix. Given the limitations on the fiscal data we have been unable to estimate

although there is a marked reduction in the precision of the estimates given the small and unbalanced nature of our panel. In what follows we therefore concentrate our discussion on the results for the case where fiscal effects are lagged.

4. Results and Discussion

We start by testing for the presence of thresholds in the impact on growth of the budget deficit and its financing using the methods developed by Hansen (1999)²⁸. First, let $S(W_1) = \hat{u}(W_1)' \hat{u}(W_1)$ be the residual sum of squares of the model defined by (17) estimated for a threshold level W_1 . The optimal threshold is then

$$\hat{W}_1 = \underset{\tilde{W}_1}{\operatorname{argmin}} S(\tilde{W}_1). \quad (19)$$

$\hat{W}_1$ is found by estimating (17) for all possible values of the deficit in the range from -2% of GDP to 6% of GDP, at quarter-point intervals. An identical procedure is used to locate the optimal seigniorage and debt-financing thresholds in (18)²⁹. Since the null that $q = 0$ is consistent with any arbitrary value of $\tilde{W}_1$, the threshold cannot be identified using standard methods of inference. We use a bootstrap method to test whether $\hat{W}_1$ is significantly different from zero. The bootstrap is used to simulate the asymptotic distribution of the following likelihood ratio statistic

$$LR_0 = (S(\tilde{W}_1^0) - S(\hat{W}_1)) / \hat{s}^2 \quad (20)$$

where $\hat{s}^2$ is the estimated error variance of the model in the presence of threshold $\hat{W}_1$, and $S(\tilde{W}_1^0)$ denotes the residual sum of squares for the model with no threshold (i.e. the linear model). Critical values for this statistic are provided by Hansen(1999). The asymptotic confidence interval for $\hat{W}_1$ is then constructed from the following statistic

our equations at the annual frequency and hence the issue of overlapping observations does not arise here.

²⁸ This procedure has recently also been applied to examine threshold effects in the relationship between inflation and growth. See Khan and Senhadji (2001).

$$LR_1 = (S(\tilde{W}_1) - S(\hat{W}_1)) / \hat{s}^2 \quad (21)$$

across the range of values for $\tilde{W}_1$. LR_1 takes the value of zero at $\tilde{W}_1 = \hat{W}_1$ and tends in distribution to the random variable Q with limiting distribution $\Pr(Q \leq x) = (1 - e^{-x/2})^2$. This distribution can be inverted to define the 100 α % critical value for the LR_1 statistic as $c(\alpha) = -2\log(1 - \sqrt{1 - \alpha})$. Plotting LR_1 for the range of $W_1 = [\tilde{W}_{1\min} \dots \tilde{W}_{1\max}]$ against this critical value traces out the confidence interval around $\hat{W}_1$.

Table 2 presents the estimates of the location and significance of the deficit threshold from equation (17) and (18) and Figure 2 plots the simulated 95% asymptotic confidence intervals around the estimated thresholds for the deficit and seigniorage.

*** Table 2 here ***

*** Fig 2 here ***

Table 2 indicates a threshold for the conventional deficit level of 1.5 percent of GDP, which, on the basis of the bootstrapped critical value for LR_0 , is statistically significant at a 5 percent, but not 1 percent critical value. Moreover, as Figure 2 shows, the bootstrapped confidence interval suggests that this threshold is located at a deficit level significantly greater than the balanced-budget position (the 95% confidence interval extends from 4.75 percent to 0.1 percent of GDP), confirming the preliminary observation from Figure 1. The remainder of the table reveals that the deficit threshold is underpinned by a threshold effect in seigniorage financing, but that there is no support for a debt-financing threshold across the examined range. The optimal debt-financing threshold occurs at a point equivalent to a net debt repayment of the order of 0.25% of GDP per period but this threshold is not statistically significant against the null that the effect is linear. By contrast, the presence of a seigniorage threshold at around 1.25 percent of GDP is strongly supported by the data³⁰.

²⁹ In this case, however, we conduct the grid-search along two axes, one for seigniorage and one for debt-financing.

³⁰ We have also examined a version of the model defined in terms of the deficit before grants-in-aid. The results are broadly similar to those reported in Table 2. We find that there is a significant threshold

Armed with these estimates we now turn to the first main set of results which are shown in Table 3. Columns [1] and [2] correspond to equation (17) above and the remainder to equation (18) where the deficit has been substituted by its sources of financing

*** Table 3 here ***

In each case the omitted fiscal category is ‘residual expenditure’ so that the coefficient on each included fiscal factor is net of the (unknown) impact of an increase in this category of expenditure. Hence the coefficients on taxes measure the impact of raising revenue to finance an equivalent value of residual expenditure, holding all other fiscal factors constant. Similarly those on *texp_y* measure the growth impact of shifting expenditure at the margin away from the residual to the non-residual expenditure, holding revenue and the deficit constant. A similar interpretation follows for the deficit. Clearly the choice of the excluded category is arbitrary, in the sense both that the model’s statistical properties are invariant to the choice of excluded categories, and that all of the net coefficients can be recovered by substitution between the different versions of the model. This can be seen from Appendix Table I where the model in column [1] of Table 3 is re-estimated over the same data sample but with each fiscal variable excluded in turn. To avoid excessive repetition we do not repeat this exercise for each regression reported in Table 3.

Column [1] reports a baseline model in which no allowance is made for the threshold effect in the fiscal deficit. The overall statistical characteristics of the model are reasonable and the results are consistent with the theory laid out in Section 2 and the coefficients are of plausible magnitudes. This equation and its counterparts reported in Appendix Table I cast light on the first three of the tax tradeoffs discussed in Section 2. Column [1] shows that higher ‘residual’ expenditure financed by tax or non-tax revenue significantly reduces average per capita growth in the next period (result (i)) and by a sizeable amount: the coefficient on *txrev_y* implies that a tax-financed increase in residual expenditure equivalent to 1% of GDP would reduce average annual *per capita* growth in the subsequent period by 0.1 percentage points.

in the deficit before grants centred on 2 percent of GDP (reflecting the positive growth effect of grants)

Result (ii) concerned the growth effects of grants. Reading across the *grants_y* row in Appendix Table 1 we see that grant-financing of lower taxes, higher ‘productive’ expenditure, higher other expenditure such as net lending and interest costs, and lower deficits is uniformly growth-enhancing and for the most part the effects are statistically significant at around the 10 percent level. By contrast, grant financing of ‘residual’ expenditure has no significant impact on growth. Result (iii) established the conditions under which tax financing of productive expenditure will be growth enhancing. As column [4] of Appendix Table 1 shows, however, this condition is only marginally satisfied and certainly with no statistical significance.

The main focus of this paper, however, is the role of the deficit. Column [1] confirms that, as expected, the average growth effect of a deficit-financed increase in ‘residual’ expenditure is negative. In column [2] we introduce the estimated threshold in the deficit from Table 2. The threshold itself is statistically significant and improves the overall fit of the model, but does not substantially alter the other coefficients. The implication is that for values of the deficit less than or equal to the threshold value of 1.5 percent of GDP a marginal increase in the deficit is locally growth-enhancing: an increase in the deficit of one percentage point (for example from a balanced budget to 1% of GDP) would increase the average annual *per capita* growth rate by around one quarter of one percent. By contrast, at levels of the deficit greater than the threshold the effect is reversed, although the semi-elasticity is of a similar order of magnitude ($0.264 - 0.473 = -0.209$). Thus not only does the threshold indicate a change in the marginal effect but this change is sufficiently large as to suggest a turning point.

The evidence from these first two columns is striking and would appear to point to the existence of a growth-maximizing budget deficit. It is important not to rush too precipitately to this conclusion, for at least two reasons. The first is that although the threshold itself is well defined, the interpretation of the coefficient on the fiscal deficit either side of the threshold remains strictly net of the effect of the excluded fiscal category. The size (and sign) of the change in the marginal effect of the fiscal deficit around the threshold will therefore necessarily depend on the expenditure increase or revenue-reduction an increase in the deficit is financing. This can be seen in the top half of Table 4 which reports the semi-elasticity of the fiscal deficit around the

although we find no evidence of a threshold in grant financing.

threshold from the model in column [2] of Table 3 for alternative excluded fiscal categories.

*** Table 4 here ***

In all three cases there is evidence of sign-switching around the threshold point. If, instead of ‘residual’ expenditure, the excluded fiscal category is total revenue – so that the coefficients on dc_y measure the effect of a deficit-financed revenue reduction – the marginal benefit of an increase in the deficit rises from 0.264 to 0.413 for a country starting from an initial deficit of less than 1.5% of GDP while the marginal cost of an increase in the deficit falls from -0.203 to -0.091 for a country starting from a deficit in excess of 1.5% of GDP. This pattern is slightly stronger if the excluded category is ‘productive’ expenditure. Given the data and our aggregation over fiscal categories, the threshold does indeed always define a turning point but this need not be so. It is entirely possible that if the excluded category was sufficiently distortionary in terms of its impact on growth, an increased deficit used to finance higher expenditure (or lower taxation) in this excluded category could increase growth for all countries, regardless of their initial deficit (although the differential effect between the groups would remain constant and the same as in Table 4). By choice of excluded category, therefore, the threshold may not represent a turning point at all but rather a (statistically significant) ‘same-sign’ change in magnitude of the marginal effect.

The second reason for caution is that the forgoing discussion implicitly assumes that the effect of the deficit is invariant to the composition of its financing. The final three columns of Table 3 therefore report the results of substituting the deficit with its financing, where we distinguish between seigniorage and debt financing. Column [3] is the counterpart to Column [1]. That the two sets of results differ reflects the differences in samples (the lack of data knocks a further 23 observations out of our already unbalanced panel). The most striking feature of this linear specification is that there is no difference in the impact at the margin of alternative deficit financing sources. Once we allow for threshold effects a distinctly different picture emerges (Columns [4] and [5]). As we noted in Table 2, there is no evidence of a threshold in debt financing; other things equal debt financing (domestic and external) of our

residual finance category is growth-reducing in a linear fashion; a 1% of GDP increase in debt financing reduces growth by an average of 0.18 percentage points. As the bottom half of Table 4 indicates, though, if a debt-financed deficit is used to fund increased productive expenditure or to lower the tax burden, the effect is (statistically) growth neutral. By contrast, the non-linearity in seigniorage financing means that seigniorage-financing up to the threshold level has, on average, no significant effects on growth but as this source of financing is driven above the threshold of 1.25% of GDP its effect is sharply growth-reducing. In fact, when used to finance ‘productive expenditure’ as opposed to our residual expenditure category, seigniorage financing appears to be significantly growth enhancing below the threshold and has no negative effect on growth above the threshold (at least locally). A similar effect results if we let the excluded fiscal category be total revenue.

Given our inability to consistently disaggregate debt financing we have been unable to directly test results (iv) to (vi) from Section 2. However our empirical results do offer broad support to the model in particular by showing that while the effect of debt financing appears to be linear (and significantly negative when financing ‘residual’ expenditure) there is a significant non-linearity in seigniorage financing consistent with growth-enhancing moderate deficit-financing, particularly when seigniorage is used to finance productive expenditure or to write down taxes.

The final step in the analysis interacts the deficit with the level of public indebtedness (Table 5)³¹. We disaggregate total government indebtedness into its interest-bearing components, namely domestic and external debt, and base money, each expressed as a share of GDP³². To focus on these stock effects we simplify the model slightly by including only total revenue and total expenditure (excluding ‘residual’ expenditure which remains the excluded fiscal category). This aggregation makes no substantial difference to the results on the effect of the deficit.

*** Table 5 here ****

³¹ Ideally we would be able to examine the consequences of the level of indebtedness on the impact of the deficit and its financing. Attempting to interact indebtedness with deficit financing we confront a dramatic reduction in usable observations; the panel shrinks from around 155 observations to less than 80. Since this severely undermines our ability to compare results, we limit ourselves in this section to examining only the relationship between the flow deficit and measures of total indebtedness.

Despite the poor quality of the data and the reduction in the usable sample size, a number of interesting features emerge from Table 5. Either on its own or interacted with the flow deficit, the degree of public indebtedness does not greatly alter the overall characteristics of the model or the values of the coefficients on total revenue and total expenditure, despite the reduction in the usable sample size. Column [0], which reports the counterpart to the basic model from Table 3 but estimated over the reduced sample, suggests that the broad characteristics of the basic model are robust to this particular change in the sample. Importantly, controlling for debt stocks does not eliminate the non-linearity on the flow effect of the deficit^{33 34}.

This aside, the level of public indebtedness matters for growth in a consistent manner, even though the measured effects are not strongly significant, particularly for domestic debt. Higher levels of interest-bearing debt (columns [1] to [3]) are associated with lower future growth, while higher real money demand is associated with higher future growth (column [4]). However it is through their interaction with the flow deficit that the external indebtedness and the stock of high-powered money, appear to have their main effect (columns [5] to [8]). To help interpret these results, Table 6 reports the effects of the fiscal deficit on growth evaluated at alternative levels of external indebtedness and real money demand.

*** Table 6 here ***

As these statistics show, changes in the levels of public debt and money have a marked impact on the relationship between the fiscal deficit and growth. Measured at their sample mean values, the implied marginal effects are, as expected, approximately equal to the corresponding values of 0.264 and -0.209 reported in

³² The measure of external indebtedness, taken from Loayza *et al* (1998), does not adjust for the degree of concessionality of external debt.

³³ Controlling for the stock of indebtedness we obtain the same estimates of the location and significance of the optimal thresholds for the deficit and seigniorage as reported in Table 2 above.

³⁴ It is also interesting to note that the inclusion of public indebtedness, principally external indebtedness, eliminates the country-specific effects. Conditional on the other variables in the model external indebtedness thus provides a summary statistic for otherwise unobservable country heterogeneity in growth patterns.

column [1] of Table 4³⁵. These effects change, however, as the level of debt increases or decreases, *ceteris paribus*. For example, an increase in the external debt ratio to 60 percent of GDP -- the average level prevailing in the 1990s -- reduces the (positive) marginal effect of an increase in the deficit by around 0.024 *per annum* for a country with a low initial deficit, and increases the (negative) marginal effect of deficit financing for a high-deficit country by the same amount.³⁶ The consequence of changes in the demand for money is no less powerful. Here, for example, a drop in domestic money demand to, say, 5 percent of GDP, similar to the level experienced in many low income countries facing stabilization problems in recent years, would reduce the beneficial effect of deficit financing on growth for low-deficit countries by 0.06 *per annum* and worsen the adverse effect in high-deficit countries by the same amount. Though apparently mild, since these stock effects are cumulative even small changes in the level of net indebtedness and real money demand will have a substantial impact on per capita income over the long-run.

5. Conclusions

This paper examines the relation between fiscal deficits and growth for a panel of 45 developing countries over the period 1970-1999. To motivate the empirical analysis we set up a simple overlapping generations (OLG) model of savings behaviour which is then embedded in an endogenous growth model with a fairly elaborated government sector. Government expenditure may be productive (in the sense of being growth-enhancing), or unproductive, and there is an output tax which is growth-inhibiting. However, the government budget need not be balanced. There are thus two types of government spending, and five ways of financing it, taxes, grants and three forms of deficit finance (by printing money, and by issuing domestic or external debt). The analysis suggests that while the impacts on growth of taxes and grants are reasonably straightforward, the impact of the deficit is likely to be complex, depending on the financing mix and the outstanding debt stock. In particular, deficits may be growth-enhancing if financed by limited seigniorage; they are likely to be

³⁵ The actual values in the case of external indebtedness are $0.305 = 0.322 - 0.068 \cdot 0.25$ for the low-deficit case and $-0.253 = 0.322 - 0.558 - 0.068 \cdot 0.25$ for the high-deficit case. If the sample data were exactly comparable between the models reported in Tables 3 and 5, this approximation would be exact.

³⁶ The equivalent adjustment to the marginal effect of the deficit carries over directly when the excluded fiscal category changes; the effect can be inferred directly from a comparison of Tables 4 and 6.

growth-inhibiting if financed by domestic debt; and to have opposite flow and stock effects if financed by external loans at market rates. In particular, two types of non-linearity may emerge, one involving the size of the deficit and the other interactions between the deficit and the public debt stock.

At a casual level, the scatterplot for our 45 countries in Figure 1 does suggest a possible non-linearity in the relation between growth and the fiscal deficit for this sample. More formally, our econometric analysis, based on a consistent treatment of the government budget constraint, confirms the existence of these stock-flow interactions and identifies a threshold effect in the deficit which is robust to their inclusion. This threshold effect is at a level of the deficit (after grants) around 1.5% of GDP. While there appears to be a growth payoff to reducing deficits to this level, this effect disappears or reverses itself for further fiscal contraction. The magnitude of this payoff, but not its general character, necessarily depends on how changes in the deficit are financed (through changes in borrowing or seigniorage) and on how the change in the deficit is accommodated elsewhere in the budget. The thresholds involve not only a change of slope but also a change of sign in the relation regardless of the budget category excluded from the model, indicating that for an economy not on its steady state growth path, there is a range over which deficit financing may be growth-enhancing. We also find evidence of interaction effects between deficits and debt stocks, with high debt stocks exacerbating the adverse consequences of high deficits.

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Appendix : Sample Countries

Argentina, Bahrain, Barbados, Bhutan, Brazil, Bulgaria, Cameroon, Chile, Colombia, Costa Rica, Dominican Republic, Egypt, El Salvador, Ghana, Honduras, Hungary, India, Indonesia, Israel, Kenya, Korea, Lebanon, Lesotho, Malaysia, Mali, Mauritius, Mexico, Morocco, Nicaragua, Peru, Panama, Paraguay, Poland, Romania, Senegal, Singapore, South Africa, Sri Lanka, Suriname, Thailand, Togo, Tunisia, Uruguay, Zambia, Zimbabwe.

*** Appendix Table I here ***

*** Appendix Table II here ***

TABLE 1. SUMMARY DATA

		1970-99	1970-74	1975-79	1980-84	1985-89	1990-94	1995-99
<u>Country Characteristics</u>	<u>Mnemonic</u>							
Annual average growth in per capital income	<i>gypc</i>	1.2%	3.6%	2.6%	0.6%	1.2%	0.1%	2.0%
Annual average population growth	<i>gpop</i>	2.2%	2.3%	2.4%	2.3%	2.1%	2.0%	1.8%
GNP per capita (constant 1995 US\$)	<i>lgnppc</i>	875	852	1031	1172	1146	1202	1186
Investment as % GDP	<i>i_y</i>	23.7%	21.0%	25.1%	24.5%	22.6%	23.7%	24.3%
Central government domestic debt as % GDP	<i>ddy</i>	12.7%	12.0%	12.5%	11.1%	13.1%	15.6%	11.0%
Central government external debt as % GDP	<i>xdy</i>	25.7%	5.1%	8.8%	24.9%	52.6%	45.2%	49.2%
Total central government debt as % GDP	<i>debt</i>	38.4%	17.1%	21.3%	36.0%	65.7%	60.8%	60.2%
Reserve Money as % GDP	<i>mb</i>	12.7%	9.6%	10.1%	11.9%	14.0%	15.7%	14.2%
<u>Fiscal Variables [% of GDP]</u>								
Total tax revenue	<i>txrev_y</i>	17.8%	13.9%	16.7%	18.4%	18.6%	18.7%	18.8%
Non-tax revenue	<i>ntax_y</i>	5.0%	2.7%	4.2%	5.3%	5.9%	5.4%	5.0%
Grants	<i>grants</i>	1.8%	0.8%	1.1%	2.0%	2.1%	1.9%	1.2%
Total Revenue and Grants	<i>trev_y</i>	24.6%	17.4%	22.0%	25.8%	26.6%	26.0%	25.0%
'Productive' Expenditure	<i>texp_y</i>	16.1%	8.9%	11.4%	12.5%	12.7%	13.1%	12.9%
'Residual' Expenditure	<i>exp2_y</i>	9.6%	11.0%	12.9%	14.2%	13.9%	11.9%	11.0%
Interest on debt	<i>int_y</i>	2.9%	1.1%	1.6%	2.8%	3.6%	3.6%	3.7%
Net Lending	<i>nl_y</i>	1.3%	1.3%	1.9%	1.8%	1.5%	0.8%	0.7%
Total Expenditure and Net Lending	<i>totexp_y</i>	29.8%	22.4%	27.8%	31.3%	31.7%	29.5%	28.4%
Overall Budget Deficit	<i>dc_y</i>	5.3%	5.1%	5.7%	5.6%	5.1%	3.5%	3.4%
Seigniorage	<i>s_y</i>	2.8%	1.8%	2.9%	3.8%	2.1%	4.4%	1.9%
Debt financing	<i>b_y</i>	1.9%	3.3%	2.8%	1.8%	3.0%	-0.9%	1.5%

Sources:

Data on per capita growth, GNP per capita, population growth, investment, and reserve money are taken from World Development Indicators (World Bank 2000)
 Data on government debt are from Loayza *et al*/ The World Saving Data Base (World Bank 1998) and fiscal data are from Government Finance Statistics (IMF CD-ROM, May 2001)

Definitions: (GFS codes reported in parentheses)

Total tax revenue = direct taxes (81ah + 81bh + 81ch + 81dh) + indirect taxes on domestic goods and services (81eh) + taxes on international trade (81fh).

Non-tax revenue = non-tax revenue plus other revenue including capital taxes (81yb + 81gh + 81ych)

Expenditure is grouped as follows:

'Productive' = education, health, housing, transport and communication, public order and safety, non-interest administration (82ch+82dh+82eh+82fh+82hih+82pah+82ach+82kh)

'Residual' = economic services plus recreation and cultural (82gh+82h-82hih)

The overall budget deficit is defined as total expenditure and net-lending less total revenue and grants;

Seigniorage is defined as the sum of inflation tax and the real growth of base money

All fiscal aggregates refer to central government fiscal accounts only.

TABLE 2. TESTS FOR THRESHOLD EFFECTS

Threshold [a]	Value [b] (% of GDP)	LR(0)	p-value (1000 bootstrap repetitions)
Deficit [Eq (17)]	1.50%	7.917	0.043
Seigniorage [Eq(18)]	1.25%	10.249	0.011
Debt-financing [Eq(18)]	-0.25%	3.711	0.456

Notes

[a] The seigniorage threshold statistics are reported conditional on the presence of the optimal threshold for debt-financing and *vice versa* for the debt-financing statistics.

[b] Thresholds are expressed in terms of the deficit so that a positive threshold denotes a budget deficit and a negative a surplus. The grid search was conducted over quarter-point intervals for the deficit, seigniorage and debt.

TABLE 3. FISCAL FACTORS AND GROWTH: NON-LINEAR FLOW EFFECTS

Fixed effects estimation with all fiscal effects lagged [a] [b]
(Excluded category: 'residual' expenditure) [c]

Sample: 5 year period averages (1970-74 to 1995-99)

Dependent Variable: average annual growth in per capita income (gypc)					
	[1]	[2]	[3]	[4]	[5]
<u>Control variables</u>					
constant	0.025 [0.96]	0.020 [0.77]	0.006 [0.25]	0.006 [0.22]	0.000 [0.01]
lgnppc_1 (x100) [d]	0.123 [0.41]	0.072 [0.25]	0.246 [0.79]	0.221 [0.70]	0.251 [0.79]
i_y	0.097 [2.75]	0.117 [3.22]	0.969 [2.69]	0.103 [2.79]	0.105 [2.84]
gpop	-0.721 [2.14]	-0.678 [2.04]	-0.636 [1.77]	-0.620 [1.72]	-0.626 [1.73]
<u>Fiscal factors</u>					
txrev_y	-0.113 [2.38]	-0.106 [2.26]	-0.112 [2.37]	-0.108 [2.24]	-0.096 [1.95]
ntax_y	-0.187 [2.95]	-0.134 [1.95]	-0.201 [3.20]	-0.194 [3.05]	-0.184 [2.88]
grants_y	-0.019 [0.23]	-0.028 [0.34]	-0.037 [0.44]	-0.041 [0.49]	-0.030 [0.35]
texp_y [c]	0.121 [1.21]	0.085 [1.64]	0.118 [1.14]	0.115 [1.11]	0.093 [1.21]
nl_y	0.203 [1.31]	0.200 [1.31]	0.352 [2.15]	0.346 [2.11]	0.343 [2.09]
int_y	0.185 [1.31]	0.221 [1.57]	0.169 [1.19]	0.186 [1.29]	0.219 [1.49]
dc_y	-0.162 [2.43]	0.264 [1.92]			
[dc_y-0.015]		-0.473 [2.17]			
s_y			-0.173 [2.21]	-0.023 [0.11]	0.069 [0.32]
[s_y-0.125]				-0.159 [1.96]	-0.313 [2.28]
b_y			-0.171 [2.16]	-0.179 [2.24]	-0.243 [2.44]
[b_y+0.025]					0.096 [0.82]
N	155	155	132	132	132
Countries	40	40	40	40	40
R-sq:	0.2638	0.2840	0.2577	0.2618	0.2691
Adjusted R-sq	0.2181	0.2343	0.2282	0.2264	0.2340
F-Pooling	1.81	1.85	1.68	1.73	1.74
Pr	0.0094	0.0078	0.0266	0.0207	0.0198

Notes:

[a] Heteroscedastic consistent t-statistics in parentheses

[b] Estimation includes time-dummy variables (not reported).

[c] texp_y excludes 'residual' expenditure (see text for description)

[d] _1 denotes start of period value.

TABLE 4. THE (NET) EFFECT OF THE FISCAL DEFICIT ON GROWTH [a]

Excluded Fiscal Category [b] [c]	[1] 'Residual' Expenditure	[2] Productive Expenditure	[3] Total Revenue
Fiscal deficit less than or equal to 1.5% of GDP [t-statistic]	0.264 [1.92]	0.444 [2.14]	0.413 [1.96]
Fiscal deficit greater than 1.5% of GDP [t-statistic]	-0.209 [2.17]	-0.06 [2.01]	-0.091 [2.19]
Seigniorage less than or equal to 1.25% of GDP [t-statistic]	-0.023 [0.11]	0.207 [2.07]	0.167 [0.86]
Seigniorage greater than 1.25% of GDP [t-statistic]	-0.182 [1.96]	0.018 [2.01]	-0.023 [2.01]
Debt-financing [no-threshold] [t-statistic]	-0.179 [2.24]	0.019 [0.34]	-0.021 [0.26]

Notes

[a] The figures reported in this table report the semi-elasticity of growth with respect to the deficit and seigniorage. A value of 0.10 implies that a one percentage-point (of GDP) increase in the deficit would increase average annual per capita growth by 0.1 percentage points.

[b] The semi-elasticities reported in column [1] are derived from Table 3, columns [2] and [4]. The coefficients in columns [2] and [3] derive from the same model estimated with an alternative fiscal category excluded.

[c] The statistical characteristics of the three models are identical and are reported in Table 3.

TABLE 5. FISCAL FACTORS, PUBLIC INDEBTEDNESS AND GROWTH

Fixed effects estimation with all fiscal effects lagged [a] [b]
(Excluded category: 'residual' expenditure) [c]

Sample: 5 year period averages (1970-74 to 1995-99)

Dependent Variable: average annual growth in per capita income (gypc)									
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Stock Effect	No debt	Total Debt	Domestic Debt	External Debt	Base Money	Total Debt	Domestic Debt	External Debt	Base Money
<u>Control variables</u>									
constant	-0.003 [0.10]	0.041 [1.55]	0.050 [1.87]	0.038 [1.46]	-0.002 [0.07]	0.042 [1.58]	0.051 [1.91]	0.039 [1.49]	0.058 [0.19]
lgnppc_1 (x100) [d]	0.264 [0.81]	-0.096 [0.35]	-0.203 [0.71]	-0.099 [0.37]	0.239 [0.72]	-0.104 [0.39]	-0.231 [0.81]	-0.124 [0.46]	0.202 [0.62]
i_y	0.100 [2.71]	0.085 [2.63]	0.074 [2.16]	0.087 [2.73]	0.101 [2.71]	0.084 [2.60]	0.077 [2.25]	0.087 [2.74]	0.105 [2.85]
gpop	-0.631 [1.62]	-1.159 [2.94]	-0.914 [2.38]	-0.993 [2.69]	-0.637 [1.62]	-1.160 [2.97]	-0.919 [2.37]	-0.975 [2.65]	-0.638 [1.65]
<u>Fiscal factors</u>									
trev_y [c]	-0.089 [1.90]	-0.054 [1.30]	-0.065 [1.53]	-0.064 [1.57]	-0.094 [1.88]	-0.057 [1.40]	-0.071 [1.67]	-0.066 [1.63]	-0.106 [2.21]
totexp_y [c]	0.126 [2.13]	0.138 [2.57]	0.141 [2.57]	0.143 [2.67]	0.124 [2.06]	0.145 [2.70]	0.142 [2.57]	0.149 [2.75]	0.123 [2.07]
dc_y	0.330 [1.23]	0.329 [1.97]	0.100 [0.39]	0.313 [1.95]	0.333 [1.23]	0.340 [2.11]	0.087 [0.34]	0.322 [1.92]	0.195 [0.70]
[dc_y-0.015]	-0.508 [1.79]	-0.584 [2.02]	-0.316 [1.15]	-0.558 [1.94]	-0.506 [1.77]	-0.581 [2.02]	-0.311 [1.13]	-0.558 [1.95]	-0.481 [1.70]
<u>Asset Stocks</u> [d]									
debt_1		-0.003 [0.69]				-0.002 [0.32]			
ddy_1			-0.014 [0.52]				-0.003 [0.26]		
xdy_1				-0.004 [1.17]				-0.002 [0.29]	
mb_1					0.019 [0.37]				0.004 [0.83]
<u>Stock Interactions</u> [e]									
D*debt_1						-0.055 [1.84]			
D*ddy_1							0.030 [0.10]		
D*xdy_1								-0.068 [1.96]	
D*mb_1									0.777 [1.86]
N	123	119	122	123	121	119	122	123	123
Countries	31	31	31	31	30	31	31	31	31
R-sq	0.2697	0.2694	0.2572	0.259	0.271	0.2744	0.2566	0.2621	0.2889
Adjusted R-sq	0.2252	0.2233	0.2116	0.2139	0.2258	0.2286	0.2110	0.2172	0.2456
F-Pooling	1.51	1.65	1.83	1.64	2.04	1.40	1.8	1.43	2.04
Pr	0.00732	0.0104	0.0025	0.0113	0.0015	0.1203	0.0202	0.1058	0.0067

Notes:

Column [0] reports basic model from Table 3 estimated over reduced sample

[a] Heteroscedastic consistent t-statistics in parentheses

[b] Estimation includes time-dummy variables (not reported).

[c] trev_y = brev_y+ntax_y+grants_y; totexp_y = taxp_y+nl_y+int_y (see text for description)

[d] _1 denotes start of period value.

[e] Interactions variables D*X denote the interaction of dc_y with variable X.

TABLE 6. THE EFFECT OF FISCAL DEFICITS ON GROWTH: STOCK-FLOW INTERACTIONS [a]

(i) External Debt (% GDP)	[1] Sample Mean 25%	[2] High External Debt 60%	[3] Low External Debt 10%
Fiscal deficit less than or equal to 1.5% of GDP	0.305	0.281	0.315
Fiscal deficit greater than 1.5% of GDP	-0.253	-0.277	-0.243
(ii) Money Demand (% Sample	Sample Mean 12.70%	High Money Demand 20%	Low Money Demand 5%
Fiscal deficit less than or equal to 1.5% of GDP	0.294	0.350	0.234
Fiscal deficit greater than 1.5% of GDP	-0.187	-0.131	-0.247

Notes:**[a]** See notes to Table 4.

APPENDIX TABLE 1: PANEL DATA ESTIMATES WITH FIXED EFFECTS (INCL. TIME DUMMY VARIABLES)

Dependent Variable (gypc)

Fiscal Effects (Lagged)

Data: 5-year period averages (1970-74 to 1995-99)

Excluded Fiscal Category	txrev_y [1]		ntax_y [2]		grants_y [3]		texp_y [4]		exp2_y [5]		int_y [6]		nl_y [7]		dc_y [8]	
	coeff.	t-value	coeff.	t-value	coeff.	t-value	coeff.	t-value	coeff.	t-value	coeff.	t-value	coeff.	t-value	coeff.	t-value
_cons	0.025	0.96	0.025	0.96	0.025	0.96	0.025	0.96	0.025	0.96	0.025	0.96	0.025	0.96	0.025	0.96
lgnppc_1	0.123	0.41	0.123	0.41	0.123	0.41	0.123	0.41	0.123	0.41	0.123	0.41	0.123	0.41	0.123	0.41
i_y	0.097	2.75	0.097	2.75	0.097	2.75	0.097	2.75	0.097	2.75	0.097	2.75	0.097	2.75	0.097	2.75
gpop	-0.721	2.14	-0.721	2.14	-0.721	2.14	-0.721	2.14	-0.721	2.14	-0.721	2.14	-0.721	2.14	-0.721	2.14
txrev_y			0.07	1.13	-0.09	1.83	0.01	0.10	-0.11	2.38	0.07	0.46	0.09	0.58	0.05	0.63
ntax_y	-0.074	1.13			-0.17	1.55	-0.07	0.74	-0.19	2.95	0.00	0.01	0.02	0.11	-0.02	0.31
grants_y	0.094	1.83	0.17	1.55			0.10	1.68	-0.02	0.23	0.17	1.00	0.18	1.07	0.14	1.85
texp_y	0.008	0.10	-0.07	0.74	0.10	1.68			0.12	1.21	-0.06	0.30	-0.08	0.47	-0.04	0.41
exp2_y	-0.113	2.38	-0.19	2.95	-0.02	0.23	-0.12	1.21			-0.19	1.31	-0.20	1.31	-0.16	2.43
int_y	0.073	0.46	0.00	0.01	0.18	1.00	0.06	0.30	0.19	1.31			-0.02	0.08	0.02	0.15
nl_y	0.090	0.58	0.02	0.11	0.17	1.07	0.08	0.47	0.20	1.31	0.02	0.08			0.04	0.29
dc_y	-0.049	0.63	0.02	0.31	-0.14	1.85	-0.04	0.41	-0.16	2.43	0.02	0.15	0.04	0.29		
n	155		155		155		155		155		155		155		155	
R-square	0.264		0.264		0.264		0.264		0.264		0.264		0.264		0.264	

Notes: See Table 3

APPENDIX TABLE II. FISCAL FACTORS AND GROWTH

Instrumental Variable fixed effects estimation with contemporaneous fiscal effects [a] [b]
(Excluded category: 'residual' expenditure) [c]

Sample: 5 year period averages (1970-74 to 1995-99)

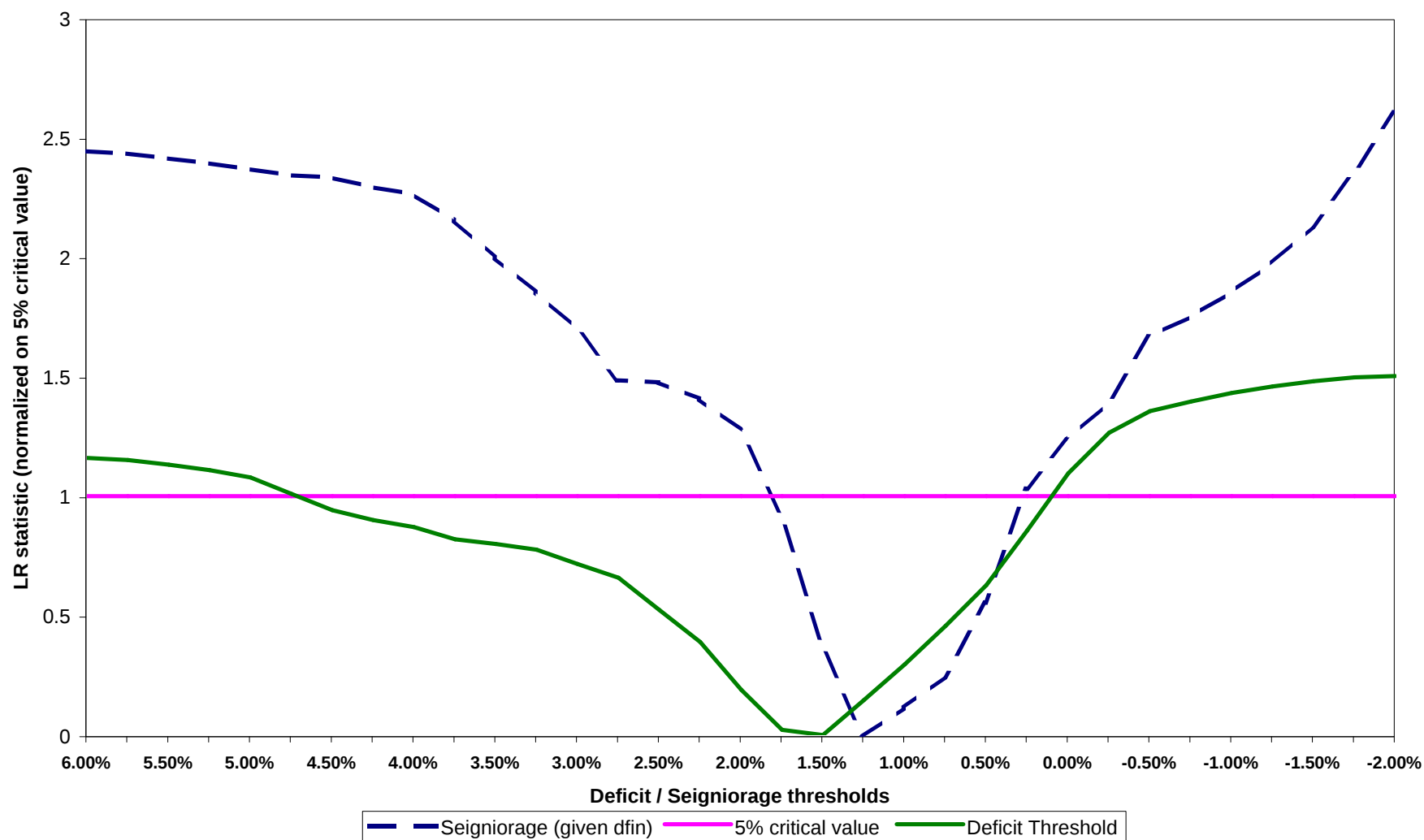
Dependent Variable: average annual growth in per capita income (gypc)						
		[1]	[2]	[3]	[4]	[5]
		IV	IV	IV	IV	IV
<u>Control variables</u>						
constant		0.021 [0.66]	0.015 [0.46]	0.001 [0.01]	0.001 [0.01]	0.005 [0.17]
lgnppc_1 (x100)	[d]	0.137 [0.40]	0.113 [0.32]	0.258 [0.73]	0.307 [0.87]	0.279 [0.79]
i_y	[e]	0.065 [1.56]	0.098 [1.89]	0.074 [1.14]	0.072 [1.42]	0.085 [1.34]
gpop		-0.612 [1.76]	-0.520 [1.38]	-0.670 [1.72]	-0.613 [1.54]	-0.561 [1.39]
<u>Fiscal factors</u>						
txrev_y	[e]	-0.105 [1.68]	-0.118 [1.84]	-0.088 [1.35]	-0.081 [1.22]	-0.081 [1.85]
ntax_y		-0.300 [3.24]	-0.257 [2.56]	-0.261 [2.73]	-0.227 [2.22]	-0.228 [2.23]
grants_y		0.012 [0.13]	-0.014 [0.13]	0.033 [0.34]	0.036 [0.35]	0.040 [0.40]
tepx_y	[c]	0.243 [1.86]	0.202 [1.61]	0.112 [1.77]	0.078 [1.48]	0.069 [1.45]
nl_y		0.570 [2.33]	0.408 [1.61]	0.757 [3.13]	0.711 [2.91]	0.778 [2.95]
int_y		-0.099 [0.48]	0.065 [0.29]	0.169 [0.71]	0.257 [1.00]	0.237 [0.91]
dc_y		-0.218 [1.66]	0.158 [1.31]			
[dc_y-0.015]			-0.422 [1.76]			
s_y				-0.239 [2.23]	-0.060 [0.22]	-0.013 [0.05]
[s_y-0.125]					-0.232 [1.86]	-0.398 [2.20]
b_y				-0.225 [1.78]	-0.211 [1.72]	-0.346 [2.61]
[b_y+0.025]						0.118 [0.71]
N		140	140	118	118	118
Countries		40	40	38	38	38
R-sq:		0.1974	0.1647	0.2529	0.2481	0.2548

Instruments: first and second lags of fiscal effects plus contemporaneous and two lags of exogenous variables.

Notes:

- [a] Heteroscedastic consistent t-statistics in parentheses
[b] Estimation includes time-dummy variables (not reported).
[c] tepx_y excludes 'residual' expenditure (see text for description)
[d] _1 denotes start of period value.
[e] Endogenous variable

Figure 2
95% confidence interval for deficit and seigniorage threshold estimates



Fiscal Deficits and Growth in Developing Countries

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Annex **Money in the OLG model**

In this annex we describe in detail the monetary component of the model laid out in the main text of the paper.

(i) Money as a productive input

Real balances are combined with the produced good in the process of capital formation. At the start of period t , the saving of the cohort who were young in period $t-1$ is S_{t-1} ; part of this is absorbed by the domestic government bond issue, D_{dt} , and the residue is available as investible funds, I_t . These funds come partly as goods held over from the previous period, x_t , and partly in the form of money balances, m_{t-1} . Firms exchange these resources with government to obtain an investment allocation of goods, z_{pt} , and nominal money balances of M_{pt} . There is perfect foresight concerning the inflation rate, p_t . Hence the real balance available to firms is $m_{pt} = M_{pt} / (1 + p_t)$. The representative firm creates capital stock for period t according to a CES function:

$$K_t = \left[a_p^{1/e} z_{pt}^{1-1/e} + (1 - a_p)^{1/e} m_{pt}^{1-1/e} \right]^{1/(1-1/e)} \quad (A1)$$

with $e > 1$. Following production, the capital is dismantled, yielding a claim on resources of K_t . This consists of the real balance, m_{pt} , and the residue of consumable/investible goods $K_t - m_{pt}$. A firm with access to investible resources of I_t will choose z_{pt}, M_{pt} to maximise $Y_t + K_t$ subject to:

$$I_t = z_{pt} + M_{pt} = z_{pt} + (1 + p_t)m_{pt} = x_t + m_{t-1}. \quad (A2)$$

The optimal choice yields a demand for real balances of a familiar form:

$$m_{pt} = ((1 - a_p) / a_p) z_{pt} (1 + p_t)^{-e}. \quad (A3)$$

However, z_{pt} is chosen simultaneously with m_{pt} . Substituting out, we obtain:

$$m_{pt} = \frac{I_t}{(1 + p_t) + (a_p / (1 - a_p))(1 + p_t)^e} \quad (A4)$$

Substituting (A3) and (A4) into (A1), the relation between installed capital and investible funds is:

$$K_t / I_t = [a_p + (1 - a_p) / (1 + p_t)^{e-1}]^{\frac{1}{e-1}} = j(p_t) = j_t \quad (A5)$$

It is easy to see that this ratio is monotonically decreasing in p_t , and that $K_t \leq I_t$ as $p_t \geq 0$. Note that the gross rate of return to savings only coincides with the marginal product of capital when inflation is zero; the ratio between the two is also given by j_t .

(ii) Money as an input to consumption

Workers in period t have to allocate wage income $(1 - t_t)(1 - a)Y_t$. Transforming this income into useable resources requires both goods and real balances, according to the same type of aggregation function as before:

$$R_t = [a_c^{1/e} z_{ct}^{1-1/e} + (1 - a_c)^{1/e} m_{ct}^{1-1/e}]^{\frac{1}{1+1/e}} \quad (A6)$$

The worker chooses z_{ct}, M_{ct} to maximise R_t subject to

$$(1 - t_t)(1 - a)Y_t = z_{ct} + M_{ct} = z_{ct} + (1 + p_t)m_{ct}^1.$$

This yields a demand for real worker balances of the same form as (A4):

$$m_{ct} = \frac{(1 - t_t)(1 - a)Y_t}{(1 + p_t) + (a_c / (1 - a_c))(1 + p_t)^e} \quad (A7)$$

where we assume for convenience that both functions share the same elasticity of substitution.

(iii) Integrating the two demands for money

This yields the aggregate quantities $M_t = M_{ct} + M_{pt}$ and $m_t = m_{ct} + m_{pt}$. The government obtains seigniorage from firms of

$$\Sigma_{pt} = x_t - z_{pt} = M_{pt} - m_{t-1} = p_t m_{pt} + (m_{pt} - m_{t-1}) \text{ and from workers of}$$

$$(1 - t_t)(1 - a)Y_t - z_{ct} = M_{ct} = (1 + p_t)m_{ct} \text{ Hence:}$$

¹Part of the goods (the first period allocation) are used up in first period consumption, with the remainder and all real balances being carried forward, as part of the investible resources transferred next period to firms.

$$\Sigma_{pt} = \frac{I_t}{1 + (a_p / (1 - a_p))(1 + p_t)^{e-1}} - \frac{I_{t-1}}{(1 + p_{t-1}) + (a_p / (1 - a_p))(1 + p_{t-1})^e} \quad (\text{A8})$$

and similarly for workers. In an economy growing at the steady rate g , with stationary inflation p , and a common distribution parameter, $a_c = a_p = a$, the ratio of seigniorage to income is:

$$\frac{\Sigma}{Y} = s = \frac{[(1+p)(1+g)-1]}{(1+g)} \frac{(I/Y + (1-t)(1-a))}{(1+p) + (a/(1-a))(1+p)^e} \quad (\text{A9})$$

This yields the rather messy condition for the seigniorage-maximising stationary inflation rate in such an economy as a solution to the equation:

$(a/(1-a))(1+p_{\max})^{e-1} [\{(1+p_{\max})(1+g)-1\}(e-1)-1] = 1$. For $p_{\max}$ to be positive, we require $e > 1$, as assumed^{2,3}.

² Strictly, this is a necessary condition and is only sufficient when $g = 0$. A solution must satisfy

$1+p > \frac{1}{(1+g)(1-1/e)}$, so $p_{\max}$ will be positive unless both g and e are fairly high (relative to 0 and 1 respectively).

³ Combined with the assumption of perfect foresight, this has the consequence that the impact effect of increasing inflation is always temporarily to reduce seigniorage, even when initial inflation is below $p_{\max}$. This is because the level of M_t demanded by agents falls when forecast inflation rises, and m_{t-1} is predetermined. This result, running from inflation to seigniorage, is rather like the Sargent and Wallace “unpleasant monetarist arithmetic” result which runs in the reverse direction, though the mechanism is somewhat different.