

International Trade in General Oligopolistic Equilibrium

J. Peter Neary
Oxford, CEPR and CESifo

Festschrift Workshop in Honour of David Greenaway
University of Nottingham
June 25, 2015

This research has been supported by the European Research Council.

Overview

- Goal:
 - Integrate imperfect competition and trade
 - Combine insights of trade theory and I.O.
 - Bring real firms into trade theory
- Has all this not been done?
 - New trade theory revolution?
- Yes, but really two revolutions:
 - ① Oligopoly in partial equilibrium
 - IIT (cross-hauling), strategic trade policy
 - ② Monopolistic competition in GE: Firms atomistic
 - IIT (love of variety), MNCs, new economic geography
 - Heterogeneous firms, endogenous organizational form
- Unfinished part of the revolution:
 - Oligopoly in general equilibrium
 - Growing evidence that large firms matter for trade

[Mayer and Ottaviano (2008), Freund and Pierola (2015)]

Why Oligopoly in General Equilibrium?

- Why General Equilibrium?
 - Interaction between goods and factor markets
- Why oligopoly not competition (perfect or monopolistic)?
 - More realistic assumptions
 - Infinitely elastic supply of atomistic firms
 - No barriers to entry or exit
 - No strategic behaviour
 - New light on central questions in trade theory:
 - Trade patterns; Gains from trade; Trade policy and income distribution
 - Adding oligopoly to GE also allows new issues to be addressed:
 - Trade and wages debate: non-price interaction
 - Trade and competition; competitive advantage
 - Effects of trade on market structure: Cross-border mergers
 - Multi-product firms

Oligopoly in GE: Theoretical Challenges

- Problems with modelling oligopoly in general equilibrium:
 - Large firms have monopsony power
 - Large firms can influence GNP
 - Reaction functions badly behaved; equilibrium may not exist
[Roberts and Sonnenschein (1977)]
 - Is profit maximization well defined?
[Gabszewicz and Vial (1972)]
- Previous attempts to embed oligopoly in GE:
 - “Perceived” versus “actual” demand curves
[Negishi (1961)]
 - Imperfect competition in goods and labour markets
[Hart (1982)]
- Resolution: Firms large in their own market, but small in the economy
 - A continuum of oligopolistic sectors
[Samuelson (1964), Dornbusch, Fischer, and Samuelson (1977)]
 - Firms take factor prices, GNP, and prices in other sectors as given
 - But: They have market power in their own sector
 - Labour market economy-wide and perfectly competitive

Related Work

- Related work:
 - Ruffin (2003)
 - Gabaix (2012), di Giovanni and Levchenko (2014)
 - Eaton, Kortum, and Sotelo (2013)
 - Edmond, Midrigan and Xu (2015)

Outline

- 1 Building Blocks
- 2 GOLE: Autarky
- 3 GOLE: Symmetric Free Trade
- 4 Conclusion

Outline

1 Building Blocks

- Continuum-Quadratic Preferences
- Measuring Welfare Change
- Specialization Patterns in an International Oligopoly
- Linking Factor and Goods Markets

2 GOLE: Autarky

3 GOLE: Symmetric Free Trade

4 Conclusion

Preferences: Additive Separability

- Additive separability:

$$U[\{x(z)\}] = \int_0^1 u[x(z)]dz, \quad u'[x(z)] > 0, \quad u''[x(z)] < 0$$

- “Frisch” demand functions: [Browning, Deaton, and Irish (1985)]

$$p(z) = \lambda^{-1} u'[x(z)]$$

- λ : Marginal utility of income
 - Taken as given by firms: “Perceived” demand function
 - Endogenous in GE: “Actual” demand function

Additive Separability plus Aggregation

- Consistent aggregation requires Gorman Polar Form preferences
- Additive separability plus Gorman Polar Form \Leftrightarrow “Pollak” preferences
[Pollak (1971)] [▶ Details](#)
- Tractable special case: Continuum-quadratic preferences

$$u[x(z)] = ax(z) - \frac{1}{2}bx(z)^2$$

- Demand functions are linear conditional on λ :

$$x(z) = \frac{1}{b}[a - \lambda p(z)]$$

- Consistent aggregation over home and foreign representative consumers:

$$\bar{x}(z) \equiv x(z) + x^*(z) = \frac{1}{b}[\bar{a} - \bar{\lambda}p(z)] \quad \bar{a} \equiv a + a^*, \quad \bar{\lambda} \equiv \lambda + \lambda^*$$

Measuring Welfare Change

- How to compare welfare in two different equilibria, A and B ?
 - Two alternative but equivalent methods, corresponding to different normalizations of utility.

1 Cardinal/quantitative: Use the expenditure function

[Dixit and Weller (1979)]

▸ Details

- An equivalent variation type of money-metric welfare change:

$$\Delta e^{AB} \equiv e(p^A, u^B) - e(p^A, u^A)$$

- With CQ preferences:

$$\Delta e^{AB} = (u^B - u^A)(\mu_2^{p^A})^{1/2} \quad \mu_2^p \equiv \int_0^1 p(z)^2 dz$$

2 Ordinal: Use the “Frisch indirect utility function”

[Mrázová and Neary (2014)]

▸ Details

- CQ utility depends on μ_2^p and marginal utility of income:

$$U = -\lambda^2 \mu_2^p$$

- In practice: Choose λ as numéraire; welfare is minus the second moment of prices

Specialization Patterns

- Cournot trade model: partial equilibrium

Cournot vs. Bertrand [Neary and Tharakan (2013)]

- Brander (1981), but with integrated rather than segmented markets.
- Given numbers of firms at home and abroad: n, n^*
- Perceived inverse demand curve:

$$p = a' - b'\bar{x}; \quad a' \equiv \frac{\bar{a}}{\lambda}, \quad b' \equiv \frac{b}{\lambda}, \quad \bar{x} = \bar{y} = ny + n^*y^*$$

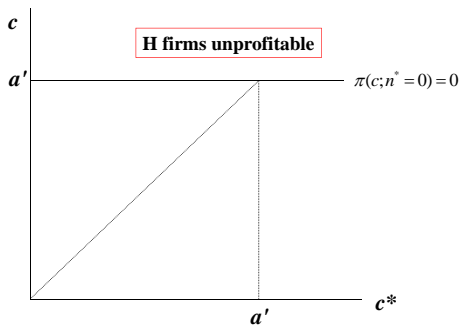
- Firms in each country have identical costs: c, c^*
- Home sales with no foreign firms:

$$y(z) = \frac{a' - c}{b'(n+1)}; \quad y(z) > 0 \Rightarrow c < a'$$

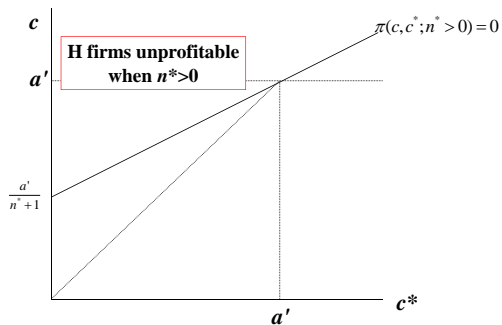
- Home sales with active foreign firms:

$$y(z) = \frac{a' - (n^*+1)c + n^*c^*}{b'(n+n^*+1)}; \quad y(z) > 0 \Rightarrow c < \frac{a' + n^*c^*}{n^*+1}$$

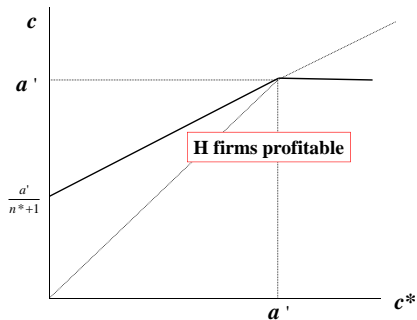
Specialization Patterns



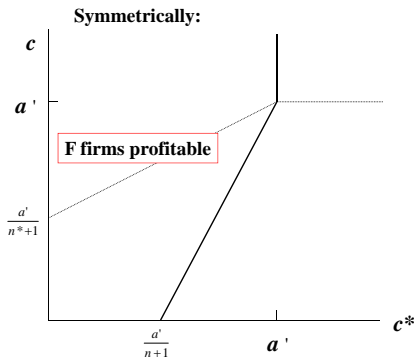
Specialization Patterns



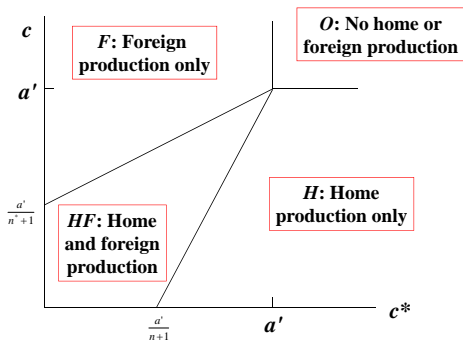
Specialization Patterns



Specialization Patterns

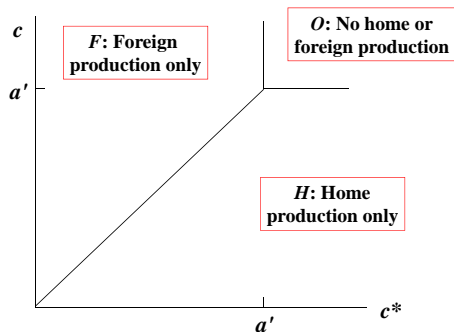


Specialization Patterns



- Equilibrium Production Patterns for Arbitrary Home and Foreign Costs

Specialization Patterns



- Compare Perfect Competition: Cone of Diversification Vanishes

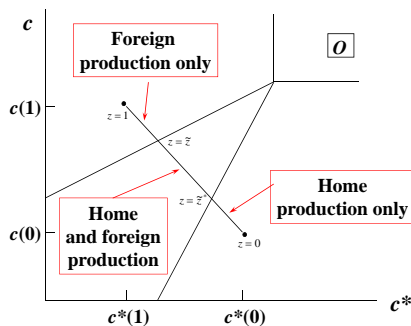
The Labour Market and Specialization Patterns

- Continuum of sectors, indexed by $z \in [0, 1]$
- Ricardian cost structure:

$$c(z) = wa(z); \quad c^*(z) = w^*a^*(z)$$

- Assume home more efficient in low- z sectors
 - Assumption: $y(z)$ decreasing, $y^*(z)$ increasing, in z
 - DFS: $a(z)/a^*(z)$ increasing in z
 - Special case: $a' > 0, a^{*'} < 0$
- Specialisation thresholds:
 - Perfect competition: $\tilde{z} : c(\tilde{z}) = c^*(\tilde{z})$
 - Here: 2 threshold sectors, \tilde{z} and \tilde{z}^* , and 3 possible regimes:
 - 1 $z \in [0, \tilde{z}^*]$: Only home firms profitable
 - 2 $z \in [\tilde{z}^*, \tilde{z}]$: Both home and foreign firms profitable
 - 3 $z \in [\tilde{z}, 1]$: Only foreign firms profitable
 - Incomplete specialization in (2): Barriers to entry allow less efficient firms to survive [▶ Recall figure](#)

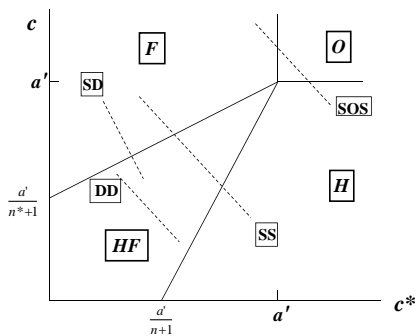
Specialization Patterns



- Equilibrium Production Patterns for a Given Cost Distribution

Formally

Specialization Patterns



- Illustrative Equilibrium Configurations

Outline

- 1 Building Blocks
- 2 GOLE: Autarky**
- 3 GOLE: Symmetric Free Trade
- 4 Conclusion

General Oligopolistic Equilibrium: Autarky

- Full employment:

$$L = \int_0^1 \alpha(z) n y(z) dz$$

- Firm output and price:

$$y(z) = \frac{a - \lambda w \alpha(z)}{b(n+1)} \quad \lambda p(z) = \frac{a + \lambda w \alpha(z)}{n+1}$$

- Equilibrium wage:

► Details

$$w_a \equiv (\lambda w)|_a = \left[a\mu_1 - \frac{n+1}{n} bL \right] \frac{1}{\mu_2}$$

$$\mu_1 \equiv \int_0^1 \alpha(z) dz \quad \mu_2 \equiv \int_0^1 \alpha(z)^2 dz$$

- Welfare:

► Details

$$U_a \equiv -(\lambda^2 \mu_2^p) = -\frac{a^2}{(n+1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1 - bL)^2}{\mu_2} \quad \sigma^2 \equiv \mu_2 - \mu_1^2$$

- Competition Effect: Welfare increasing in n
- But: Only if sectors differ: $\sigma^2 > 0$

[Lerner (1933)]

Outline

1 Building Blocks

2 GOLE: Autarky

3 **GOLE: Symmetric Free Trade**

- Symmetric Free Trade: Wages
- Gains from Trade
- Winners and Losers from Trade
- Volume of Trade

4 Conclusion

Symmetric Free Trade: Wages

- Full employment:

$$L = \int_0^1 \alpha(z) n y(z) dz \quad y(z) = \frac{a - (n+1)\lambda w \alpha(z) + n\lambda w \alpha^*(z)}{b(2n+1)}$$

- **Equilibrium wage:**

$$w_f \equiv (\lambda w)_f \equiv \left[a\mu_1 - \frac{2n+1}{2n} bL \right] \frac{1}{\mu_2 + n\delta}$$

- Compare with autarky: $w_a = \left[a\mu_1 - \frac{n+1}{n} bL \right] \frac{1}{\mu_2}$

① Market Size Effect: $w_f > w_a$

② Competition Effect: $w_f < w_a$

③ Comparative Advantage Effect: $w_f < w_a$

- δ : International technological dissimilarity; i.e., comparative advantage

$$\delta \equiv \mu_2 - \int_0^1 \alpha(z) \alpha^*(z) dz$$

Gains from Trade

- Welfare in free trade:

$$U_f = -\frac{a^2}{(2n+1)^2} \frac{2\sigma^2 - \delta}{2\mu_2 - \delta} - (a\mu_1 - bL)^2 \frac{2\mu_2 - \delta}{2(\mu_2 + n\delta)^2}$$

- Compare with autarky: $U_a = -\frac{a^2}{(n+1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1 - bL)^2}{\mu_2}$
 - Zero in a featureless world: $\sigma^2 = \delta = 0$ [Lerner (1933)]
 - Strictly positive if $\delta = 0$ but some technological heterogeneity across sectors: $\sigma^2 > 0$ (competition effect)
 - i.e., pro-competitive gains even when no trade, and all sectors identical ex ante and ex post
 - Compare Brander (1981): Here, gains even when markets are integrated
 - Increasingly positive the greater is comparative advantage δ
 - All this, despite complete symmetry and incomplete specialisation

Winners and Losers from Trade

- Recall:
 - Market size effect tends to raise wage
 - Competition and comparative advantage effects tend to reduce it
 - Latter may dominate for large δ
- But: Aggregate welfare always rises
- Implication: Profits may increase because of comparative advantage
 - Contrary to partial equilibrium [Anderson-Donsimoni-Gabszewicz (1989)]
- Even stronger result: Share of wages in GDP is decreasing in δ

Volume of Trade

- Volume of Trade
 - Import volumes $m(z)$ are increasing in n
 - Import shares $m(z)/x(z)$ are increasing in n on average
 - So, oligopoly may explain the “missing trade” mystery

[Trefler (1995), Davis and Weinstein (2001), Ruffin (2003)]

Outline

- 1 Building Blocks
- 2 GOLE: Autarky
- 3 GOLE: Symmetric Free Trade
- 4 Conclusion**

Conclusion

- Model: General Oligopolistic Equilibrium [GOLE]
- Details:
 - Continuum-quadratic preferences
 - Cournot + Ricardo, or Brander + Samuelson
- Results, in contrast with perfect and monopolistic competition:
 - Production patterns more diverse, incomplete specialization
 - Gains from trade even if countries identical ex post and ex ante
 - Competition effects operate only if sectors heterogeneous
 - Profits may rise with free trade
 - Volume of trade is lower (missing trade)
- Extensions and Applications ...
- Broader implications:
 - For some questions, oligopoly richer than competition
 - (either perfect or monopolistic)

Thanks and Acknowledgements

Thank you for listening. Comments welcome!

The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013), ERC grant agreement no. 295669. The contents reflect only the authors' views and not the views of the ERC or the European Commission, and the European Union is not liable for any use that may be made of the information contained therein.

Measuring Welfare Change: Details

- $u^B > u^A \Leftrightarrow e(p, u^B) > e(p, u^A)$ for any fixed p [▶ Back to text](#)
- In particular, for $p = p^A$: $u^B > u^A \Leftrightarrow e(p^A, u^B) > e(p^A, u^A)$
- So, money metric measure of welfare change:

$$\Delta e^{AB} = e(p^A, u^B) - e(p^A, u^A)$$
- Relate this to equivalent variation: $EV_{AB} = e(p^A, u^B) - e(p^B, u^B)$
 - Subtract $I^B = e(p^B, u^B)$ from both sides: $\Leftrightarrow EV_{AB} > I^A - I^B$
 - EV: the amount that someone who currently has income I^A and prices p^A would be willing to pay in order to avoid a change such that the new price vector is p^B and her income is I^B .
 - Dixit and Weller (1979): “basic test for utility increase in going from A to B : the gain in consumer’s surplus should exceed any loss in lump-sum income.”
- With Gorman (GPF) preferences, $e(p, u) = f(p) + ug(p)$, this simplifies:

$$\Delta e^{AB} = (u^B - u^A)g(p^A)$$

CQ Preferences: The Frisch Indirect Utility Function

[▶ Back to text](#)

- Utility: $U [\{x(z)\}] = \int_0^1 \left[ax(z) - \frac{1}{2}bx(z)^2 \right] dz$
- Frisch demands: $\rightarrow x(z) = \frac{1}{b} [a - \lambda p(z)]$
- Substitute back into U to get Frisch indirect utility function:

[Mrázová and Neary (2014)]

$$\begin{aligned}
 V^F [\lambda, \{p(z)\}] &= \int_0^1 x(z) \left[a - \frac{1}{2}bx(z) \right] dz \\
 &= \frac{1}{2b} \int_0^1 [a - \lambda p(z)] [a + \lambda p(z)] dz = \frac{1}{2b} \int_0^1 \left[a^2 - \lambda^2 p(z)^2 \right] dz \\
 &\rightarrow V^F = \frac{1}{2b} (a^2 - \lambda^2 \mu_2^p) \quad \mu_2^p \equiv \int_0^1 p(z)^2 dz \\
 &\rightarrow \tilde{V}^F = -\lambda^2 \mu_2^p \quad \text{where:} \quad \tilde{V}^F \equiv 2bV^F - a^2
 \end{aligned}$$

Pollak Preferences

▶ [Back to text](#)

- Gorman Polar Form preferences
 - $e(p, u) = f(p) + ug(p)$, f and g linear homogeneous in p
 - “Quasi-homothetic”: Linear Engel curves from $f(p)$
 - $f(p)$ is the price index of the reference indifference curve; $g(p)$ is the marginal price index
 - Consistent aggregation
- Additive separability plus Gorman Polar Form \Leftrightarrow “Pollak” preferences

$$u[x(z), z] =$$

Demand functions are “translated CES” conditional on λ :

$$x(z) =$$

CQ Preferences: The Expenditure Function

- Solve for λ as a function of $\{p(z)\}$ and I :
 - Multiply demand function by $p(z)$, then integrate over all goods:

[▶ Back to text](#)

$$\int_0^1 p(z) x(z) dz = \frac{1}{b} \int_0^1 [ap(z) - \lambda p(z)^2] dz \rightarrow I = \frac{1}{b} (a\mu_1^p - \lambda\mu_2^p)$$

$$\rightarrow \lambda = \frac{a\mu_1^p - bI}{\mu_2^p}$$

- Substitute into Frisch indirect utility function to get Marshallian:

$$V[I, \{p(z)\}] = \frac{1}{2b} \left[a^2 - \left(\frac{a\mu_1^p - bI}{\mu_2^p} \right)^2 \mu_2^p \right]$$

- Rewrite in Gorman Polar Form, $\tilde{V}[\{p(z)\}, I] = \frac{I - f(p)}{g(p)}$:

$$\rightarrow \tilde{V}[\{p(z)\}, I] \equiv -\frac{1}{b} (a^2 - 2bV)^{1/2} = \frac{I - \frac{a}{b}\mu_1^p}{(\mu_2^p)^{1/2}}$$

- Invert to get expenditure function $e[\{p(z)\}, u] = f(p) + ug(p)$:

$$e[\{p(z)\}, u] = \frac{a}{b}\mu_1^p + \tilde{u}(\mu_2^p)^{1/2} \quad \text{where:} \quad \tilde{u} = -\frac{1}{b} [(a^2 - 2bu)]^{1/2}$$

Detailed Derivations: Autarky Wage

- Full employment:

▶ [Back to text](#)

$$L = \int_0^1 \alpha(z) n y(z) dz \quad y(z) = \frac{a - \lambda w \alpha(z)}{b(n+1)}$$

- Evaluate integral:

$$L = n \int_0^1 \alpha(z) \frac{a - \lambda w \alpha(z)}{b(n+1)} dz = \frac{n}{b(n+1)} \left[\int_0^1 \alpha(z) a - \lambda w \alpha(z) dz \right]$$

- Solve for equilibrium wage:

$$w_a \equiv (\lambda w)|_a = \left[a\mu_1 - \frac{n+1}{n} bL \right] \frac{1}{\mu_2}$$

$$\mu_1 \equiv \int_0^1 \alpha(z) dz \quad \mu_2 \equiv \int_0^1 \alpha(z)^2 dz$$

Detailed Derivations: Autarky Welfare

- Price:

▶ [Back to text](#)

$$\lambda p(z) = \frac{a + \lambda w \alpha(z)}{n + 1}$$

- Welfare:

$$\begin{aligned}
 U_a &\equiv -(\lambda^2 \mu_2^p) = -\frac{1}{(n+1)^2} (a^2 + 2an\mu_1 w_a + n^2 \mu_2 w_a^2) \\
 &= -\frac{a^2}{(n+1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1 - bL)^2}{\mu_2} \quad \sigma^2 \equiv \mu_2 - \mu_1^2
 \end{aligned}$$

References I

- BRANDER, J. A. (1981): "Intra-Industry Trade in Identical Commodities," *Journal of International Economics*, 11(1), 1–14.
- BROWNING, M., A. DEATON, AND M. IRISH (1985): "A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle," *Econometrica*, 53(3), 503–544.
- DIXIT, A. K., AND P. A. WELLER (1979): "The Three Consumer's Surpluses," *Economica*, pp. 125–135.
- DORNBUSCH, R., S. FISCHER, AND P. A. SAMUELSON (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 67(5), 823–839.
- EATON, J., S. KORTUM, AND S. SOTELO (2013): "International Trade: Linking Micro and Macro," in D. Acemoglu, M. Arellano, and E. Dekel (eds.): *Advances in Economics and Econometrics Tenth World Congress*, Volume II: Applied Economics, Cambridge University Press.
- MRÁZOVÁ, M., AND J. P. NEARY (2014): "Together at Last: Trade Costs, Demand Structure, and Welfare," *American Economic Review, Papers and Proceedings*, 104(5), 298–303.
- POLLAK, R. A. (1971): "Additive Utility Functions and Linear Engel Curves," *Review of Economic Studies*, 38(4), 401–414.
- SAMUELSON, P. A. (1964): "Theoretical Notes on Trade Problems," *Review of Economics and Statistics*, 46(2), 145–154.