International Trade in General Oligopolistic Equilibrium

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Introduction

Overview

Goal:
- Integrate imperfect competition and trade
- Combine insights of trade theory and I.O.
- Bring real firms into trade theory

Has all this not been done?
- New trade theory revolution?

Yes, but really two revolutions:
1. Oligopoly in partial equilibrium
   - IIT (cross-hauling), strategic trade policy
2. Monopolistic competition in GE: Firms atomistic
   - IIT (love of variety), MNCs, new economic geography
   - Heterogeneous firms, endogenous organizational form

Unfinished part of the revolution:
- Oligopoly in general equilibrium
- Growing evidence that large firms matter for trade

[Mayer and Ottaviano (2008), Freund and Pierola (2015)]
Why General Equilibrium?
- Interaction between goods and factor markets

Why oligopoly not competition (perfect or monopolistic)?
- More realistic assumptions
  - Infinitely elastic supply of atomistic firms
  - No barriers to entry or exit
  - No strategic behaviour
- New light on central questions in trade theory:
  - Trade patterns; Gains from trade; Trade policy and income distribution
- Adding oligopoly to GE also allows new issues to be addressed:
  - Trade and wages debate: non-price interaction
  - Trade and competition; competitive advantage
  - Effects of trade on market structure: Cross-border mergers
  - Multi-product firms
Oligopoly in GE: Theoretical Challenges

- Problems with modelling oligopoly in general equilibrium:
  - Large firms have monopsony power
  - Large firms can influence GNP
    - Reaction functions badly behaved; equilibrium may not exist
  - Is profit maximization well defined?

- Previous attempts to embed oligopoly in GE:
  - “Perceived” versus “actual” demand curves
  - Imperfect competition in goods and labour markets

- Resolution: Firms large in their own market, but small in the economy
  - A continuum of oligopolistic sectors
  - Firms take factor prices, GNP, and prices in other sectors as given
  - But: They have market power in their own sector
  - Labour market economy-wide and perfectly competitive
Related work:

- Ruffin (2003)
- Eaton, Kortum, and Sotelo (2013)
- Edmond, Midrigan and Xu (2015)
Outline

1. Building Blocks
2. GOLE: Autarky
3. GOLE: Symmetric Free Trade
4. Conclusion
Outline

1 Building Blocks
   - Continuum-Quadratic Preferences
   - Measuring Welfare Change
   - Specialization Patterns in an International Oligopoly
   - Linking Factor and Goods Markets

2 GOLE: Autarky

3 GOLE: Symmetric Free Trade

4 Conclusion
Preferences: Additive Separability

Additive separability:

\[ U[\{x(z)\}] = \int_0^1 u[x(z)]dz, \quad u'[x(z)] > 0, \quad u''[x(z)] < 0 \]

“Frisch” demand functions:

\[ p(z) = \lambda^{-1} u'[x(z)] \]

\( \lambda \): Marginal utility of income

- Taken as given by firms: “Perceived” demand function
- Endogenous in GE: “Actual” demand function

[Browning, Deaton, and Irish (1985)]
Additive Separability plus Aggregation

- Consistent aggregation requires Gorman Polar Form preferences
- Additive separability plus Gorman Polar Form ⇔ “Pollak” preferences

\[ u[x(z)] = ax(z) - \frac{1}{2} bx(z)^2 \]

- Demand functions are linear conditional on \( \lambda \):
  \[ x(z) = \frac{1}{b}[a - \lambda p(z)] \]

- Consistent aggregation over home and foreign representative consumers:
  \[ \bar{x}(z) = x(z) + x^*(z) = \frac{1}{b}[\bar{a} - \bar{\lambda} p(z)] \quad \bar{a} \equiv a + a^*, \quad \bar{\lambda} \equiv \lambda + \lambda^* \]
Building Blocks
Measuring Welfare Change

Measuring Welfare Change

- How to compare welfare in two different equilibria, \( A \) and \( B \)?
  - Two alternative but equivalent methods, corresponding to different normalizations of utility.

1. **Cardinal/quantitative**: Use the expenditure function

   \[ \Delta e^{AB} \equiv e(p^A, u^B) - e(p^A, u^A) \]

   - An equivalent variation type of money-metric welfare change:

     \[ \Delta e^{AB} = (u^B - u^A)(\mu_2^p)^{1/2} \]

     - With CQ preferences:

       \[ \mu_2^p \equiv \int_0^1 p(z)^2 \, dz \]

2. **Ordinal**: Use the “Frisch indirect utility function”

   \[ U = -\lambda^2 \mu_2^p \]

   - CQ utility depends on \( \mu_2^p \) and marginal utility of income:

   - In practice: Choose \( \lambda \) as numéraire; welfare is minus the second moment of prices
Specialization Patterns

- Cournot trade model: partial equilibrium
  - Cournot vs. Bertrand [Neary and Tharakan (2013)]
  - Brander (1981), but with integrated rather than segmented markets.
- Given numbers of firms at home and abroad: \( n, n^* \)
- Perceived inverse demand curve:
  \[ p = a' - b'\bar{x}; \quad a' \equiv \frac{a}{\lambda}, \quad b' \equiv \frac{b}{\lambda}, \quad \bar{x} = \bar{y} = ny + n^*y^* \]
- Firms in each country have identical costs: \( c, c^* \)
- Home sales with no foreign firms:
  \[ y(z) = \frac{a' - c}{b'(n+1)}; \quad y(z) > 0 \Rightarrow c < a' \]
- Home sales with active foreign firms:
  \[ y(z) = \frac{a' - (n^* + 1)c + n*c^*}{b'(n+n^*+1)}; \quad y(z) > 0 \Rightarrow c < \frac{a' + n*c^*}{n^*+1} \]
Specialization Patterns

\[ \pi(c; n^* = 0) = 0 \]

H firms unprofitable
Specialization Patterns

H firms unprofitable when \( n^* > 0 \)

\[ \pi(c, c^*; n^* > 0) = 0 \]
Specialization Patterns

H firms profitable

\[
\frac{d'}{n^* + 1}
\]
Symmetrically:

\[ a' \frac{a'}{n^* + 1} \]

F firms profitable
Specialization Patterns

- **HF**: Home and foreign production
- **F**: Foreign production only
- **O**: No home or foreign production
- **H**: Home production only

Equilibrium Production Patterns for Arbitrary Home and Foreign Costs
Specialization Patterns

- **F**: Foreign production only
- **O**: No home or foreign production
- **H**: Home production only

**Compare Perfect Competition: Cone of Diversification Vanishes**
The Labour Market and Specialization Patterns

- Continuum of sectors, indexed by \( z \in [0, 1] \)
- Ricardian cost structure:
  \[
  c(z) = wa(z); \quad c^*(z) = w^*a^*(z)
  \]
- Assume home more efficient in low-\( z \) sectors
  - Assumption: \( y(z) \) decreasing, \( y^*(z) \) increasing, in \( z \)
  - DFS: \( a(z)/a^*(z) \) increasing in \( z \)
  - Special case: \( a' > 0, a^* < 0 \)
- Specialisation thresholds:
  - Perfect competition: \( \tilde{z} : c(\tilde{z}) = c^*(\tilde{z}) \)
  - Here: 2 threshold sectors, \( \tilde{z} \) and \( \tilde{z}^* \), and 3 possible regimes:
    1. \( z \in [0, \tilde{z}^*] \): Only home firms profitable
    2. \( z \in [\tilde{z}^*, \tilde{z}] \): Both home and foreign firms profitable
    3. \( z \in [\tilde{z}, 1] \): Only foreign firms profitable
  - Incomplete specialization in (2): Barriers to entry allow less efficient firms to survive
- Recall figure
Specialization Patterns

Equilibrium Production Patterns for a Given Cost Distribution

Formally
Illustrative Equilibrium Configurations
General Oligopolistic Equilibrium: Autarky

- Full employment:
  \[ L = \int_0^1 \alpha(z)ny(z)dz \]

- Firm output and price:
  \[ y(z) = \frac{a-\lambda w\alpha(z)}{b(n+1)} \quad \lambda p(z) = \frac{a+\lambda w\alpha(z)}{n+1} \]

- Equilibrium wage:
  \[ w_a \equiv (\lambda w)|_a = [a\mu_1 - \frac{n+1}{n} bL] \left( \frac{1}{\mu_2} \right) \]
  \[ \mu_1 \equiv \int_0^1 \alpha(z)dz \quad \mu_2 \equiv \int_0^1 \alpha(z)^2dz \]

- Welfare:
  \[ U_a \equiv -\left( \lambda^2 \mu_2^p \right) = -\frac{a^2}{(n+1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1-bL)^2}{\mu_2} \]
  \[ \sigma^2 \equiv \mu_2 - \mu_1^2 \]

- Competition Effect: Welfare increasing in \( n \)
- But: Only if sectors differ: \( \sigma^2 > 0 \)

[Lerner (1933)]
Outline

1 Building Blocks

2 GOLE: Autarky

3 GOLE: Symmetric Free Trade
   - Symmetric Free Trade: Wages
   - Gains from Trade
   - Winners and Losers from Trade
   - Volume of Trade

4 Conclusion
Symmetric Free Trade: Wages

- **Full employment:**
  \[
  L = \int_0^1 \alpha(z)ny(z)dz \\
y(z) = \frac{a-(n+1)\lambda w\alpha(z)+n\lambda w\alpha^*(z)}{b(2n+1)}
  \]

- **Equilibrium wage:**
  \[
  w_f \equiv (\lambda w)_f = \left[ a\mu_1 - \frac{n+1}{2n} bL \right] \frac{1}{\mu_2+n\delta}
  \]

- Compare with autarky: \( w_a = \left[ a\mu_1 - \frac{n+1}{n} bL \right] \frac{1}{\mu_2} \)
  
  1. **Market Size Effect:** \( w_f > w_a \)
  2. **Competition Effect:** \( w_f < w_a \)
  3. **Comparative Advantage Effect:** \( w_f < w_a \)

- \( \delta \): International technological dissimilarity; i.e., comparative advantage
  \[
  \delta \equiv \mu_2 - \int_0^1 \alpha(z)\alpha^*(z)dz
  \]
Welfare in free trade:

\[ U_f = -\frac{a^2}{(2n+1)^2} \frac{2\sigma^2-\delta}{2\mu_2-\delta} - (a\mu_1 - bL)^2 \frac{2\mu_2-\delta}{2(\mu_2+n\delta)^2} \]

Compare with autarky: \( U_a = -\frac{a^2}{(n+1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1-bL)^2}{\mu_2} \)

- Zero in a featureless world: \( \sigma^2 = \delta = 0 \) \[ \text{[Lerner (1933)]} \]
- Strictly positive if \( \delta = 0 \) but some technological heterogeneity across sectors: \( \sigma^2 > 0 \) (competition effect)
  - i.e., pro-competitive gains even when no trade, and all sectors identical ex ante and ex post
  - Compare Brander (1981): Here, gains even when markets are integrated

- Increasingly positive the greater is comparative advantage \( \delta \)
  - All this, despite complete symmetry and incomplete specialisation
Recall:
- Market size effect tends to raise wage
- Competition and comparative advantage effects tend to reduce it
- Latter may dominate for large $\delta$

But: Aggregate welfare always rises
Implied: Profits may increase because of comparative advantage
- Contrary to partial equilibrium

Even stronger result: Share of wages in GDP is decreasing in $\delta$

[Anderson-Donsimoni-Gabszewicz (1989)]
Volume of Trade

- Import volumes $m(z)$ are increasing in $n$
- Import shares $m(z)/x(z)$ are increasing in $n$ on average
- So, oligopoly may explain the “missing trade” mystery

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Model: General Oligopolistic Equilibrium [GOLE]

Details:
- Continuum-quadratic preferences
- Cournot + Ricardo, or Brander + Samuelson

Results, in contrast with perfect and monopolistic competition:
- Production patterns more diverse, incomplete specialization
- Gains from trade even if countries identical ex post and ex ante
- Competition effects operate only if sectors heterogeneous
- Profits may rise with free trade
- Volume of trade is lower (missing trade)

Extensions and Applications ...

Broader implications:
- For some questions, oligopoly richer than competition
- (either perfect or monopolistic)
Thank you for listening. Comments welcome!

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Measuring Welfare Change: Details

- $u^B > u^A \iff e(p, u^B) > e(p, u^A)$ for any fixed $p$
- In particular, for $p = p^A$: $u^B > u^A \iff e(p^A, u^B) > e(p^A, u^A)$
- So, money metric measure of welfare change:
  \[ \Delta e^{AB} = e(p^A, u^B) - e(p^A, u^A) \]
- Relate this to equivalent variation: $EV_{AB} = e(p^A, u^B) - e(p^B, u^B)$
  - Subtract $I^B = e(p^B, u^B)$ from both sides: $\iff EV_{AB} > I^A - I^B$
  - EV: the amount that someone who currently has income $I^A$ and prices $p^A$ would be willing to pay in order to avoid a change such that the new price vector is $p^B$ and her income is $I^B$.
  - Dixit and Weller (1979): “basic test for utility increase in going from $A$ to $B$: the gain in consumer’s surplus should exceed any loss in lump-sum income.”
- With Gorman (GPF) preferences, $e(p, u) = f(p) + ug(p)$, this simplifies:
  \[ \Delta e^{AB} = (u^B - u^A)g(p^A) \]
Utility:  \( U \{x(z)\} = \int_0^1 \left[ ax(z) - \frac{1}{2} bx(z)^2 \right] dz \)

Frisch demands:  \( x(z) = \frac{1}{b} [a - \lambda p(z)] \)

Substitute back into  \( U \) to get Frisch indirect utility function:

\[
V^F[\lambda, \{p(z)\}] = \int_0^1 x(z) \left[ a - \frac{1}{2} bx(z) \right] dz \\
= \frac{1}{2b} \int_0^1 [a - \lambda p(z)] [a + \lambda p(z)] dz = \frac{1}{2b} \int_0^1 \left[ a^2 - \lambda^2 p(z)^2 \right] dz \\
\rightarrow V^F = \frac{1}{2b} \left( a^2 - \lambda^2 \mu^p_2 \right) \quad \mu^p_2 \equiv \int_0^1 p(z)^2 dz \\
\rightarrow \tilde{V}^F = -\lambda^2 \mu^p_2 \quad \text{where: } \tilde{V}^F \equiv 2bV^F - a^2
\]
Pollak Preferences

- Gorman Polar Form preferences
  - \( e(p, u) = f(p) + u g(p) \), \( f \) and \( g \) linear homogeneous in \( p \)
  - "Quasi-homothetic": Linear Engel curves from \( f(p) \)
  - \( f(p) \) is the price index of the reference indifference curve; \( g(p) \) is the marginal price index
  - Consistent aggregation

- Additive separability plus Gorman Polar Form \( \Leftrightarrow \) "Pollak" preferences

\[
  u [x(z), z] = \]

Demand functions are "translated CES" conditional on \( \lambda \):

\[
x(z) = \]
CQ Preferences: The Expenditure Function

- Solve for $\lambda$ as a function of $\{p(z)\}$ and $I$:
  - Multiply demand function by $p(z)$, then integrate over all goods:
    \[ \int_0^1 p(z) x(z) \, dz = \frac{1}{b} \int_0^1 \left[ a p(z) - \lambda p(z)^2 \right] \, dz \quad \rightarrow \quad I = \frac{1}{b} \left( a \mu_1^p - \lambda \mu_2^p \right) \]
    \[ \rightarrow \quad \lambda = \frac{a \mu_1^p - bI}{\mu_2^p} \]

- Substitute into Frisch indirect utility function to get Marshallian:
  \[ V[I, \{p(z)\}] = \frac{1}{2b} \left[ a^2 - \left( \frac{a \mu_1^p - bI}{\mu_2^p} \right)^2 \right] \mu_2^p \]

- Rewrite in Gorman Polar Form, $\tilde{V}[\{p(z)\}, I] = \frac{I - f(p)}{g(p)}$:
  \[ \rightarrow \quad \tilde{V}[\{p(z)\}, I] = -\frac{1}{b} \left( a^2 - 2bV \right)^{1/2} = \frac{I - \frac{a}{b} \mu_1^p}{(\mu_2^p)^{1/2}} \]

- Invert to get expenditure function $e[\{p(z)\}, u] = f(p) + u g(p)$:
  \[ e[\{p(z)\}, u] = \frac{a}{b} \mu_1^p + \tilde{u} (\mu_2^p)^{1/2} \quad \text{where:} \quad \tilde{u} = -\frac{1}{b} \left[ (a^2 - 2bu) \right]^{1/2} \]
Detailed Derivations: Autarky Wage

- Full employment:

\[ L = \int_0^1 \alpha(z)ny(z)\,dz \quad y(z) = \frac{a - \lambda w\alpha(z)}{b(n+1)} \]

- Evaluate integral:

\[ L = n \int_0^1 \alpha(z) \frac{a - \lambda w\alpha(z)}{b(n+1)}\,dz = \frac{n}{b(n+1)} \left[ \int_0^1 \alpha(z)a - \lambda w\alpha(z)\,dz \right] \]

- Solve for equilibrium wage:

\[ w_a \equiv (\lambda w)\bigg|_{\substack{a \mu_1 - \frac{n+1}{n} \, bL}} = \left[ a\mu_1 - \frac{n+1}{n} \, bL \right] \frac{1}{\mu_2} \]

\[ \mu_1 \equiv \int_0^1 \alpha(z)\,dz \quad \mu_2 \equiv \int_0^1 \alpha(z)^2\,dz \]
Detailed Derivations: Autarky Welfare

- **Price:**
  \[ \lambda p(z) = \frac{a + \lambda w \alpha(z)}{n + 1} \]

- **Welfare:**
  \[ U_a \equiv -(\lambda^2 \mu_2^p) = -\frac{1}{(n + 1)^2} \left( a^2 + 2an\mu_1w_a + n^2\mu_2w_a^2 \right) \]

  \[ = -\frac{a^2}{(n + 1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1 - bL)^2}{\mu_2} \quad \sigma^2 \equiv \mu_2 - \mu_1^2 \]


