Not So Demanding: Preference Structure, Firm Behavior and Welfare

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Motivation

Assumptions about Demand matter for Comparative Statics: Skip examples

1. Competition Effects: Does globalization reduce mark-ups?
   - Yes IFF demand elasticity falls with sales
     - Krugman (*JIE* 1979), Zhelobodko-Kokovin-Parenti-Thisse (*Em* 2013)
2. Pass-Through: Do firms raise prices by more than costs increase?
   - Yes IFF demand function is more than log-convex
     - Bulow-Pfleiderer (*JPE* 1983), Weyl-Fabinger (*JPE* 2013)
3. Selection Effects: Do more productive firms export or engage in FDI?
   - FDI IFF demand elasticity and convexity sum to more than three
     - Helpman-Melitz-Yeaple (*AER* 2004), Mrázová-Neary (2011)
4. Price Discrimination: Does it raise welfare? Section 4.1
   - Yes IF demand convexity falls as price rises
     - Schmalensee (*AER* 1981), Aguirre-Cowan-Vickers (*AER* 2010)
5. Welfare: Is monopolistic competition efficient? Section 4.2
   - Yes IFF preferences are CES
     - Dixit-Stiglitz (*AER* 1977), Dhingra-Morrow (2012)

Background

- Assumptions about demand matter for comparative statics: Why?
  - Perfect competition: Supply shocks $\Rightarrow$ movement along demand curve
    - So: Demand elasticity matters for comparative statics
  - Monopolistic competition: Supply shocks $\Rightarrow$ movement along *marginal revenue* curve
    - Elasticity of MR matters; depends on demand elasticity *and* curvature
- Previous approaches:
  - [IO] Weyl-Fabinger (2013): Pass-through
  - [Macro] Kimball (1995): "Superelasticity"
  - [Trade] Mrázová-Neary (2011): Elasticity of marginal revenue
- Proliferation of alternatives to CES:
  - Quadratic preferences: Melitz-Ottaviano (2008)
  - Stone-Geary LES: Simonovska (2010)
  - Translog: Feenstra-Weinstein (2010)
  - CARA: Behrens-Murata (2007)
  - QMOR: Feenstra (2014)
  - Bulow-Pfleiderer: Atkin-Donaldson (2012)

Our Contribution

- Develop a common framework for these and other questions:
  - Illustrate existing results in a simple and compact way
  - Relate functional form to comparative statics
  - Accommodate a wide range of demand behavior
  - Explore implications for monopolistic competition in GE
- Extensions:
  - Variable pass-through
  - Welfare effects of trade liberalization

Outline

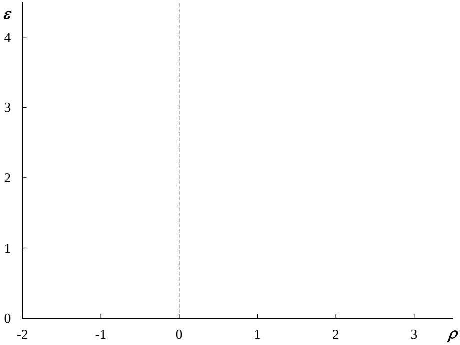
Outline

A Firm's-Eye View of Demand

- Perceived inverse demand function:

$$p = p(x) \qquad p' < 0$$

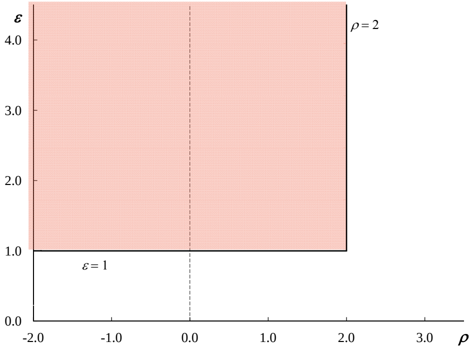
- Two key demand parameters:
  1. Slope/Elasticity:
     $\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)} > 0$
  2. Curvature/Convexity:
     $\rho(x) \equiv -\frac{xp''(x)}{p'(x)}$



Alternative measures of slope and curvature

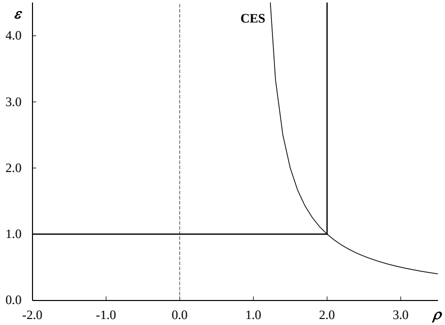
The Admissible Region

- For a monopoly firm:
  - First-order condition:
    $p + xp' = c \geq 0 \quad \Rightarrow \quad \varepsilon \geq 1$
  - Second-order condition:
    $2p' + xp'' < 0 \quad \Rightarrow \quad \rho < 2$



CES Demands

- In general, both ε and ρ vary with sales
- Exception: CES/iso-elastic case:
  - $p = \beta x^{-1/\sigma}$
  - $\Rightarrow \quad \varepsilon = \sigma, \quad \rho = \frac{\sigma+1}{\sigma} > 1$
  - $\Rightarrow \quad \varepsilon = \frac{1}{\rho-1}$



- Cobb-Douglas: $\varepsilon = 1, \rho = 2$; just on boundary of both FOC and SOC

Competition Effects: Superconvexity

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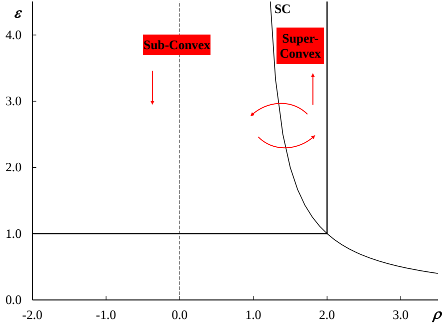
Competition Effects: Superconvexity

- Globalization lowers mark-ups IFF:
  - Demand is subconvex ▸ Details



Super-Pass-Through

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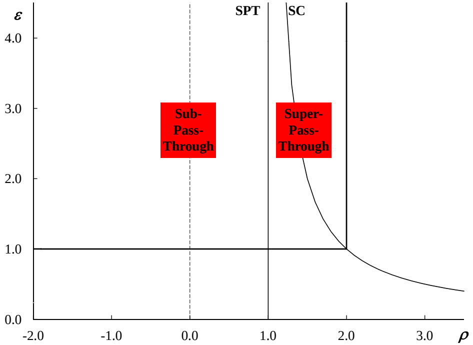
Pass-Through: Log-Convexity

- Globalization lowers mark-ups IFF:
  - Demand is subconvex
- "Super Pass-Through" $\frac{dp}{dc} \geq 1$ IFF:
  - $\rho \geq 1$

Details



Supermodularity

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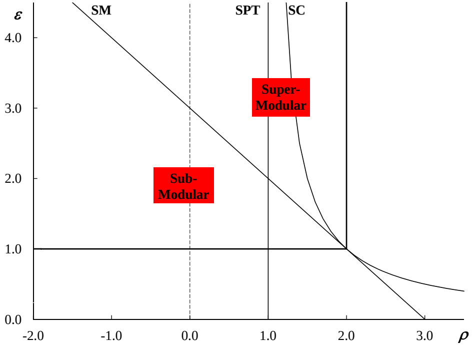
Selection Effects: Supermodularity

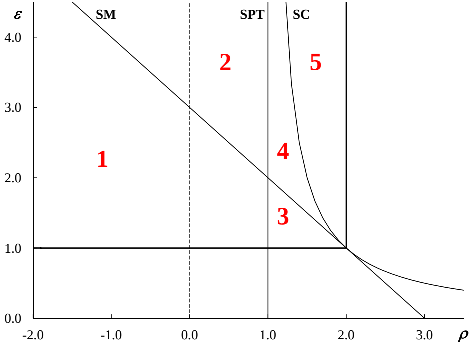
- Globalization lowers mark-ups IFF:
  - Demand is subconvex
- "Super Pass-Through" $\frac{dp}{dc} \geq 1$ IFF:
  - $\rho \geq 1$
- Lower-c firms choose FDI IF:
  - $\varepsilon + \rho > 3$

  Intuition1 Intuition2 Proof



Summary

Region	SC	SPT	SM
1			
2			✓
3		✓	
4		✓	✓
5	✓	✓	✓



- 8 logically possible regions:
    - 3 ruled out by: SC $\Rightarrow$ SPT and SC $\Rightarrow$ SM

Outline

Definition and Existence

Given a demand function $p = p_0(x)$ defined over a range $X(p_0) \subseteq R_1^+$:

$$\Omega(p_0) \equiv \left[\varepsilon, \rho : \; \varepsilon = -\frac{p_0(x)}{x p_0'(x)}, \; \rho = -\frac{x p_0''(x)}{p_0'(x)}, \; \forall x \in X(p_0) \right]$$

Proposition

For every continuous, three-times differentiable, strictly-decreasing demand function, $p_0(x)$, other than the CES, the set $\Omega(p_0)$ corresponds to a smooth curve in $\{\varepsilon, \rho\}$ space.

- Intuition of proof: For any non-CES demand function: ▸ Proof
  - At any point, at least one of $\varepsilon(x)$ and $\rho(x)$ can be inverted and substituted into the other to give:

$$\varepsilon = \bar{\varepsilon}(\rho) \equiv \varepsilon \left[X(\rho) \right] \quad \text{or} \quad \rho = \bar{\rho}(\varepsilon) \equiv \rho \left[X(\varepsilon) \right]$$

  - CES: Manifold is a point

Manifold Invariance

- When is the demand manifold invariant to shocks?
  - Linear: Manifold is invariant to both parameters
  - CES: Manifold is invariant to ϕ, not to σ:

$$x\left(p,\phi\right)=\delta\left(\phi\right)p^{-\sigma} \quad \Leftrightarrow \quad p\left(x,\phi\right)=\beta(\phi)x^{-1/\sigma}$$

- More general conditions for invariance with respect to ϕ:
  - Multiplicatively Separable Direct or Inverse Demands:

Skip

$$x\left(p,\phi\right)=\delta\left(\phi\right)\tilde{x}\left(p\right) \quad \text{or} \quad p\left(x,\phi\right)=\beta(\phi)\tilde{p}(x)$$

  - "Bipower" or "Double CES" Direct or Inverse Demands:

$$x(p,\phi)=\gamma(\phi)p^{-\nu}+\delta(\phi)p^{-\sigma} \quad \text{or} \quad p(x,\phi)=\alpha(\phi)x^{-\eta}+\beta(\phi)x^{-\theta}$$

Demand Functions Separable in a Parameter

Proposition

The Demand Manifold is invariant to shocks in a parameter ϕ if either the direct or inverse demand function is multiplicatively separable in ϕ:

$$\text{(a) } x(p,\phi) = \delta(\phi)\,\tilde{x}(p)\,; \quad \text{or} \quad \text{(b) } p(x,\phi) = \beta(\phi)\tilde{p}(x)$$

Why? In both cases, ε and ρ are invariant to ϕ. Proof

- Corollary 1: With *either* direct or indirect additivity, the Demand Manifold is invariant to changes in the price index.
- Corollary 2: The Demand Manifold is invariant to neutral changes in market size: $x(p,s) = s\tilde{x}(p)$. Example: Logistic Demand
- Corollary 3: The Demand Manifold is invariant to neutral changes in quality: $p(x,\phi) = \phi\tilde{p}(x)$.
  - Baldwin-Harrigan (2011): "Box-size quality"

Bipower Direct Demands

Proposition

The Demand Manifold is such that ρ is linear in the inverse and squared inverse of ε if and only if the direct demand function takes a bipower form:

$$x(p) = \gamma p^{-\nu} + \delta p^{-\sigma} \quad \Leftrightarrow \quad \bar{\rho}(\varepsilon) = \frac{\nu + \sigma + 1}{\varepsilon} - \frac{\nu\sigma}{\varepsilon^2}$$

Proof

- Manifold is invariant with respect to γ and δ

Proposition

Bipower direct demand functions are superconvex if and only if $\gamma\delta > 0$

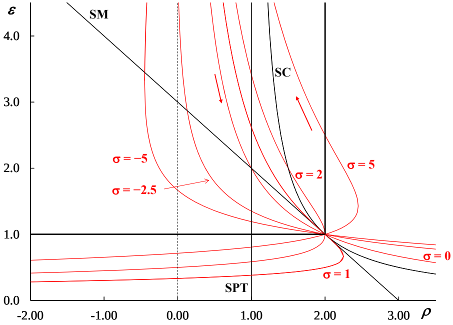
- Special cases:
  - Diewert-Feenstra "Quadratic Mean of Order r" family Details
  - Muellbauer (1975) "PIGL" family: $\nu = 1$
  - Pollak (1971) family: $\nu = 0$ Details

PIGL and Translog

- "PIGL": $\nu = 1$:

$$x(p) = \frac{1}{p}\left(\gamma + \delta p^{1-\sigma}\right)$$

$$\bar{\rho}(\varepsilon) = \frac{(\sigma+2)\varepsilon - \sigma}{\varepsilon^2}$$



PIGL and Translog

- "PIGL": $\nu = 1$:

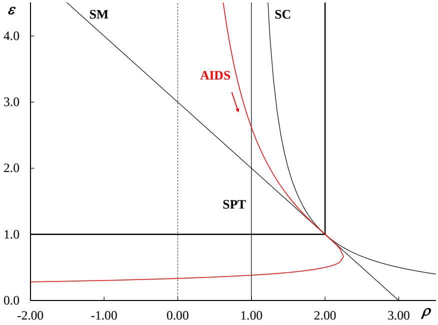
$$x(p) = \frac{1}{p}\left(\gamma + \delta p^{1-\sigma}\right)$$

$$\bar{\rho}(\varepsilon) = \frac{(\sigma+2)\varepsilon - \sigma}{\varepsilon^2}$$

  - Translog: $\nu = 1$, $\sigma \to 1$:

$$x(p) = \frac{1}{p}\left(\gamma' + \delta' \log p\right) \quad \bar{\rho}(\varepsilon) = \frac{3\varepsilon - 1}{\varepsilon^2}$$

  - Includes both:
    - AIDS: Not homothetic [Deaton-Muellbauer]
    - Homothetic Translog [Feenstra (2003)]
  - Uniquely, both subconvex *and* supermodular



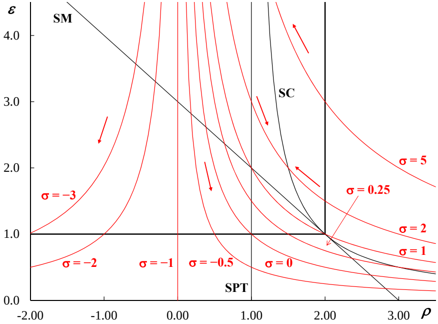
Details

The Pollak Family

$\nu = 0$:

$$x(p) = \gamma + \delta p^{-\sigma}, \ \bar{\varepsilon}(\rho) = \frac{\sigma + 1}{\rho}$$

- $\sigma = -1$: Linear
- $\sigma \to 0$: CARA
- $\sigma = 1$: Stone-Geary LES



Details
Compare Bulow-Pfleiderer
Compare Kimball Superelasticity
Skip to GE section

Bipower Inverse Demands

Proposition

The Demand Manifold is linear in ε and ρ if and only if the inverse demand function takes a bipower form:

$$p(x) = \alpha x^{-\eta} + \beta x^{-\theta} \quad \Leftrightarrow \quad \bar{\rho}(\varepsilon) = \eta + \theta + 1 - \eta\theta\varepsilon$$

▸ Proof

- Manifold is invariant with respect to α and β; however:

Proposition

Bipower inverse demand functions are superconvex if and only if $\alpha\beta > 0$

- Special cases:
  - "Inverse PIGL" family: $\eta = 1$
    - Inverse Translog: $\eta = 1$, $\theta \to 1$
  - Bulow-Pfleiderer (1983) family: $\eta = 0$

▸ Skip

The Inverse PIGL Family

- "PIGL": "Price-Independent Generalized Linearity"

  [Muellbauer (1975)]

$$\eta = 1 \quad \Rightarrow \quad p(x) = \frac{1}{x}\left(\alpha + \beta x^{1-\theta}\right)$$

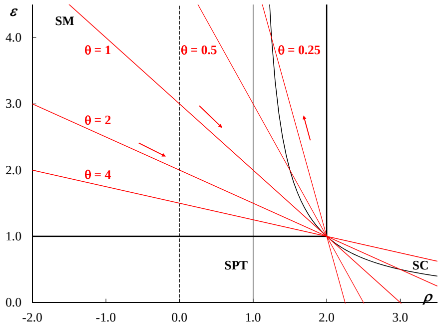
$$\bar{\varepsilon}(\rho) = \frac{\theta + 2 - \rho}{\theta}$$

- θ: Elasticity of marginal revenue

- Inverse "PIGLOG"/Translog:

$$\theta \to 1 \quad \Rightarrow \quad p(x) = \frac{1}{x}\left(\alpha' + \beta' \log x\right)$$

- Coincides with the supermodularity locus: $\varepsilon + \rho = 3$



The Bulow-Pfleiderer or Iso-Convex Family

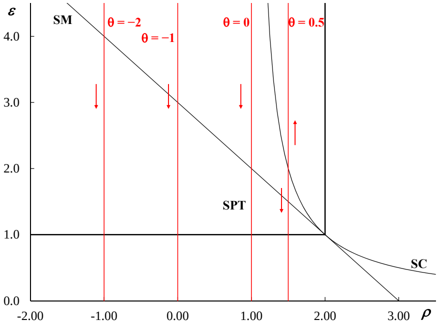
$\eta = 0$:

$$p(x) = \alpha + \beta x^{-\theta}, \; \rho = \theta + 1$$

- Constant pass-through: $\frac{1}{1-\theta}$
- Special cases:
  - $\theta = -1$: linear demand
  - $\theta \to 0$: log-linear demand

$$\log x(p) = \gamma + \delta p$$

  - $\alpha = 0$, $\theta > 0$: iso-elastic



Details
Compare Pollak Family
Skip to GE section

Demand Functions that are Not Manifold-Invariant

Demand Functions whose manifolds can be written in closed form, though they depend on all the parameters, and so are not manifold invariant:

1. The "Doubly-Translated CES" super-family: Details

$$p(x) = \alpha + \beta \left(x - \gamma\right)^{-\theta}$$

   - Nests both the Pollak and iso-convex families

2. The "Translated Bipower-Inverse" super-family

$$p(x) = \alpha_0 + \alpha x^{-\eta} + \beta x^{-\theta}$$

   - Nests the Fabinger-Weyl (2012) "Apt" (Adjustable pass-through) system and a new family: the "inverse iso-temperance" system.

3. Exponential Inverse Demand Details

Outline

Equilibrium in the Global Economy

[Krugman (1979), Neary (2009), ZKPT (2013), Bertoletti-Epifani (2014)]

Recall Introduction

- k identical countries; n homogeneous firms in each; $N = kn$
- Symmetric preferences: $x_i = x$
- 5 variables: $p, w; x, y; n;$ set $w = 1$ by choice of numeraire;
- Free trade [Extension to trade costs: Mrázová-Neary (2014)]
- 4 equations:

Profit Maximization (MR=MC): $$p = \frac{\varepsilon(x)}{\varepsilon(x) - 1} c$$

Free Entry (AR=AC): $$p = \frac{f}{y} + c$$

Goods-Market Equilibrium (GME): $$y = kLx$$

Labour-Market Equilibrium (LME): $$L = n(f + cy)$$

Effects of Globalization

$$\hat{k} > 0 \qquad [\hat{x} \equiv d \log x, \; x \neq 0]$$

$$\text{MR=MC:} \;\; \hat{p} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(\varepsilon - 1)}\hat{x}$$

- Slope of MR=MC:
  - $\frac{\hat{p}}{\hat{x}} > 0$ IFF $\rho < \frac{\varepsilon+1}{\varepsilon}$
  - i.e., IFF demand is subconvex

$$\text{AR=AC:} \;\; \hat{p} = -(1-\omega)\hat{y}$$

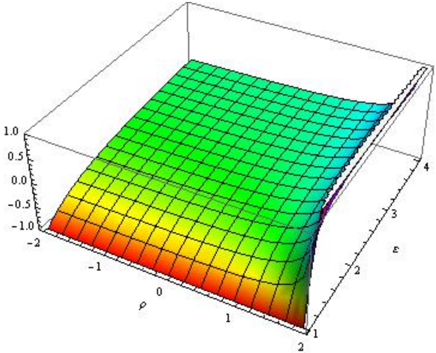
$$\text{GME:} \;\; \hat{y} = \hat{k} + \hat{x}$$

$$\text{LME:} \;\; 0 = \hat{n} + \omega\hat{y}$$

- $\omega \equiv \frac{cy}{f+cy} = \frac{c}{p}$
  - Share of variable in total costs
  - Inverse measure of returns to scale
  - In equilibrium: $\omega = \frac{\varepsilon-1}{\varepsilon}$
  - Comparative statics depend *only* on ε and ρ

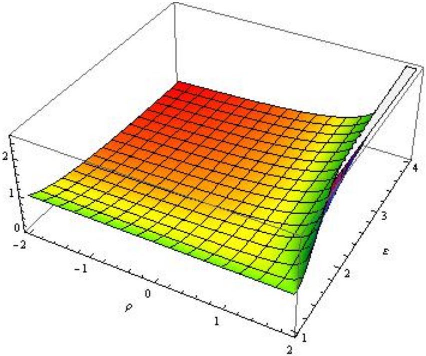
Details

Changes in Price and Number of Varieties



$$\hat{p} = -\frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon^2 (2 - \rho)} \hat{k}$$

- Negative IFF $\rho < 1 + \frac{1}{\varepsilon}$ ‣ Hetero
- Always increasing in ρ
- Increasing in ε IFF $\rho < 1 + \frac{2}{\varepsilon}$



$$\hat{N} = \frac{(\varepsilon - 1)^2 + (2 - \rho)\,\varepsilon}{\varepsilon^2 (2 - \rho)} \hat{k}$$

- Always positive ‣ BP ‣ Pollak
- Always increasing in ρ ‣ NPOLLAK
- Decreasing in ε IFF $\rho < \frac{2}{\varepsilon}$

Heterogeneous Firms

Maximum operating profits:

$$\pi(\underset{-}{c}, \underset{-}{\lambda}, \underset{+}{k})$$

- c: Firm marginal cost: Distributed as $g(c)$, with support $[\underline{c}, \overline{c}]$
- λ: Market aggregate, endogenous to industry, exogenous to firms
  - Interpretation: "competition"
  - Additive separability: Marginal utility of income

Free-entry condition:

$$c_0: \quad \pi(c_0, \lambda, k) = f$$

Zero-expected-profit condition:

$$\bar{\pi}(\lambda, k) \equiv \int_{\underline{c}}^{\overline{c}} \pi(c, \lambda, k) g(c) \, \mathrm{d}c = f + f_e$$

Globalization, Competition and Profits

- Determination of firm profits:

$$\hat{\pi} = \frac{\lambda \pi_\lambda}{\pi} \hat{\lambda} + \frac{k \pi_k}{\pi} \hat{k}$$

- Determination of the level of competition:

$$\bar{\pi}(\lambda, k) = f + f_e \quad \Rightarrow \quad \hat{\lambda} = -\frac{\bar{\pi}}{\lambda \bar{\pi}_\lambda} \frac{k \bar{\pi}_k}{\bar{\pi}} \hat{k}$$

- Combine:

$$\hat{\pi} = \Bigg(\underbrace{\frac{k \pi_k}{\pi}}_{(1)} - \underbrace{\frac{\lambda \pi_\lambda}{\pi} \frac{\bar{\pi}}{\lambda \bar{\pi}_\lambda} \frac{k \bar{\pi}_k}{\bar{\pi}}}_{(2)} \Bigg) \hat{k}$$

1. Direct effect: Tends to *raise* each firm's profits
2. Indirect effect: Raises *all* firms' profits $\Rightarrow$ Increases competition $\Rightarrow$ Tends to *reduce* each firm's profits

Globalization and the Extensive Margin

- With additive separability: ▸ Details

$$\hat{\pi} = \left(1 - \frac{\varepsilon}{\bar{\varepsilon}}\right)\hat{k} \qquad \hat{c}_0 = \frac{1}{\varepsilon_0 - 1}\left(1 - \frac{\varepsilon_0}{\bar{\varepsilon}}\right)\hat{k}$$

  - $\varepsilon_0 \equiv \varepsilon(c_0)$: Elasticity faced by the marginal firm
  - $\bar{\varepsilon} \equiv \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \varepsilon(c) g(c) \, \mathrm{d}c$: Elasticity faced by the average firm

1. CES case: $\varepsilon_0 = \bar{\varepsilon} = \sigma \quad \Rightarrow \quad$ No change in threshold firm
2. Subconvex case: Marginal firm has lowest x and so highest ε
   $\Rightarrow \quad$ Competition effect dominates: least efficient firms exit
3. Superconvex case: $\varepsilon_0 < \bar{\varepsilon} \quad \Rightarrow \quad$ Threshold firm *more* profitable

Globalization and the Intensive Margin

- Firm output: $y(c, \lambda, k)$ ▸ Details

$$\hat{y} = \left[\underbrace{\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(2-\rho)}}_{*} + \frac{\bar{\varepsilon}-\varepsilon}{\bar{\varepsilon}}\frac{\varepsilon-1}{\varepsilon(2-\rho)}\right]\hat{k}$$

- Prices:

$$\hat{p} = -\frac{\varepsilon}{\bar{\varepsilon}}\underbrace{\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon^2(2-\rho)}}_{*}\hat{k}$$

- Same as in homogeneous-firms case ($*$), adjusted for $\varepsilon \neq \bar{\varepsilon}$ ▸ Recall
  - Subconvexity: Outputs $\uparrow$ more, prices $\downarrow$ less in larger firms

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Variable Pass-Through

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Globalization and Welfare

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5. Welfare: Is monopolistic competition efficient?
   - Yes IFF preferences are CES
     - Dixit-Stiglitz (*AER* 1977), Dhingra-Morrow (2012)

Additive Separability and the Utility Manifold

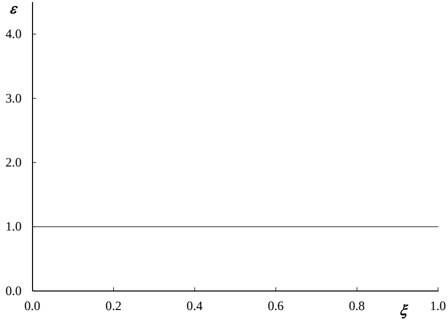
- $U = F\left[\int_{i\in\Omega} u\{x(i)\}di\right], \;\; F' > 0,\, u' > 0,\, u'' < 0$
- With symmetry and no trade costs: $U = F\left[Nu(x)\right]$
  1. Extensive margin: $N = kn$: $\begin{cases} k\text{: International (Exogenous)} \\ n\text{: Intranational (Endogenous)} \end{cases}$
  2. Intensive margin: x
- Change in Real Income: Details

$$\hat{Y} = \frac{1-\xi}{\xi}\hat{N} - \hat{p}$$

- Elasticity of Utility: $\xi \equiv \frac{xu'}{u}$ Skip to BF
  - Taste for variety requires $0 < \xi < 1$
  - ξ an *inverse* measure of taste for variety

Superconcavity of Utility

- Admissible region:
  - $0 < \xi < 1$: Elasticity of $u(x)$
  - $\varepsilon^{-1} < 1$: Concavity of $u(x)$
- CES benchmark:
  - $\varepsilon = \sigma$, $\xi = \frac{\sigma-1}{\sigma}$
  - $\Rightarrow \quad \varepsilon = \frac{1}{1-\xi}$
- $u(x)$ is *superconcave* IFF:
  - $\log u(x)$ is concave in $\log x$
  - $u(x)$ more concave than CES
  - $\varepsilon < \frac{1}{1-\xi} \quad \Leftrightarrow \quad \xi > \frac{\varepsilon-1}{\varepsilon}$



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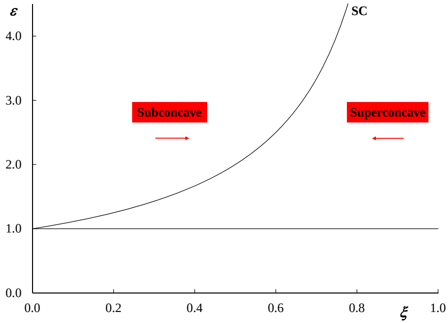
  - $\varepsilon < \frac{1}{1-\xi} \quad \Leftrightarrow \quad \xi > \frac{\varepsilon-1}{\varepsilon}$
  - ξ is decreasing in consumption



Superconcavity and Efficiency

- Change in Real Income in GE:

$$\hat{Y} = \frac{1-\xi}{\xi}\hat{k} + \frac{\omega-\xi}{\xi(1-\omega)}\hat{p}$$

- Gains equal $\frac{1-\xi}{\xi}$ when $\omega = \xi$; i.e., equilibrium is *efficient*:
  - Preferences are CES: $\frac{1-\xi}{\xi} = \frac{1}{\sigma-1}$ and $\omega = \xi = \frac{\sigma-1}{\sigma}$; *or:*
  - Optimal competition policy chooses the welfare-maximizing levels of n (and hence p) for given k
- $\xi < \omega$ [Varieties *under*-supplied] $\Leftrightarrow$ $\xi < \frac{\varepsilon-1}{\varepsilon}$ [Utility subconcave]

Globalization and Welfare

$$\hat{Y} \quad = \quad \frac{1-\xi}{\xi}\hat{N} - \hat{p} \quad = \quad \frac{1-\xi}{\xi}\hat{k} + \frac{\omega-\xi}{\xi(1-\omega)}\hat{p}$$

- Gains from globalization:
  1. Direct: Exceeds CES benchmark $\frac{1}{\varepsilon-1}$ IFF utility is subconcave
  2. Indirect: Positive IFF efficiency increases
     - Higher efficiency sufficient, *not* necessary, for positive gains
- Alternative sufficient conditions for positive gains:
  1. Demand *subconvex* ($\frac{\varepsilon+1}{\varepsilon} > \rho$):
     - Prices fall, so consumers reap a double dividend: welfare rises at both intensive and extensive margins
     - Curvature of utility is irrelevant for sign of welfare change.
  2. Utility *subconcave* ($\xi < \frac{\varepsilon-1}{\varepsilon} \;\Leftrightarrow\; \xi < \omega$):
     - Varieties are under-supplied;
     - Even if prices rise, the gain at the extensive margin offsets the losses at the intensive margin.

Globalization and Welfare

- Necessary and sufficient condition: Substitute for $\omega = \frac{\varepsilon - 1}{\varepsilon}$ and $\hat{p}$:

$$\hat{Y} = \frac{1}{\xi\varepsilon}\left[1 - \left(\xi - \frac{\varepsilon - 1}{\varepsilon}\right)\frac{\varepsilon - 1}{2 - \rho}\right]\hat{k}$$

- Now: Three sufficient statistics for change in welfare
- Gains from globalization always decreasing in ξ: greater, the greater the taste for diversity
- In general:
  - $\hat{Y}/\hat{k} = G(\xi, \varepsilon, \rho)$
  - $\varepsilon = \bar{\varepsilon}(\rho, \phi)$ Demand Manifold
  - $\xi = \bar{\xi}(\varepsilon, \phi)$ Utility Manifold
    - 3 equations in $4 + m$ unknowns, where m is the dimension of ϕ

Bipower Inverse Utility

- Demands:

$$p(x,\phi) = \alpha x^{-\eta} + \beta x^{-\theta} \quad \Leftrightarrow \quad \rho = \eta + \theta + 1 - \eta\theta\varepsilon$$

- Utility function:

$$u(x) = \tfrac{1}{1-\eta}\alpha x^{1-\eta} + \tfrac{1}{1-\theta}\beta x^{1-\theta}, \qquad u(0) = 0$$

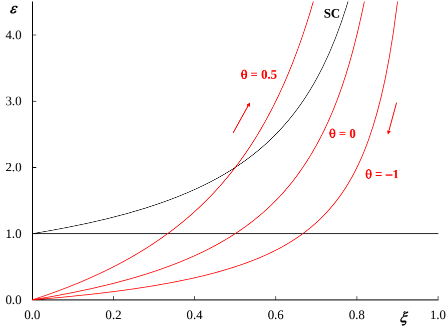
$$\rightarrow \;\; \xi = \tfrac{(1-\eta)(1-\theta)\varepsilon}{(1-\eta-\theta)\varepsilon+1}$$

- Demand subconvex if and only if $(\eta\varepsilon - 1)(\theta\varepsilon - 1) \geq 0$;
- Utility subconcave if and only if $\frac{(\eta\varepsilon-1)(\theta\varepsilon-1)}{(1-\eta-\theta)\varepsilon+1} \leq 0$
- Corollary: Gains from trade guaranteed if $(1 - \eta - \theta)\varepsilon + 1 > 0$
- Bulow-Pfleiderer: $\eta = 0$, $\rho = \theta + 1 \quad \rightarrow \quad \xi = \frac{(1-\theta)\varepsilon}{(1-\theta)\varepsilon+1} = \frac{(2-\rho)\varepsilon}{(2-\rho)\varepsilon+1}$

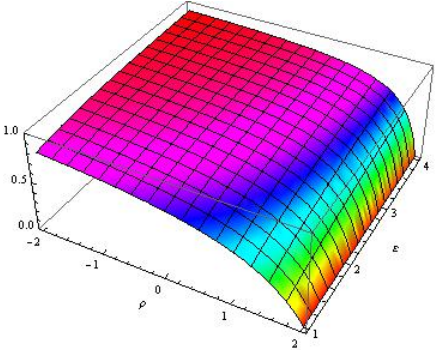
Globalization and Welfare: Bulow-Pfleiderer Utility

$$u(x) = \alpha x + \frac{1}{1-\theta}\beta x^{1-\theta}$$

- To the right of SC locus:
    - Utility is superconcave
    - Demand is subconvex
    - Equilibrium moves rightward
    - Efficiency rises ...
    - ... but so does ξ

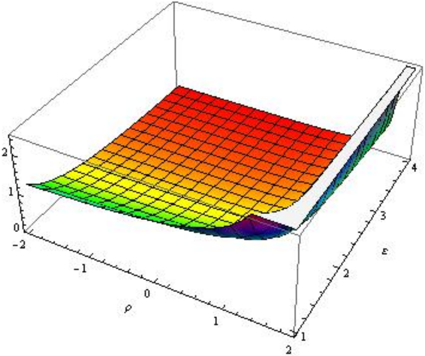


Globalization and Welfare: Bulow-Pfleiderer Utility



$$\xi = \frac{(2-\rho)\,\varepsilon}{(2-\rho)\,\varepsilon + 1}$$

- Always between zero and one
- Increasing in ε
- Decreasing in ρ: greater taste for diversity at high ρ



$$\hat{Y} = \frac{1}{\xi\varepsilon}\left[1 - \left(\xi - \frac{\varepsilon-1}{\varepsilon}\right)\frac{\varepsilon-1}{2-\rho}\right]\hat{k}$$

- Always positive
- Decreasing in ε Recall $\hat{p}$ and $\hat{N}$
- Increasing in ρ Pollak NPOLLAK

Globalization and Welfare: Pollak Utility

$$x(p) = \gamma + \delta p^{-\sigma} \quad \Leftrightarrow \quad \bar{\varepsilon}(\rho) = \frac{\sigma+1}{\rho}$$

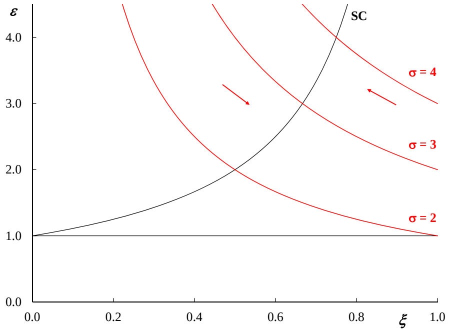
- Inverse demand: $p(x) = \beta(\sigma x + \zeta)^{-\frac{1}{\sigma}}$, $\zeta \equiv -\gamma\sigma$ and $\beta \equiv (\delta/\sigma)^{1/\sigma}$
- Utility function: $u(x) = \frac{\beta}{\sigma-1}(\sigma x + \zeta)^{\frac{\sigma-1}{\sigma}}$

$$\rightarrow \ \xi = \frac{\sigma-1}{\sigma}\frac{x}{x-\gamma} = \frac{\sigma-1}{\varepsilon} = \frac{\varepsilon\rho-2}{\varepsilon}$$

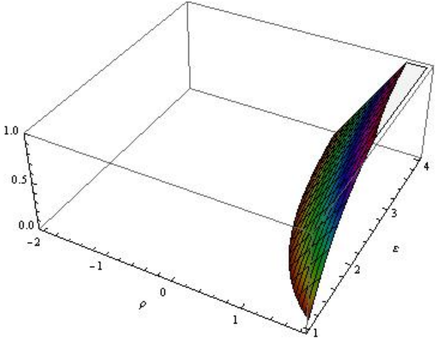
- Confine attention to: $0 < \xi < 1$ (nests CES):
  - $\xi > 0 \quad \Leftrightarrow \quad \rho > \frac{2}{\varepsilon}$ (i.e., more convex than LES)
  - $\xi < 1 \quad \Leftrightarrow \quad \rho < 1 + \frac{2}{\varepsilon}$

Globalization and Welfare: Pollak Utility

- To the right of SC locus:
  - Utility is superconcave
  - So demand is superconvex
  - Equilibrium moves rightward
  - Efficiency falls ...
  - ... and so may welfare

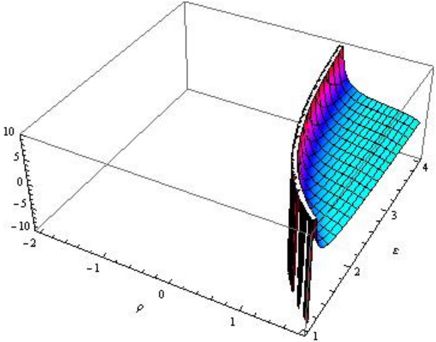


Globalization and Welfare: Pollak Utility



$$\xi = \frac{\varepsilon\rho - 2}{\varepsilon}$$

- Increasing in ε
- *Increasing* in ρ: lower taste for diversity at high ρ



$$\hat{Y} = \frac{1}{\xi}\left[1 - \frac{(\varepsilon - 1)^2}{\varepsilon^2 (2 - \rho)}\right]\hat{k}$$

- Decreasing in ε *and* ρ
- *Negative* for high ε and ρ

Recall $\hat{p}$ and $\hat{N}$ | BP | NP1 | NP2

Outline

Conclusion

- A "firms'-eye view" of demand:
  - Focus on elasticity ε and convexity ρ
- "Demand Manifold" $\varepsilon = \bar{\varepsilon}(\rho)$ or $\rho = \bar{\rho}(\varepsilon)$:
  - A parsimonious representation of demand functions ...
  - ...and a sufficient statistic for many comparative statics results
- Allows easy calibration of monopolistically competitive trade models
  - "Average" firm behaves as in homogeneous-firm model
- Extensions:
  - Variable pass-through: "Demand-Slope Manifold"
  - Gains from trade: "Utility Manifold"

Thanks and Acknowledgements*

Thank you for listening. Comments welcome!

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Globalization and Welfare: Summary

- For any inverse demand function:
  - Diversity always rises
  - Prices rise if and only if demands are superconvex
- With iso-convex demands:
  - Welfare always rises
  - ... and by *more* the more convex are demands
- With Pollak demands:
  - Welfare rises by *less* the more convex are demands
  - ... and *falls* if preferences are sufficiently superconvex
    - Diversity rises a lot, but price increase offsets this
- Conclusion: We need to know ε, ρ, *and* ξ to predict welfare change
  - Note: Free trade, so in all cases budget share of home goods is $\frac{1}{k}$
- Compare Arkolakis et al. (2012 mimeo):
  - ε and budget share of home goods are sufficient statistics for welfare change in CES case
  - Gains from trade are smaller but still positive with subconvex demands

Alternative Measures of Slope and Curvature

Back to text

Alternative measure of slope:

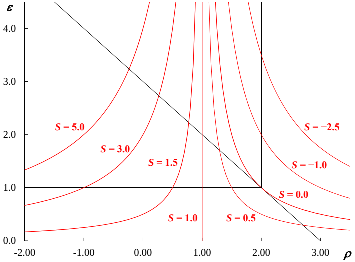
- Inverse elasticity: $e = \varepsilon^{-1} = -\frac{xp'}{p}$
  - Elasticity of marginal utility: $e = -\frac{d \log u'(x)}{d \log x}$
  - "Relative love for variety": Zhelobodko et al. (2012)
  - (In equilibrium) Profit margin/Lerner Index: $e = \frac{p-c}{p}$

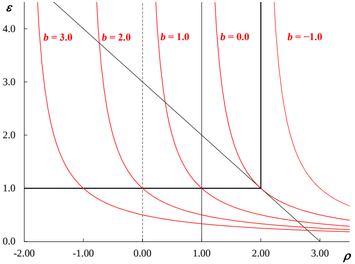
Alternative measures of curvature:

- "Convexity" = Elasticity of inverse demand slope: $\rho(x) = -\frac{d \log p'(x)}{d \log x}$
  - Seade (1985), Bulow-Geanakoplos-Klemperer (1985), Shapiro (1989)
- Convexity of direct demand $x(p)$: $r(p) \equiv -\frac{px''(p)}{x'(p)}$
  - e and r dual to ε and ρ:
    - $e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon} \qquad r \equiv -\frac{px''}{x'} = \frac{pp''}{(p')^2} = \varepsilon\rho$
    - $\varepsilon \equiv -\frac{p}{xp'} = \frac{1}{e} \qquad \rho \equiv -\frac{xp''}{p'} = \frac{xx''}{(x')^2} = er$
- Kimball (1995) "Superelasticity":

$$\frac{d \log \varepsilon}{d \log p} = \frac{d \log \varepsilon}{d \log x}\frac{d \log x}{d \log p} = \left(\frac{x\varepsilon_x}{\varepsilon}\right)(-\varepsilon) = \varepsilon + 1 - \varepsilon\rho$$

Kimball Superelasticity





$$S \equiv \frac{d \log \varepsilon}{d \log p} = \varepsilon + 1 - \varepsilon\rho$$

Klenow-Willis (2006): $S = b\varepsilon$

$$\Rightarrow \quad \bar{\rho}\,(\varepsilon) = \frac{(1-b)\varepsilon + 1}{\varepsilon}$$

Back to Firm's-Eye View   Back to Pollak Family

Oligopoly

- Now: need to distinguish market demand X from firm sales x_i:

  (Assume homogeneous goods)

  - $p(X), \quad \varepsilon \equiv -\frac{p}{Xp'}, \quad \rho \equiv -\frac{Xp''}{p'}$
- Admissible region expands: (Assume Cournot competition)
  - First-order condition: $p + x_i p' = c_i \geq 0 \quad \Rightarrow \quad \varepsilon \geq \max_i(\omega_i) \leq 1$

    $(\omega_i \equiv \frac{x_i}{X})$
  - Second-order condition: $2p' + x_i p'' < 0 \quad \Rightarrow \quad \rho < 2\min_i(\frac{1}{\omega_i}) \geq 2$
- Many comparative statics results hinge on strategic substitutability:
  - $\Leftrightarrow p' + x_i p'' < 0 \quad \Rightarrow \quad \rho < \min_i(\frac{1}{\omega_i}) \geq 1$
- Translated iso-elastic demand functions can be illustrated in diagram:
  - $\max_{x_i} \left[p(X) - c_i\right] x_i, \quad p(X) = \beta\left(x_i + X_{-i} - \gamma\right)^{-(1-\theta)}$

Back to Text

Competition Effects and Superconvexity: Details

[Mrázová-Neary (2011)]

Back to Text

- $p(x)$ is superconvex at x^0 IFF:
  - $\log p(x)$ is convex in $\log x$
  - $p(x)$ is more convex than a CES demand function with the same elasticity: $\rho > \frac{\varepsilon+1}{\varepsilon}$
  - ε is increasing in sales:
    - $\varepsilon_x = \frac{\varepsilon}{x}\left[\rho - \frac{\varepsilon+1}{\varepsilon}\right]$

Example

- "Globalization" $[x \downarrow]$ lowers mark-ups $\left[\frac{p}{c} \downarrow\right]$ IFF demands are subconvex
  - Because the mark-up is decreasing in elasticity: $\frac{p}{c} = \frac{\varepsilon}{\varepsilon-1} = 1 + \frac{1}{\varepsilon-1}$

Proportional and Absolute Pass-Through: Details

Back

- Proportional Pass-Through:
  - $p + xp' = c \quad \Rightarrow \frac{p}{c} = \frac{\varepsilon+1}{\varepsilon}$
  - $\Rightarrow \frac{d\log p}{d\log c} = \frac{\varepsilon+1}{\varepsilon}\frac{1}{2-\rho} > 0$
  - $\frac{d\log p}{d\log c} - 1 = -\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(2-\rho)} \gtreqless 0$
  - ≥ 1 IFF superconvex demand
- Absolute Pass-Through:
  - $\frac{dp}{dc} = \frac{1}{2-\rho} > 0$
  - $\Rightarrow \frac{dp}{dc} - 1 = \frac{\rho-1}{2-\rho} \gtreqless 0$

Sketch of Supermodularity Proof

Back to Text

- FDI avoids trade cost t: $\Pi^F = \pi(0, c) - f_F$
- Exports incur lower fixed cost: $\Pi^X = \pi(t, c) - f_X$, $f_X < f_F$
- *Tariff-jumping gain*: $\gamma(t, c, f) \equiv \Pi^F - \Pi^X = \pi(0, c) - \pi(t, c) - f$
  - $f \equiv f_F - f_X > 0$: the excess fixed cost of FDI relative to exporting
- We want to sign the difference between firms with different production costs c of this profit difference between access modes with different costs t
- Since γ measures the incentive to engage in exporting relative to FDI, a sufficient condition for less productive firms to export is $\gamma(t, c_1, f) > \gamma(t, c_2, f)$, $c_1 > c_2$.
- Mrázová-Neary: This is equivalent to π being supermodular in t and c
  - If π is twice differentiable, this is equivalent to $\pi_{tc} > 0$.

Supermodularity with Multiplicative Costs: Proofs

Back

- $\pi(t,c) \equiv \max_x \tilde{\pi}(x,t,c)$
- $\tilde{\pi}\left(x;t,c\right) = \left[p(x) - tc\right]x \quad \Rightarrow \quad \pi_t = \tilde{\pi}_t = -cx$
- $\Rightarrow \pi_{tc} \;=\; -x - c\frac{dx}{dc} \;=\; -x - \frac{tc}{2p'+xp''} \;=\; -x + \frac{\varepsilon-1}{2-\rho}x$
  $= \frac{\varepsilon+\rho-3}{2-\rho}x$
- Elasticity of marginal revenue $\varepsilon_{MR,x}$:

$$r(x) \equiv xp(x)$$

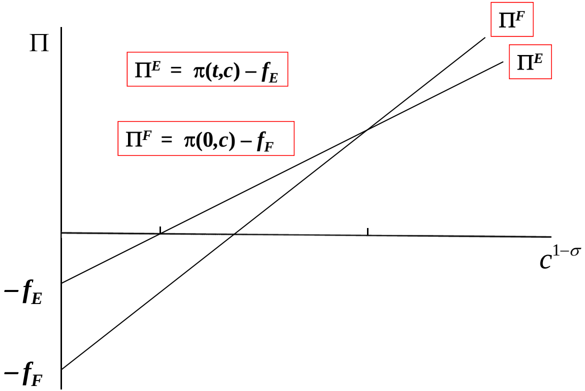
$$\Rightarrow \quad r' = xp' + p = xp'(1-\varepsilon) > 0$$

$$\Rightarrow \quad r'' = 2p' + xp'' = p'(2-\rho) < 0$$

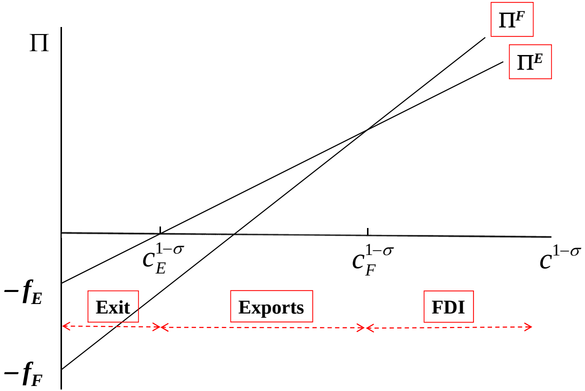
$$\Rightarrow \quad \varepsilon_{MR,x} \equiv -\frac{xr''}{r'} = \frac{2-\rho}{\varepsilon-1}$$

$$\Rightarrow \quad \varepsilon_{MR,x} - 1 = -\frac{\varepsilon+\rho-3}{\varepsilon-1}$$

First- and Second-Order Selection Effects

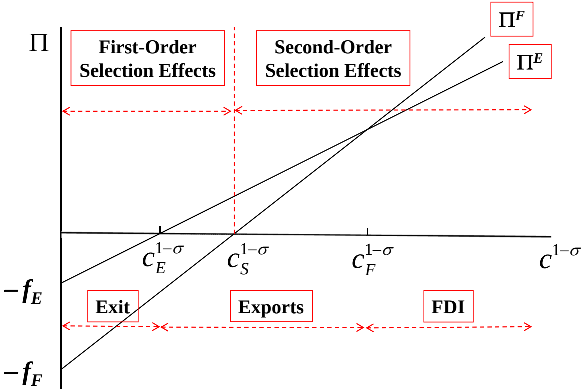


First- and Second-Order Selection Effects



First- and Second-Order Selection Effects

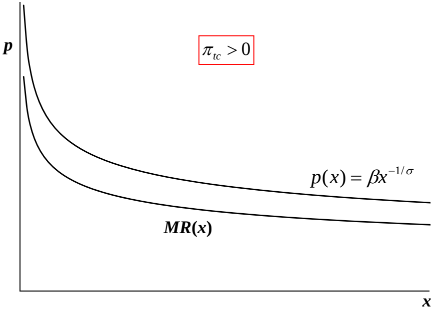
Back



Supermodularity Intuition (a): CES

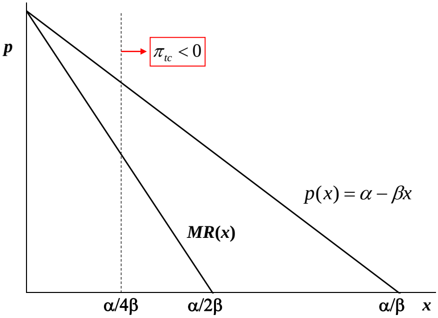
Back

- MR highly inelastic:
  - Same elasticity as the inverse demand function: $1/\sigma$
    - Here: $1/4$
- This implies a very large response of sales to costs:
  - More productive firms sell a lot more than less productive ones,
  - and so gain more from a reduction in trade costs, enjoying a strong Matthew Effect.
- Hence the profit function is always supermodular



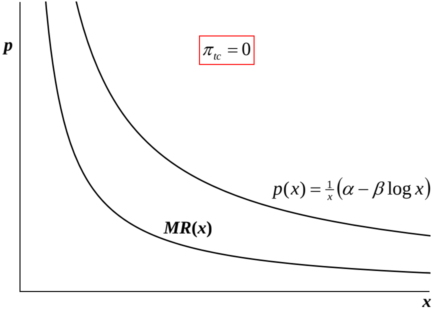
Supermodularity Intuition (b): Quadratic

- Both demand and MR are linear
  - Both elasticities (as a function of sales) rise as sales rise.
- Profits must be submodular for very efficient firms:
  - $c \to 0 \quad \Rightarrow \quad \tau c \to 0$
    $\Rightarrow$ no gain to FDI
- Elasticity of marginal revenue equals one, and so profits switch from super- to sub-modularity, at half the maximum output.
- So, profits are *submodular* for larger lower-cost exporters.

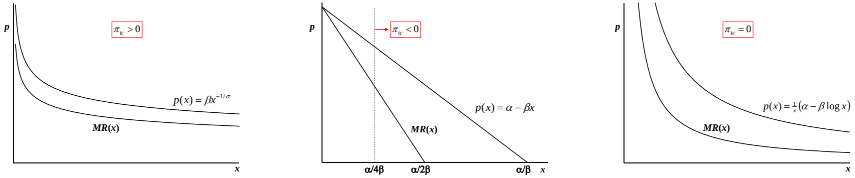


Supermodularity Intuition (c): Inverse Translog

- Used in both:
  - Theory: Diewert (1976)
  - Empirics: Agricultural economics
- Necessary and sufficient for π to be *modular* in (t, c) at all levels of output
- $MR(x) = \frac{\beta}{x}$: Rectangular hyperbola
- So total variable costs are the same for all firms, irrespective of tariffs
- Hence, no selection will be observed.



Supermodularity Intuition: Summary



(a) CES (b) Quadratic (c) Inverse Translog

- Theory ambiguous:
  - Different widely-used demand functions imply different selection effects

Back

- Empirical evidence?
  - No direct evidence on ρ
  - Indirect evidence ambiguous but favours sub-convexity
    - Pass-through, etc.
  - Some evidence that largest firms engage less in low-marginal-cost activities than a CES model predicts:
    - Yeaple (2009) on FDI, Spearot (2013) on investment

Existence of the Demand Manifold: Proof

Back

- We wish to prove that, except in CES case, only one of ε_x and ρ_x can be zero at any x:

$$\varepsilon_x = \frac{\varepsilon}{x}\left[\rho - \frac{\varepsilon+1}{\varepsilon}\right]$$

$$\rho_x = \frac{\rho}{x}\left(1 + \rho - \chi\right), \quad \chi \equiv -\frac{xp'''}{p''}$$

- Two cases where $\rho_x = 0$:
  1. $\rho = 0 \quad \Rightarrow \quad \varepsilon_x = -\frac{\varepsilon}{x}\frac{\varepsilon+1}{\varepsilon} \quad \neq 0$
  2. $1 + \rho - \chi = 0$; this implies that demand is $p(x) = \alpha + \beta x^{-\theta}$
     - Intersection of this with $\varepsilon_x = 0$ is the CES

Proof of Proposition 1 and Corollaries

(a) Separable Direct Demands: $x(p,\phi) = \delta(\phi)\,\tilde{x}(p)$

- Both ε and ρ are independent of ϕ:
  - $\varepsilon = -\frac{p x_p(p,\phi)}{x(p,\phi)} = -\frac{p\tilde{x}'(p)}{\tilde{x}(p)}, \quad \rho = \frac{x(p,\phi)x_{pp}(p,\phi)}{[x_p(p,\phi)]^2} = \frac{\tilde{x}(p)\tilde{x}''(p)}{[\tilde{x}'(p)]^2}$
- Corollary: Indirect Additivity:
  - $\int v\,[p(\omega)/I]\,d\omega \quad \Rightarrow$ Roy's Identity: $v'\,[p(\omega)/I] = -\lambda x(\omega)$
  - $\Rightarrow \quad x(p,\phi) = -\lambda^{-1}(\phi)\tilde{x}(p/I)$

(b) Separable Inverse Demands: $p(x,\phi) = \beta(\phi)\tilde{p}(x)$

- Both ε and ρ are independent of ϕ:
  - $\varepsilon = -\frac{p(x,\phi)}{x p_x(x,\phi)} = -\frac{\tilde{p}(x)}{x\tilde{p}'(x)}, \quad \rho = -\frac{x p_{xx}(x,\phi)}{p_x(x,\phi)} = -\frac{x\tilde{p}''(x)}{\tilde{p}'(x)}$
- Corollary: Direct Additivity:
  - $\int u\,[x(\omega)]\,d\omega \quad \Rightarrow$ FOC: $u'\,[x(\omega)] = \lambda p(\omega)$
  - $\Rightarrow \quad p(x,\phi) = \lambda^{-1}(\phi)\tilde{p}(x)$

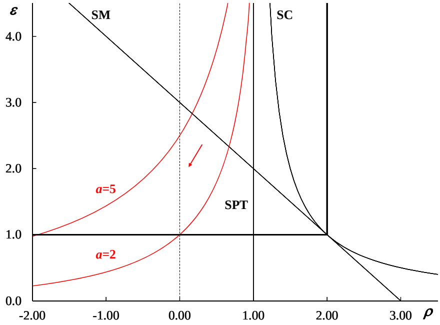
Back to Text

Log-Concave Demand Functions

- Logistic:

  [Cowan (2012)]

  - $x(p) = \left(1 + e^{p-a}\right)^{-1} s \in [0, s]$
  - $\Leftrightarrow \quad p(x) = a - \log \frac{x}{s-x}$
  - $\Rightarrow \quad \varepsilon = p\frac{s-x}{s}, \ \rho = \frac{s-2x}{s-x} < 1$
  - $\Rightarrow \quad \varepsilon = \frac{a-\log(1-\rho)}{2-\rho}$
  - Interpretation of a:
    - $p = a \ \Rightarrow \ x = \frac{s}{2}$
      $\Rightarrow \ \rho = 0 \ \Rightarrow \ \varepsilon = \frac{a}{2}$



- Other log-concave distributions: Bagnoli-Bergstrom (2005)
- Summary: All Sub-C and Sub-PT; SM for low x, Sub-M for high x

Back to Text

Proof of Proposition 2: Bipower Inverse Demands

Sufficiency follows by calculating the manifold directly:

1. Calculate the derivatives of $p(x) = \alpha x^{-\eta} + \beta x^{-\theta}$:

$$p' = -\eta\alpha x^{-\eta-1} - \theta\beta x^{-\theta-1}$$

$$p'' = \eta(\eta+1)\alpha x^{-\eta-2} + \theta(\theta+1)\beta x^{-\theta-2}$$

2. To eliminate terms in $\alpha x^{-\eta}$, add η times p to x times p' to obtain:

$$\eta p + xp' = (\eta - \theta)\beta x^{-\theta}$$

3. Similarly, add $(\eta+1)$ times p' to x times p'' to obtain:

$$(\eta+1)p' + xp'' = (\theta-\eta)\theta\beta x^{-\theta-1}$$

4. To eliminate terms in $\beta x^{-\theta}$, multiply (2) by θ and (3) by x and add:

$$\eta\theta p + \theta xp' + (\eta+1)xp' + x^2 p'' = 0$$

5. Dividing by xp' gives the desired result:

$$\bar{\rho}(\varepsilon) = \eta + \theta + 1 - \eta\theta\varepsilon$$

Proof of Proposition 2: Necessity

Necessity follows by setting $\rho(x) = a + b\varepsilon(x)$, where a and b are constants, and solving the resulting Euler-Cauchy differential equation:

1. Substituting for $\rho(x)$ and $\varepsilon(x)$ and collecting terms yields:
$$x^2 p''(x) + axp'(x) - bp(x) = 0$$
2. Change variables as follows: $t = \log x$ and $p(x) = g(\log x) = g(t)$.
3. Substituting for $p(x) = g(t)$, $p'(x) = \frac{1}{x} g'(t)$ and $p''(x) = \frac{1}{x^2}\left[g''(t) - g'(t)\right]$:
$$g''(t) + (a-1)g'(t) - bg(t) = 0$$
4. Assuming a trial solution $g(t) = e^{\lambda t}$ gives the characteristic polynomial: $\lambda^2 + (a-1)\lambda - b = 0$, with roots $\lambda = \frac{1}{2}\left[-(a-1) \pm \sqrt{(a-1)^2 + 4b}\right]$.
   - If $(a-1)^2 > -4b$, the roots are distinct: $g(t) = \alpha e^{\lambda_1 t} + \beta e^{\lambda_2 t}$, where α and β are constants of integration.
   - If $(a-1)^2 = -4b$, the roots are equal: $g(t) = (\alpha + \beta t)e^{\lambda t}$.
5. Switch back from t and $g(t)$ to $\log x$ and $p(x)$, recalling that $e^{\lambda \log x} = x^\lambda$:
$$p(x) = \alpha x^{\lambda_1} + \beta x^{\lambda_2} \quad \text{or} \quad p(x) = (\alpha + \beta \log x) x^\lambda$$
6. The sum of the roots is $\lambda_1 + \lambda_2 = 1 - a$ and their product is $\lambda_1 \lambda_2 = b$.

Back to Text

Bulow-Pfleiderer Demands: Details

- Restrictions: $\varepsilon = \frac{1}{\theta\beta}\frac{p}{x^{-\theta}} = \frac{1}{\theta}\frac{p}{p-\alpha} \quad \Rightarrow \quad \theta\beta > 0, \quad \theta(p-\alpha) > 0$
- Superconvex IFF: $\theta\varepsilon > 1$
- Possible parameter configurations:
  1. $\theta < 0 \;\Rightarrow\; p(x)$ is log-concave and subconvex, and $\alpha > p$
  2. $\theta > 0$, $\alpha < 0 \;\Rightarrow\; p(x)$ is log-convex and subconvex
  3. $\theta > 0$, $\alpha > 0 \;\Rightarrow\; p(x)$ is log-convex and superconvex, and $p > \alpha$
- Nice properties:
  - Necessary and sufficient for marginal revenue to be affine in price
  - Discrete choice interpretation: Cumulative demand from a Generalized Pareto Distribution
    - Bulow-Klemperer (2012)
  - Allows division of surplus between consumers and producer to be calculated without knowledge of quantities
    - Weyl-Fabinger (2013), Atkin-Donaldson (2013)

Compare Pollak | Back to Text

Proof of Proposition 3: Bipower Direct Demands

This proposition follows immediately from Proposition 5 by exploiting the duality between direct and inverse demand functions.

1. The direct demand function in Proposition 3, $x(p) = \gamma p^{-\nu} + \delta p^{-\sigma}$, is the dual of the inverse demand function in Proposition 5, $p(x) = \alpha x^{-\eta} + \beta x^{-\theta}$.
2. Hence Proposition 5 with appropriate relabeling implies that the direct demand function $x(p) = \gamma p^{-\nu} + \delta p^{-\sigma}$ is necessary and sufficient for a linear *dual* manifold, i.e., an equation linking the dual parameters r and e:

$$r(e) = \nu + \sigma + 1 - \nu\sigma e$$

3. Recalling that $e = \frac{1}{\varepsilon}$ and $r = \varepsilon\rho$ gives the desired result.

Back to Text

QMOR Demand Functions

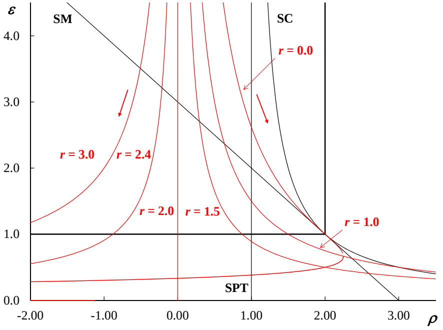
- "Quadratic Mean of Order r"
  [Diewert (1976), Feenstra (2014)]

$$\nu = 1 - r, \;\; \sigma = \frac{2-r}{2}$$

$$\Rightarrow \quad x(p) = \gamma p^{-(1-r)} + \delta p^{-\frac{2-r}{2}}$$

$$\bar{\rho}(\varepsilon) = \frac{(2-r)(3\varepsilon - 1 + r)}{2\varepsilon^2}$$

- $r \to 0$: Translog



Back

Translog: Details

- Sub-C: $\rho - \frac{\varepsilon+1}{\varepsilon} = -\frac{(\varepsilon-1)^2}{\varepsilon^2} \leq 0$
- SM: $\varepsilon + \rho - 3 = \frac{(\varepsilon-1)^3}{\varepsilon^2} \geq 0$ for $\varepsilon \geq 1$
- SPT/Sub-PT: $\rho \lesseqgtr 1$ as $\varepsilon \gtreqless \frac{1}{2}(3+\sqrt{5}) \approx 2.62.$
- Decreasing PT: $E_\rho < 0$ and $\varepsilon_x < 0 \;\Rightarrow\; \rho_x > 0$

Return

Pollak Demands: Details

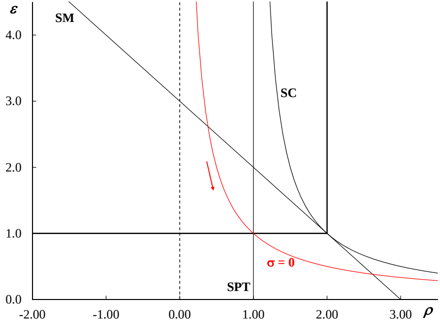
Compare Bulow-Pfleiderer   Back to Text

- The only demand functions consistent with both additive separability and quasi-homotheticity/Gorman Polar Form
- Restrictions: $\varepsilon = \sigma\delta\frac{p^{-\sigma}}{x} = \sigma\frac{x-\gamma}{x} \quad \Rightarrow \quad \sigma\delta > 0, \ \ \sigma(x-\gamma) > 0$
- Possible parameter configurations:
  1. $\sigma < 0 \ \Rightarrow \ p(x)$ is log-concave and subconvex, and $\gamma > x$
  2. $\sigma > 0, \gamma < 0 \ \Rightarrow \ p(x)$ is log-convex and subconvex
  3. $\sigma > 0, \gamma > 0 \ \Rightarrow \ p(x)$ is log-convex and superconvex, and $x > \gamma$
- Dual to the Bulow-Pfleiderer family $p(x) = \alpha + \beta x^{-\theta}$:
  - BP: Marginal revenue affine in price: $p + xp' = \theta\alpha + (1-\theta)p$.
  - Pollak: Marginal *loss* in revenue following an increase in price is affine in sales: $x + px' = \sigma\gamma + (1-\sigma)x$.
  - Hence: "HARA" ("hyperbolic absolute risk aversion") [Merton (1971)]
    - Arrow-Pratt absolute risk aversion: $A(x) \equiv -\frac{u''(x)}{u'(x)} = -\frac{p'(x)}{p(x)}$
    - So: $A(x) = -\frac{1}{px'(p)} = \frac{1}{\sigma(x-\gamma)}$, which is hyperbolic in x.

The Pollak Family

$$x(p) = \gamma + \delta p^{-\sigma}, \quad \bar{\varepsilon}(\rho) = \frac{\sigma+1}{\rho}$$

- $\sigma \to 0$: Inverse log-linear
  $$x = \gamma - \xi \ln p \quad \Rightarrow \quad \varepsilon = \frac{1}{\rho}$$
  - Chipman (1975), Bertoletti (2006)
  - Behrens-Murata (2007): "CARA"



Back to Text

The "Doubly-Translated CES" Super-Family

$$p(x) = \alpha + \beta \left(x - \gamma\right)^{-\theta}$$

- Elasticity and convexity:

$$\varepsilon(x) = \frac{1}{\theta} \frac{p}{p - \alpha} \frac{x - \gamma}{x} \qquad \rho(x) = (\theta + 1) \frac{x}{x - \gamma}$$

- Assuming $\gamma \neq 0$, invert $\rho(x)$: $x = \frac{\rho}{\rho - (\theta+1)} \gamma$.
- Substituting into $\varepsilon(x)$:

$$\bar{\varepsilon}(\rho) = \left[1 + a_1 \left(\frac{1}{\rho - a_2}\right)^{a_3}\right] \frac{a_4}{\rho}$$

  - where: $a_1 = \frac{\alpha}{\beta} \left\{(\theta + 1)\gamma\right\}^{\theta}$, $a_2 = \theta + 1$, $a_3 = \theta$, and $a_4 = \frac{\theta+1}{\theta}$.
  - A closed-form expression for the manifold; depends on all 4 parameters.

Relationship with ρ-Concavity

Back to Text

The Doubly-Translated CES Family and ρ-Concavity

[Avriel (1972), Caplin-Nalebuff (1991a,b), Anderson-Renault (2003)]

- Definition: A function f is ρ-concave if f^{ρ} is concave
- Pollak Demands: $x = \gamma + \alpha p^{\frac{1}{\theta-1}}$, $(x-\gamma)(1-\theta) > 0$
  - "Translated" iso-elastic direct demand functions
  - Inverse demands are both ρ-concave and ρ-convex, with $\rho = \frac{1}{\theta-1}$:

$$p^{\frac{1}{\theta-1}} = \alpha^{-1}(x-\gamma)$$

- Iso-Convex Demands: $p(x) = \alpha + \beta x^{-\theta}$, $\beta\theta > 0$
  - "Translated" iso-elastic inverse demand functions
  - Direct demands are both ρ-concave and ρ-convex, with $\rho = -\theta$: $x^{-\theta} = \beta^{-1}(p-\alpha)$

Back to Text

"Translated Bipower-Inverse" Demand Manifold

$$p(x) = \alpha_0 + \alpha x^{-\eta} + \beta x^{-\theta}$$

- Elasticity and convexity:

$$\varepsilon(x) = \frac{\alpha_0 x^{\eta} + \alpha + \beta x^{\eta-\theta}}{\eta\alpha + \theta\beta x^{\eta-\theta}} \qquad \rho(x) = \frac{\eta(\eta+1)\alpha + \theta(\theta+1)\beta x^{\eta-\theta}}{\eta\alpha + \theta\beta x^{\eta-\theta}}$$

- Assuming $\rho \neq \theta + 1$ and $\eta \neq \theta$, invert $\rho(x)$: $x = \left[\frac{\eta\alpha}{\theta\beta}\frac{(\eta+1)-\rho}{\rho-(\theta+1)}\right]^{\frac{1}{\eta-\theta}}$.
- Substituting into $\varepsilon(x)$ gives the demand manifold:

$$\bar{\varepsilon}(\rho) = \frac{\rho - a_1}{a_2} + (a_3 - \rho)^{a_4}(\rho - a_5)^{a_6} a_7$$

$$a_1 = \eta + \theta + 1,\ a_2 = -\eta\theta,\ a_3 = \eta + 1,\ a_4 = \tfrac{\eta}{\eta-\theta},\ a_5 = \theta + 1,\ a_6 = -\tfrac{\theta}{\eta-\theta}$$

$$a_7 = \left(\tfrac{\eta}{\beta}\right)^{\frac{\eta}{\eta-\theta}}\left(\tfrac{\theta}{\alpha}\right)^{-\frac{\theta}{\eta-\theta}}\tfrac{\alpha_0}{\eta\theta(\eta-\theta)}.$$

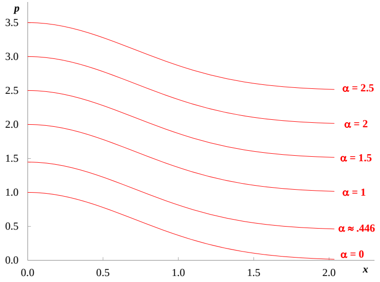
- Special cases:
  1. Iso-Temperance: $\bar{\varepsilon}(\rho) = -\frac{\theta-\rho}{\theta} - \left(\frac{\rho}{\beta}\right)^{\frac{1}{1+\theta}}\left[\frac{\theta}{\alpha}\{\rho-(\theta+1)\}\right]^{\frac{\theta}{1+\theta}}\frac{\alpha_0}{\theta(1+\theta)}$
  2. Apt: $\bar{\varepsilon}(\rho) = \frac{1+3\theta-\rho}{2\theta^2} - [(2\theta+1)-\rho]^2[\rho-(\theta+1)]^{-1}\frac{2\alpha\alpha_0}{(\beta\theta)^2}$

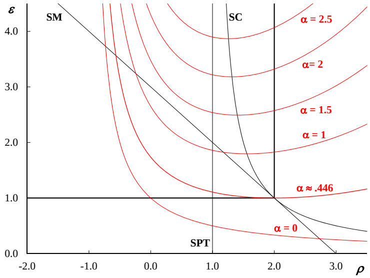
Back

Simultaneously Sub- and Superconvex Demands

- Exponential Inverse Demand:

Back1 Back2





$$p(x) = \alpha + \beta \exp(-\gamma x^{\delta})$$

$$\bar{\varepsilon}(\rho) = \frac{1+\frac{\alpha}{\beta}\exp\left(\frac{\rho+\delta-1}{\delta}\right)}{\rho+\delta-1}$$

Effects of Globalization with Subconvex Demands

$\hat{k} > 0$:

$$\hat{y} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(2-\rho)}\hat{k} \quad > \quad 0^*$$

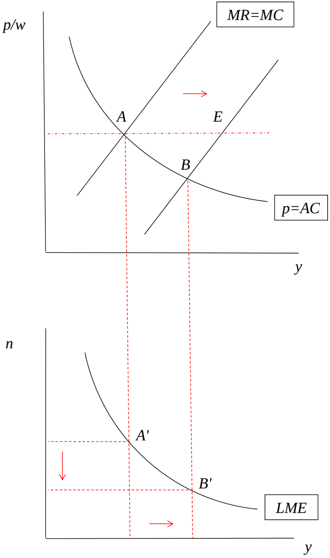
$$\hat{p} = -\frac{1}{\varepsilon}\hat{y} \quad < \quad 0^*$$

$$\hat{n} = -\frac{\varepsilon - 1}{\varepsilon}\hat{y} \quad < \quad 0^*$$

$$\hat{x} = \hat{y} - \hat{k} = -\frac{\varepsilon - 1}{\varepsilon(2-\rho)}\hat{k} \quad < \quad 0$$

$$\hat{N} = \hat{k} + \hat{n} = \frac{(\varepsilon-1)^2 + (2-\rho)\varepsilon}{\varepsilon^2(2-\rho)}\hat{k} > 0$$

* IFF demands are subconvex



CES Case | Superconvex Demands | Back to text

Globalization: The CES Special Case

- $\hat{k} > 0$, $\varepsilon = \sigma$:

$$\hat{p} = 0$$

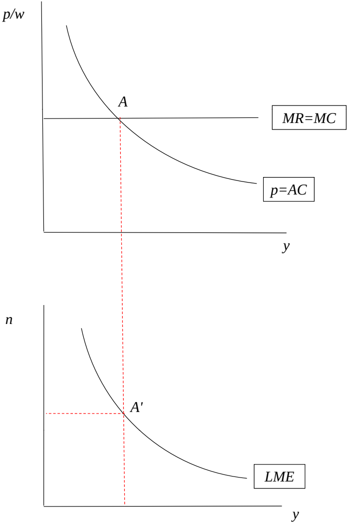
$$\hat{y} = 0$$

$$\hat{n} = 0$$

$$\hat{x} = \hat{y} - \hat{k} = -\hat{k} \quad < \quad 0$$

$$\hat{N} = \hat{k} + \hat{n} = \hat{k} > 0$$

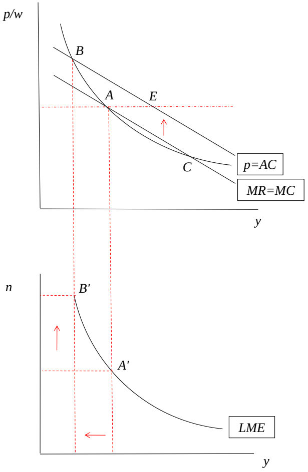
Back to Text



Globalization with Superconvex Demands

- MR=MC now slopes downwards
  - Multiple equilibria are possible
  - Second-order condition is violated at C: $\rho > 2$
- Globalization shifts MR=MC to right
- Comparative statics reversed for p, y, n

Back to Text



Additive Separability: Detailed Derivations

Calculate the elasticities of profits, using the envelope theorem:

Back to Text

$$\pi(c,\lambda,k) \equiv \max_y \Big[p(y,\lambda,k) - c\Big] y \quad \text{where:} \quad p(y,\lambda,k) = \lambda^{-1} u'(y/kL)$$

$$\Rightarrow \quad \pi_c = -y, \quad \pi_\lambda = -\lambda^{-2} u' y = -\lambda^{-1} p y, \quad \pi_k = -\frac{y^2 u''}{\lambda k^2 L} = -\frac{y^2}{k} p_y$$

- So:

$$\frac{c\pi_c}{\pi} = -\frac{cy}{\pi} = -\frac{c}{p-c} = -(\varepsilon - 1) \qquad \frac{\lambda \pi_\lambda}{\pi} = -\frac{py}{\pi} = -\frac{p}{p-c} = -\varepsilon$$

$$\frac{k\pi_k}{\pi} = -\frac{y^2 p_y}{(p-c)y} = \frac{y p_y}{y p_y} = 1$$

- Similarly for the elasticities of aggregate profits:

$$\frac{\lambda \bar{\pi}_\lambda}{\bar{\pi}} = \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \frac{\lambda \pi_\lambda(c)}{\pi(c)} g(c)\,\mathrm{d}c = -\int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \varepsilon(c) g(c)\,\mathrm{d}c = -\bar{\varepsilon}$$

$$\frac{k\bar{\pi}_k}{\bar{\pi}} = \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \frac{k\pi_k(c)}{\pi(c)} g(c)\mathrm{d}c = \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} g(c)\mathrm{d}c = 1$$

Globalization and the Intensive Margin: Details

▸ Back to Text

- Firm output:

$$y(c,\lambda,k) = -\pi_c(c,\lambda,k)$$

$$\rightarrow \quad \hat{y} = \left(\frac{ky_k}{y} + \frac{\lambda y_\lambda}{y}\frac{k}{\lambda}\frac{d\lambda}{dk}\right)\hat{k} = \left(\frac{ky_k}{y} + \frac{\lambda y_\lambda}{y}\frac{1}{\bar{\varepsilon}}\right)\hat{k} = \left(1 - \frac{\varepsilon-1}{2-\rho}\frac{1}{\bar{\varepsilon}}\right)\hat{k}$$

$$\rightarrow \quad \hat{y} = \left[\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(2-\rho)} + \frac{\bar{\varepsilon}-\varepsilon}{\bar{\varepsilon}}\frac{\varepsilon-1}{\varepsilon(2-\rho)}\right]\hat{k}$$

- Prices:

$$\hat{p} = -\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(\varepsilon-1)}\hat{x}$$

$$\hat{x} = \hat{y} - \hat{k} = \left(1 - \frac{\varepsilon-1}{2-\rho}\frac{1}{\bar{\varepsilon}} - 1\right)\hat{k} = -\frac{\varepsilon-1}{2-\rho}\frac{1}{\bar{\varepsilon}}\hat{k}$$

$$\rightarrow \quad \hat{p} = -\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(\varepsilon-1)}\frac{\varepsilon-1}{2-\rho}\frac{1}{\bar{\varepsilon}}\hat{k} = -\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon\bar{\varepsilon}(2-\rho)}\hat{k} = -\frac{\varepsilon}{\bar{\varepsilon}}\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon^2(2-\rho)}\hat{k}$$

Gains from Trade: The Story So Far

- Krugman (1979): Gains from variety
- Melitz (2003): Gains from selection
- Melitz and Ottaviano (2007): Gains from competition

  Though also Brander (1981)
- ACR (*AER* 2012): NO! All these models (plus AvW, EK) imply the same simple expression for gains from trade
  - BUT: All with CES preferences, implying an iso-elastic gravity equation
- ACDR (2012 mimeo): Similar results with non-CES preferences
  - BUT: All with Pareto distribution of firm productivities, implying an iso-elastic gravity equation
- Simonovska-Waugh (2012), Melitz-Redding (2013): Structure versus reduced form

Change in Real Income: Details

Back

- Additive separability: $U = F\left[Nu(x)\right]$
- Budget constraint: $I = \Sigma_i^N p_i x_i = Npx \quad \Rightarrow \quad x = \frac{I}{Np}$
- Taste for variety requires $\xi < 1$
  - Proof: $Npx = I$, $\hat{p} = \hat{I} = 0 \Rightarrow \hat{x} = -\hat{N} \Rightarrow \hat{U} = (1-\xi)\hat{N}$
- Indirect utility function: $V(N,p,I) = F\left[Nu\left(\frac{I}{Np}\right)\right]$
- Define equivalent income $Y(N,p)$: $\quad V\left(N,p,\frac{I}{Y}\right) = U_0$
  - So: $Nu\left(\frac{I}{NpY}\right)$ is fixed by U_0
  - With $I = wL$ fixed: $\hat{N} = \xi(\hat{N} + \hat{p} + \hat{Y})$
- $\Rightarrow$ Change in Real Income:

$$\hat{Y} = \frac{1-\xi}{\xi}\hat{N} - \hat{p}$$

Table 1: Implications for Gains from Trade

Demand		Utility: Subconcave $\xi \leq \frac{\varepsilon-1}{\varepsilon} \Leftrightarrow \varepsilon \geq \frac{1}{1-\xi}$ Varieties under-supplied: Direct gain exceeds CES benchmark	Utility: Superconcave $\xi \geq \frac{\varepsilon-1}{\varepsilon} \Leftrightarrow \varepsilon \leq \frac{1}{1-\xi}$ Varieties over-supplied: Direct gain less than CES benchmark
Subconvex: $\rho \leq \frac{\varepsilon+1}{\varepsilon}$	$\hat{p} < 0$: GT OK $\Leftrightarrow \varepsilon \leq \frac{1}{\rho-1}$	Indirect gain negative GT OK (even though efficiency falls)	Indirect gain positive GT OK
Superconvex: $\rho \geq \frac{\varepsilon+1}{\varepsilon}$	$\hat{p} > 0$: GT? $\Leftrightarrow \varepsilon \geq \frac{1}{\rho-1}$	Indirect gain positive GT OK	Indirect gain negative Losses possible!!!

Table 2: Examples

Demand		Utility: Subconcave $\xi \leq \frac{\varepsilon-1}{\varepsilon} \Leftrightarrow \varepsilon \geq \frac{1}{1-\xi}$ Varieties under-supplied	Utility: Superconcave $\xi \geq \frac{\varepsilon-1}{\varepsilon} \Leftrightarrow \varepsilon \leq \frac{1}{1-\xi}$ Varieties over-supplied
Subconvex: $\rho \leq \frac{\varepsilon+1}{\varepsilon}$	$\Leftrightarrow \varepsilon \leq \frac{1}{\rho-1}$	Pollak/HARA: $\gamma > x$ or $\gamma < 0$	Bulow-Pfleiderer: $\alpha > p$ or $\alpha < 0$
Superconvex: $\rho \geq \frac{\varepsilon+1}{\varepsilon}$	$\Leftrightarrow \varepsilon \geq \frac{1}{\rho-1}$	Bulow-Pfleiderer: $p > \alpha > 0$	Pollak/HARA: $x > \gamma > 0$

Globalization and Welfare: Normalized Pollak Utility

- Problem with unnormalized Pollak utility: $u(0) \neq 0$
  - Pettengill (1979) versus Dixit-Stiglitz (1977, 1979)
- Normalization avoids this ...
  - Though it imposes subconvexity

$$u(x) = \frac{\beta}{\sigma - 1}\left[(\sigma x + \zeta)^{\frac{\sigma-1}{\sigma}} - \zeta^{\frac{\sigma-1}{\sigma}}\right]$$

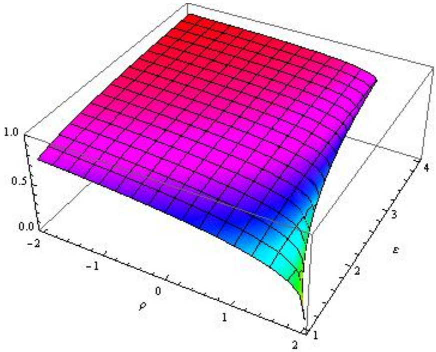
  - $\zeta = -\gamma\sigma$: Must be positive, so only subconvex demand allowed

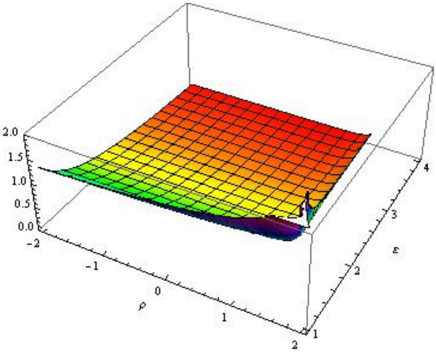
$$\Rightarrow \quad \xi^N = H\xi^P$$

$$\xi^N(\varepsilon, \sigma) = \frac{1}{1 - \left(\frac{\varepsilon - \sigma}{\varepsilon}\right)^{\frac{\sigma-1}{\sigma}}} \frac{\sigma - 1}{\varepsilon}, \quad \xi^N(\varepsilon, \rho) = \frac{1}{1 - \left(\frac{\varepsilon - \varepsilon\rho + 1}{\varepsilon}\right)^{\frac{\varepsilon\rho - 2}{\varepsilon\rho - 1}}} \frac{\varepsilon\rho - 2}{\varepsilon}$$

- More like Bulow-Pfleiderer than unnormalized Pollak:
  - $0 < \xi^N < 1$ throughout the subconvex region
  - Utility always superconcave

Globalization and Welfare: Normalized Pollak Utility





- ξ^N Increasing in ε
- *Decreasing* in ρ in subconvex region only

- $\hat{Y}$ *negative* for high ε and ρ

Recall $\hat{p}$ and $\hat{N}$ | Compare Pollak
Compare Bulow-Pfleiderer | Conclusion