A modeling and control approach to magnetic levitation system based on state-dependent ARX model

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\begin{abstract}
Magnetic levitation (Maglev) systems are usually strongly nonlinear, open-loop unstable and fast responding. In order to control the position of the steel ball in a Maglev system, a data-driven modeling approach and control strategy is presented in this paper. A state-dependent AutoRegressive with eXogenous input (SD-ARX) model is built to represent the dynamic behavior between the current of electromagnetic coil and the position of the ball. State-dependent functional coefficients of the SD-ARX model are approximated by Gaussian radial basis function (RBF) neural networks. The model parameters are identified offline by applying the structured nonlinear parameter optimization method (SNPOM). Based on the model, a predictive controller is designed to stabilize the magnetic levitation ball to a given position or to make it track a desired trajectory. The real-time control results of the proposed approach and the comparisons with other two approaches are given, which demonstrate that the modeling and control method presented in this paper are very effective and superior in controlling the fast-responding, strongly nonlinear and open-loop unstable system. This paper gives the real experimental evidence that the RBF-ARX model is capable of not only globally, but also locally capturing and quantifying a nonlinear and fast-response system’s behavior, and the model-based predictive control strategy is able to work quite well in a wide working-range of the nonlinear system.
\end{abstract}

1. Introduction

Magnetic levitation (Maglev) technique has been widely applied into many engineering systems such as frictionless bearings and high-speed Maglev trains, because of its contactless, low noise and low friction characteristics. For improving the control performance of Maglev systems, the Maglev ball system is often used as an important experimental platform. It is a kind of typically nonlinear and open-loop unstable system, and its modeling and control strategies attract a great deal of attention around the world.

In recent years, a lot of works have been reported in the literature for controlling Maglev systems. These works were mainly focused on the controller design based on the system’s physical model. To control a nonlinear Maglev system, Ahmadi and Javaid \cite{1} designed a linear controller by linearizing the model around a nominal operating gap, and some scholars designed a nonlinear controller on the basis of feedback linearization \cite{2–4} or feedforward linearization \cite{5}. A piecewise linearization technique \cite{6} was also used to establish a set of piecewise linear model of the Maglev system in every (sufficiently small) grid region and the calculation of each sub-model depended on the topology of the partitions. Some scholars \cite{7–10} directly built a nonlinear model to describe the dynamic behavior of Maglev system. These treatments depend on an accurate physical model structure and/or parameters of a device in this system. However, it is not an easy task in practice to obtain the accurate physical model of a Maglev system.

PID controller is still widely applied to industry fields, because it has a simple structure and is easy to be understood by engineers; however, manually tuning PID parameters is its drawback in practice. For overcoming the problem, a fuzzy-PID compound controller \cite{11,12} was designed for the Maglev system, which used fuzzy inference for self-regulating PID parameters in order to gain a good control result, but the implementation may depend on empirical knowledge. Baranowski and Piatek \cite{13} used PID controller, an observer and

\begin{rowenv}
\item Article history:
\item Received 30 March 2013
\item Received in revised form 17 August 2013
\item Accepted 27 October 2013
\item Available online 14 December 2013
\end{rowenv}

\begin{keywords}
Magnetic levitation system
RBF-ARX model
Parameter optimization
Predictive control
Real-time control
\end{keywords}

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\begin{doi}
0959-1524/S – see front matter © 2013 Elsevier Ltd. All rights reserved.
\end{doi}

\begin{url}
http://dx.doi.org/10.1016/j.jprocont.2013.10.016
\end{url}
feedforward technique to easily tune parameters and extend the range of stable operation of a Maglev system. A generalized proportional integral (GPI) controller for a Maglev system control was also designed to compensate the errors in the integral part, which can guarantee an asymptotically exponentially stable behavior and enhance its robustness [25,26]. Besides, the variable structure or sliding mode control [10,14,15], the adaptive control [3] and the robust control [16–18] were also considered to effectively improve the Maglev system’s robustness and were successfully implemented in experimental platforms of the Maglev system.

In this paper, a new modeling and control strategy for a Maglev system is presented on the basis of data-driven [19] modeling technique to overcome the influence of the inaccurate physical model structure and/or parameters of the system. To capture and quantify nonlinear dynamics a Maglev system, the state-dependent autoregressive with eXogenous variable (ARX) model could be used. Using a set of RBF networks to approximate the coefficients of a state-dependent ARX model yields an RBF-ARX model [20]. The RBF-ARX model, which combines the advantages of the RBF network and the ARX model, is a globally nonlinear model and also a locally linear model around each working-point. It may excellently characterize a nonlinear system. The orders and parameters of this model are identified offline by the structured nonlinear parameter optimization method (SNPOM) [20], which can improve modeling accuracy and obtain a faster convergence rate. Meanwhile the failure of online parameter estimation during real-time control can be avoided.

Linear model-based predictive control (MPC) [21–24] has more and more applications in industry because of its easy implementation, while nonlinear model-based predictive control (NMPC) [6] technology is also gradually implemented in practice. A constrained generalized model predictive control based on a linearized Maglev system model was investigated to guarantee the stability [25,26]. Wang et al. [27] applied a networked feedback linearization predictive control into the Maglev system in order to compensate the network-induced delay. Ulbig et al. [6] proposed an explicit nonlinear predictive control law constructed upon an approximating solution of a constrained finite-time optimal control (CFTOC) problem to improve its real-time performance for Maglev system. And the methodology only evaluated the control law on-line and greatly reduced computational burden. In this paper, a predictive controller based on the RBF-ARX model is designed to control the nonlinear Maglev system.

The off-line identified RBF-ARX model-based NMPC has been simulated and applied successfully in real slow-response industries system, such as the control of a thermal power plant [28–30] and the tracking control of a ship [30]. Its stability has also been analyzed and verified [29]. Its optimization problem with constraints was solved by quadratic programming (QP) routines, which is computationally expensive and suitable for the type of slow-changing system. By utilizing the ex post facto input constraint rule, in this paper, the optimization time of the MPC is greatly reduced to achieve the control of the fast-response Maglev system. The real-time control results demonstrate that the off-line identified RBF-ARX model can globally characterize the nonlinear Maglev system, and the model-based predictive controller exhibits excellent control performance to the fast-responding, strongly nonlinear and open-loop unstable system by comparing with some other control schemes. On the basis of the previous works on the RBF-ARX model-based MPC capable of controlling some slow-response plants, the main contribution of this paper is to further demonstrate a strong real-time experimental evidence in detail, that is, the RBF-ARX model is able to not only globally, but also locally capture and quantify a strongly nonlinear and fast-response system's behavior, and the model-based predictive control strategy is able to work quite well in a wide working-range of the nonlinear system.

2. Modeling of the Maglev ball system

The study object of this paper is a Maglev ball system at our laboratory, which consists of two parts: the Maglev device and a control computer. The Maglev device is composed of the electromagnet, the power amplifier, photoelectric sensors, LED light source and control objects, as shown in Fig. 1. The experimental device is a single axis of Maglev system and only able to control the object to move up and down. The control purpose is to keep the magnetic levitation ball stable in a given position or to make the ball track a desired trajectory.

The structure diagram of Maglev ball system is depicted in Fig. 2, where y denotes the position of ball and also represents the gap between the ball and the electromagnet surface. The electromagnet surface is defined as the origin and the downward motion is negative axis. When the current flows through the winding, which is decided by control strategy, the electromagnetic force $F$ is generated to control the position of the Maglev ball.

2.1. Physical model

In order to build the physical model of the system, one has to make several assumptions as follows:

(i) An ideal magnetic field exists between the electromagnet and the ball.

![Fig. 1. The platform of Maglev ball system.](image-url)
(ii) The ball is a homogeneous sphere.
(iii) Magnetic resistance between the electromagnet and the ball, and the leakage of magnetic flux between windings are ignored.

On the basis of Newton's second law, Biot–Savart law, the fundamental law of energy conservation and Kirchhoff's law, the motion equations of the Maglev ball system can be given as follows:

\[
\begin{align*}
\frac{m}{2} \frac{d^2 y(t)}{dt^2} &= F(i, y) + mg \\
F(i, y) &= K \left( \frac{i(t)}{y} \right)^2 \\
u(t) &= Ri(t) + L_1 \frac{di(t)}{dt}
\end{align*}
\]

where \(m\) denotes the mass of the steel ball, \(y(t)\) is a instantaneous gap between the ball and the electromagnet surface, \(g\) is the acceleration due to gravity, \(F(i, y)\) is the electromagnetic attraction, \(u(t)\) is the voltage of electromagnetic coil, \(i(t)\) is the instantaneous current through the coil, \(R\) is the equivalent resistance of electromagnetic coil, \(L_1\) is the self-inductance of the coil, \(K\) is a constant coefficient of the electromagnetic coil related to the mutual inductance. The prototype parameters of the system are given in Table 1.

In model (1), the first equation characterizes the dynamics of the Maglev system, and the second equation relates to electricity and mechanics and shows a strong nonlinear relation between \(F\), \(i\) and \(y\). If the ball is expected to be in equilibrium, i.e., stable levitation state, the electromagnetic force must be equal to the gravity of the ball, i.e. \(mg + F(i_0, y_0) = 0\), where the point \((i_0, y_0)\) is regarded as an equilibrium-point or working-point. Once the ball is disturbed, the previous balance will be broken, and the closed-loop control system must ensure that the ball can move to the equilibrium-point. From model (1), one can also see that the equilibrium-point varies with the variables \(i\) and \(y\), and the system is highly nonlinear. However, at a certain equilibrium-point, this system may be regarded as a locally linear system. This feature can be caught and quantified by a RBF-ARX model.

2.2. RBF-ARX model

Model (1) is just an approximate equation under the assumptions (i)–(iii), and some parameters in Table 1 such as \(K\), \(R\) and \(L\) are not so accurate, even they are not constants actually. Therefore, it is not easy to obtain a good control result using a model (1) based controller. In order to overcome the drawback of the physical modeling above, we may try to use data-driven modeling technique to get a model that may globally capture and quantify the nonlinear behavior of the Maglev ball system. The RBF-ARX model [20] may be the one that is able to represent the nonlinear dynamic characteristics of the Maglev system. The experimental Maglev system is a single input and single output (SISO) system, so its SISO RBF-ARX model can be designed as below.

\[
y(t) = \phi_0(W(t-1)) + \sum_{i=1}^{k_y} \phi_{y,i}(W(t-1))y(t-i) + \sum_{i=1}^{k_u} \phi_{u,i}(W(t-1))u(t-i) + \xi(t)
\]
where \( y(t) \) is the output, i.e. the position of ball; \( u(t) \) is the input, i.e. the electromagnet control voltage; \( x(t) \) denotes the white noise sequence; \( n_y, n_u, m \) and \( n_w = \dim(W(t - 1)) \) are the corresponding orders; \( W(t - 1) \) is the state vector of working point of the system; \( Z_k \) are the center of the RBF network, where \( Z_k = [z_{k,1}, z_{k,2}, \ldots, z_{k,n_w}]^T \); \( i = y, u \); \( \phi_0, \phi_{y,i} \) and \( \phi_{u,i} \) are the Gaussian nonlinear state-dependent coefficients and vary with \( W(t - 1) \); \( c_{i,0}^y \) and \( c_{i,k}^y \) (with \( i = y, u \); \( j = y, u \); \( k = 0, 1, 2, \ldots, m \)) are the weight coefficient; \( \lambda_k^y \) (with \( k = 1, 2, \ldots, m \); \( i = y, u \)) are the scaling factors; \( \| \cdot \| \) denotes the vector 2-norm.

In fact, model (2) with Gaussian RBF network-style coefficients has an autoregressive structure, which is similar to a linear ARX model structure at each working-point by fixing \( W(t - 1) \). It deals with a nonlinear process by dividing the parameter space into many small segments, each segment is regarded as a locally stationary process. Furthermore, coefficients \( \phi_{y,i}, \phi_{u,i} \) and a local mean \( \phi_0 \) also depend on the working-point of the process at time \( t \). The RBF-ARX model is also regarded as a global nonlinear model which is capable of globally describing nonlinear dynamics of the system.

### 2.3. Identification of RBF-ARX model

The identification of RBF-ARX model mainly includes determining the model orders and estimating the model parameters. In this paper, all parameters are estimated off-line by using the observed data generated from the experiment platform in Fig. 1 to avoid the potential problem caused by the failure of on-line parameter estimation during real-time control. The observation data are sampled by applying a traditional PID control algorithm showed in Fig. 3. The ‘Gain’ is used to convert the output voltage (unit: V) into the position of ball (unit: mm), which is obtained by calibrating the photonic sensor. The sensor and the current driver are connected via the PCI1711 converter card to a Windows XP-based PC with a MATLAB/SIMULINK 2010b environment. The sampling period is chosen as 5 ms for all of the experiments. To make the Maglev system identifiable, it is necessary to make the ball move stably within a large range as possible. We select a sine wave (i.e. \( 3.6 \sin t - 5 \) (unit: mm)) as the reference trajectory and add a white noise signal to the input of the plant for inspirng its dynamic modes. Fig. 4 shows 8000 observed data, in which the first 4000 data points are used to optimize the parameters of model (2) and the last 4000 data points are used to test the modeling performance.

The structured nonlinear parameter optimization method (SNPOM) [20], which estimates nonlinear parameters using the Levenberg–Marquardt method (LMM) and estimates linear parameters using the least-squares method (LSM) at each searching iteration, is applied to estimate parameters of the RBF-ARX model by using the observed data in Fig. 4. This is a very efficient optimization method, especially for the system whose linear parameters number is greater than the number of nonlinear parameters. In model (2), \( Z_k \) and \( \lambda_k^y \) are nonlinear parameters, \( c_i \) are linear parameters. Hence, model (2) has \( (m + 1)(n_y + n_u + 1) \) linear parameters and \( (m + m + n_w) \) nonlinear parameters to be estimated. A set of appropriate model orders and parameters are obtained by changing the model orders and the state vector repetitively under the same condition and by observing the modeling AIC (Akaike information criterion) value, the modeling residual distribution and the step response trend of the model. Generally, the final model should have a small AIC value, small modeling residual and a good dynamic performance.

In this paper, a set of historical series of the ball position (i.e. \( y(t) \)) is selected as the state \( W(t - 1) \) at time \( t - 1 \) in (2). A subset of \( W(t - 1) \) may be chosen randomly as the initial RBF centers \( Z_k \), and the initial scaling factors are set as \( \lambda_k = -\log 0.0001/\max_{t-1} W(t - 1) - Z_k^2 \).
The detailed optimization process of parameters can be seen in [20]. The model orders of this Maglev ball system are confirmed finally as $n_y = 6$, $n_u = 2$, $m = 1$ and $n_w = 2$ after repetitively training and comparison.

Figs. 5 and 6 show the comparison between the observation data and the one-step-ahead prediction output generated by the RBF-ARX model, as well as modeling residuals and residual histogram for training data and testing data, respectively. It can be seen from the figures that the output of the identified RBF-ARX model are very close to the actual output, no matter for training data or for testing data. The modeling errors range from only $-0.04$ to $+0.04$, and show almost a Gaussian distribution. It can be seen that the RBF-ARX model applied to the Maglev system can achieve a sufficiently high modeling accuracy. Fig. 7 plots the poles of the estimated model (2) changing with the variation of the position signal of the ball shown in Figs. 5 and 6, from which one may see that the model's dynamic behavior changes with the ball state. It can be also seen that there is a pole outside the unit circle, which explains that the Maglev system is nonlinear and open-loop unstable.
3. Design of the predictive controller

To design a predictive controller, it is necessary to transform the identified model (2) into a state-space expression. Based on the built model of the magnetic levitation system, define a state vector

\[
\begin{align*}
\mathbf{x}(t) &= [\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_{k_n}^T]^T \\
x_{1,t} &= y(t) \\
x_{k,t} &= \sum_{i=1}^{k_n-k+1} \phi_{y,i}(\mathbf{W}(t-1))y(t-i) + \sum_{i=1}^{k_n-k+1} \phi_{u,i}(\mathbf{W}(t-1))u(t-i) \\
k = 2, 3, \ldots, k_n; \quad k_n = \max(n_y, n_u)
\end{align*}
\]

(3)

where \(k_n = 6\) for the identified RBF-ARX model. Then a state-space model of the Maglev ball system (2) can be given as follows:

\[
\begin{align*}
\mathbf{x}(t+1) &= \mathbf{A}_t \mathbf{x}(t) + \mathbf{B}_t u(t) + \mathbf{F}_t + \mathbf{E}(t+1) \\
y(t) &= \mathbf{C}_t \mathbf{x}(t)
\end{align*}
\]

(4)
where

$$A_t = \begin{bmatrix} \phi_{y,1}(\bullet) & 1 & 0 & 0 & 0 \\ \phi_{y,2}(\bullet) & 0 & 1 & 0 & 0 \\ \phi_{y,3}(\bullet) & 0 & 0 & 1 & 0 \\ \phi_{y,4}(\bullet) & 0 & 0 & 0 & 1 \\ \phi_{y,6}(\bullet) & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_t = \begin{bmatrix} \phi_{u,1}(\bullet) \\ \phi_{u,2}(\bullet) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Phi_t = \begin{bmatrix} \phi(\bullet) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Xi(t+1) = \begin{bmatrix} \xi(t+1) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1^T \end{bmatrix}$$  \hspace{1cm} (5)

Note that $(\bullet) = (W(t))$ denotes the coefficients are all related to $W(t)$ at time $t$. The state vector $x(t)$ in (4) can be easily obtained by (3) according to the estimated RBF-ARX model (2), the present output $y(t)$ and the past input/output data.

On the basis of the state space model (4) of the Maglev ball system, a predictive controller can be designed based on a SISO locally linear ARX model obtained from the globally nonlinear RBF-ARX model. At first, define the following vectors:

$$\begin{align*}
\hat{x}(t) &= [\hat{x}(t+1)^T \hat{x}(t+2)^T \cdots \hat{x}(t+N_p(t)^T]^T, \\
\hat{y}(t) &= [\hat{y}(t+1) \hat{y}(t+2) \cdots \hat{y}(t+N_p(t)^T]^T, \\
\hat{u}(t) &= [u(t) u(t+1) \cdots u(t+N_c-1)^T]^T, \\
\hat{\Phi}_t &= [\Phi_T \Phi_{t+1} \cdots \Phi_{t+N_p-1}^T]^T,
\end{align*}$$  \hspace{1cm} (6)

where $\hat{x}(t)$ denotes the predictive state vector, $\hat{y}(t)$ denotes the predictive output. Note that in (6), $N_p$ denotes the prediction horizon and $N_c$ denotes the control horizon. Based on model (4) at time $t$, the optimal predictive state and output may be obtained as follows.

$$\begin{align*}
\hat{x}(t) &= \hat{A}_t \hat{x}(t) + \hat{B}_t \hat{u}(t) + \hat{G}_t \hat{\Phi}_t, \\
\hat{y}(t) &= \hat{C}_t \hat{x}(t) + \hat{D}_t \hat{u}(t) + \hat{F}_t \hat{\Phi}_t,
\end{align*}$$  \hspace{1cm} (7)

$$\hat{A}_t = \begin{bmatrix} 0 \\ \| A_{t+j} \|_{j=0} \\ \| A_{t+j} \|_{j=0} \\ \| A_{t+j} \|_{j=0} \\ \| A_{t+j} \|_{j=0} \end{bmatrix},$$  \hspace{1cm} (8)

$$\begin{align*}
k \| A_{t+i} \|_{i=0} &= \begin{cases} A_{t+i} A_{t+k-1} \cdots A_{t+i}, & i \leq k \\ I, & i > k. \end{cases}
\end{align*}$$  \hspace{1cm} (9)
using QP routines whose computation time may exceed the sampling time 5 ms in this experiment, we do not consider output constraints.

Note that, in this paper, we compute ̂A, ̂B, ̂c and ̂Γt with the working-point state ̂W(t).

From (7)-(10), a new prediction output may be also expressed to

\[
\hat{y}(t) = G_t \hat{u}(t) + y_0(t),
\]

\[
\Gamma_t = \tilde{c} \hat{B}_t,
\]

\[
y_0(t) = \tilde{c} \hat{A} \tilde{x}(t) + \tilde{c} \hat{r}_t \Phi_t,
\]

where  \( \hat{A} \) and  \( \hat{r} \) are the upper bounds of the output error,  \( \hat{u} \) and  \( \Delta \hat{u} \) respectively. In the real application, to avoid using QP routines whose computation time may exceed the sampling time 5 ms in this experiment, we do not consider output constraints and just keep input constraints. According to the ex post facto input constraint rule [29], substitute (11) into (13) and the predictive optimal control  \( \hat{u}(t) \) may be derived as follows.

\[
\min_{\hat{u}(t)} \begin{bmatrix} ||x(t) - ̂x(t)||_Q^2 + ||\hat{u}(t)||_H^2 + ||\Delta \hat{u}(t)||_R^2 \end{bmatrix}
\text{s.t.} \begin{bmatrix} y_{\min} \leq \hat{y}(t) \leq y_{\max}, \, u_{\min} \leq \hat{u}(t) \leq u_{\max} \end{bmatrix},
\]

\[
\Delta u_{\min} \leq \Delta \hat{u}(t) \leq \Delta u_{\max}
\]

where  \( ||x(t)||_Q^2 = X^T \Omega X \) and  \( R_1 \) and  \( R_2 \) are the positive weight matrices of the output error,  \( \hat{u} \) and  \( \Delta \hat{u} \) respectively. In the real application, to avoid using QP routines whose computation time may exceed the sampling time 5 ms in this experiment, we do not consider output constraints and just keep input constraints. According to the ex post facto input constraint rule [29], substitute (11) into (13) and the predictive optimal control  \( \hat{u}(t) \) may be derived as follows.

\[
\hat{u}(t) = \arg \min_{\hat{u}(t)} ||G_t \hat{u}(t) + y_0(t) - \hat{y}(t)||_Q^2 + ||\hat{u}(t)||_H^2 + ||\Delta \hat{u}(t)||_R^2(t) R_1 \hat{u}(t) + \Delta \hat{u}^T(t) R_2 \Delta \hat{u}(t)
\]

\[
\text{s.t.} \begin{bmatrix} u_{\min} \leq \hat{u}(t) \leq u_{\max}, \, \Delta u_{\min} \leq \Delta \hat{u}(t) \leq \Delta u_{\max} \end{bmatrix}
\]

where

\[
\hat{u}(t) = \left[ \hat{u}(t) \, \hat{u}(t + 1) \cdots \hat{u}(t + N_c - 1) \right]^T
\]

\[
\hat{u}(t) = u_0(t - 1) + E \Delta \hat{u}(t)
\]

\[
u_0(t - 1) = \left[ u(t - 1) \, u(t - 1) \cdots u(t - 1) \right]^T_{1 \times N_c}
\]

\[
E = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}_{N_c \times N_c}
\]
Let \( \frac{df}{du}(t) = 0 \), then the optimal control input \( \tilde{u}(t) \) may be obtained directly as follows:

\[
\tilde{u}(t) = \left( G_t^T Q y_r(t) + G_t^T Q y_0(t) + E^{-T} R_2 E^{-1} \right)^{-1} \left( G_t^T Q y_r(t) - G_t^T Q y_0(t) + E^{-T} R_2 E^{-1} \tilde{u}_0(t-1) \right)
\]

\[s.t. \quad \tilde{u}(t) = \tilde{u}(t) \quad \text{if} \quad \tilde{u}_\text{min} \leq \tilde{u}(t) \leq \tilde{u}_\text{max} \quad \text{and} \quad \Delta \tilde{u}_\text{min} \leq \Delta \tilde{u}(t) \leq \Delta \tilde{u}_\text{max}
\]

\[
u(t) = u(t-1) + \Delta \tilde{u}_\text{max} \quad \text{if} \quad \Delta \tilde{u}(t) > \Delta \tilde{u}_\text{max}
\]

\[
u(t) = u(t-1) + \Delta \tilde{u}_\text{min} \quad \text{if} \quad \Delta \tilde{u}(t) < \Delta \tilde{u}_\text{min}
\]

\[
u(t) = \tilde{u}_\text{max} \quad \text{if} \quad \tilde{u}(t) > \tilde{u}_\text{max}
\]

\[
u(t) = \tilde{u}_\text{min} \quad \text{if} \quad \tilde{u}(t) < \tilde{u}_\text{min}
\]

In the optimal control \( \tilde{u}(t) \) from (15), only the first component \( \tilde{u}(t) \) is used as an effective control input. In this paper, the limit of the control input of the Maglev system is \( 0 \leq u(t) \leq 10 \) (V). From the formulas above, one can see that the RBF-ARX model-based predictive controller does not require estimating parameters on-line, because its inherent RBF-ARX model is an off-line estimated local linear and global nonlinear model.

The structure diagram of the predictive control for the Maglev ball system is shown in Fig. 8. The dashed box in Fig. 8 is the designed predictive controller (MPC). At the time \( t \), the control performance can be optimized and the optimal control variable \( \tilde{u}(t) \) can be calculated on the basis of given \( \hat{y}_r \), historical output information and the actual position output \( y(t) \).

### 4. Real-time controls

In this section, real-time experiments are carried out to evaluate the performance of the proposed RBF-ARX MPC used to control the actual Maglev ball system in Fig. 1. Corresponding to Fig. 8, we design a MPC control system block to the Maglev ball system under the MATLAB/SIMULINK 2010b environment. The sampling period is still chosen as 5 ms. The diagram is shown in Fig. 9.

In Fig. 9, the real control module completes the input signal sampling and the A/D or D/A converter of signals. The MPC module denotes the dashed box part in Fig. 8 and is an S-function written in C language. There are six inputs: the control weighting matrix \( R_1 \) and \( R_2 \), the
output error weighting matrix \( Q \) (i.e. \( R_1 = R_1 \cdot I_{N_y \times N_y} \), \( R_2 = R_2 \cdot I_{N_y \times N_y} \) and \( Q = Q \cdot I_{N_y \times N_y} \) in (15)), the given desired output \( y_r \) (i.e. \( y_r \) in (15)), the state \( W(t) \) (i.e. \( W(t) \) in (2)) of working-point and the system output \( y \) at the current time, where we select the output \( y \) as the state \( W(t) \). The MPC module outputs the optimal control \( u \) by Eq. (15). By experimenting repeatedly to ensure that the Maglev ball can be levitate stably, finally the RBF-ARX model-based MPC parameters are selected as \( N_p = 12, N_c = 10, R_1 = 0.03, R_2 = 0.41, Q = 1.425 \times 10^{-2} \), and \( R_2 = 0.41 \times 10^{-2} \).

In the following subsection, the designed predictive controller is applied to control the movement of the Maglev ball. In order to illustrate what improvements can be obtained under control of the RBF-ARX model-based MPC (RBF-ARX-MPC), a linear ARX model is also built and the ARX model-based predictive control (ARX-MPC) with the same predictive controller structure is also used to the Maglev system. The linear ARX model to be used is as follows:

\[
y(t) = a_0 + \sum_{i=1}^{n_a} a_i y(t-i) + \sum_{j=1}^{n_b} b_j u(t-j) + \xi(t)
\]

(16)

where \( a_0, a_i \) and \( b_j \) are constant coefficients; \( \xi(t) \) denotes the white noise sequence; \( n_a \) and \( n_b \) are the corresponding orders where \( n_a = 6 \) and \( n_b = 2 \) that are the same with the orders of the identified RBF-ARX model. Here, the identification method presented in Section 2.3 is also adopted to identify the ARX model. In this paper, three sets of observation data are generated by giving functions \( y_r = 0.56 \sin t - 6 \) (mm), \( y_r = 0.56 \sin t - 9 \) (mm), \( y_r = 0.56 \sin t - 12 \) (mm) as the reference value under the control of PID in Fig. 3, respectively; and three ARX
models are then obtained around the three positions, i.e. $-6$ mm (top), $-9$ mm (middle), and $-12$ mm (bottom). The linear ARX model's corresponding state-space representation can be also obtained by Eqs. (3) and (4), and its coefficient matrices are as follows:

\[
A = 
\begin{bmatrix}
    a_1 & 1 & 0 & 0 & 0 & 0 \\
    a_2 & 0 & 1 & 0 & 0 & 0 \\
    a_3 & 0 & 0 & 1 & 0 & 0 \\
    a_4 & 0 & 0 & 0 & 1 & 0 \\
    a_5 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix},
\quad
B = 
\begin{bmatrix}
    b_1 \\
    b_2 \\
\end{bmatrix},
\quad
\Phi = 
\begin{bmatrix}
    a_0 \\
    0 \\
    0 \\
    0 \\
\end{bmatrix},
\quad
\Xi(t + 1) = 
\begin{bmatrix}
    \xi(t + 1) \\
    0 \\
    0 \\
    0 \\
\end{bmatrix},
\quad
C = 
\begin{bmatrix}
    1 \\
    0 \\
    0 \\
    0 \\
    0 \\
\end{bmatrix}
\]  

(17)
The above coefficient matrices coming from one of the three linear ARX models are constant in all control process. The identified three linear ARX models’ coefficients are shown in Table 2. Then the linear ARX model-based predictive control (ARX-MPC) can be also calculated by Eqs. (6)–(15).

For the Maglev ball system, the step response experiments are designed to test the dynamic performance of the system and the move range of the ball under different control strategy. In the experiments, the ball is controlled to follow a given rectangular wave, which can test the dynamic response capability of the control system.

4.1. Small jump-amplitude step response

In order to test the control performance of the RBF-ARX-MPC, the ARX-MPC and a classical well-tuned PID controller for the Maglev ball system, we implement the small jump-amplitude step response to the Maglev system around the positions: −6 mm, −9 mm and −12 mm, respectively. The step amplitudes are all 1.1 mm, and the data obtained in those ranges are used to identify the three linear ARX models. Through trial and error, the well-tuned parameters of each controller used for controlling the Maglev ball system are given in Table 3.
As seen in Table 3, in all experiments, which are showed in Figs. 10–23, only one RBF-ARX model built in Section 2 and one set of parameters of RBF-ARX-MPC are used, but three linear ARX models and three sets of different parameters of ARX model-based MPC and PID control are used for getting the best control performance at the positions around the top (−6 mm), middle (−9 mm), and bottom (−12 mm). Comparisons of control results between the three control strategies are demonstrated in Figs. 10–23.

From Figs. 10–12 one can see that the overshoots of the RBF-ARX-MPC and the ARX-MPC are much smaller than that of the PID control; besides, the control inputs of two MPCs are also much smoother than that of the PID control, thus, the control performance of the predictive control strategy is much better than that of the PID controller for the Maglev ball system. Comparing two MPC results in Figs. 10–12, it is clear that at the top (−6 mm) and the bottom (−12 mm) positions, both MPC results are similar, and at the middle (−9 mm) position the RBF-ARX-MPC performance is better than the ARX-MPC performance. Therefore, it can be verified that the RBF-ARX model is capable to capture and quantify the global behavior of the nonlinear Maglev system, and the one global RBF-ARX model-based MPC behaves like the multi-local linear ARX model-based MPCs.

Fig. 15. Step response of MPC and PID control to 3.8 mm jump amplitude (center: −9 mm).

Fig. 16. Step response of RBF-ARX-MPC and ARX-MPC to 4.7 mm jump amplitude (center: −9 mm).
4.2. Large jump-amplitude step response

In a relatively small range, the three control methods are all able to make the ball to move well. For illustrating the control effects and operating range of three control strategies, the amplitude of given step signal is gradually widened in the next experiments. Note that the controller parameters of the following experiments are the same as that in Table 3.

First of all, the central position is set at −9 mm which is about the middle position of the operating range of ball, so there is enough space to test the range of stable operation. The different jump-amplitude step response results of the real-time control of the MPCs and PID controller are shown in Figs. 13–17 around the central position −9 mm where the linear ARX model used in Figs. 13–16, also in Fig. 11, is the same with the model identified by the data obtained at around the −9 mm central position, but the RBF-ARX model is the same with the one used in Figs. 10–12. Actually, just one RBF-ARX model is used in all cases in the paper.

Comparing Fig. 13 with Fig. 14, the overshoots, adjustment time and rise time of three controllers all increase with the extension of step amplitude. Especially when the ball jumps downward from −7.0 mm to −10.6 mm under the PID control, the overshoot is close to 80% as showed in Fig. 14. The main reason is that, when the ball is controlled to jump downward quickly and deeply, the PID controller is not able to respond quickly to instantaneous and large decline of the electromagnetic force. This situation is also reflected in the rising stage. From
Fig. 14, it is also clearly seen that the ball under the PID control has an obvious and frequent oscillation when jumping upward. This is also caused by the reason mentioned above. Moreover, due to magnetic saturation and eddy current effects, especially when the ball is close to the electromagnet so that the electromagnetic force becomes very large, the PID controller has to take some time to adjust the control variable repetitively so as to make the ball enter a new steady state. On the contrary, MPC is capable to predict the future state based on historical information of the running process and is able to control the large variation in time. Therefore, the dynamic performance exhibited by the MPCs is much better than that of the PID control as seen in Figs. 13–15.

To test the controllable range of three control methods, the jump amplitude of step style set-value is further enlarged. It can be seen in Fig. 15 that when the jump span reaches to 3.8 mm, the results of the PID control become quite worse. When the required position of the ball is switched upward 3.8 mm, the ball is still capable to keep stable after large oscillating, but it is out of control when set-value jumps downwards 3.8 mm under the PID control. The control performances under the MPC based on RBF-ARX model or ARX model are still quite good in case of 3.8 mm jump amplitude of set-value in Fig. 15.

In Fig. 16, the ARX-MPC may control the ball to step upwards 4.7 mm, but it does not make the ball to descend successfully from −6.3 mm to −11.0 mm (4.7 mm jump). The main reason for this phenomenon is that the magnetic field generated by the electromagnet shown in Fig. 1 is strongly nonlinear and asymmetric. It causes that the magnetic lines are dense when the ball is close to the electromagnet,
while they are sparse and the nonlinearity is stronger when the ball is at greater distance from the electromagnet. When the step range is enlarged, the system’s nonlinearity is strengthened; this will directly degrade the control performance of the ARX-MPC. This is why the ball is out of control when descending from −6.3 mm to −11.0 mm (4.7 mm jump). On the contrary, the RBF-ARX-MPC can control the ball to follow the set-value with large positive or negative step-jump easily as seen in Figs. 16 and 17, and the overshoot and adjustment time of the RBF-ARX-MPC are much less than those of the ARX-MPC or PID control in almost all cases. The primary reason is due to the strong nonlinearity of the Maglev ball system, the local linear ARX model-based MPC and PID controller can give a ‘not bad’ control only at a local working-point but not for a large working-range. But the RBF-ARX model-based MPC can work well in a large range, because the RBF-ARX model can characterize the nonlinear Maglev system in a large range; it is a global model.

Fig. 17 shows the step response for large amplitude under the RBF-ARX-MPC. When the jump span is set from −12.1 mm to −5.4 (i.e. 6.7 mm step amplitude, which is the variation range of the observation data used to identify the RBF-ARX model in Fig. 4), the control performance is still very good. Whether the set-value is switched from −5.4 mm downward to −12.1 mm or from −12.1 mm upward to −5.4 mm, the proposed RBF-ARX model-based MPC strategy is able to make the ball quickly to return to the stable state, but the fluctuations of jumping downwards or upwards becomes greater and the adjustment time become longer comparing the other cases in Figs. 13–16.
The jump range of step response under RBF-ARX-MPC is so large, this illustrates that the RBF-ARX model can globally describe the system behaviors very well, so that the designed controller based on this model may obtain the excellent control performance.

In addition, to further verify effectiveness of the RBF-ARX-MPC, we also did step response experiments with the three controllers around the top (−6 mm) and bottom (−12 mm) positions in the ball’s movable range, respectively, and the control results are given in Figs. 18–23.

Comparing Figs. 10 and 18–20, it is clear that the control performance of the RBF-ARX-MPC is much better than that of the ARX-MPC and PID controller around the top position −6 mm and for different jump amplitude of step setting, and the controllable jump-step range of PID or the ARX-MPC is smaller than 2.9 mm or 3.8 mm, respectively, and the RBF-ARX-MPC can control the ball in all range.

Figs. 12 and 21–23 show the similar results with above, one can see that the control performance of the RBF-ARX-MPC is also much better than that of the ARX-MPC and PID controller around the bottom position −12 mm and for different jump amplitude of step setting, and the controllable jump-step range of PID or the ARX-MPC is smaller than 2.2 mm or 2.5 mm, respectively, and the RBF-ARX-MPC can control the ball also in all range.

From the results given in Figs. 10–23, we can conclude that the MPC is much better than PID control, and the former controlled output and control input are much smoother than the latter. Although local linear ARX model-based MPC works well just around a small range that is the valid range of the model, its control performance will be quickly getting worse as the increase of step’s jump amplitude due to strong nonlinearity of the Maglev ball system. Because the proposed RBF-ARX model can globally capture and quantify the nonlinear feature of the Maglev ball system quite well, the model-based MPC exhibits very good step response control performance in case of larger
set-value jump variation, and it can make the ball quickly track the setting signals with fast and large changes. Therefore, the RBF-ARX-MPC is a feasible and effective control method for the underlying system.

5. Effectiveness of different identification data for RBF-ARX modeling

To check the influence of different type identification data for RBF-ARX modeling to MPC design, a set of combination data with various waves is selected as the reference trajectory to generate the identification data that are given in Fig. 24, in which the first 4000 data points are used to optimize the parameters of model (2) and the last 4000 data points are used to test the modeling performance. We use the same identification method to select the best model orders, which are $n_y = 7$, $n_u = 3$, $m = 1$ and $n_w = 2$. The identification results are shown in Figs. 25 and 26.

Then we use the same RBF-ARX-MPC to control the ball and the corresponding control parameters are set as $N_p = 12$, $N_c = 10$, $Q = I_{12 \times 12}$, $R_1 = 0.03 I_{10 \times 10}$ and $R_2 = 0.16 I_{10 \times 10}$. A step response to 4.7 mm jump is depicted in Fig. 27. In order to compare with the previous results (RBF-ARX-MPC(1) in Fig. 27) that is shown in Fig. 16, the two control results are depicted in Fig. 27. From Fig. 27, one can see that the two results are quite similar, and the MPC result (RBF-ARX-MPC(2) in Fig. 27) based on the RBF-ARX model identified using the data in Fig. 24 is a little better than that of the one shown in Fig. 16 (RBF-ARX-MPC(1) in Fig. 27). It means that if the identification data have rich spectrum and their variation range cover almost all working-range of the system, the identified RBF-ARX model will be effective for MPC design.

Fig. 25. Outputs, modeling residuals and histogram of residuals of RBF-ARX model for training data (standard deviation: 0.0120).

Fig. 26. Outputs, modeling residuals and histogram of residuals of RBF-ARX model for testing data (standard deviation: 0.0118).
6. Conclusions

This paper showed how a data-driven modeling technique can be used to build an ARX-type model for capturing and quantifying the global nonlinear behavior of an open-loop unstable and fast-response magnetic levitation system. The proposed modeling approach to the underlying system was different to general physical model-based procedures, which may overcome the drawbacks such as a lack of accurate math model structure and/or parameters of a device in the system. The system state-dependent RBF-ARX model identified off-line from real sample input/output data was capable of characterizing the nonlinear system in a wide range of state changing. The RBF-ARX model-based MPC strategy presented in this paper was suitable for the ball position control of the fast-response magnetic levitation system, because which worked like a time-varying locally linear ARX model-based linear MPC process. Through the successful application to a real magnetic levitation system and the detailed comparisons of real-time control results with the classical PID controller and the linear ARX-MPC, it was illustrated that the RBF-ARX-MPC is effective and feasible in modeling and controlling the fast-responding, strongly nonlinear and open-loop unstable system. Therefore, one could say that this paper may provide an optional modeling and control approach to such a nonlinear and fast-changing system. Another main contribution of this paper was to clearly reveal the strong evidence by real-time experiments and the detailed comparisons with some other control approaches, that is, the RBF-ARX model could completely and accurately characterizing the strongly nonlinear and fast-response system, and the model-based predictive control strategy could provide very good performance in a wide working-range of the nonlinear system.

Acknowledgments

This work is supported by the International Science & Technology Cooperation Program of China (2011DFA10440), and the National Natural Science Foundation of China (71271215 and 71221061) and Hunan Provincial Innovation Foundation for Postgraduate of China (CX2012B067).

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jprocont.2013.10.016.

References