Abstract

Empirical evidence strongly suggests that R&D increases a firm’s "absorptive capacity" (its ability to absorb spillovers from other firms) as well as contributing directly to profitability. We explore the theoretical implications of this. We specify a general model of the absorptive capacity process and show that costly absorption both raises the effectiveness of own R&D and lowers the effective spillover coefficient. This weakens the case for encouraging research joint ventures, even if there is complete information sharing between its members. It also implies an additional strategic pay-off to policies that raise the level of extra-industry knowledge.

Keywords: Absorptive capacity of R&D; competition policy; industrial policy; R&D spillovers; research joint ventures.

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1. Introduction

At least since Nelson (1959) and Arrow (1962), it has been widely held that information is almost completely non-appropriable and costless to acquire.\(^1\) As emphasised in those papers, the resulting free-rider problem reduces the incentive to invest in research and development (R&D) in a market economy, since individual firms incur the costs but cannot fully appropriate the benefits. One possible resolution to this dilemma was highlighted by Spence (1984), who pointed to research joint ventures as a way of internalising the positive externality which one firm’s R&D confers on its rivals. Work by Katz (1986), d’Aspremont and Jacquemin (1988) and Leahy and Neary (1997) among others cautioned that research joint ventures also have a negative impact on welfare, which dominates for low R&D spillovers. Research joint ventures serve in effect as a partial surrogate for product-market collusion, so lowering output and welfare below the social optimum. Nevertheless, it is widely held that R&D spillovers are sufficiently high that the net effect of research joint ventures is positive, especially when information sharing within research joint ventures leads to full technology transfer between the members of the research consortium. In practice, competition authorities in both the EU and the U.S. tend to tolerate if not actively encourage research joint ventures.

Yet even if information cannot be appropriated, it need not be a free good to other firms. As emphasised by Cohen and Levinthal (1989), acquiring the results of R&D requires effort by the recipient firm. Rather than thinking of R&D spillovers as exogenous, they argued that a firm needs to invest in its "absorptive capacity" if it is to realize R&D spillovers from other firms.\(^2\)

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\(^1\) "... no amount of legal protection can make a thoroughly appropriable commodity of something so intangible as information. The very use of the information in any productive way is bound to reveal it, at least in part." (Arrow (1962), p. 615.)

\(^2\) An older literature in development economics used the term "absorptive capacity" in a macroeconomic context to refer to the idea that higher rates of investment lower the productivity
Cohen and Levinthal themselves also presented some empirical evidence in favour of the absorptive capacity hypothesis using observations on business units, and subsequent work has produced considerably more. The importance of absorptive capacity has been confirmed using observations on a panel of industries across thirteen OECD countries by Griffith, Redding and Van Reenen (2004), and on individual firms by Girma (2005). Cockburn and Henderson (1998) have shown it to be crucial in a case study of the pharmaceutical industry, while Blomström and Kokko (1998) find that spillovers from foreign-owned firms are greatest when the technological gap between them and local firms is least, and interpret this as consistent with the absorptive capacity hypothesis. Perhaps most persuasively, Wakelin (1998) used one data set to classify exporting firms as either innovators or non-innovators, and then showed that the exports of innovators were less sensitive to costs and more sensitive to R&D spillovers, as measured by a different data set.

Partly stimulated by this accumulation of empirical evidence in favour of the absorptive capacity hypothesis, increasing attention has been devoted to its theoretical implications, and in particular its implications for the efficacy of research joint ventures. Kamien and Zang (2000) and Grunfeld (2003) consider this issue assuming that investment both reduces costs directly and contributes to absorptive capacity. Martin (2002) covers similar ground in a tournament model of R&D, where the winner of the innovation race licenses the new technology to the loser. Kamien and Zang (2000), Molto et al. (2005) and Weithaus (2005) explore the implications of assuming that firms initially choose the "direction" of their R&D efforts, and that the difference of new investment. See Eckaus (1987) for a review and Keller (1996) for an extension. Cohen and Levinthal appear to have been the first to apply the concept of absorptive capacity in the microeconomic context to individual firms.
between the directions chosen by two firms affects their ability to absorb spillovers. However, all these papers have used special functional forms and have concentrated on the case of Cournot duopoly. In this paper we consider the desirability of research joint ventures in a model which admits general forms for both the demand and absorptive capacity functions, and encompasses both Cournot and Bertrand competition with many firms. We also explore the implications of the absorptive capacity hypothesis for the view that research joint ventures are always desirable when information is fully shared between firms, and for the efficacy of public provision of research. Throughout, we concentrate on the "two faces of R&D" case, assuming that, in addition to its direct effect on costs, a firm’s own R&D enhances its capacity to absorb the benefits of rivals’ R&D. We also confine attention to the case of incremental process R&D, since this seems most appropriate to the industries in which absorptive capacity has been found to be important, from automobiles to microelectronics to pharmaceuticals.3

In Section 2 we specify a general model of the absorptive capacity process and explore its implications for the incentives to engage in R&D and for the effective level of spillovers. We also show how our model relates to some special cases which have been considered by Cohen and Levinthal (1989), Kamien and Zang (2000), Martin (2002) and Grunfeld (2003). Section 3 turns from the firm to the market to consider how absorptive capacity alters R&D incentives and welfare with and without cooperation by firms. Section 4 extends this analysis to the case where there is complete information sharing between firms. As already noted, the literature to date has suggested that research joint ventures are unambiguously welfare-improving in this case. Finally, Section 5 considers how, when building absorptive capacity is important, knowledge from outside

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3 See Kim (1998), Lim (2000), and Cockburn and Henderson (1998) respectively.
the industry affects the strategic interactions between R&D-intensive firms.

2. Absorptive Capacity

We follow the usual treatment of R&D spillovers in assuming that a typical firm’s marginal cost depends negatively on both its own and its rivals’ R&D. The novel feature is that the level of usable rival R&D may be less than the actual level of R&D carried out by other firms in the industry. Following Cohen and Levinthal (1989), the ratio of usable to actual rival R&D is the firm’s absorptive capacity and it depends on its own level of investment in R&D.

To formalise these ideas, write the typical firm’s marginal production cost $c$ as a negative function of own and usable rival R&D, denoted by $x$ and $y$ respectively:\textsuperscript{4}

$$c = c(x,y) \quad (1)$$

Own R&D $x$ reduces marginal cost in a standard fashion, with its effectiveness measured by the partial derivative of $c$ with respect to $x$, which we denote $\theta$:

$$\theta = -c_x(x,y) > 0 \quad (2)$$

Similarly, usable rival R&D $y$ gives rise to spillovers, whose extent is measured by the ratio of the partial derivatives of $c$ with respect to $y$ and $x$, assumed to lie between zero and one:

$$\beta = c_y(x,y)/c_x(x,y) \geq 0, \quad 0 \leq \beta \leq 1. \quad (3)$$

Note that $\beta$ and $\theta$ are not necessarily constant.

\textsuperscript{4} To avoid over-burdening the paper with additional notation, we assume that other firms are symmetric, so usable rival R&D $y$ can be treated as a scalar. The analysis can easily be extended to the general case, but this yields no additional insights.
So far, this specification is standard. Now, let $X$ denote the actual level of R&D carried out by other firms in the industry. The ratio of usable to actual rival R&D, $y/X$, is the firm’s absorptive capacity. Crucially, usable rival R&D $y$ depends on the firm’s own investment $x$. In addition, to facilitate comparative statics, we assume it depends on an exogenous parameter $\delta$, which represents the "difficulty" of absorbing rival R&D:\footnote{This difficulty has many different dimensions. On the one hand is the complexity of the R&D, which influences the ease with which it can be absorbed. On the other hand are the capabilities of the firm, which, for a given level of research effort $x$, influence its ability to reverse engineer and otherwise benefit from the research efforts of other firms. Finally, there is also the "direction" of R&D, which matters for absorptive capacity to the extent that it differs from the directions chosen by other firms. As noted in the introduction the latter interpretation has been endogenized by Kamien and Zang (2000), Molto et al. (2005) and Weithaus (2005).}

$$y = y(x, X, \delta) \quad (4)$$

We normalise $\delta$ to lie between zero and one, corresponding to the extreme cases of full ($y=X$) and zero absorptive capacity ($y=0$) respectively.

To flesh out the concept of absorptive capacity, we must place some restrictions on equation (4). Trivially, we assume that usable rival R&D cannot exceed actual rival R&D, so that absorptive capacity lies between zero and one: $y \leq X$, with a strict inequality when $\delta$ is positive.

Next, we make two mild assumptions about the marginal responsiveness of $y$ to $x$ and $X$:

**Assumption 1:** $y_x \geq 0$, with a strict inequality if and only if $0 < \delta < 1$ and $X > 0$.

**Assumption 2:** $0 \leq y_x \leq 1$; when $\delta = 0$, $y_x = 1$; when $\delta = 1$, $y_x = 0$; and, when $0 < \delta < 1$, $y_x < 1$. 
These assumptions imply that both \( x \) and \( X \) increase usable rival R&D at the margin, except in the extreme cases of zero and full absorptive capacity.

To see the implications of this approach, combine equations (1) and (4) into a single reduced-form marginal cost function:

\[
\bar{c}(x,X,\delta) = c[x,y(x,X,\delta)]
\]

(5)

Now we can define two new coefficients:

\[
(a) \quad \bar{\theta} = -\bar{c}_x = (1+\beta y)_x \theta \\
(b) \quad \bar{\beta} = \frac{\bar{c}_X}{\bar{c}_x} = \frac{\beta y_x}{1+\beta y_x} 
\]

(6)

Here \( \bar{\theta} \) measures the full impact of own R&D on unit costs. Equation (6a) shows that this cannot be less than the direct impact, \( \theta \), and will typically be more than it. Expenditure on R&D has an added pay-off because it allows the firm to avail of spillovers from rivals’ R&D. As for \( \bar{\beta} \), it measures the effective spillover coefficient, which gives the ratio of the marginal returns to rival and own R&D. The key implication of (6b) is that \( \bar{\beta} \) cannot be more than the direct spillover coefficient \( \beta \) and will typically be less than it. Because rival R&D is costly to absorb, its attractiveness relative to own R&D is reduced. To sum up:

**Proposition 1:** Given Assumptions 1 and 2: (i) \( \bar{\theta} \geq \theta \), with a strict inequality for \( \beta y_x > 0 \); and (ii) \( \bar{\beta} \leq \beta \), with a strict inequality for either \( y_x < 1 \) or \( y_x > 0 \).

Thus the dependence of absorptive capacity on own R&D raises the effectiveness of own R&D but lowers effective spillovers.

Next, consider the shift parameter \( \delta \). We need to make some further assumptions to allow
us to interpret $\delta$ as a measure of the difficulty of absorbing rival knowledge:

**Assumption 3**: $y_\delta < 0$.

Naturally, usable rival R&D, and so absorptive capacity $y/X$ itself, falls as the difficulty of absorbing rival R&D increases.

**Assumption 4**: $y_{X\delta} \leq 0$.

This implies that usable rival R&D is decreasing in $\delta$ at the margin.

**Assumption 5**: $d(y_X/y_{x\delta})/d\delta \leq 0$.

This implies that the marginal rate of substitution between own and rival R&D in producing $y$ is decreasing in $\delta$. Finally:

**Assumption 6**: $d\beta/dy \geq 0$.

This implies that the direct spillover coefficient is not decreasing in usable rival R&D.\footnote{Differentiating (3), $d\beta/dy = (c_x c_{yy} - c_y c_{xy})/c_x^2$. Hence, Assumption 6 is equivalent to assuming that $c_x c_{yy} - c_y c_{xy}$ is non-negative.} Since, from Assumption 3, $y$ is decreasing in $\delta$, Assumption 6 also implies that an increase in the
difficulty of absorbing rival R&D does not raise the direct spillover coefficient.

We can now state how the key coefficients \( \beta \) and \( \theta \) are affected by changes in \( \delta \):

**Proposition 2**: Given Assumptions 1 to 6: (i) \( \theta - \theta \) is increasing in \( \delta \) at \( \delta = 0 \) and decreasing in \( \delta \) at \( \delta = 1 \); and (ii) \( \beta \) is decreasing in \( \delta \). All these derivatives are strict when \( \beta \) is strictly positive.

The proof is in Appendix 1. Proposition 2 states that greater difficulty of absorbing rival R&D reduces the effective spillover coefficient \( \beta \), but has an ambiguous effect on the effectiveness of own R&D, paradoxically raising it if the level of difficulty is initially low.\(^7\) Intuitively, part (i) of the result arises because own R&D "has only one face", i.e., cannot raise usable rival R&D, if absorption is either effortless or impossible (i.e., \( \delta \) equals either zero or one respectively). At intermediate values of \( \delta \), R&D is more effective because it raises absorptive capacity. Hence the effectiveness of R&D is increasing in \( \delta \) when absorption is effortless but decreasing in it when absorption is impossible.

Proposition 2 has an insightful corollary. Consider an industry composed of \( n \) symmetric firms, each with technology given by (1) and (4). Suppose that all firms increase their R&D by a small amount. Let \( \theta \mu^\rho \) denote the resulting fall in the unit cost of production of each firm,

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\(^7\) Both \( \theta - \theta \) and \( \beta \) depend on the levels of R&D, as well as on \( \delta \), in general. To find conditions under which they are independent of \( x \), assume that the absorptive capacity function \( y \) is homogeneous of degree \( v \) in \( \{ x, X \} \). Then, with \( X = (n - 1)x \) in symmetric equilibria, it can be shown that \( d\ln(\theta - \theta)/d\ln x = v - 1 + d\ln(\beta\theta)/d\ln x \) and \( (\theta/\theta)d\ln\beta/d\ln x = v - 1 + d\ln\beta/d\ln x \). Hence \( \theta - \theta \) (respectively \( \beta \)) is independent of \( x \) if and only if (i) \( y \) is linearly homogeneous in \( \{ x, X \} \), and (ii) \( \beta \theta \) (respectively \( \beta \)) is independent of \( x \). These restrictions are satisfied by the Kamien-Zang specification, (9) below, but not by the Cohen-Levinthal one (8) (for which \( v = 1 + \gamma_\gamma, \gamma > 1 \)).
where $\mu^O$ equals the marginal social return to R&D per unit output, normalised by the marginal private return to R&D $\tilde{\theta}$. This must equal:

$$
\mu^O = \frac{\tilde{c}_x + (n-1)\tilde{c}_x}{\tilde{c}_x} = 1 + (n-1)\tilde{\theta}.
$$

(7)

Proposition 2 (ii) immediately implies:

*Corollary 1:* An increase in the difficulty of absorbing rival R&D reduces the marginal social return to R&D relative to the marginal private return.

This shows that, the more important is the need to invest in absorptive capacity, the smaller is the externality associated with R&D.

The general specification of absorptive capacity given in (5) encompasses some special cases which have been considered in the literature. Two in particular are worth noting. The first, due to Cohen and Levinthal (1989), assumes that average and marginal absorptive capacity ($y/X$ and $y_x$) are equal:

$$
c = c(x+\beta y), \quad c' < 0, \quad \text{where:} \quad y = X\gamma(x, \delta)
$$

(8)

where $\beta$ is a constant and $\gamma$ is increasing in $x$ and decreasing in $\delta$. An unsatisfactory feature of this specification is that absorptive capacity $y/X$ depends only on own R&D and is independent of the extent of rival R&D, $X$. A further problem is that Cohen and Levinthal assume that the marginal responsiveness of usable rival R&D to own R&D is increasing in $\delta$: $\gamma_{x\delta} > 0$, implying that $y_{x\delta} > 0$. This guarantees that Assumption 5 is satisfied but it also implies that $d\tilde{\theta}/d\delta$ is always positive. The latter is plausible for low $\delta$ but, as Proposition 2 (i) shows, is inconsistent with our
assumptions as $\delta$ approaches one.\(^8\)

The second special case, due to Kamien and Zang (2000), assumes that marginal cost depends linearly on own and usable rival R&D, and that usable rival R&D in turn is a Cobb-Douglas function of own and actual rival R&D:

$$c = c_0 - \theta(x + \beta y) \quad \text{where:} \quad y = (1 - \delta)x^\delta X^{1 - \delta}$$

where $\theta$, $\beta$ and $\delta$ are constants.\(^9\) This specification satisfies Assumptions 1 to 6 for most reasonable parameter configurations.\(^{10}\) Figure 1 illustrates the implied values of $\tilde{\theta}$ and $\tilde{\beta}$ as functions of the primitive parameters $\beta$ and $\delta$ in a symmetric two-firm case (with $\theta$ set equal to unity). The effect of $\delta$ in first raising and then lowering $\tilde{\theta}$ is evident from panel (i); while panel (ii) shows that the effective spillover coefficient $\tilde{\beta}$ falls off very rapidly as $\delta$ increases.

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\(^8\) Martin (2002) uses a variant of (8) which avoids this difficulty. In our notation, his specification is $\gamma(x, \delta) = -x \ln \delta / (1 - x \ln \delta)$. This implies that $\gamma_{x \delta} = -(1 + x \ln \delta) / (1 - x \ln \delta)^3$, which approaches zero as $\delta$ approaches zero, is positive for low values of $\delta$, and falls to $-1$ when $\delta$ equals one. Grunfeld (2003) uses a different variant, with $\beta \gamma$ equal to $(\beta + sx) / (1 + sx)$ and $s$ interpreted as a "learning" parameter. A potential problem with this specification is that it is not always possible to distinguish between spillovers and absorptive capacity. A value of $\beta$ equal to one is sufficient to render the absorptive capacity motive redundant: rival R&D is then freely absorbed.

\(^9\) This functional form was introduced in an earlier unpublished version of Kamien and Zang (2000). In their published paper the right-hand side of (9) is multiplied by an additional term $(1 - \delta^*)$ where $\delta^*$ is the "direction" of the rival firm’s R&D, and both $\delta$ and $\delta^*$ are chosen by their respective firms.

\(^{10}\) Assumptions 1, 2, 5 and 6 are always satisfied. In extreme cases, (9) may violate the restriction that absorptive capacity cannot exceed unity ($y \leq X$), and the assumptions that usable rival R&D is decreasing in $\delta$ on average (Assumption 3) and at the margin (Assumption 4). Violations are at risk when the firm spends considerably more on R&D than all its rivals put together (so $x / X$ is much greater than one) and $\delta$ is relatively low. Direct calculation shows that: $y / X = (1 - \delta)(x / X)^\delta$ and $d\ln y / d\delta = \ln(x / X) / (1 - \delta)$. In symmetric equilibria, when $x / X = (n - 1)^{-1}$, all the restrictions must hold.
3. Research Joint Ventures and Absorptive Capacity

So far, we have considered only the firm-level implications of the need to invest in absorptive capacity. We now want to consider its implications for industry performance. A central issue in evaluating performance in R&D-intensive industries is the welfare implications of research joint ventures. To explore the implications of absorptive capacity for this issue, we build on Leahy and Neary (1997), which extends the model of d’Aspremont and Jacquemin (1988) to many firms and a general specification of functional form and market structure.

We consider a two-period model of symmetric oligopoly, in which \( n \) firms first invest in cost-reducing R&D and then engage in either output (Cournot) or price (Bertrand) competition. R&D takes time to affect costs, and R&D spending cannot be concealed from rivals. Hence, it is natural to confine attention to two-stage sub-game perfect Nash equilibria, with decisions on R&D in period 1 anticipating subsequent decisions in period 2. In period 2, each firm chooses the level of an action \( a_i \), which corresponds to either output in Cournot competition or price in Bertrand competition.

The typical firm’s profits equal sales revenue \( R \) less variable costs \( cq \) less fixed costs \( \Gamma \):

\[
\pi^i = R(a_i, A_{-i}) - \tilde{c}(x_i, X_{-i}, \delta) q(a_i, A_{-i}) - \Gamma(x_i). \tag{10}
\]

Here \( x_i \) and \( X_{-i} \) denote the levels of R&D by firm \( i \) and by all other firms respectively; similarly \( a_i \) and \( A_{-i} \) denote the period-2 action chosen by firm \( i \) and the vector of actions by all other firms, respectively. Production costs depend on the level of output \( q \), which we write as a function of own and rivals’ actions. In Cournot competition, \( q(a_i, A_{-i}) \) equals \( a_i \), while in Bertrand competition it equals the demand facing firm \( i \), which depends on own and rivals’ prices. As for the cost terms, marginal cost \( \tilde{c} \) is given by (5) and fixed costs \( \Gamma \) depend only on own R&D.
We first solve for the non-cooperative equilibrium. Once R&D levels are determined, the representative firm maximises profits by setting to zero the partial derivative of (10) with respect to its action $a_i$. This in turn implies that higher levels of R&D are associated with higher levels of output in symmetric industry equilibrium irrespective of market structure:\footnote{See Lemma A1 in the Appendix for more details. The first-order condition is $\pi = R - c q = 0$, where $\pi$ denotes $\partial\pi / \partial a_i$. Totally differentiating and imposing symmetry, so that $da_j = da_i$, and $dX_{-i} = (n-1)dx_i$, gives equation (11). The denominator $\Delta$ equals $-(\pi_{ii} + (n-1)\pi_{ij})$ and must be positive from stability of the period-2 sub-game.}

$$\frac{dq}{dx} = \mu^0 \frac{\bar{q}_i q_a}{\Delta} > 0. \quad (11)$$

Here we use $x$ and $q$ without subscripts to denote the values of R&D and output in symmetric equilibria; $q_i$ denotes $\partial q(a_i, A_{-i}) / \partial a_i$; $q_a$ denotes $dq/da$ in symmetric equilibria; and both $q_i$ and $q_a$ are positive (equal to one) in Cournot competition and negative in Bertrand competition. Recall from Section 2 that $\mu^0$ is the marginal social return to R&D per unit output, normalised by the private return $\bar{\theta}$. In symmetric equilibria, this equals both the effect on one firm’s marginal cost of a unit increase in R&D spending by all firms, and the effect on the marginal costs of all firms of a unit increase in R&D spending by a single firm.

Consider next the choice of R&D levels. Without cooperation, each firm chooses its R&D to maximise its own profits only:

$$\frac{d\pi^i}{dx_i} = \frac{\partial \pi^i}{\partial x_i} + \sum_{j \neq i} \frac{\partial \pi^i}{\partial a_j} \frac{da_j}{dx_i} = \mu^0 \bar{\theta} q - \Gamma^i = 0, \quad (12)$$

As in all two-stage oligopoly games (see Fudenberg and Tirole (1984)), the firm takes account
of both the direct or "non-strategic" effect of its R&D, given by $\partial \pi / \partial x$, and also of the "strategic" effect which works by affecting the rival firms’ outputs in period 2. This implies equating the marginal cost of R&D, $\Gamma'$, to the marginal return, where the normalised marginal return per unit output (with a superscript "N" for "non-cooperation") is given by:\footnote{The notation is similar to that in Leahy and Neary (1997). The term $\alpha$ equals $h \pi_q (\pi_q - \pi_j)$, where $h$ in turn equals $-\pi_q / q \Delta$. The term $\pi_j$ denotes $\partial \pi / \partial q_j$ and is negative in Cournot competition and positive in Bertrand competition. Since $\Delta$ is positive as already noted in footnote 8, $h$ must be positive. The second derivative $\pi_{ij}$ in the numerator of $\alpha$ is negative from the firm’s second-order condition for output. We make the natural assumption that $\pi_{ij} < \pi_{jj}$. This ensures that $\alpha$ is always positive, and that the threshold spillover parameter, $\beta$, is always less than one.}

$$\mu^N = 1 + (n-1) \alpha (\tilde{\beta} - \bar{\beta}), \quad \text{where: } \alpha > 0 \text{ and } \tilde{\beta} = \frac{\pi_q}{\pi_{ij}}. \quad (13)$$

This shows that with spillovers lower than the threshold level $\tilde{\beta}$, the firm "over-invests" in R&D, relative to the non-strategic benchmark marginal return of $\bar{\theta}$; while with spillovers higher than $\bar{\beta}$, the fear of providing costless benefits to rivals leads to under-investment. (Note that $\tilde{\beta}$ is positive if and only if period-2 actions are strategic substitutes (i.e., $\pi_{ij} < 0$).) Since, from Proposition 2, greater difficulty of absorbing rival R&D reduces $\tilde{\beta}$, it follows that, other things equal, it raises the marginal return to R&D when firms do not cooperate.

The situation is very different if the firms form an industry-wide research joint venture. To model this, we follow d’Aspremont and Jacquemin (1988) and assume that firms choose their R&D levels cooperatively to maximise joint profits (denoted by $\Pi$), while period-2 actions continue to be chosen non-cooperatively. This seems to match the regulatory environment in both Europe and the U.S., in which R&D joint ventures are permitted by anti-trust authorities. (Until the next section, we assume that a research joint venture does not directly raise the
The first-order condition for firm $i$ is:

$$
\frac{d\Pi}{dx_i} = \frac{\partial \Pi}{\partial x_i} + \frac{\partial \Pi}{\partial a} \left[ \frac{da_i}{dx_i} + (n-1) \frac{da_j}{dx_j} \right] = \mu_C^\varnothing q - \Gamma' = 0,
$$

where the normalised marginal return to R&D per unit output (with a superscript "C" for "cooperation") is now:\(^{13}\)

$$
\mu_C = \phi \mu^\varnothing, \quad \phi < 1.
$$

Higher spillovers raise the impact of one firm’s R&D on industry profits and so tend to encourage investment when firms cooperate. (Though even with high spillovers, $\mu_C$ is less than $\mu^\varnothing$: the incentive to engage in R&D is lower than its marginal social return.) Hence, other things equal, greater difficulty of absorbing rival R&D lowers the marginal return to R&D in cooperative equilibria, since it lowers the effective spillover coefficient $\beta$.

Whether cooperation leads to more R&D than non-cooperation depends on the magnitude of the effective spillover coefficient, $\beta$. Combining (13) and (15), evaluated at the same levels of R&D and actions:

$$
\mu_C - \mu^\varnothing = \alpha'(\beta - \beta'), \quad \alpha' = (n-1) (\alpha + \phi) > 0 \quad \text{and} \quad \beta' = \frac{\alpha}{\alpha - \phi} > 0
$$

So, cooperation implies a greater incentive to invest in R&D if and only if the effective spillover

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\(^{13}\) To derive (15), note first that $\partial \Pi/\partial x_i$ is just $\mu^\varnothing \delta q - \Gamma'$, the net marginal social return to R&D. The next term in (14), $\partial \Pi/\partial a$, gives the effect on industry profits of a change in any firm’s action in a symmetric equilibrium. It equals $\pi_i + (n-1) \pi_j$, which reduces to $(n-1) \pi_j$. Finally, the term in parentheses in (14) is equal, in symmetric equilibria, to $d \Pi/dx$, which is just $\mu^\varnothing \delta q/\Delta$ from (11). Hence, $\phi$ equals $1 - (n-1)h$, which is less than one. We assume that the second-order conditions hold in symmetric equilibrium: see Leahy and Neary (2005) for further discussion.
coefficient exceeds a new threshold value $\bar{\beta}$. Equation (16) implies that the difference between $\mu^C$ and $\mu^N$ depends on $\delta$ only through $\bar{\beta}$.\textsuperscript{14} Hence we can again make use of Proposition 2:

**Corollary 2**: An increase in the difficulty of absorbing rival R&D reduces the marginal private return to R&D cooperation relative to non-cooperation.

Under mild regularity and stability conditions (see Leahy and Neary (1997), Proposition 3), this ranking of the incentives to engage in R&D translates into an equivalent ranking of output and R&D levels in symmetric equilibria.

Finally, we need to consider the effect of a small increase in $\delta$ on the relative levels of welfare in the two regimes. Consider first the special case where the demand and marginal cost functions are linear, the non-cooperative equilibrium is stable in the sense of Seade (1980), and the absorptive capacity function satisfies the restrictions in footnote 5. In this benchmark case, it is straightforward to show that an increase in $\delta$ reduces the welfare advantage of R&D cooperation. When the benchmark assumptions (other than stability) are relaxed, the result continues to hold provided we retain some mild regularity conditions: the second-best welfare function is quasi-concave in R&D, and the demand and absorptive capacity functions are well-behaved (analogous to ruling out "too much" convexity of the demand function, as is standard in oligopoly models). Subject to these conditions, which, along with the proof, are set out in

\textsuperscript{14} Inspecting the expressions for $\alpha'$ and $\bar{\beta}'$, it can be seen that they depend only on the levels of actions and on the derivatives of the profit function with respect to actions. The latter derivatives include terms in $\bar{c}$, which in turn depends on $\delta$. However, these terms can be eliminated using the first-order condition for actions from footnote 8. Hence $\alpha'$ and $\bar{\beta}'$ are independent of $\delta$. 

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Appendix 2, we can state the following:

**Proposition 3**: Given the conditions in Appendix 2, the threshold value of $\beta$, above which welfare with cooperation on R&D exceeds welfare without, is increasing in $\delta$.

This result implies that cooperation is less likely to yield welfare benefits as the difficulty of absorbing rival R&D increases: a reduction in absorptive capacity undermines the welfare case for encouraging research joint ventures.

The implications of this result can be seen more clearly by concentrating on a special case. This combines the functional form for marginal production costs already given in (9) with Cournot behaviour, a linear demand function and a quadratic cost of R&D function.\(^{15}\) Figure 2 shows how welfare without and with R&D cooperation vary with $\beta$ and $\delta$. Panel (i) shows that welfare without cooperation reflects the asymmetric effect of higher difficulty on $\tilde{\theta}$, the full effectiveness of R&D, which we noted in Figure 1; while both panels (i) and (ii) show that the benefits of higher values of $\beta$ are quickly eroded as difficulty increases. Panel (iii) compares cooperation and non-cooperation directly, the region above $AB$ showing the size of the parameter space in which cooperation fails to raise welfare. The result of d’Aspremont and Jacquemin applies when $\delta$ is zero: cooperation leads to higher welfare when the spillover parameter $\beta$ exceeds 0.5. However, as $\delta$ rises, this advantage is rapidly eroded.

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\(^{15}\) The demand and R&D cost functions are $p(q+Q)=a-b(q+Q)$ and $\Gamma(x)=\frac{1}{2}\chi x^2$. The diagrams are drawn for the case of $n=2$ and $\eta=0.4$, where $\eta=\tilde{\theta}^2/b\chi$ is a measure of the relative effectiveness of R&D. In addition, $a-c_0$ and $b$ are normalised to equal unity. A Gauss program to draw the diagrams for arbitrary values of $n$ and $\eta$ is available at: http://www.economics.ox.ac.uk/members/peter.neary/neary.htm.
4. Information Sharing and Effective Spillovers

So far we have assumed that a decision by firms to engage in a research joint venture does not in itself affect the spillover coefficient \( \beta \). However, as emphasised by Kamien et al. (1992), Motta (1996), Katsoulacos and Ulph (1998) and others, cooperation is likely to involve information sharing and so an increase in \( \beta \). This increases the presumption that research joint ventures will increase welfare. Nevertheless, making information freely available does not guarantee that it can be freely absorbed: an increase in \( \beta \) does not imply a fall in \( \delta \) (the difficulty of absorbing rival R&D) and may have limited benefits if \( \delta \) is high. In this section we consider how the need to invest in absorptive capacity qualifies the benefits of information sharing.

To emphasise the contrast, we assume that a research joint venture leads to full information sharing (so \( \beta \) is unity) whereas non-cooperation implies no information sharing (so \( \beta \) is less than one). (For convenience we assume in this section that \( \beta \) is parametric.) Writing \( W^I(\delta) \) for the level of welfare attained with full information sharing, and \( W^C(\beta, \delta) \) for the level of welfare attained with cooperation in the absence of information sharing, we have by definition that:

\[
W^I(\delta) = W^C(1, \delta)
\]  

We can be sure that full information sharing does not lower welfare. Formally:

*Lemma 1*: Full information sharing cannot reduce the level of welfare when firms cooperate on R&D:
We now wish to show that, even with full information sharing, cooperation in R&D may lead to lower welfare than non-cooperation. A sufficient condition for this outcome is that, in the absence of spillovers, non-cooperation leads to higher welfare than cooperation:

\[ W^N(0, \delta) > W^C(0, \delta), \quad \forall \delta \]  

(19)

This condition is intuitively plausible, since with no spillovers, there are no R&D externalities to internalise, so the only role of cooperation is to act as a partial surrogate for product-market collusion. It therefore leads to unambiguously lower levels of R&D and output, which presumptively lowers welfare. The condition is satisfied by all the specific functional forms used in the literature. (Compare for example panels (i) and (ii) in Figure 2 at \( \beta=0 \).) It could conceivably be violated in cases where, in the absence of spillovers, non-cooperation leads to socially excessive levels of R&D, and cooperation offsets this. However, such cases must be

\[ W^L(\delta) \geq W^C(\beta, \delta), \quad \forall \beta \]  

(18)

The proof is immediate.\(^{16}\)

\(^{16}\) From equation (33) in Appendix 2, \( W^C(\beta, \delta) = W^S[x^C(\beta, \delta), \beta, \delta] \), where \( W^S \) is the second-best welfare function, conditional on oligopolistic behaviour in the second stage. To show that this is increasing in \( \beta \), note that \( \partial W^S/\partial x \) is positive for reasons given in Appendix 2 (following equation (34)); \( dx^C/d\beta \) is positive from Lemma A1 in Appendix 2; and \( \partial W^S/\partial \beta \) is positive by inspection.
considered unlikely. Ruling them out leads to the next result:

**Proposition 4:** Given (19), then, for every value of the spillover coefficient $\beta$ in the unit interval, there exists a threshold value of the difficulty of absorption coefficient $\delta$, $\delta(\beta)$, with $1 > \delta(\beta) \geq 0$, such that non-cooperation leads to higher welfare than cooperation with full information sharing, for all $\delta$ greater than $\delta(\beta)$.

**Proof:** Since (19) holds for all values of $\delta$, it holds when $\delta$ equals one. But when $\delta$ equals one, effective spillovers are zero and so both $W^C$ and $W^N$ are independent of $\beta$. Hence:

$$W^N(\beta, 1) > W^C(1, 1) = W^F(1), \quad \forall \beta \tag{20}$$

where the last equality follows from (17). This proves that, for every value of $\beta$, non-cooperation leads to higher welfare than full information sharing when $\delta$ takes its maximum value. Proposition 4 follows provided welfare in each regime is continuous in $\delta$.

*Q.E.D.*

Proposition 4 is illustrated by the region above $DB$ in Figure 3 for the special functional forms used earlier. As in panel (iii) of Figure 2, non-cooperation leads to higher welfare than

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17 "Socially" is crucial here. Absent spillovers, investment in R&D is greater without cooperation than with cooperation in many models, such as Cournot competition with linear demands or models of patent races. However, this does not imply that investment is socially excessive; on the contrary, cooperation in the absence of spillovers, by reducing investment and output, is likely to lower welfare. (On patent races see Reinganum (1989, p. 850).) Martin (2002) finds that cooperation raises welfare in a two-firm model of an R&D race even if input spillovers are zero; however, he assumes that both firms use the post-innovation technology, so in effect output spillovers are high.
cooperation without information sharing at all points above the AB locus. With full information sharing, the advantages of cooperation are naturally greater. However, if absorbing rival R&D is sufficiently difficult, then even cooperation with full information sharing is dominated by non-cooperation. This occurs for points in the region above DB. In this region, high difficulty reduces the effective spillover coefficient \( \hat{\beta} \) to such an extent that even full information sharing does not justify cooperation.

5. Extra-Industry Knowledge

A further implication of the absorptive capacity perspective is that firms must engage in R&D before they can reap the benefits of knowledge from outside the industry. This can be formalised by augmenting equation (1) as follows:

\[
c = c(x, y, k) \quad \text{where:} \quad k = k(x, K)
\]  

and \( y \) is determined by (4) as before. Own investment is needed to transform actual extra-industry knowledge \( K \) into usable extra-industry knowledge \( k \). Crucially, \( K \) is exogenously given, independent of the actions of firms in the industry, whereas \( k \) depends positively on both \( x \) and \( K \), and in turn reduces marginal cost \( c \).

An obvious implication of this specification is that the welfare levels in all the equilibria considered in the paper are increased if there is an exogenous increase in actual extra-industry knowledge \( K \). A more subtle implication is the effect of higher \( K \) on the strategic incentives to invest in R&D. As in Section 2, we can combine the components in (21) into a single reduced-form marginal cost function:
\[ \ddot{c}(x,X,K) = c[x,y(x,X),k(x,K)] \]  

(22)

In this augmented framework, the effectiveness of own R&D and the effective spillover coefficient from rival firms’ R&D become, instead of equation (6), the following:

\[ \begin{align*}
(a) \quad \tilde{\theta} &= -\tilde{c}_x = (1 + \beta y_x + \kappa k_x) \theta \\
(b) \quad \tilde{\beta} &= \frac{\tilde{c}_x}{\tilde{c}_x} = \frac{\beta y_x}{1 + \beta y_x + \kappa k_x}
\end{align*} \]  

(23)

where \( \kappa \) equals \( \frac{c_x}{c_x} \), the effectiveness of extra-industry knowledge relative to own R&D. Equation (23) implies that the effectiveness of R&D is now further increased, and the effective spillover coefficient \( \tilde{\beta} \) is now further reduced, relative to their direct counterparts, \( \theta \) and \( \beta \).

Proposition 1 can therefore be strengthened. With both \( \kappa \) and \( k_x \) strictly positive, \( \tilde{\theta} \) is strictly greater than \( \theta \), and \( \tilde{\beta} \) is strictly less than \( \beta \), even if \( y_x \) is zero and \( y_x \) is one.

Finally, we can show that more outside knowledge reduces the extent of effective spillovers. As in Section 2, we need some mild restrictions:

**Assumption 7:** \( \frac{d\beta}{dk} \leq 0 \).

**Assumption 8:** \( \frac{d(\beta/\kappa)}{dk} \leq 0 \).

In words, an increase in usable extra-industry knowledge does not raise the inter-firm spillover coefficient, either absolutely or relative to the extra-industry spillover coefficient \( \kappa \).

**Assumption 9:** \( k_{k_x} \geq 0 \).
This implies that an increase in own R&D does not reduce $k_K$, which can be interpreted as the marginal rate of absorption of extra-industry R&D.

We can now state:

*Proposition 5:* Given Assumptions 7 to 9, $\beta$ is decreasing in $K$.

(The proof is in Appendix 3.) Increasing external knowledge has an extra strategic effect, diluting the disincentive to refrain from investment which will benefit competitors. The policy message is clear (though, of course, the direct costs of increasing $K$ would have to be included in a complete cost-benefit calculation). Measures to raise the general level of research expertise in the economy are presumably desirable in themselves for a variety of reasons, not least because, unlike direct subsidies to R&D, they avoid the need for governments to pick winners and are less prone to capture. Our results show that they have the additional advantage of diluting the strategic disincentive to engage in research with unappropriable spillovers.

6. Conclusion

In this paper we have peeped inside the black box of R&D spillovers. A growing body of empirical evidence strongly supports the view that R&D increases a firm’s "absorptive capacity" (its ability to absorb spillovers from other firms) as well as contributing directly to profitability. To explore the theoretical and policy implications of this insight, we first specified a general

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18 For a discussion of the issues surrounding publicly-funded R&D, as well as a review of the econometric evidence for its effects on private R&D, see David and Hall (2000).
model of the absorptive capacity process which encompasses a number of special cases considered in previous work. We showed in this framework that costly absorption both raises the effectiveness of a firm’s own R&D and lowers the effective spillovers which it obtains from rival firms.

We then turned to consider the implications of the absorptive capacity perspective for the stance of public policy towards R&D. It is well-known that spillovers dilute the strategic incentive for competing firms to engage in R&D. It is also well-known that cooperation between firms has the effect of internalising the strategic externality between them, which, at least for high spillovers, leads to higher R&D and welfare. By contrast, we show in this paper that, when firms have an incentive to engage in R&D to build up absorptive capacity, the non-cooperative incentives to carry out R&D are enhanced, and so the ability of R&D cooperation to raise welfare is reduced. Surprisingly, this effect operates even when R&D cooperation leads to full information sharing between firms. These results weaken the case for encouraging research joint ventures.19

The final contribution of this paper is to examine the possibility that a firm's own R&D may help it to absorb knowledge from outside the industry as well as from rival firms. We show that the need to engage in R&D to absorb external knowledge further reduces the effective spillover coefficient between rival firms. This means that an increase in external knowledge has an extra strategic effect, over and above its obvious direct effect. This in turn implies an additional strategic pay-off to policies that raise the general level of research in the economy.

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19 It also weakens the case for subsidising R&D in open economies. See Leahy and Neary (1999).
Of course, inside the black box of spillovers, we find more black boxes. Our model simplifies by assuming that R&D spending is homogeneous, whereas many applications of the absorptive capacity concept have viewed it as privileging basic research ("R") at the expense of applied research ("D"). (See, for example, Cassiman, Pérez-Castrillo and Veugelers (2002), and see Hammerschmidt (2006) for a theoretical model which distinguishes between investment in R&D and investment in absorptive capacity.) Provided there is some substitutability between different kinds of R&D, the qualitative results of our paper still go through. Our parameterisation of extra-industry knowledge in Section 5 invites further refinement, though it may be the first attempt to formalise the notion of a firm’s "connectedness" to external knowledge, the importance of which is suggested by case studies of the pharmaceutical and semiconductor industries by Cockburn and Henderson (1998) and Lim (2000) respectively. Finally, our theoretical framework takes no account of the spatial dimension of R&D spillovers. Even within countries, Audretsch and Feldman (1996) have shown that spillovers tend to be very localised, so firms have an incentive to locate close to their rivals in order to reduce their need to invest in absorptive capacity. On a larger scale, Branstetter (2001) has shown that spillovers are more important intra- than internationally. This effect gives multinational firms an incentive to perform their R&D more centrally than their production in order to minimise technology transfer to rivals. Further work is needed to explore the implications of the absorptive capacity perspective for all these issues.
Appendix 1: Proof of Proposition 2

To prove part (i), differentiate $\tilde{\theta}$ from (6):

$$\frac{d\tilde{\theta}}{d\delta} = -(c_{xy} + c_{yy}y_x) y_{\delta} - c_y y_{x\delta}$$  \hspace{1cm} (24)

The first term in this expression, $-c_{xy} y_{\delta}$, is the derivative of the direct effectiveness coefficient with respect to $\delta$, $d\theta/d\delta$. We therefore need to sign the remainder of the expression, $d(\tilde{\theta}-\theta)/d\delta$, at the end-points $\delta=0$ and $\delta=1$. From Assumption 1, $y_x$ is zero at $\delta=0$ and $\delta=1$, and strictly positive for $0<\delta<1$. Hence it follows that $y_{\delta}$ and therefore $d(\tilde{\theta}-\theta)/d\delta$ itself, must be positive at $\delta=0$ and negative at $\delta=1$. (For example, this is true in the simplest case of the Kamien-Zang specification, equation (9), with two symmetric firms, so $x=X$: $c_{yx}=0$, and $y_{\delta}=1-2\delta$).

To prove part (ii), differentiate $\tilde{\beta}$ from (6):

$$\frac{d\tilde{\beta}}{d\delta} = \left[ (c_x c_{yy} - c_y c_{x\delta}) y_x y_{\delta} + c_x c_y y_{x\delta} + c_y^2 (y_x y_{x\delta} - y_y y_{x\delta}) \right] \frac{1}{\varepsilon_x^3}$$  \hspace{1cm} (25)

The three terms inside the square brackets are non-positive from Assumptions 3 plus 6, 4 and 5 respectively. The first term is zero when $\beta$ is fixed independent of $y$, and the third term is zero when $y$ is separable in $\{x,X\}$ and $\delta$.

Q.E.D.

Appendix 2: Proof of Proposition 3

We simplify in this appendix relative to the text by assuming that the direct spillover coefficient $\beta$ can be treated as a coefficient. In addition, we confine attention to comparisons between symmetric equilibria. Hence we can define a new cost function $\tilde{c}$ as a function of
symmetric $x$ as follows:
\[
\bar{c}(x, \beta, \delta) = c[x + \beta y \{x, (n - 1)x\}]
\]  
(26)

The derivatives of this are:
\[
\bar{c}_x = -\mu \tilde{\Theta} < 0; \quad \bar{c}_\beta = -\Theta y < 0; \quad \bar{c}_\delta = -\beta \Theta y_\delta > 0,
\]  
(27)

making use of (6), (2) and Assumption 3.

The first step in the proof is to show that equilibrium output is positively related to R&D and is shifted in the expected direction by changes in $\beta$ and $\delta$:

**Lemma A1:** Irrespective of the mode of competition, equilibrium output is increasing in R&D, and is shifted upwards by a rise in $\beta$ and downwards by a rise in $\delta$.

**Proof:** The period-2 first-order condition for a typical firm can be written as a function of symmetric $x$ and $a$ and of $\beta$ and $\delta$:
\[
\pi_i(x, a, \beta, \delta) = 0.
\]  
(28)

where the subscript "i" denotes a derivative with respect to an individual $a_i$. Differentiate and rearrange:
\[
(-\pi_{ia})da = \pi_{ix}dx + \pi_{i\beta}d\beta + \pi_{i\delta}d\delta
\]  
(29)

where the subscript "a" denotes a derivative with respect to symmetric $a$. The coefficient of $da$ (which equals $\Delta$ from footnote 8) is positive from the stability condition for the period-2 game. The term $\pi_{ix}$ equals $\tilde{\Theta}q_d\rho$ and is always positive under Cournot and negative under Bertrand.
competition. The other two terms can be signed from (27): \( \pi_{i\beta} = -q_i c_{i\beta} \) is positive under Cournot and negative under Bertrand competition; while \( \pi_{i\delta} = -q_i c_{i\delta} \) is negative under Cournot and positive under Bertrand competition. Finally, these ambiguous effects on equilibrium actions imply unambiguous effects on equilibrium outputs given that \( q_a = dq/da \) is positive under Cournot competition and negative under Bertrand competition, as stated in the text.

\[ Q.E.D. \]

Next, we need to sign the comparative statics of R&D in the cooperative equilibrium.

**Lemma A2:** The cooperative level of \( x \) is increasing in \( \beta \) and decreasing in \( \delta \), provided \( \mu / q_a \) and \( \theta_\beta \) are not too negative and \( \theta_\delta \) is not too positive.

**Proof:** The first-order condition for R&D of a typical firm under cooperation, equation (14), can be written in terms of industry profits as a function of symmetric \( x \) and \( a \) and of \( \beta \) and \( \delta \):

\[ \Pi_i(x, a, \beta, \delta) = 0 \]  

where the subscript "\( i \)" here denotes the derivative with respect to \( x_i \). Differentiating totally and using (29) to eliminate \( da \) gives:

\[ -\left[ \Pi_{ix} - \Pi_{ia} \pi_{ia}^{-1} \pi_{i\beta} \right] dx^C = \left[ \Pi_{i\beta} - \Pi_{ia} \pi_{ia}^{-1} \pi_{i\beta} \right] d\beta + \left[ \Pi_{i\delta} - \Pi_{ia} \pi_{ia}^{-1} \pi_{i\delta} \right] d\delta \]

On the left-hand side, the coefficient of \( dx^C \) is positive from the second-order condition for maximisation of industry profits by choice of R&D. Turning to the right-hand side, the term \( \Pi_{ia} = q_a (\mu + q_{\mu_a} / q_a) \theta \) is positive under Cournot competition and negative under Bertrand correction.
competition, provided \( \mu_C/q_a \) is not too negative. This term is zero if demands are linear, while under homogeneous-product Cournot competition it has the same sign as \( r' \), the marginal curvature of demand (where \( p(nq) \) is the inverse demand function and \( r=nqp''/p' \)). The term \( \Pi_{i\beta} \) in the coefficient of \( d\beta \) equals \( q(\mu_C^{\hat{\theta}+\mu_C^{\theta}}) \), which is positive provided \( \hat{\theta}_p \) is not too negative. Combined with the terms already signed in Lemma A1, this implies that \( dx_C/d\beta \) is positive. Finally, the term \( \Pi_{i\delta} \) in the coefficient of \( d\delta \) is \( q(\mu_C^{\hat{\delta}+\mu_C^{\theta}}) \), which is negative provided \( \hat{\theta}_\delta \) is not too positive. This implies that \( dx_C/d\delta \) is negative.

\[ Q.E.D. \]

Next, we must compare the levels of welfare with and without cooperation. In symmetric equilibria the welfare function can be written as \( W=W(x,a,\beta,\delta) \). As in Suzumura (1992), we can use Lemma A1 to eliminate \( a \) to obtain the second-best welfare function, \( W^S \):

\[
W^S(x,\beta,\delta) = W[x,a(x,\beta,\delta),\beta,\delta]
\]  

This can be used to evaluate different R&D levels given second-period competition in actions. The actual R&D levels under cooperation and non-cooperation depend on the spillover and difficulty parameters. Using \( x^h(\beta,\delta) \) and \( x^C(\beta,\delta) \) in (32) gives:

\[
W^h(\beta,\delta) = W^S[x^h(\beta,\delta),\beta,\delta], \quad h=N,C.
\]

Let \( W^D(\beta,\delta)=W^C(\beta,\delta)-W^N(\beta,\delta) \) denote the difference between the cooperative and the non-cooperative levels of welfare at given values of \( \beta \) and \( \delta \). Similarly, we define \( \mu^D(x,a,\beta,\delta) \) as \( \mu^C(x,a,\beta,\delta)-\mu^N(x,a,\beta,\delta) \), etc. We can now state Proposition 3 in full:
Proposition 3: Assume the following: (1) The second-best welfare function is quasi-concave in R&D; (2) the Seade (1980) stability condition holds at the non-cooperative R&D equilibrium; (3) the conditions of Lemma A2 apply; and (4) the product $\mu^D_i q_i$ and the derivative $\tilde{\beta}_x$ are not too negative. Then the threshold value of $\beta$, above which welfare with cooperation in R&D exceeds welfare without cooperation, is increasing in $\delta$.

Proof: The strategy of the proof is to show that $W^D_\beta > 0$ and $W^D_\delta < 0$ at all $W^D = 0$. Since $d\beta/d\delta$ conditional on $W^D = 0$ equals $-W^D_\delta / W^D_\beta$, this suffices to prove the proposition. Consider first the total differential of $W^D(\beta, \delta) = 0$. We assume that the R&D levels under cooperation and non-cooperation are identical when the welfare levels are equal: $W^D = 0$ implies that $x^D = 0$. (We rule out pathological cases in which $W^D = 0$ but $x^C \neq x^N$: these can only occur if $x^N > x^S$.) Hence:

$$dW^D = W^D_\beta d\beta + W^D_\delta d\delta = W^S_x \left( x^D_\beta d\beta + x^D_\delta d\delta \right) = 0$$

(34)

$W^S_x$ is positive given the quasi-concavity of the second-best welfare function $W^S$ in R&D and the fact that the cooperative level of R&D is below the second-best optimal level. Therefore it must be the case that $W^D_\beta$ has the same sign as $x^D_\beta$ and that $W^D_\delta$ has the same sign as $x^D_\delta$. All we then have to show is that $x^D_\beta$ is positive and $x^D_\delta$ is negative.

The cooperative and non-cooperative R&D levels satisfy first-order conditions of the form: $\mu^h(x^h, a^h, \beta, \delta) \tilde{\theta}^h(x^h, \beta, \delta) q^h(a^h) = \Gamma^h(x^h)$, where $h = C, N$. Since they also satisfy the first-order condition for actions in (28), we can use Lemma A1 to eliminate $a^h$. The cooperative and non-cooperative first-order conditions for R&D can then be written in compact form as:
\[ m^h(x^h, \beta, \delta) = \Gamma'(x^h) \quad h=C,N \]  

(35)

where the left-hand side is: \( m^h(x^h, \beta, \delta) \equiv \mu^h[x^h, a(x^h, \beta, \delta), \beta, \delta] \bar{\theta}(x^h, \beta, \delta) q^h[a(x^h, \beta, \delta)] \). Totally differentiate (35) with respect to \( \beta \):

\[ \left( m^h_x - \Gamma'' \right) x^h_\beta + m^h_\beta = 0, \quad h=C,N. \]  

(36)

Subtracting the two equations in (36), writing \( m^D(x,\beta,\delta) \) for \( m^C(x,\beta,\delta) - m^N(x,\beta,\delta) \), and manipulating gives:

\[ x^D_\beta = -\frac{1}{m^N_x - \Gamma''} \left( m^D_x x^C_\beta + m^D_\beta \right) \]  

(37)

To determine the sign of the right-hand side of (37), first note that the denominator \( m^N_x - \Gamma'' \) is negative from the Seade (1980) stability condition for the non-cooperative R&D equilibrium. The sign of the right-hand side therefore depends on the sign of the expression in square brackets. We consider the individual terms in this expression in turn.

The derivative \( x^C_\beta \) is positive from Lemma A2. Since \( m^C_x \) and \( m^N_x \) are evaluated at the same point, we can write:

\[ m^D_x = \left( \alpha^D \beta_x + \mu^D a_i q_i \frac{\mu^D \delta}{\Delta} \right) \delta q \]  

(38)

The first term in brackets has the same sign as \( \beta_x \), while the second has the same sign as \( \mu^D a_i q_i \). Assumption 4 of the Proposition requires that these two terms are not sufficiently negative that \( x^D_\beta \) in (37) becomes negative. This is guaranteed in many plausible special cases. (For example, from footnote 5, \( \beta_x \) is strictly positive for the Cohen-Levinthal specification and zero for the Kamien-Zang specification of the absorptive capacity function. As for \( \mu^D a_i q_i \), with linear demands,
whether in Cournot or Bertrand competition, it is zero. In homogeneous-product Cournot
competition it has the same sign as the marginal curvature of demand, $r'$. Finally, since $m^C_\beta$ and
$m^N_\beta$ are evaluated at the same point, we can write:

$$m^D_\beta = \left( \mu^D_\theta + \mu^D_\Theta q_i \frac{\theta y}{\Delta} \right) \tilde{q}$$  \hspace{1cm} \text{where:} \hspace{1cm} \mu^D_\theta = \alpha' \bar{\beta} > 0 \tag{39}$$

Once again, this expression, and hence (37) as a whole, is positive provided $\mu_i^D q_i$ is not too
negative. Substituting (37) into (34) completes the proof that $W^D_\beta$ is positive. A similar chain
of reasoning can be used to show that $W^D_\delta$ is negative. This completes the proof that the
threshold value of $\beta$ is monotonically increasing in $\delta$.

\textit{Q.E.D.}

\textbf{Appendix 3: Proof of Proposition 5}

Differentiating $\beta$ from (23):

$$\frac{d\bar{\beta}}{dK} = \left[ (c_x c_y - c_y c_{xy}) k_x k - (c_x c_y - c_y c_{kk}) k_x k - c_y c_{kx} k \right] \frac{y_x}{x^2} \tag{40}$$

The three terms inside the square brackets are non-positive, from Assumptions 8, 7 and 9
respectively. The first and second terms are zero respectively when $\beta$ and $\kappa/\beta$ are fixed and
independent of $K$.  

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References


Fig. 1: Effects of the Difficulty of Absorption

(i) Full Effectiveness of R&D

(ii) Effective Spillover Parameter
Fig. 2: Welfare Without and With Cooperation

(i) Without Cooperation

(ii) With Cooperation
Fig. 3: Welfare With and Without Information Sharing