ONLINE APPENDICES TO: TOGETHER AT LAST: TRADE COSTS, DEMAND STRUCTURE, AND WELFARE*

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Appendices

A Notes on the Literature

Because of pressures on space, many relevant references have had to be omitted from the text. Further details can be found in Mrázová and Neary (2013).

Introduction: Quantifying the Gains from Trade with CES Preferences: The results of Arkolakis, Costinot and Rodríguez-Clare (2012) have been further considered by Simonovska and Waugh (2011), and Melitz and Redding (2013). Ossa (2012) explores the implications of elasticities that differ exogenously across industries in a CES framework, whereas we focus on how they differ endogenously between home and foreign markets with non-CES demands.

Introduction: Alternatives to the CES: In discussing papers that have gone beyond the CES, we mention in the text only those that look at broad classes of preferences or demands, such as additive separability, and that explore comparative statics in the presence of trade costs. Many important papers have explored the implications of particular alternatives to the CES, such as quadratic (Melitz and Ottaviano (2008)), translog (Novy (2013) and Feenstra and Weinstein (2010)), or Stone-Geary (Simonovska (2010)). The case of general additive preferences first considered by Dixit and Stiglitz (1977) and Krugman (1979) has been reexamined by Neary (2009), Zhelobodko et al. (2012), Dhingra and Morrow (2011), and Mrázová and Neary (2013), but without trade costs. Related results have been independently presented in Russian by Evgeny Zhelobodko and Sergey Kokovin with Maxim Goryunov and Alexey Gorn. Dhingra and Morrow (2011) also explore how different assumptions about demand affect efficiency.

Section I: The terms "superconvexity" and "superconcavity" were first used in this context in Mrázová and Neary (2011) and Mrázová and Neary (2013) respectively. They will not come as a surprise to the careful reader of Dixit and Stiglitz (1977): see for example their equation (45). Our contribution, apart from the labels, is to present a framework which throws light on the implications of a wide range of assumptions about preferences and demand for comparative statics and calibration of general-equilibrium models with monopolistic competition.

Section II: We follow Jones (1965) in using "hats" (circumflexes) to denote proportional changes.

Section III: Corden (1960) discusses the "expenditure-reduction" and "expenditure-switching" effects of devaluation.

Section V: We present some preliminary calibration exercises in Mrázová and Neary (2013).

B The Change in Compensating Income

B.1 The Direct Utility Function

We wish to express the change in utility in expenditure units. The first step is to totally differentiate the utility function (1), ignoring the transformation function F. This yields:

$$\hat{U} = \hat{N}_u + \omega_u \xi \hat{x} + (1 - \omega_u)\, \xi^* \hat{x}^* \tag{15}$$

where $\hat{N}_u \equiv \hat{n} + (1 - \omega_u)\, \hat{\kappa}$ is the extensive margin change in utility.

B.2 Frisch Demands and the Frisch Indirect Utility Function

The consumer's optimization problem yields the Frisch demand functions: $u'(x) = \lambda p$, $u'(x^*) = \lambda p^*$. Totally differentiating these:

$$\hat{x} = -\varepsilon \left(\hat{\lambda} + \hat{p}\right), \; \hat{x}^* = -\varepsilon^* \left(\hat{\lambda} + \hat{p}^*\right) \tag{16}$$

Substituting the Frisch demands into the direct utility function yields what we can call the "Frisch indirect utility function":

$$V^F(N, p, p^*, \lambda) \equiv U[N, x(p, \lambda), x^*(p^*, \lambda)] \tag{17}$$

In differential form, the change in utility becomes:

$$\hat{U} = \hat{N}_u - \Omega\left[\hat{\lambda} + \omega_\Omega \hat{p} + (1 - \omega_\Omega)\hat{p}^*\right] \tag{18}$$

The coefficient of $\hat{\lambda}$ is a utility-share-weighted average of the home and foreign price-elasticities of V^F:

$$\Omega \equiv \omega_u \xi \varepsilon + (1 - \omega_u)\xi^* \varepsilon^* = \bar{\varepsilon}_u \bar{\xi}_u \tag{19}$$

where: $\bar{\varepsilon}_u \equiv \omega_u \varepsilon + (1 - \omega_u)\varepsilon^*$, $\bar{\xi}_u \equiv \omega'_u \xi + (1 - \omega'_u)\xi^*$, and $\omega'_u \equiv \frac{\omega_u \varepsilon}{\bar{\varepsilon}_u}$. The coefficients of price changes are shares in this: $\omega_\Omega \equiv \frac{\omega_u \xi \varepsilon}{\Omega}$. Ω itself is the elasticity of the Frisch indirect utility function with respect to λ; i.e., it tells us how much the consumer would gain from a unit reduction in the marginal utility of income. In the CES case it reduces to: $\Omega = \xi\varepsilon = \frac{\sigma-1}{\sigma}\sigma = \sigma - 1$.

B.3 Solve for the Marginal Utility of Income

Totally differentiating the budget constraint, $I = n[px(p, \lambda) + \kappa p^* x^*(p^*, \lambda)]$, yields:

$$\hat{I} = \hat{N}_I + \omega_z(\hat{p} + \hat{x}) + (1 - \omega_z)(\hat{p}^* + \hat{x}^*) \tag{20}$$

where $\hat{N}_I \equiv \hat{n} + (1 - \omega_z)\hat{\kappa}$ is the extensive margin change in the budget constraint. Substitute from the Frisch demands to solve for $\hat{\lambda}$:

$$\bar{\varepsilon}_z \hat{\lambda} = \hat{N}_I - \omega_z(\varepsilon - 1)\hat{p} - (1 - \omega_z)(\varepsilon^* - 1)\hat{p}^* - \hat{I} \qquad \bar{\varepsilon}_z \equiv \omega_z \varepsilon + (1 - \omega_z)\varepsilon^* \tag{21}$$

This is $\lambda\left(N, p, p^{*}, I\right)$ in changes.

B.4 Solve for the Change in Real Income

Eliminating λ from the Frisch indirect utility function gives the familiar Marshallian indirect utility function. In levels this is:

$$
\begin{aligned}
V\left(n, \kappa, p, p^{*}, I/Y\right) &= V^{F}\left[N, p, p^{*}, \lambda\left(N, p, p^{*}, I/Y\right)\right] && (22)\\
&= U\left[n, \kappa, x\left\{p, \lambda\left(N, p, p^{*}, I/Y\right)\right\}, x^{*}\left\{p^{*}, \lambda\left(N, p, p^{*}, I/Y\right)\right\}\right] && (23)
\end{aligned}
$$

In terms of changes:

$$
\begin{aligned}
\hat{Y} &= \bar{\varepsilon}_{z}\hat{\lambda}-\hat{N}_{I}+\omega_{z}\left(\varepsilon-1\right)\hat{p}+\left(1-\omega_{z}\right)\left(\varepsilon^{*}-1\right)\hat{p}^{*} && (24)\\
&= \frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\left[\hat{N}_{u}-\omega_{u}\xi\varepsilon\hat{p}-\left(1-\omega_{u}\right)\xi^{*}\varepsilon^{*}\hat{p}^{*}\right]-\hat{N}_{I}+\omega_{z}\left(\varepsilon-1\right)\hat{p}+\left(1-\omega_{z}\right)\left(\varepsilon^{*}-1\right)\hat{p}^{*} && (25)
\end{aligned}
$$

which can be written more compactly as follows:

$$
\hat{Y}=\hat{N}_{Y}-\omega_{Y}\hat{p}-\omega_{Y}^{*}\hat{p}^{*} \tag{26}
$$

where:

$$
\hat{N}_{Y}\equiv\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\hat{N}_{u}-\hat{N}_{I}=\left(\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}-1\right)\hat{n}+\left[\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\left(1-\omega_{u}\right)-\left(1-\omega_{z}\right)\right]\hat{\kappa} \tag{27}
$$

$$
\omega_{Y}\equiv\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\omega_{u}\xi\varepsilon-\omega_{z}\left(\varepsilon-1\right)\quad\omega_{Y}^{*}\equiv\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\left(1-\omega_{u}\right)\xi^{*}\varepsilon^{*}-\left(1-\omega_{z}\right)\left(\varepsilon^{*}-1\right) \tag{28}
$$

Alternatively, we can write the weights as follows:

$$
\omega_{Y}=\omega_{z}+\left(\omega_{u}\frac{\bar{\varepsilon}_{z}\xi}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}-\omega_{z}\right)\varepsilon\quad\omega_{Y}^{*}=\left(1-\omega_{z}\right)+\left[\left(1-\omega_{u}\right)\frac{\bar{\varepsilon}_{z}\xi^{*}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}-\left(1-\omega_{z}\right)\right]\varepsilon^{*} \tag{29}
$$

Note finally that the weights sum to unity, as they must since the consumer is rational:

$$\omega_Y + \omega_Y^* = 1 + \omega_u \frac{\bar{\varepsilon}_z \xi}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon + (1 - \omega_u) \frac{\bar{\varepsilon}_z \xi^*}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon^* - \omega_z \varepsilon - (1 - \omega_z) \varepsilon^* \tag{30}$$

$$= 1 + \left[\omega_u \frac{\xi}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon + (1 - \omega_u) \frac{\xi^*}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon^* - 1 \right] \bar{\varepsilon}_z \tag{31}$$

$$= 1 + \left[\frac{\omega_u \varepsilon \xi + (1 - \omega_u) \varepsilon^* \xi^*}{\bar{\varepsilon}_u \bar{\xi}_u} - 1 \right] \bar{\varepsilon}_z = 1 \tag{32}$$

Hence (26) is equation (13) in the text.

C Solving for Welfare Change With Trade Costs

We want to evaluate the change in real income given by (26), where the change in varieties is given by (27). To do this, we need to use equations (9), (10), and (11) in the text for the changes in prices, firm output, and firm numbers respectively. Evaluating the latter at initial free trade gives:

$$\hat{p} = \frac{\varepsilon + 1 - \varepsilon \rho}{\varepsilon (\varepsilon - 1)} \hat{x} \qquad \hat{p}^* = \frac{\varepsilon^* + 1 - \varepsilon^* \rho^*}{\varepsilon^* (\varepsilon^* - 1)} \hat{x}^* + \hat{\tau} \tag{33}$$

$$\hat{y} = [\omega \hat{x} + (1 - \omega) \hat{x}^*] + (1 - \omega) (\hat{\kappa} + \hat{\tau}) \qquad \hat{n} = -\psi \hat{y} \tag{34}$$

From (33) we can calculate the average change in prices:

$$\omega \hat{p} + (1 - \omega) \hat{p}^* = \frac{\varepsilon + 1 - \varepsilon \rho}{\varepsilon (\varepsilon - 1)} [\omega \hat{x} + (1 - \omega) \hat{x}^*] + (1 - \omega) \hat{\tau} \tag{35}$$

From the free-entry condition, equation (6) in the text, we can calculate the change in total sales:

$$\varepsilon \eta [\omega \hat{x} + (1 - \omega) \hat{x}^*] = - (1 - \omega) (\hat{\kappa} + \hat{\tau}) \tag{36}$$

Add this to $\hat{y}$ to express the change in total sales as a function of the change in output only:

$$\omega \hat{x} + (1-\omega)\,\hat{x}^* = -\frac{1}{\varepsilon\eta - 1}\hat{y} \tag{37}$$

Hence the average change in prices becomes:

$$\omega \hat{p} + (1-\omega)\,\hat{p}^* = -\frac{1}{\varepsilon}\hat{y} + (1-\omega)\,\hat{\tau} = \frac{1}{\varepsilon\psi}\hat{n} + (1-\omega)\,\hat{\tau} \tag{38}$$

where we make use of the fact that: $\varepsilon\eta - 1 = \varepsilon\frac{2-\rho}{\varepsilon-1} - 1 = \frac{\varepsilon+1-\varepsilon\rho}{\varepsilon-1}$.

We now have all we need to calculate the change in real income. At initial free trade, equation (26) simplifies to:

$$\hat{Y}\Big|_{\tau=1} = \left(\frac{1}{\xi} - 1\right)\hat{n} + \left(\frac{1}{\xi} - 1\right)(1-\omega)\,\hat{\kappa} - \left[\omega\hat{p} + (1-\omega)\,\hat{p}^*\right] \tag{39}$$

Substituting for the changes in prices from (38) gives:

$$\hat{Y}\Big|_{\tau=1} = \frac{\psi - \xi}{\xi\psi}\hat{n} + (1-\omega)\left(\frac{1-\xi}{\xi}\hat{\kappa} - \hat{\tau}\right) \tag{40}$$

where we use the definition of ψ to simplify the coefficient of $\hat{n}$: since $\psi = \frac{\varepsilon-1}{\varepsilon}$, and so $\varepsilon = \frac{1}{1-\psi}$, it follows that $\frac{1-\xi}{\xi} - \frac{1}{\varepsilon\psi} = \frac{1-\xi}{\xi} - \frac{1-\psi}{\psi} = \frac{\psi-\xi}{\xi\psi}$. This is equation (14) in the text. If desired, we can substitute from equation (11) for the change in firm numbers to calculate $\hat{Y}$ explicitly:

$$\hat{n} = -\psi\hat{y} = -\psi\,(1-\omega)\left(1 - \frac{1}{\varepsilon\eta}\right)(\hat{\kappa} + \hat{\tau}) \tag{41}$$

This gives:

$$\hat{Y}\Big|_{\tau=1} = (1-\omega)\left[\frac{1}{\xi}\left\{1 - \psi + \frac{\psi-\xi}{\varepsilon\eta}\right\}\hat{\kappa} - \left\{1 + \frac{\psi-\xi}{\xi}\left(1 - \frac{1}{\varepsilon\eta}\right)\right\}\hat{\tau}\right] \tag{42}$$

This is harder to interpret, though it shows clearly that efficiency (the sign of $\psi - \xi$) matters for the welfare effects of both shocks, while super- versus subconvexity (the sign of $\varepsilon\eta - 1$) matters for the welfare effects of trade liberalization. In the CES case this reduces to:

$$\hat{Y}\Big|_{\tau=1,CES} = (1-\omega)\left(\frac{1}{\sigma-1}\hat{\kappa} - \hat{\tau}\right) \tag{43}$$

as in Arkolakis et al. (2012).

It can be checked that the coefficient of $\hat{\kappa}$ in (42) is identical to the expression given in Mrázová and Neary (2013) for the effect of globalization on welfare. (To see this, note that if $k = \kappa + 1$ is the total number of countries, then $(1-\omega)\hat{\kappa} = \hat{k}$; also recall that $\psi = \frac{\varepsilon-1}{\varepsilon}$ and $\eta = \frac{2-\rho}{\varepsilon-1}$.) That paper discusses the implications of the expression in detail, and presents a quantitative analysis of the welfare impact of globalization, as a function of ε and ρ, for two widely-used families of demand functions, due to Bulow and Pfleiderer (1983) and Pollak (1971) respectively. It is straightforward to repeat these exercises for the coefficient of $\hat{\tau}$ in (42) which gives the welfare effects of changes in trade costs in the neighborhood of free trade.

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