CROSS-BORDER MERGERS AS INSTRUMENTS OF COMPARATIVE ADVANTAGE* †

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Abstract

A two-country model of oligopoly in general equilibrium is used to show how changes in market structure accompany the process of trade and capital market liberalisation. The model predicts that bilateral mergers in which low-cost firms buy out higher-cost foreign rivals are profitable under Cournot competition. As a result, trade liberalisation can trigger international merger waves, in the process encouraging countries to specialize and trade more in accordance with comparative advantage. With symmetric countries, welfare is likely to rise, though the distribution of income always shifts towards profits.

Keywords: Comparative advantage; cross-border mergers; GOLE (General Oligopolistic Equilibrium); market integration; merger waves.

JEL Classification: F10, F12, L13

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1 Introduction

Cross-border mergers are an increasingly important phenomenon in the world economy. They comprise well over half of all foreign direct investment (FDI), considerably more than greenfield investment. They also constitute an increasing proportion of all mergers.\(^1\) Moreover, there is considerable anecdotal and other evidence suggesting that cross-border merger waves coincide with episodes of trade liberalization and market integration. A study by the European Commission concluded that cross-border mergers were the dominant form of adjustment by European firms to the extension of the EU Single Market, and similar patterns have been found for the Mercosur economic union in Latin America.\(^2\) Yet the theoretical literature on cross-border mergers is tiny, both in absolute terms and relative to the enormous literature on greenfield FDI.\(^3\)

How to explain cross-border merger waves? Both international trade theory and the theory of industrial organization might be expected to throw some light on them, but neither provides a fully satisfactory explanation.\(^4\) Consider first the theory of industrial organization. The enormous I.O. literature on the topic suggests two broad motives for mergers: an efficiency motive and a strategic motive. Efficiency gains can arise from a variety of sources, such as cost savings via internal technology transfer, economies in the use of firm-specific assets, managerial synergies, or the integration of pricing and marketing decisions on differentiated products. However, mergers may also raise costs, as different managerial and production structures and different corporate cultures have to be integrated. In any case, the empirical evidence on efficiency gains is far from conclusive.\(^5\) As for the strategic motive, the benchmark case where symmetric firms engage in Cournot competition and there are no efficiency gains yields what is sometimes called the “Cournot merger paradox”, due to Salant, Switzer and Reynolds (1983). They showed that mergers between identical firms are unprofitable unless the merged firms produce a very high proportion of pre-merger industry output: over 80% when demand is linear. Subsequent work has shown that mergers may be profitable if Cournot competition is extended in various ways, such as allowing cost synergies, convex demand or union-firm bargaining. It

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\(^1\) UNCTAD (2000) documents the increasing share of cross-border mergers in FDI, rising on their measure to 78% in the late 1990s. Gugler et al. (2003) study 2,753 mergers worldwide from 1981 to 1998, and find an upward trend in the percentage of mergers which are cross-border, a trend which is particularly pronounced for EU countries in the 1990’s. For example, the percentage of all mergers in Continental Europe which were cross-border rose from 24.2% in 1991-92 to 39.8% in 1997-98.


\(^3\) For models of greenfield FDI, see for example Helpman (1984), Markusen (2002), Barba-Navaretti et al. (2004) and Helpman, Melitz and Yeaple (2004).

\(^4\) A third approach to merger waves is to relate them to major technological innovations. For example, Jovanovic and Rousseau (2003) construct an endogenous growth model in which merger activity is driven by pressure to reallocate capital, stimulated by the diffusion of new general-purpose technologies. This effect is likely to be important in explaining many real-world mergers: Jovanovic and Rousseau argue that four of the five major merger waves of the twentieth century can be explained by the diffusion of electricity and information technology. However, it does not explain why they should intensify in periods of adjustment to trade liberalization.

\(^5\) The study by Gugler et al. (2003) mentioned in footnote 1 finds that, relative to the median firm in the same 2-digit industry, acquiring firms had significantly higher profits but lower sales in each of the five years after the merger. They interpret these results as implying that the average profitable merger in their sample increased market power but did not yield significant cost savings.
has also shown that the opposite problem, of too many mergers rather than too few, holds in Bertrand competition.6

While the industrial organization approach throws a lot of light on mergers, it provides an incomplete basis for understanding cross-border mergers. Especially if we want to relate cross-border mergers to an economy-wide shock such as trade liberalization, partial equilibrium models which take demands and factor prices as given cannot tell the whole story.7 This suggests that a more promising route might start from the theory of international trade, with its almost two-centuries-old tradition of studying trade liberalization in general equilibrium. But here there is a different problem. The two dominant paradigms in trade theory assume either perfect or monopolistic competition. While these differ in their assumptions about returns to scale and product differentiation, they model firms in the same way: as atomistic agents, which are in infinitely elastic supply, face no barriers to entry or exit, and do not engage in strategic interaction. This framework leaves relatively little scope for discussing mergers.8 A fully satisfactory theory linking trade liberalization and mergers requires a theory of oligopoly in general equilibrium, but progress in this direction has been held back by the generally negative results of Gabszewicz and Vial (1972) and Roberts and Sonnenschein (1977).

In this paper I use a model of oligopoly in general equilibrium introduced elsewhere to show how trade liberalization can lead to cross-border merger waves. (The model is presented in Neary (2002); for a non-technical overview see Neary (2003).) The model draws on the traditions of both industrial organization and international trade theory. It allows for strategic interaction between firms, so permitting a game-theoretic approach to explaining merger activity. At the same time, it is a completely specified general equilibrium model, so making it possible to track the full effects of trade liberalization on trade and production patterns. The problems of modelling oligopolistic interaction in general equilibrium highlighted by Gabszewicz and Vial (1972) and Roberts and Sonnenschein (1977), among others, are avoided by assuming a continuum

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6 From a large literature, see Perry and Porter (1985) and Farrell and Shapiro (1990) on cost synergies, Fauli-Oller (1997) and Leahy (2002) on demand convexities, Lommerud, Straume and Sørgard (2005) on union-firm bargaining, and Deneckere and Davidson (1985) on Bertrand competition. Recent papers by Fauli-Oller (2000), Macho-Stadler, Pérez-Castrillo and Porteiro (2006) and Toxvaerd (2002) model sequential mergers. Of these, the model of Fauli-Oller is closest in spirit to that of the present paper, but like all the papers mentioned it is cast in partial equilibrium.

7 There is also a small literature which examines international mergers in partial equilibrium, much of it concerned with issues of merger policy design. See Bertrand and Zitouna (2006), Falvey (1998), Francois and Horn (1998), Head and Ries (1997), Horn and Levinsohn (2001) and Long and Voussel (1995). An alternative approach, taken by Horn and Persson (2001), is to explore international mergers as the outcome of a cooperative game.

8 Three interesting recent contributions are partial exceptions. Marin and Verdier (2002) use a principal-agent framework to model the internal organisation of firms in general equilibrium with monopolistic competition. They show that a rise in the intensity of competition, modelled as an increase in the elasticity of substitution between goods, can induce a shift from an equilibrium with mostly agent-oriented organisations towards one with mostly principal-oriented organisations, which they interpret as a merger wave. However, the mergers they predict are solely vertical. Barbas-Navares et al. (2004) show how two firms in monopolistic competition have an incentive to merge, but do not explore the implications of a first merger for subsequent ones, since this would eventually invalidate the “large-group” assumption. Finally, Nocke and Yeaple (2007) model cross-border mergers as international purchases and sales of country-specific firm capabilities (e.g., distribution networks), and show how inter-firm differences in capabilities affect the choice between greenfield and acquisition FDI. However, they assume that the market for capabilities is perfectly competitive.
of oligopolistic sectors with economy-wide factor markets, so that firms have market power in their own sector but cannot influence factor prices or national income. In addition, since the model builds on the standard Ricardian framework, it permits an exploration of the process by which specialization according to comparative advantage may or may not be helped by the rational decisions of individual oligopolistic firms. Finally, by concentrating on the case where mergers take place for strategic motives only, the model allows a clear focus on the possibility that mergers which do not yield efficiency gains may nonetheless lead to improved resource allocation.

The initial equilibrium in the paper is one where trade is free but cross-border mergers are not permitted. This can be thought of as a hypothetical step on the way from autarky to complete liberalization of trade and foreign investment. Alternatively, it can be viewed as representing a situation such as prevailed in the European Union before the Single Market Act of 1992, in which trade barriers have been eliminated but restrictions on cross-border mergers and acquisitions remain in force. This free-trade equilibrium is then disturbed by an exogenous and unanticipated abolition of restrictions on cross-border mergers.

The first part of the paper considers mergers in a single industry, in which there are exogenous cost differences between home and foreign firms. Section 2 introduces the model, while Sections 3 and 4 show how trade liberalization can induce cross-border mergers, both when mergers take place in response to myopic incentives, and when firms contemplating a merger anticipate its effects on their own and other firms’ future actions. The second part of the paper turns to general equilibrium to show how costs are determined endogenously. Section 5 reviews the model of general oligopolistic equilibrium introduced in Neary (2002), and Sections 6 and 7 explore its implications for the likelihood of cross-border mergers and for their effects on resource allocation, income distribution and welfare. Proofs of all propositions, along with other technical details, are given in the Appendix.

2 Specialization Patterns in the Absence of Mergers

Until Section 5, we consider a single sector in partial equilibrium. Hence, we can suppress the sectoral index, and, like the participating firms themselves, we treat the cost and demand parameters as exogenous.

Consider a sector with a small number of firms located in two countries, called “home” and “foreign”. The firms produce identical products and, following the liberalization of trade, engage in Cournot competition on an integrated world market. The world inverse demand function they face is:

\[ p = a' - b' \bar{x} \] (1)
where $\bar{x}$ is world demand. This function is linear from the perspective of firms, though the primes attached to the coefficients $a'$ and $b'$ are there to remind us that it is highly non-linear in general equilibrium, as we will see in Section 5. All firms in a given location have the same unit cost: $c$ for home firms and $c^*$ for foreign. In autarky there are $n$ firms at home and $n^*$ abroad, with $\bar{n}$ equal to $n$ plus $n^*$. Because of exogenous barriers to entry, trade liberalization cannot induce entry of new firms. However, it can induce exit, either because some firms become unprofitable and produce zero output or (as we will see in the next section) because they are taken over. In free-trade equilibrium world demand equals world output: $\bar{x} = ny + n^* y^*$, where the output of either a typical home or foreign firm, $y$ or $y^*$, may be zero.

Following Neary (2002), it is convenient to illustrate the possible regimes in $\{c, c^*\}$ space. The condition for home firms to be profitable is easily stated. We abstract from fixed costs, since they would provide a trivial justification for mergers. Hence, profits are proportional to the square of output, $\pi = b'y^2$, and a positive level of profits is equivalent to a positive level of output. From the expression for home output, given by equation (23) in the Appendix, profits of a home firm will be positive if and only if its unit cost is less than or equal to a weighted average of the demand intercept and the unit cost of foreign firms, where the weight attached to the former is decreasing in the number of foreign firms:

$$c \leq \xi_0 a' + (1 - \xi_0)c^* \quad \text{where:} \quad \xi_0 = \frac{1}{n^* + 1} \leq 1$$ (2)

This condition continues to hold when the number of active foreign firms $n^*$ is zero; in that case, the weight $\xi_0$ equals one, so home profitability requires that the unit cost of a home firm cannot exceed the demand intercept, the maximum price which consumers are willing to pay: $c \leq a'$. These two conditions ((2) with $n^* > 0$ and $n^* = 0$), along with corresponding profitability conditions for foreign firms (expressed in terms of $\xi_0^* = 1/(n + 1)$), define four regions in $\{c, c^*\}$ space, as illustrated in Fig. 1. (We defer discussion of the downward-sloping locus until Section 5.) If the costs of all firms exceed $a'$, as in region $O$, then the good is not produced. If firms in only one country have relatively low costs, then only they will produce it, as in regions $H$ and $F$ (where only home and foreign firms respectively can compete). Finally, region $HF$ is a “cone of diversification” where both home and foreign firms can coexist: barriers to entry act as a surrogate for tariffs, allowing high-cost firms in one country to survive in the face of low-cost rivals in the other. In the competitive limit (as $\xi_0$ and $\xi_0^*$ approach zero), this region collapses to the 45° line, and the two countries specialize completely according to comparative advantage.

The discussion so far assumes that, in both countries, all firms which are profitable in free trade continue

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9 Variables referring to the foreign country are denoted by an asterisk, and (where necessary to avoid ambiguity) variables referring to the world as a whole are denoted by an over-bar.

10 By contrast, sunk costs of entry cannot be recouped and so have no effect on merger decisions.
to produce. However, opening markets to international competition may generate incentives for mergers. The next sections show how the analysis must be amended when these incentives are taken into account.

3 Myopic Merger Incentives

We need to impose some structure on how firms decide to merge. I first make the plausible assumption that only two firms merge at any one time:

*Assumption 1:* Only bilateral mergers can occur.

This does not preclude sequential mergers, but it implies that they must consist of a sequence of bilateral mergers, each of which is desired by both parties. Assumption 1 could be justified by an underlying assumption that simultaneous negotiations between more than two firms face prohibitive transactions costs. However, its main justification is empirical: If simultaneous mergers between any number of firms were possible, then there would always be an incentive for all firms to merge, and the model could not capture the empirical phenomenon of merger waves that stop short of monopoly.\(^{11}\)

My next assumption is that a merger must yield a surplus which is sufficient to compensate both participating firms. Since firms produce identical products and there are no tariffs or transport costs, there is no incentive for a firm to operate more than one plant, and so a merger implies that one of the participating firms is closed down. Hence the surplus following a merger equals the profits of the surviving firm less the initial profits of both firms. To fix ideas, consider the case where there are initially \(n\) home and \(n^*\) foreign firms, and a home firm is taken over. The surplus or net gain from such a merger is:

\[
G_{FH}(n, n^*) \equiv \pi^* (n-1, n^*) - \pi^* (n, n^*) - \pi(n, n^*)
\]

where \(\pi(n, n^*)\) and \(\pi^*(n, n^*)\) denote the profits of a home and foreign firm respectively, when there are \(n\) active home firms and \(n^*\) active foreign firms. Henceforward I assume that this surplus must be strictly positive for a merger to occur:\(^{12}\)

*Assumption 2:* A merger will not take place if the gain defined in (3) is zero or negative.

Since the home firm ceases to produce, this merger can be thought of as a takeover of the home firm by the foreign firm, and I will use the terms “merger” and “takeover” interchangeably from now on. Assumption 2 should be interpreted as reflecting the inability of firms to borrow against future mergers. If such

\(^{11}\)Macho-Statler et al. (2006) make the same assumption and present empirical evidence in support of its realism.

\(^{12}\)The acquired firm must be paid at least its initial profits \(\pi(n, n^*)\) plus an infinitesimal amount if it is to agree to a merger, which is why \(G_{FH}(n, n^*)\) must be strictly positive for a merger to take place.
borrowing were allowed, then multilateral mergers would effectively be possible and so Assumption 1 could be circumvented. In particular, a merger to monopoly, since it ensures a sufficient surplus to compensate all firms, would always be possible.

Assumption 2 is only a necessary condition for a merger to take place, since it does not rule out forward-looking behaviour. However, I defer consideration of such behaviour until the next section. In the remainder of this section, I explore the implications of assuming that a positive value of $G_{FH}(n, n^*)$ is sufficient as well as necessary for a merger to take place:

**Assumption 3:** A merger will take place if the gain defined in (3) is strictly positive.

As we will see in Section 4, Assumption 3 is consistent with cases where firms would earn higher profits by refraining from merging. Hence, I describe a positive value for $G_{FH}(n, n^*)$ as a myopic merger incentive.

Armed with Assumptions 1 to 3, I can now proceed to explore in more detail the circumstances when a merger will occur. Equation (3) provides some intuition for the Salant-Switzer-Reynolds result: if the two participating firms have the same costs, and hence the same initial outputs and profits, the profits of the acquiring firm (and of all firms like it) would have to double for a takeover to be profitable. This result continues to hold if other firms have costs which differ from those of the two firms involved in the merger. For completeness I state this formally, though this result is closely related to the central result in Salant et al. (1983):

**Proposition 1:** A merger between two firms with the same unit cost (whether two home or two foreign firms), is never profitable, provided $n + n^* > 2$.

(Proofs of all propositions are in the Appendix.) This result implies that there are no incentives to merge in the $H$ and $F$ regions of Fig. 1 (except in the trivial case of only two firms, when a merger to monopoly is always profitable). Hence the remainder of our discussion of the partial equilibrium case concentrates on the cone of diversification region, $HF$. Proposition 1 also implies that there are no incentives to merge in autarky (except in the case of only two firms in one or other country, which we exclude by assumption). Hence our starting point of $n$ and $n^*$ firms in each country is an equilibrium industrial structure in the absence of trade.

By contrast with Proposition 1, the next result shows that, when free trade is opened up, and provided the cost differential between the two participating firms is sufficiently large, the gain to a takeover is strictly positive:

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13Relative to Salant et al., Proposition 1 is less general since it considers only bilateral mergers, but more general since it allows the unit costs of uninvolved firms to differ from those of the involved firms. Falvey (1998) shows that Proposition 1 holds for an arbitrary distribution of the marginal costs of non-participating firms.
Proposition 2: A takeover of a home firm by a foreign firm meets the myopic merger criterion \( G_{FH} > 0 \) if and only if:

\[
c > \xi_1 a' + (1 - \xi_1) c^*
\]  

where:

\[
\xi_1 = \frac{n^2 - 2n - 1}{2n\bar{n} + (n^* + 1)(\bar{n}^2 - 1)}, \quad \xi_0 > \xi_1 > 0
\]

Proposition 2 states that a takeover of a home firm will be profitable provided its unit cost exceeds a weighted average of the demand intercept and the unit cost of the acquiring firm. Key features of the result are that the condition for the myopic merger criterion to hold has the same form as the profitability condition, equation (2), and that the weight \( \xi_1 \) defined in (5) is positive but smaller than the weight \( \xi_0 \) in (2). Hence, though \( \xi_1 \) is highly non-linear in \( n \) and \( n^* \), equation (4) with equality defines for given \( n \) and \( n^* \) a linear upward-sloping locus in Fig. 2 which lies strictly below that implied by (2). (This figure repeats the regions of specialization from Fig. 1, with the boundaries of the \( HF \) region corresponding to zero profitability of home and foreign firms denoted by dotted lines). Introducing the possibility of takeovers has the effect of expanding the \( F \) region, adding to it a sub-region in Fig. 2 in which high-cost home firms can earn positive profits, but are vulnerable to a bilateral takeover by low-cost foreign rivals.

Intuitively, the reason for Proposition 2 is that outputs are strategic substitutes in Cournot competition, at least when demands are linear. (See Gaudet and Salant (1991) for further discussion.) Hence, a takeover which closes down one firm raises output and therefore operating profits for all surviving firms (including the acquiring firm). Provided the cost differential is sufficiently great, the profits of a low-cost foreign firm increase sufficiently to justify its taking over a high-cost home one. A large cost differential is only a sufficient condition however, since Proposition 2 shows that the numbers of firms of both types also matter, through their effect on \( \xi_1 \). The easiest way to see the effect of \( n \) and \( n^* \) is to consider the difference \( \xi_0 - \xi_1 \), which from Fig. 2 measures the size of the mergers region along the vertical axis (where \( c^* = 0 \)) when \( a' \) is normalized to one. Fig. 4 shows that \( \xi_0 - \xi_1 \) is decreasing in both \( n \) and \( n^* \), though it is more sensitive to the latter.

The implications of this can be summarized as follows:

Corollary: For given cost and demand parameters \( (c, c^* \text{ and } a') \), takeovers are more likely the more concentrated the industry both in the home, target, country, and especially in the foreign, acquiring, country.

The discussion of foreign takeovers so far can also be applied, \textit{mutatis mutandis}, to home takeovers. Similar derivations to those which yield Proposition 2, with the roles of the countries reversed, show that a
takeover of a foreign firm by a domestic one is profitable if and only if:

\[ c^* > \xi_1^* a' + (1 - \xi_1^*) c \]  

(6)

where \( \xi_1^* \) is defined in the same way as in (5), except that \( n \) replaces \( n^* \) and vice versa. Hence there is a similar region in Fig. 2 in which high-cost foreign firms can earn positive profits, but are vulnerable to a bilateral takeover by low-cost home rivals.

Next, we can ask what will be the effect of such a bilateral takeover on the incentives for further takeovers in the same industry. Because all outputs are strategic substitutes, the outputs, and hence the profits, of all surviving firms, both home and foreign, increase. Indeed, with linear demands, the outputs of all surviving firms rise by the same absolute amount.\(^{14}\) But the low-cost foreign firms have larger output to begin with. Hence, since profits are proportional to the square of output, a further takeover increases their profits by more. Formally, whenever \( G_{FH} \) is positive, it is decreasing in \( n \):

**Proposition 3**: A profitable takeover of a home firm increases the gain to a takeover of one of the remaining home firms by a foreign firm.

It follows that a fall in \( n \) as a result of a takeover raises the incentive for a further takeover. This is the basis for the prediction of merger waves. A profitable takeover of one home firm by a foreign firm increases the gain to a takeover of one of the remaining home firms by a foreign firm in the same sector.

Because all home firms have identical costs, Proposition 3 implies that, if one of them is taken over, then all of them will be. Within sectors where a first takeover is profitable, further takeovers are even more profitable. Hence no high-cost firms survive. However, because of Assumption 2, the cone of diversification \( HF \) does not vanish altogether. In that region, bilateral mergers would be profitable if there were fewer high-cost firms to begin with, i.e., if some bilateral mergers had already taken place. But, given the initial number of high-cost firms, the first bilateral takeover is not profitable, and so a merger wave is not initiated. A final implication of Proposition 3 is that, in the absence of cost synergies, encouraging “national champions” by promoting domestic mergers in high-cost sectors makes foreign takeovers more rather than less likely.

### 4 Forward-Looking Merger Incentives

So far, I have worked only with the myopic merger criterion embodied in Assumption 3. Putting this differently, the equilibria in the previous section are not sub-game perfect. This matters because it means

\[^{14}\text{The increase equals the initial output of the takeover target divided by the initial number of firms in both countries: } y(n - 1, n^*) - y(n, n^*) = y^*(n - 1, n^*) - y^*(n, n^*) = y(n, n^*)/n. \text{ See Lemma 1 in the Appendix, with } \bar{n} \text{ set equal to } n - 1.\]
that firms ignore the “after-you” or free-rider problem, which arises for both low- and high-cost firms. In the words of Stigler (1950), “Firms will do everything possible to encourage a merger, short of participating in one themselves.” For low-cost foreign firms, all face the same takeover incentive $G_{FH}(n, n^*)$, but uninvolved firms reap the gains, $\pi^*(n-1, n^*) - \pi^*(n, n^*)$, without having to incur the cost of acquiring a home firm, $\pi(n, n^*)$. Hence no individual foreign firm may opt to engage in a takeover, if it believes that another foreign firm will do so first. High-cost home firms face a different free-rider problem. Since $\pi(n, n^*)$ is decreasing in $n$, each high-cost firm that is made a takeover offer has a higher outside option than high-cost firms which have already been taken over. Hence, if payoffs are determined on the basis of the myopic merger criterion, (3), every high-cost firm would like to be taken over after rather than before other firms like it.

Two different responses can be given to this problem, and both have some validity. On the one hand, the model of the last section can be interpreted as making the empirical prediction that trade liberalization increases the conditional probability of cross-border merger waves. A positive value of $G_{FH}(n, n^*)$ does not guarantee that a takeover will take place, partly because of free-riding but also because there are many other reasons why takeovers which promise net gains may not take place (especially if the gains are small), such as transactions costs, uncertainty, or managerial hubris. Nevertheless, it provides a clear-cut prediction which can be confronted with the data. On the other hand, it would be very desirable to construct a properly specified extensive-form game which is consistent with cross-border mergers taking place, even if this requires making some further assumptions. In the remainder of this section I outline one approach which meets this requirement and show that it yields the same qualitative predictions as the model of Section 3. Readers less interested in this extension may wish to skip to the general-equilibrium discussion in Section 5.

I retain Assumptions 1 and 2 from the previous section, but drop Assumption 3. In its place, I put more structure on the way in which firms interact as follows:  

**Assumption 4**: (a) The pre-production phase of the game lasts for $n$ stages; (b) in each stage a randomly selected pair of firms, one high-cost and the other low-cost, meet; (c) at every such meeting, the low-cost firm makes a take-it-or-leave-it offer to the high-cost firm; and (d) all firms are risk-neutral.

Part (a) of Assumption 4 ensures that firms do not have an incentive to postpone indefinitely; part (b) borrows from the literature on search and matching the idea that friction is equivalent to the random arrival of trading partners; and parts (c) and (d) allow the pay-offs to both parties to be calculated explicitly.  

With forward-looking behaviour, we need to solve for the full sequence of mergers at once. Consider

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15 For simplicity, I assume that no production takes place before all mergers are completed, and that the $n$ stages in the pre-production phase are sufficiently short so discounting can be ignored.
16 Alternative sharing rules could be used instead of (c): for example, the Nash bargaining outcome is a natural candidate. This will alter the equilibrium but not significantly affect the conclusions reached below.
the case where there are $s$ stages and $\hat{n}$ high-cost home firms remaining in the pre-production phase of the game. (From Assumption 4, at most one high-cost firm is taken over in each stage. Hence $s$ cannot exceed $\hat{n}$: $s \leq \hat{n}$.) Define the expected returns to the two types of firms in these circumstances as follows:

**Definition 1:** $R(s, \hat{n}, n^*)$ is the minimum reward a high-cost firm requires to agree to a takeover when $s$ stages and $\hat{n}$ high-cost home firms remain.

**Definition 2:** $R^*(s, \hat{n}, n^*)$ is the expected ex ante payoff to a low-cost firm entering the $s$'th last stage when $\hat{n}$ high-cost home firms remain.

We can now state the criterion for a forward-looking merger wave:

**Proposition 4:** Given Assumptions 1, 2 and 4, a merger wave will take place if the following expression is strictly positive:

$$G_{FH}^0(n, n^*) = R^*(n-1, n-1, n^*) - R^*(n-1, n, n^*) - R(n, n, n^*)$$

(7)

The expression for $G_{FH}^0(n, n^*)$ in (7) emphasizes the parallel with the myopic merger criterion (3) in the last section, with the forward-looking functions $R^*$ and $R$ replacing the myopic pay-offs $\pi^*$ and $\pi$ respectively, and (7) reduces to (3) when $s$ equals one. It is easier to explain intuitively when the terms in $R^*$ are expanded using equation (36) in the Appendix, which gives:

$$G_{FH}^0(n, n^*) = \pi^*(0, n^*) - \pi^*(1, n^*)$$

$$- \frac{1}{n^*} \sum_{j=1}^{n-1} [R(n-j, n-j, n^*) - R(n-j, n-j+1, n^*)] - R(n, n, n^*)$$

(8)

Consider a low-cost firm contemplating a takeover in the first stage ($s = n$), assuming that all future $n-1$ takeover offers are made and accepted. Equation (8) shows that making a takeover offer now affects its current and expected future profits in three different ways. First, by reducing the final number of high-cost firms from one to zero, it tends to raise expected profits. This effect is given by the two terms on the first line of the right-hand side of (8). Second, it changes the payoffs to being matched in all future $n-1$ pre-production stages: with one less high-cost firm in each stage, the low-cost firm has a $1/n^*$ probability of having to pay an amount which, from Corollary 3 in the Appendix, is greater. This effect is given by the sum of $n-1$ terms in the second line. Finally, of course, the high-cost firm has to be paid its outside option, as shown by the final term in the second line. The first effect tends unambiguously to raise $G_{FH}^0(n, n^*)$ relative to $G_{FH}(n, n^*)$, but the second and third tend to reduce it. So it is not surprising that the myopic and
forward-looking conditions cannot be uniquely ranked, though simulations show that $G'_{FH}(n, n^*)$ is larger in almost all cases.\footnote{From Corollary 2 in the Appendix, $R(n, n, n^*)$ is less than $\pi(1, n^*)$. Hence, the sum of the first and third effects, $\pi^*(0, n^*) - \pi^*(1, n^*) - R(n, n, n^*)$, is greater than $G'_{FH}(1, n^*)$, which from Proposition 3 is greater than $G_{FH}(n, n^*)$. However, this ranking can be reversed by the second effect.}

We can now investigate the implications of Proposition 4. A similar series of derivations to those which led to Proposition 2 in the last section now yields:

**Proposition 5:** A takeover of a home firm by a foreign firm meets the forward-looking merger profitability criterion $G'_{FH}(n, n^*) > 0$ if and only if:

$$c > \xi_2 a' + (1 - \xi_2) c^*, \quad 0 < \xi_2 < \xi_0$$

(9)

The complexity of the recursive equations defining $R(s, \tilde{n}, n^*)$ and $R^*(s, \tilde{n}, n^*)$ precludes deriving an explicit expression for $\xi_2$. However, the Appendix shows that the forward-looking merger profitability criterion exhibits the same form as the myopic criterion in the last section: highly non-linear in $n$ and $n^*$, but linear in $a' - c$ and $a' - c^*$. Once again, the requirement that a takeover be profitable implies that the unit cost of the home firm exceed a weighted average of the demand intercept and the unit cost of the foreign firm. It can therefore be illustrated in $\{c, c^*\}$ space just as in the previous section. As for the likelihood of mergers under the two alternative assumptions about behaviour, I have already noted that simulations show that the forward-looking criterion $G'_{FH} > 0$ is usually though not always less stringent than the myopic one $G_{FH} > 0$. If we retain Assumption 2, which implicitly means that firms face a myopic capital-market constraint that each merger must be profitable even if no further mergers take place, then the two criteria have identical implications. Alternatively, if we drop Assumption 2, then more mergers will usually take place when firms are forward-looking than when they are myopic. In both cases, the $H$ region expands at the expense of the $HF$ region when mergers are allowed.

A final issue which needs to be addressed is that, although firms are too small to influence economy-wide variables, if they are forward-looking they should be able to predict how such variables will be affected by takeovers occurring in many sectors. Hence it makes sense to assume that only ex post profitable takeovers will take place. This implies that firms anticipate the wage changes which the economy-wide takeovers will induce. So, the next step is to solve for the change in wages, which requires us to move from partial to general equilibrium.
5 Oligopoly in General Equilibrium

The model in the absence of mergers is similar to that presented in Neary (2002). Consider first the demand side. Utility is an additive function of a continuum of goods, with each sub-utility function quadratic:

\[ U \left[\{x(z)\}\right] = \int_0^1 \left[ ax(z) - \frac{1}{2} bx(z)^2 \right] dz \]  \hspace{1cm} (10)

In each country there is a single representative consumer who maximizes (10) subject to the budget constraint:

\[ \int_0^1 p(z) x(z) dz \leq I \]  \hspace{1cm} (11)

where \( I \) is aggregate income. This leads in each country to inverse demand functions for each good which are linear in own price, conditional on the marginal utility of income, which is the Lagrange multiplier attached to the budget constraint:

\[ p(z) = \frac{1}{\lambda} [a - bx(z)], \quad \lambda \left[\{p(z)\}, I\right] = \frac{a\mu_1^p - bI}{\mu_2^p} \]  \hspace{1cm} (12)

The effects of prices on \( \lambda \) are summarized by the first and second moments of the distribution of prices:

\[ \mu_1^p = \int_0^1 p(z) dz \quad \text{and} \quad \mu_2^p = \int_0^1 p(z)^2 dz \]  \hspace{1cm} (13)

Finally, in a free-trade world equilibrium, where the behaviour of each of two countries can be described by an aggregate utility function like (10), the world inverse demand curve for each good comes from combining equation (12) for home country demand \( x(z) \) and the corresponding equation for foreign country demand \( x^*(z) \) (assumed to have the same demand slope \( b \), but a possibly different intercept \( a^* \)):

\[ p(z) = a' - b' \bar{x}(z) \quad \text{where:} \quad a' \equiv \frac{\bar{a}}{\bar{\lambda}} = \frac{a + a^*}{\lambda + \lambda^*} \quad \text{and} \quad b' \equiv \frac{b}{\lambda} \]  \hspace{1cm} (14)

Here \( \bar{\lambda} \) is the world marginal utility of income, which provides the link between the actual demand functions in (14) and the demand functions perceived by firms as given by equation (1) in Section 2.

Next, to close the model, we need to explain how costs are determined in general equilibrium. Following Neary (2002), we adopt Ricardian assumptions about technology and factor markets: labour is the sole factor of production and is intersectorally but not internationally mobile; each country can potentially produce a continuum of goods indexed by \( z \in [0,1] \); while sectors differ in their unit labour requirements, denoted by
\(\alpha(z)\) and \(\alpha^*(z)\) at home and abroad respectively. Hence the unit costs of firms in sector \(z\) are as follows:

\[
c(z) = w\alpha(z), \quad c^*(z) = w^*\alpha^*(z)
\]

where \(w\) and \(w^*\) denote the home and foreign wages respectively. We assume that \(\alpha\) and \(\alpha^*\) are continuous in \(z\), and that sectors are ordered such that \(z\) is an index of foreign comparative advantage. To help the exposition it is often convenient to strengthen this last assumption and assume that \(\alpha(z)\) is increasing and \(\alpha^*(z)\) is decreasing in \(z\), though this is not necessary for many of the results.\(^{18}\)

Inspecting equations (1), (2) and (15), as well as the expressions for sectoral outputs in the Appendix, equation (26), it can be seen that the values of real variables are homogeneous of degree zero in three nominal variables: the home and foreign wage rates \(w\) and \(w^*\), and the inverse of the world marginal utility of income, \(\lambda^{-1}\). Putting this differently, the absolute values of these and all other nominal variables (e.g., prices and costs) are indeterminate. This is a standard property of real models, and it implies that we can choose an arbitrary numeraire or normalization of nominal variables. It is most convenient to choose the world marginal utility of income as numeraire, so we work henceforward with wages normalized by it: \(W \equiv \lambda w\), \(W^* \equiv \lambda w^*\). These are not the same as real wages in the conventional sense (since tastes are not homothetic, so “real wages” cannot be defined independent of the level of utility), though they come close. A better interpretation is “marginal real wages”, since they equal nominal wages deflated by the marginal cost of utility.

We can now state the equations which define the free-trade equilibrium in the absence of mergers. First, the threshold sectors at home and abroad, denoted \(\tilde{z}\) and \(\tilde{z}^*\) respectively, are determined implicitly by setting (2) and the corresponding equation for the foreign country to equalities.\(^{19}\) This is illustrated in Fig. 1, where the downward-sloping line denotes an arbitrary distribution of home and foreign costs, conditional on wages in the two countries. As shown, the home country produces positive outputs in all sectors for which \(z\) is below \(\tilde{z}\), while the foreign country specializes in sectors for which \(z\) is above \(\tilde{z}^*\). Next, wages are determined by the conditions for full employment at home and abroad. That for the home country is as follows:

\[
L = \int_0^{\tilde{z}} \alpha(z) ny(W, z; n, 0) dz + \int_{\tilde{z}}^{\tilde{z}^*} \alpha(z) ny(W, W^*, z; n, n^*) dz
\]

\(^{18}\)As discussed in more detail in Neary (2002), necessary and sufficient conditions for the model to be well-behaved are that \(y\) is decreasing in \(z\) at \(z = \tilde{z}\), which implies that \(\omega(\tilde{z}) > (1 - \xi_0)w^*\alpha^*(\tilde{z})\); and that \(y^*\) is increasing in \(z\) at \(z = \tilde{z}^*\), which implies that \((1 - \xi_0^*)\omega(\tilde{z}^*) > w^*\alpha^*(\tilde{z}^*)\). In the competitive limit, these conditions collapse to an assumption made by Dornbusch, Fischer and Samuelson (1977) that \(\alpha(z)/\alpha^*(z)\) is increasing in \(z\). Geometrically, they imply that the cost locus intersects each of the boundaries of the HF region at most once.

\(^{19}\)Equation (2) and the corresponding equation for the foreign country could hold as strict inequalities for all \(z\). This would happen for example if all home and foreign sectors were identical. However, since the focus of the paper is on how cross-border mergers shift the threshold of specialisation, we will assume throughout that both \(\tilde{z}\) and \(\tilde{z}^*\) lie strictly within the \([0, 1]\) interval.
where, from equation (26) in the Appendix, the levels of output in sectors which do not and which do face foreign competition equal respectively:

\[ y(W, z; n, 0) = \frac{\bar{a} - W\alpha(z)}{b(n + 1)} \]
\[ y(W, W^*, z; n, n^*) = \frac{\bar{a} - (n^* + 1)W\alpha(z) + n^*W^*\alpha^*(z)}{b(n + n^* + 1)} \] (17)

The full-employment condition for the foreign country, analogous to (16), completes the specification of the model.

6 Effects of Mergers on Specialization Patterns and Income Distribution

To solve the model explicitly, I concentrate on the case of countries which are the same size but exhibit symmetric inter-sectoral differences. This is analytically convenient, and also gives a stylized representation of an empirical paradigm sketched by Davis and Weinstein (2002): significant differences in sectoral relative efficiencies prevail even between high-income countries with similar factor endowments. Equal size and symmetry imply that countries have the same endowments: \( L = L^* \); the same tastes: \( a = a^* = \frac{1}{2}\bar{a} \); the same industrial structure: \( n = n^* \); and technology distributions which are “mirror images” of each other: \( \alpha(z) = \alpha^*(1 - z) \), for all \( z \). In equilibrium they therefore have the same marginal utility of income: \( \lambda = \lambda^* = \frac{1}{2}\bar{\lambda} \); the same wage: \( W = W^* \); and symmetric threshold sectors: \( \tilde{z} = 1 - \tilde{z}^* \).

The effects of mergers on equilibrium wages can be explained intuitively by considering how they affect the demand for labour at initial wages. In some sectors, high-cost firms in the home country are bought out by low-cost foreign rivals, while in other sectors the converse happens. In both countries there are expanding and contracting sectors. However, at the initial wages, expanding firms increase their output by only a fraction of the output of the firms which are taken over. (Recall footnote 13.) In addition, the expanding firms have lower labour requirements per unit output than the contracting ones. Aggregating over all sectors, the total demand for labour therefore contracts in both countries at the initial wages. Hence, wages must fall to restore labour-market equilibrium.

To see this formally, consider the two equations which define the equilibrium. (To help the exposition we refer to these as equations for the home country, though nothing hinges on this. Similar equations apply to the foreign country but, with symmetry, they are identical to (18) and (19).) First is the equation for the equilibrium home threshold sector, or the “extensive margin”. Whether this is the threshold for zero profits, or for profitable takeovers under either myopic or forward-looking assumptions, we have seen in equations (2), (4) and (9) that it takes the same form, highly non-linear in \( n \) and \( n^* \), but linear in \( c \) and \( c^* \) and hence
in \( \alpha \) and \( \alpha^* \). With or without mergers, it can therefore be written as follows:

\[
G(W, \tilde{z}; \xi) = W\alpha(\tilde{z}) - \xi \tilde{a} - (1 - \xi) W\alpha^*(\tilde{z}) = 0
\]

where \( \xi \) depends only on \( n \) and takes different values depending on the context. The signs under the arguments of \( G(.) \) indicate the signs of the corresponding partial derivatives. Higher values of both \( W \) and \( \tilde{z} \) make specialization or profitable takeovers more likely, while higher values of \( \xi \), the weight attached to the demand intercept, make them less likely.\(^{20}\) Hence the loci corresponding to equation (18) for different values of \( \xi \) are downward-sloping in \( \{W, \tilde{z}\} \) space, as illustrated in Fig. 4, and shift downwards as \( \xi \) falls.

The second equation is the labour-market equilibrium condition. With symmetric countries, this is simply (16) with \( n = n^* \) and with the indexes of the home and foreign threshold sectors summing to unity: \( \tilde{z} + \tilde{z}^* = 1 \). This can be written as follows:

\[
L(W, \tilde{z}) = L
\]

where once again the signs under the arguments indicate the signs of the corresponding partial derivatives. These are justified formally in the Appendix, and are intuitively plausible. A higher wage both at home and abroad encourages many sectors to shed labour at the intensive margin. Admittedly this is not necessarily true of all: some low-cost home sectors may gain more from the higher wages which their foreign rivals have to pay than they lose from higher home wages.\(^{21}\) However, summing across all sectors, the aggregate demand for labour is unambiguously decreasing in \( W \). As for a change in \( \tilde{z} \), it is intuitively obvious that labour demand increases when the extensive margin expands, though only in the region below full employment.\(^{22}\) Hence the labour-market equilibrium locus is upward-sloping to the left of the \( G(\cdot; \xi_0) \) locus and horizontal where it cuts it, as shown in Fig. 4.

The effects of cross-border mergers on wages and on the extensive margin can now be deduced from Fig.

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\(^{20}\)The signs of the derivatives of \( G \) can be justified more formally as follows. With symmetry, \( \tilde{z} \) is always greater than \( \frac{1}{2} \), and so \( \alpha(\tilde{z}) \) is always greater than \( \alpha^*(\tilde{z}) \). Hence \( G_W \), which equals \( \alpha(\tilde{z}) - (1 - \xi)\alpha^*(\tilde{z}) \), is positive. As for \( G_{\tilde{z}} \), which equals \( W\alpha'(\tilde{z}) - (1 - \xi)W\alpha^{''}(\tilde{z}) \), this must be positive at \( \xi = \xi_0 \), from the assumption in footnote 17, and from analogous assumptions at other values of \( \xi \). Finally, \( G_{\xi} \), which equals \( -[\tilde{a} - W\alpha^*(\tilde{z})] \), must be negative from the requirement that foreign firms are profitable in the post-merger equilibrium.

\(^{21}\)Extending the argument, the labour-market equilibrium locus is downward-sloping at points to the right of the \( G(\cdot; \xi_0) \) locus. Such points correspond to negative values of home output in sectors with \( z > \tilde{z} \), and so are irrelevant to equilibrium analysis. They are of interest, however, from the perspective of establishing the stability of equilibrium. Out of equilibrium, such points generate incentives for a reduction in \( z \); and (in the no-mergers case where \( \xi = \xi_0 \)) the converse holds for points to the left of the \( G(\cdot; \xi_0) \) locus. Similarly, points above the \( L \) locus correspond to unemployment and generate incentives for reductions in \( W \), and conversely for points below the \( L \) locus. Combining these dynamics shows that the intersection point of the \( L \) locus and the \( G(\cdot; \xi_0) \) locus is a stable equilibrium, as shown by the arrows in in Fig. 4. A similar argument applies to the intersection point of the \( L \) locus and each of the other \( G(\cdot; \xi) \) loci.
4. Free-trade equilibrium in the absence of mergers is at point \( A_0 \), the intersection of the \( L \) and \( G(\xi_0) \) loci. When mergers take place, the \( G \) locus shifts leftward to \( G(\xi_1) \), since \( \xi_1 \) is less than \( \xi_0 \), but the \( L \) locus is unaffected. At initial wages (the partial equilibrium case considered in Sections 3 and 4), this reduces the demand for labour in both countries. In general equilibrium, assuming labour markets are perfectly flexible, wages are bid down, which raises the profitability of marginal high-cost firms, putting them out of reach of takeovers. Hence the general-equilibrium repercussions working through labour markets dampen, though they cannot reverse, the tendency towards merger waves. The new equilibrium is at \( A_1 \), with a lower wage and mergers in the \( \{\xi_1, \xi_0\} \) interval. The same outcome is illustrated from a different perspective in Fig. 5.

The dashed line indicates the cost distribution corresponding to the equilibrium in the absence of mergers. As wages fall, this cost locus shifts inwards as shown, and so the range of sectors which remain in the \( HF \) region is greater than an analysis based on fixed wages would predict, though still less than if no mergers had occurred. For example, in the sector represented by point \( B \) in Fig. 5, home firms are just on the margin of being taken over at initial wages. The fall in wages shifts the point corresponding to this sector inwards along a ray towards the origin from \( B \) to \( B' \), where home firms are now too profitable to be taken over.

Wages fall not only relative to the numeraire but also relative to profits. Profits rise even if wages do not change, since this is a necessary condition for mergers to take place. The fall in wages raises profits further. (See the Appendix for a formal demonstration.) Hence:

**Proposition 6**: When the two countries are symmetric, both forward-looking and myopic mergers lower the share of wages in national income.

So mergers have an unambiguous effect on the functional distribution of income.

7 Mergers and Welfare

Finally, we wish to establish the effect of mergers on aggregate welfare. It is instructive to begin with a partial-equilibrium perspective. In this view, only changes in sectors where mergers occur need be taken into account; the elimination of some firms raise prices in those sectors, which in turn lowers consumer surplus; but since aggregate profits rise, the net effect on welfare is ambiguous. In fact, it can be shown that this ambiguity can be resolved in the present case. Since only high-cost firms are eliminated, the increase in production efficiency ensures that the rise in profits dominates the fall in consumer surplus. Hence, if aggregate welfare is measured by the sum of profits and consumer surplus at given wages, it unambiguously rises.\(^2\)

\(^2\)This effect was first noted by Lahiri and Ono (1988). Adapting their terminology to the present context, closing down minor firms raises welfare. A full derivation is given in a supplementary appendix to this paper, available at:
However, this partial-equilibrium approach is misleading when we consider an economy-wide shock such as the elimination of restrictions on trade and cross-border mergers. In general equilibrium only consumers matter, and the benefits if any from more efficient resource allocation show up not in higher profits but in lower prices throughout the economy. We can see this by calculating explicitly the indirect utility function corresponding to the direct utility function (10). Substituting from the direct demand functions (and ignoring a constant) gives:

\[ U = a^2 - \lambda^2 \mu^2 \]

Thus welfare depends inversely on the second moment of the marginal-utility-of-income-weighted price distribution, and we need to determine the effects of cross-border mergers on this expression.

Because of symmetry, it is sufficient to consider the level of welfare evaluated over half of the technology distribution only:

\[ U(W, z^+) = a^2 - 2 \int_0^{z^+} [\lambda p(W, z; n, n)]^2 dz - 2 \int_{z^+}^1 [\lambda p(W, z; 0, n)]^2 dz \]

Since prices are increasing in wages, it follows that welfare is decreasing in \( W \). To establish how welfare varies with the extensive margin \( z^+ \), partial equilibrium intuition is perfectly adequate: at given wages, mergers must raise prices in all sectors where they occur, so welfare is increasing in \( z^+ \). The only exception is when the target firms are infinitesimally small so prices are unaffected, i.e., when \( z^+ \) equals \( z_0 \). Iso-welfare loci are therefore upward-sloping to the left of the \( G(.; \xi_0) \) locus in Fig. 6 and horizontal where they meet it. This is exactly the same qualitative configuration as the labour market equilibrium locus considered in the previous section. Hence, we can state a necessary and sufficient condition for welfare to rise following a “small” merger wave, in the sense of the elimination of high-cost firms in a marginal sector; i.e., an infinitesimal reduction in the parameter \( \xi \) in (18), which induces an infinitesimal leftward shift in the \( G(.; \xi) \) locus in Fig. 6.\(^{24}\) Such a change raises welfare if and only if the labour-market equilibrium locus is more steeply sloped than the iso-welfare locus at the initial equilibrium, the case illustrated in Fig. 6.

Intuitively, it is easy to see why mergers have an ambiguous effect on welfare in general equilibrium, though the source of the ambiguity is very different from that in partial equilibrium. On the one hand, mergers lead to more concentration in sectors where they occur, which (at initial wages) raises prices in those sectors and tends to lower welfare. On the other hand, mergers reduce wages, as we have seen, which tends to reduce prices in all sectors and so to raise welfare.

The Appendix explores this further and shows that the ambiguity cannot be resolved in general. 

\(^{24}\) This is analogous to, though different from, the approach adopted in Farrell and Shapiro (1990).
reason is that the cost distribution $\alpha(z)$ is unrestricted (other than being monotonically increasing in $z$), and it is possible to find examples of $\alpha(z)$ under which a small merger would lower welfare. To see how this works, we introduce two measures of the difference between home and foreign technology in the diversified sectors:

$$H(\tilde{z}) \equiv (n + 1) \alpha(\tilde{z}) - n\alpha(1 - \tilde{z}) \text{ and } J \equiv (n + 1) \mu_2^D - n\gamma^2$$

(22)

$H(\tilde{z})$ and $J$ can be interpreted as measures of home comparative disadvantage at the margin (i.e., at $\tilde{z}$) and on average (i.e., over all the diversified sectors $z \in [1 - \tilde{z}, \tilde{z}]$) respectively. It is easily checked (from equation (63) in the Appendix) that:

$$H(\tilde{z}) \mu_1^D - J > 0$$

(23)

reflecting the fact that, of all the diversified sectors, home comparative disadvantage is greatest in sector $\tilde{z}$. However, $H(\tilde{z}) \mu_1^D - J$ can be arbitrarily close to zero, and in that case (as shown in the appendix) the anti-competitive effect of mergers on prices dominates the effect of lower wages. It is possible for this net welfare reduction in the diversified sectors to dominate in turn the welfare increase in the specialized sectors (which always benefit from the fall in wages: see Section 9.10.1) such that welfare overall falls. Intuitively, a high value of $H(\tilde{z})$ means that labour demand is highly responsive to a change in the marginal sector $\tilde{z}$, while a high value of $J$ means that it is highly responsive to a change in the wage rate. When $J$ is large relative to $H(\tilde{z})$, the labour-market equilibrium locus is relatively shallow (see equation (55) in the Appendix), with only a small fall in wages needed to offset the reduction in labour demand from the elimination of firms in the marginal sector.

However, such a configuration is likely to be rare.\textsuperscript{25} The Appendix supports this by showing that a welfare increase is guaranteed in two cases. The first involves placing restrictions on the two measures of comparative disadvantage:

**Proposition 7**: An infinitesimal merger wave raises welfare if home comparative disadvantage is sufficiently greater at the margin than on average so that:

$$H(\tilde{z}) \mu_1^D > \Xi J, \text{ where: } \Xi \equiv 1 + \frac{1}{n + 1} \frac{2n^2 (2n + 1)}{4n^3 + 8n^2 - n - 1}$$

(24)

The term $\Xi$ equals 1.179 when $n$ equals 3, and falls asymptotically towards one as $n$ rises. Hence only a modest excess of comparative disadvantage at the margin relative to that on average is sufficient to ensure

\textsuperscript{25}The extreme case where $H(\tilde{z}) \mu_1^D$ and $J$ are equal implies that $\alpha(z)$ is a step function between $1 - \tilde{z}$ and $\tilde{z}$, with only a single vertical jump at $z = \frac{1}{2}$. Such a configuration must be considered highly implausible.
that the wage effect dominates. The second sufficient condition for mergers to raise welfare comes from restricting the cost distribution directly:

**Proposition 8**: An infinitesimal merger wave raises welfare if the distribution of sectoral unit labour requirements \( a(z) \) is linear in \( z \).

This case too ensures that the reduction in labour demand from the marginal contracting sector is sufficiently large relative to the increase in labour demand induced by a fall in wages to ensure that the fall in wages dominates the reduction in competition.

To sum up, it is possible for the reduced competition in the sectors where mergers occur to raise prices there by enough to offset the welfare gain from lower prices throughout the economy induced by the fall in wages. However, this requires that the distribution of costs take an idiosyncratic form, relatively steep close to \( z = \frac{1}{2} \) and relatively flat close to \( z = \tilde{z}_1 \). (Recall the previous footnote.) Bearing in mind that a merger in the neighbourhood of \( z = \tilde{z}_0 \) always raises welfare (see equation (65) in the Appendix), and that the effects on specialized sectors always tend to raise welfare, we can conclude that there is a strong presumption that a merger wave leads to a welfare gain in our model.

## 8 Conclusion

In this paper I have used a new model of oligopoly in general equilibrium to throw light on the phenomenon of cross-border mergers. The key to the prediction of mergers in my framework is not general equilibrium per se, but rather cost differences between firms. However, the two go together, since the cost differences arise from international differences in technology between sectors in a Ricardian trade model.\(^{26}\)

The model predicts that international differences in technology generate incentives for bilateral mergers in which low-cost firms located in one country acquire high-cost firms located in the other. As a result, cross-border mergers serve as “instruments of comparative advantage”. They facilitate more specialization in the direction of comparative advantage, so moving production and trade patterns closer to what would prevail in a competitive Ricardian world. They also have implications for income distribution, putting downward pressure on wages, and so tilting the distribution of income towards profits at the expense of wages in both countries. As for aggregate welfare, the fall in wages puts downward pressure on prices in all sectors, which tends to increase the gains from trade in both countries. Potentially offsetting this, the sectors in which

\(^{26}\)This paper allows only for firm heterogeneity between countries in each sector, unlike the model of within-country firm heterogeneity of Melitz (2003), which has been applied to greenfield and acquisition FDI by Helpman, Melitz and Yeaple (2004) and Nocke and Yeaple (2007) respectively. All these papers assume a large-group monopolistically competitive market structure, whereas the present paper assumes that markets are oligopolistic without any entry by new firms. Modelling endogenous entry by heterogeneous firms in oligopolistic (small-group) markets is an important research challenge for the future.
mergers occur become less competitive, so their prices tend to rise. On balance, the net effect on welfare is likely to be positive.

The results of this paper lend themselves to empirical testing. By construction, the model predicts that trade liberalization encourages cross-border merger waves, consistent with the evidence cited in the Introduction. In addition, the model makes a further empirical prediction: absent cost synergies, the pattern of cross-border mergers which results from market integration follows that of comparative advantage, in the sense that low-cost firms acquire high-cost foreign rivals. As a corollary, the model predicts that cross-border mergers and exports are complements rather than substitutes, in the sense that exporting sectors tend to be sources of rather than hosts for foreign direct investment. Finally, the model predicts that cross-border merger waves tend to reduce factor demands and so put downward pressure on the returns to productive factors. These predictions are very different from those of standard models of greenfield foreign direct investment, and more consistent with the available empirical evidence.27

There is ample anecdotal evidence that cross-border mergers tend to reflect comparative advantage.28 In addition, two recent papers provide more formal evidence in favour of the sectoral-level predictions of the model.29 Feliciano and Lipsey (2002) use U.S. Bureau of Economic Analysis data for individual foreign acquisitions of U.S. firms from 1989 to 1997, and find that acquisitions tend to be in industries in which the investing country has a comparative advantage in exporting. Brakman, Garretsen and van Marrewijk (2005) explicitly test the “instruments of comparative advantage” theory of cross-border mergers using data on individual firms. They look at cross-border mergers between five OECD countries in twenty sectors over the period 1980-2004 and find strong evidence of a role for comparative advantage: acquiring firms come disproportionately from sectors which have a revealed comparative advantage, as measured by the standard Balassa index. They also find evidence that mergers are positively autocorrelated within sectors, consistent with the hypothesis that mergers occur in waves. Of course, these results may be consistent with other theories too, so further work is needed to test their robustness.

As for the general-equilibrium effects of cross-border mergers, they have not been subject to empirical scrutiny as yet, though they have been discussed in the policy literature. For example, UNCTAD (2000, p. 18) voices concerns that cross-border mergers may be less attractive than greenfield FDI because they

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27 Baldwin and Ottaviano (2001) is a notable exception.
28 This is even true of the increasingly important phenomenon of South-North acquisitions. Developing-country acquiring firms tend to be in sectors in which those countries have a comparative advantage, such as steel and generic pharmaceuticals. See, for example, “Steel the prize: CSN v Tata,” The Economist, 23 November 2006, and “Marauding maharajahs: India’s acquisitive companies,” The Economist, 29 March 2007. Of course, the model does not predict that all exporting firms in less-developed countries should acquire targets in developed countries. As the corollary to Proposition 2 shows, comparative advantage is a necessary condition for mergers but not a sufficient one: in addition the industry in the acquiring (low-cost) country must be relatively concentrated.
29 The results of Breinlich (2006) are more mixed. He finds that U.S. acquiring firms tend to have higher total factor productivity than their Canadian target firms, but the converse does not hold.
tend to reduce rather than increase employment. This confirms the positive predictions of the model in this paper, but the model suggests a very different normative interpretation: reductions in employment through takeovers of existing firms may be a means of realizing the gains from trade liberalization, just like firm closures in traditional trade theory.\textsuperscript{30}

Of course the model presented here is subject to many qualifications. The assumption of no efficiency gains may seem especially unrealistic at first blush. It can be defended by referring to the empirical evidence, which suggests that efficiency gains are frequently not realized in practice (see footnote 5) and that many takeovers are followed by closure of the acquired firm’s plants.\textsuperscript{31} However, it is probably best seen as a methodologically convenient simplification, which highlights the finding that, in this model, mergers motivated by purely strategic (and so anti-competitive) motives nonetheless lead to a rationalization of production in the socially desirable direction, and are likely to increase welfare. There are many possible sources of efficiency gains - and losses - from mergers, and therefore many ways of modelling them, each with potentially different implications. Further work is needed to explore the robustness of the conclusions presented here to relaxing this and other assumptions. The present paper is unlikely to be the last word on the theory of cross-border mergers in general equilibrium, but at least it appears to be the first.

\textsuperscript{30} Margolis (2006) presents some microeconometric evidence on the employment effects of mergers drawn from a linked employer-employee data set for France. He finds that workers in acquired firms are less likely to stay with the new entity in the short run following a takeover, but this effect disappears after three years, and the characteristics of employees who leave both acquiring and acquired firms after a takeover are broadly similar.

\textsuperscript{31} Jensen and Bernard (2005), in a very large sample of U.S. plants, find that the probability of plant closure, controlling for plant characteristics, is significantly and “dramatically” higher when the parent firm has been taken over in the previous five years. They speculate that this may be because the plant was acquired in order to shut it down.
Appendix

9.1 Preliminaries

The setting is a homogeneous-good Cournot oligopoly where all firms have constant marginal costs, and face 
the (perceived) linear inverse demand curve (1). In the proofs of Propositions 1 to 5, we consider a single 
industry in partial equilibrium. Hence the arguments \( W, W^* \) and \( z \) can be suppressed. In the pre-takeover 
equilibrium, there are \( n = n + n^* \) firms, of which \( n \) home firms have marginal cost \( c \) and \( n^* \) foreign firms have 
marginal cost \( c^* \). The first-order conditions are \( b' y(n, n^*) = p(n, n^*) - c \) and \( b'y^*(n, n^*) = p(n, n^*) - c^* \). 
Solving these for industry output and price gives:

\[
y(n, n^*) = \frac{n(a' - c) + n^*(a' - c^*)}{b'(n + n^* + 1)}, \quad p(n, n^*) = \frac{a' + nc + n^*c^*}{n + n^* + 1} \tag{25}
\]

These in turn can be used to solve for the outputs of each firm:

\[
y(n, n^*) = \frac{a' - (n^* + 1)c + n^*c^*}{b(n + n^* + 1)}, \quad y^*(n, n^*) = \frac{a' - (n + 1)c^* + nc}{b(n + n^* + 1)} \tag{26}
\]

We can now state a key lemma, which gives the effects on the surviving firms’ outputs of takeovers which 
eliminate a subset of home firms. Let \( \tilde{n} \) be the number of surviving home firms. Then:

**Lemma 1**: Closing down \( n - \tilde{n} \) home firms increases the output of all remaining firms (both home and 
foreign) by the same amount, equal to a constant times the initial output of each home firm:

\[
y(\tilde{n}, n^*) - y(n, n^*) = y^*(\tilde{n}, n^*) - y^*(n, n^*) = \frac{n - \tilde{n}}{n + n^* + 1} y(n, n^*) \tag{27}
\]

The proof is immediate from (26).

9.2 Proof of Proposition 1

Consider first a takeover of a home firm by another home firm. From Lemma 1, with \( \tilde{n} \) equal to \( n - 1 \), the 
post-takeover output of each remaining home firm, \( y(n - 1, n^*) \), is proportional to their pre-takeover output 
\( y(n, n^*) \). Hence, recalling that profits are proportional to the square of output, the gain to such a takeover 
can be written as follows:

\[
\frac{1}{b'} G_{HH}(n, n^*) = y(n - 1, n^*)^2 - 2y(n, n^*)^2 = \left( \frac{n + 1}{n} \right)^2 - 2 \right] y(n, n^*)^2 \propto -n^2 + 2\tilde{n} + 1 \tag{28}
\]
This is negative for $\bar{n} > 2$, which proves that a takeover of a home firm by another home firm is never profitable, irrespective of the level of foreign firms’ costs. Similar reasoning shows that a takeover of a foreign firm by another foreign firm is never profitable. This proves Proposition 1.

9.3 Proof of Proposition 2

Next, consider a takeover of a home firm by a foreign firm. The myopic merger criterion (3) can be expressed in terms of outputs as follows:

$$\frac{1}{b} G_{FH}(n, n^*) = y^*(n - 1, n^*)^2 - y^*(n, n^*)^2 - y(n, n^*)^2$$  (29)

The key step is to factorize the difference between the pre- and post-takeover squared output levels of the foreign firm so that (29) can be rewritten as follows:

$$\frac{1}{b} G_{FH}(n, n^*) = [y^*(n - 1, n^*) + y^*(n, n^*)][y^*(n - 1, n^*) - y^*(n, n^*)] - y(n, n^*)^2$$  (30)

Using Lemma 1, with $\bar{n}$ equal to $n - 1$, to express $y^*(n - 1, n^*)$ in terms of $y^*(n, n^*)$ and $y(n, n^*)$ gives an explicit expression for $G_{FH}$ in terms of pre-takeover outputs only:

$$\frac{1}{b} G_{FH}(n, n^*) = \frac{1}{\bar{n}} y(n, n^*) \left[2\bar{n}y^*(n, n^*) - (\bar{n}^2 - 1) y(n, n^*)\right]$$  (31)

The sign of $G_{FH}$ depends only on the expression in square brackets, which is linear in outputs. Using (26) to eliminate $y(n, n^*)$ and $y^*(n, n^*)$ from this expression yields:

$$G_{FH}(n, n^*) = \frac{2n\bar{n} + (n^* + 1)(\bar{n}^2 - 1)}{\bar{n}^2(n + 1)} y(n, n^*) [c - \xi_1 a' - (1 - \xi_1) c^*]$$  (32)

where $\xi_1$ is defined in equation (5) and is positive for $\bar{n} \geq 3$. Hence equation (4) in Proposition 2 follows immediately. The final step in the proof is to show that $\xi_1$ is less than $\xi_0$. Substituting for $\xi_0$ from (2), this follows by direct calculation.

9.4 Proof of Proposition 3

To prove the proposition, we treat the number of home firms as a continuous variable. Differentiate (26) with respect to $n$:

$$\frac{dy(n, n^*)}{dn} = \frac{dy^*(n, n^*)}{dn} = -\frac{1}{\bar{n} + 1} y(n, n^*), \quad \frac{dy^*(n - 1, n^*)}{dn} = \frac{\bar{n} + 1}{\bar{n}^2} y(n, n^*)$$  (33)
The first equation in (33) restates Lemma 1 for \( \tilde{n} = n - 1 \) in continuous form: to a first-order approximation, a takeover of a home firm (a fall in \( n \)) leads to identical increases in the pre-takeover output of home and foreign firms. The second equation in (33) shows that it leads to a larger increase in the post-takeover output of a foreign firm. To see the implications of this for the profits of all three types of firms and hence for the gains from a takeover, differentiate \( G_{FH} \) from (30) with respect to \( n \):

\[
\frac{1}{2b'} \frac{dG_{FH}}{dn} (n, n^*) = y^*(n - 1, n^*) \frac{dy^*(n - 1, n^*)}{dn} - y^*(n, n^*) \frac{dy^*(n, n^*)}{dn} - y(n, n^*) \frac{dy(n, n^*)}{dn} = \frac{1}{\tilde{n}^2 (\tilde{n} + 1)} y(n, n^*) \left[ (\tilde{n} + 1)^2 y^*(n - 1, n^*) - \tilde{n}^2 y^*(n, n^*) - \tilde{n}^2 y(n, n^*) \right]
\]

As in (31), the sign of this depends only on the expression in square brackets, which is linear in outputs. Using Lemma 1, with \( \tilde{n} \) equal to \( n - 1 \), to eliminate \( y^*(n - 1, n^*) \) yields:

\[
\frac{dG_{FH}}{dn} (n, n^*) \propto \tilde{n} (2\tilde{n} + 1) y^*(n, n^*) + \left\{ \tilde{n}^3 - (\tilde{n} + 1)^2 \right\} y(n, n^*)
\]

This is ambiguous in sign in general. However, we wish to evaluate it only at points where the gain to a takeover is strictly positive. From (31), this is equivalent to:

\[
G_{FH} (n, n^*) > 0 \iff 2\tilde{n} y^*(n, n^*) = (\tilde{n}^2 - 1) y(n, n^*) + Z
\]

where \( Z > 0 \). Substituting into (34) gives:

\[
\frac{dG_{FH}}{dn} (n, n^*) \propto - (3\tilde{n}^2 + 2\tilde{n} + 1) y(n, n^*) - (2\tilde{n} + 1) Z
\]

This is negative, which proves the result. Note also that, from (??), \( d\xi_1/dn \) is proportional to the right-hand side of (36) with \( Z = 0 \). Hence, an increase in the gain to a takeover shifts downwards the boundary between the \( F \) and \( HF \) regions, as asserted in the text.

9.5 Proof of Proposition 4

The proof proceeds by calculating the payoffs to both types of firms when they expect the game to follow an equilibrium path along which a merger takes place at each stage, and then showing that such expectations are validated. Consider first the high-cost firms. We can calculate their minimum required return, which because of Assumption 4 (c) is also their supply price, as follows:

\textit{Lemma 2:} Given Assumption 4, when a high-cost firm expects a takeover offer to be made and accepted
in each future stage, its minimum required return \( R(s, \tilde{n}, n^*) \) equals:

\[
\begin{align*}
(a) & \quad \pi^*(\tilde{n}, n^*), \quad s = 0; \\
(b) & \quad R^*(s - 1, \tilde{n} - 1, n^*) - \frac{1}{n^*} R(s - 1, \tilde{n}, n^*), \quad 0 < s \leq \tilde{n}
\end{align*}
\]  

(38)

Part (b) of (37) is a bivariate recursive equation with initial conditions given by (a). The proof is straightforward by backward induction. (Fig. 7 illustrates the reasoning for the last three stages.) In the final stage \( s = 1 \), with \( \tilde{n} \) remaining high-cost firms, forward-looking behaviour is identical to myopic behaviour. If the high-cost firm refuses to sell it will receive \( \pi(\tilde{n}, n^*) \). Because of part (c) of Assumption 4, the low-cost firm has a first-mover advantage in each meeting with a high-cost firm and so it appropriates all the surplus from a merger. Hence the high-cost firm will actually sell for \( \pi(\tilde{n}, n^*) \).\(^{32}\) This proves part (a) of Lemma 2. By contrast, in earlier stages (when \( s \) exceeds \( 1 \)), the high-cost firm will not be willing to sell for only its “current” profits \( \pi(\tilde{n}, n^*) \). If it rejects a takeover offer in stage \( s \) it will face in the subsequent stage \( s - 1 \) a \( 1/\tilde{n} \) probability of being made an offer and a \( (\tilde{n} - 1)/\tilde{n} \) probability of not being made an offer. Recalling from Assumption 4 (d) that all firms are risk-neutral, the expected value of this lottery must equal the price at which the high-cost firm is willing to sell in period \( s \), which proves part (b) of the Lemma.

For later use, we note some useful properties of the function \( R(s, \tilde{n}, n^*) \):\(^{33}\)

**Corollary 1:** For all \( s \), \( R(s, \tilde{n}, n^*) \) is a weighted average of \( \pi(m, n^*) \), \( m = \tilde{n} - s + 1, \ldots, \tilde{n} \).

**Corollary 2:** For all \( \tilde{n} \geq 2 \) and all \( s, 1 < s \leq \tilde{n} \), \( \pi(\tilde{n}, n^*) < R(s, \tilde{n}, n^*) < \pi(\tilde{n} - s + 1, n^*) \).

**Corollary 3:** \( R(s, \tilde{n}, n^*) \) is (a) increasing in \( s \); (b) decreasing in \( \tilde{n} \); and (c) decreasing in \( m \), where \( s = \tilde{n} = m \).

Consider next the low-cost firms. Their expected ex ante payoff is given by the following:

**Lemma 3:** Given Assumption 4, when a low-cost firm expects a takeover offer to be made and accepted in each future stage, its expected pay-off \( R^*(s, \tilde{n}, n^*) \) equals:

\[
\begin{align*}
(a) & \quad \pi^*(\tilde{n}, n^*), \quad s = 0; \\
(b) & \quad R^*(s - 1, \tilde{n} - 1, n^*) - \frac{1}{n^*} R(s - 1, \tilde{n}, n^*), \quad 0 < s \leq \tilde{n}
\end{align*}
\]

\(^{32}\)Strictly speaking, recalling footnote 10, it will only sell for \( \pi(\tilde{n}, n^*) \) plus an infinitesimal amount. For convenience I ignore qualifications of this kind in the rest of this section.

\(^{33}\)The proofs are straightforward, and are given in the supplementary appendix.
As with Lemma 2, this is proved by backward induction. With no remaining pre-production stages \((s = 0)\) and \(\tilde{n}\) remaining high-cost firms, a low-cost firm will earn profits of \(\pi^* (\tilde{n}, n^*)\) with certainty, which proves part (a) of Lemma 3. In earlier stages, a low-cost firm entering stage \(s\) faces a \(1/n^*\) chance of being matched with a high-cost partner and an \((n^* - 1)/n^*\) chance of not being matched. In the former case, its expected future payoff assuming it makes an offer is \(R^* (s - 1, \tilde{n} - 1, n^*)\) less what it must pay to the high-cost firm, \(R(s, \tilde{n}, n^*)\); in the latter case, assuming that a different low-cost firm makes a successful offer in this stage, it moves to the next stage with expected payoff \(R^* (s - 1, \tilde{n} - 1, n^*)\). Summing the expected payoffs weighted by the appropriate probabilities gives part (b) of the Lemma.

The final step in the proof is the forward-looking analogue of Proposition 3:34

**Lemma 4:** If \(G'_{FH} (n, n^*)\) is positive, then \(G'_{FH} (\tilde{n}, n^*)\) is also positive, for all \(\tilde{n} < n\).

This ensures that, when \(G'_{FH} (n, n^*)\) is positive at the initial value of \(n\), each subsequent takeover (reducing \(n\) by one) leads in the next stage to a value for \(G'_{FH} (n, n^*)\) which is also positive, thus validating the expectations (which we assumed in proving Lemmas 2 and 3 are held by both types of firms) that all future merger offers will be made and accepted. While an analytic proof of Lemma 4 is not available, an exhaustive search of parameter space confirms that it holds for all admissible cost levels and for all values of \(n\) and \(n^*\) up to fifty (beyond which the empirical relevance of mergers is moot, as Fig. 4 shows).35 Since this result guarantees that a merger wave occurs provided the initial value of \(G'_{FH} (n, n^*)\) is positive, it completes the proof of Proposition 4.

### 9.6 Proof of Proposition 5

The proof is similar to that of Proposition 2. Consider the expression for \(G'_{FH} (n, n^*)\) in (??). By expressing the difference in profits as the difference in squared outputs, the first two terms in this expression can be written as follows:

\[
\pi^* (0, n^*) - \pi^* (1, n^*) = \frac{y(1, n^*)}{n^* + 1} \left[ y^* (0, n^*) + y^* (1, n^*) \right]
\]  

(39)

Using Lemma 1, \(y (1, n^*)\) can be shown to be proportional to \(y (n, n^*)\). As for the remaining terms in (8), from Corollary 1 to Lemma 2, they are linear in \(y(\tilde{n}, n^*)\), for \(\tilde{n} = 1, \ldots n\). Repeated use of Lemma 1 allows all of these to be expressed as multiples of \(y (n, n^*)\). Hence this common factor can be extracted from the expression, leaving (after substituting for outputs from (23)) a term which is linear in \(a^* - c\) and \(a' - c^*\). Hence the requirement that \(G'_{FH} (n, n^*)\) be positive can be expressed in the linear form given by (9), which proves the proposition.

34 The analogy is only partial, since \(G'_{FH} (n, n^*)\) does not always increase as \(n\) falls.
35 A Gauss program for the simulations is available at http://www.economics.ox.ac.uk/Members/peter.neary/papers/crossbor.htm.
9.7 Properties of the Labour-Market Equilibrium Condition

We first derive an explicit expression for total labour demand in the home country (16), by substituting home and foreign outputs from (17) into (16) and evaluating the integrals at the symmetric equilibrium (where $\tilde{z}^* = 1 - \tilde{z}$ and $\tilde{a} = 2a$):

$$L = \frac{n}{b} \frac{2\alpha \mu^S_1 - W \mu^S_2}{n + 1} + \frac{n}{b} \frac{2\alpha \mu^D_1 - W [(n + 1) \mu^D_2 - n\gamma^2]}{2n + 1}$$ (40)

Here “$S$” and “$D$” denote the specialized sectors ($0 \leq z \leq 1 - \tilde{z}$) and the diversified sectors ($1 - \tilde{z} \leq z \leq \tilde{z}$) in the home country respectively; $\mu^S_1$ and $\mu^D_1$ denote the average labour requirements in the specialized and diversified sectors, respectively; $\mu^S_2$ and $\mu^D_2$ denote their average squared labour requirements; and $\gamma^2$ denotes the truncated uncentred covariance of the distribution of labour requirements (which is relevant only for the diversified sectors). These are defined as follows:

$$\mu^S_1 \equiv \int^{\tilde{z}^*}_0 \alpha (z) \, dz = \int^{1-\tilde{z}}_0 \alpha (z) \, dz; \quad \mu^D_1 \equiv \int^{\tilde{z}}_0 \alpha (z) \, dz = \int^{\tilde{z}}_{1-\tilde{z}} \alpha (z) \, dz$$

$$\mu^S_2 \equiv \int^{\tilde{z}^*}_0 \alpha (z)^2 \, dz = \int^{1-\tilde{z}}_0 \alpha (z)^2 \, dz; \quad \mu^D_2 \equiv \int^{\tilde{z}}_0 \alpha (z)^2 \, dz = \int^{\tilde{z}}_{1-\tilde{z}} \alpha (z)^2 \, dz$$

$$\gamma^2 \equiv \int^{\tilde{z}}_{\tilde{z}^*} \alpha (z) \alpha (1-z) \, dz = \int^{\tilde{z}}_{1-\tilde{z}} \alpha (z) \alpha (1-z) \, dz$$

For later use, note that the two terms on the right hand side of (40) correspond to total employment in the specialized and diversified sectors respectively.

To confirm the signs under the arguments of (19), consider first the derivative of (40) with respect to $W$:

$$L_W = -\frac{n}{b} \left[ \frac{\mu^S_2}{n + 1} + \frac{(n + 1) \mu^D_2 - n\gamma^2}{2n + 1} \right]$$ (44)

From symmetry we have:

$$\mu^D_2 - \gamma^2 = \frac{1}{2} \int^{\tilde{z}}_{1-\tilde{z}} \left[ \alpha (z)^2 + \alpha (1-z)^2 - 2\alpha (z) \alpha (1-z) \right] \, dz = \frac{1}{2} \int^{\tilde{z}}_{1-\tilde{z}} [\alpha (z) - \alpha (1-z)]^2 \, dz > 0$$ (45)

from which it follows that $L_W$ in (19) must be negative as claimed.

Consider next the effect on labour demand of a change in the extensive margin. Differentiate (16) with

---

36 In each case the definition for the general (non-symmetric) case is given first, followed by the values in the symmetric case.
respect to $\tilde{z}$ bearing in mind that $\tilde{z}^* = 1 - \tilde{z}$ with symmetry:

$$L_{\tilde{z}} = n \left[ \alpha (\tilde{z}) y (W, W, \tilde{z}; n, n) + \alpha (1 - \tilde{z}) \left\{ y (W, W, 1 - \tilde{z}; n, n) - y (W, 1 - \tilde{z}; n, 0) \right\} \right]$$

(46)

Now use symmetry to replace $y (W, W, 1 - \tilde{z}; n, n)$ by $y^* (W, W, \tilde{z}; n, n)$ and $y (W, 1 - \tilde{z}; n, 0)$ by $y^* (W, \tilde{z}; 0, n)$. The expression in curly brackets in (46) can then be written in terms of home output $y (W, W, \tilde{z}; n, n)$, using Lemma 1 with $\bar{n}$ set equal to zero. Hence (46) simplifies to give:

$$L_{\tilde{z}} = n \left[ \alpha (\tilde{z}) - \frac{n}{n + 1} \alpha (1 - \tilde{z}) \right] y (W, W, \tilde{z}; n, n)$$

(47)

With symmetry, and recalling that $\tilde{z} > \frac{1}{2}$, the expression in square brackets is positive. As for $y (W, W, \tilde{z}; n, n)$, it is positive at points to the left of the $G_{FH} (\cdot; \xi)$ locus in Fig. 4 and zero where it meets the locus. This proves that $L_{\tilde{z}}$ is positive in the relevant region.

### 9.8 Proof of Proposition 6

Consider the effects of mergers on total profits in general equilibrium. From symmetry, total profits in the home country equal half of total profits in the world, which in turn equal total profits for all firms in sectors where $z$ exceeds $\frac{1}{2}$:

$$\Pi (w, \tilde{z}) = n \int_{\frac{1}{2}}^{\tilde{z}} \pi (W, W, z; n, n) dz + n \int_{\frac{1}{2}}^{\tilde{z}} \pi^* (W, W, z; n, n) dz + n \int_{\tilde{z}}^{1} \pi^* (W, z; 0, n) dz$$

(48)

This is clearly decreasing in $W$. To determine how it varies with $\tilde{z}$, differentiate with respect to $\tilde{z}$:

$$\Pi_{\tilde{z}} = -n \left[ \pi^* (W, \tilde{z}; 0, n) - \pi^* (W, W, \tilde{z}; n, n) \right] - \pi (W, W, \tilde{z}; n, n)$$

(49)

The expression in brackets is written in a way which facilitates comparison with the myopic gain from a single merger, $G_{FH} (n, n^*)$, in (3). From Proposition 3, it is greater than $G_{FH} (n, n^*)$, since every subsequent merger (at given wages and $\tilde{z}$) further raises the profits of remaining firms. Hence we can conclude that $\Pi$ is decreasing in $\tilde{z}$ whenever $G_{FH} (n, n^*)$ is positive, which means for all $\tilde{z}$ in the range $\tilde{z}_1 \leq \tilde{z} < \tilde{z}_0$. Since $\Pi$ is decreasing in both $W$ and $\tilde{z}$, both of which fall as a result of merger waves, it follows that total profits must increase, which proves Proposition 6.
9.9 Effects of Wages and the Extensive Margin on Welfare

Differentiating (21) with respect to \( W \) yields:

\[ U_W = -n \left[ \frac{2a \mu_1^S + nW \mu_2^S}{(n+1)^2} + \frac{2a \mu_1^D + nW (\mu_2^D + \gamma^2)}{(2n+1)^2} \right] \]  

Next, differentiate (21) with respect to the threshold sector, \( \tilde{z} \):

\[ U_{\tilde{z}} = 2 \left[ \lambda p (W, \tilde{z}; 0, n) \right]^2 - 2 \left[ \lambda p (W, W, \tilde{z}; n, n) \right]^2 \]  

This difference in squares can be simplified by factorizing it and by using the Cournot equilibrium prices from (25), which in general equilibrium can be written as:

\[ \lambda p (W, \tilde{z}; 0, n) = \frac{2a + nW \alpha (1 - \tilde{z})}{2(n+1)} \quad \lambda p (W, W, \tilde{z}; n, n) = \frac{2a + nW [\alpha (\tilde{z}) + \alpha (1 - \tilde{z})]}{2(2n+1)} \]  

Substituting from these into (51) gives:

\[ U_{\tilde{z}} = \frac{n}{n+1} \left[ \frac{3n+2}{2(n+1)(2n+1)} \Theta (\tilde{z}) + W \alpha (\tilde{z}) \right] \text{by} (W, W, \tilde{z}; n, n) \]  

where \( \Theta (\tilde{z}) \equiv 2a - W [(n+1) \alpha (\tilde{z}) - n \alpha (1 - \tilde{z})] \) is proportional to home output in sector \( \tilde{z} \). By inspection, \( U_{\tilde{z}} \) is zero along the \( G(\cdot; \xi_0) = 0 \) locus in Fig. 6 and negative to the left of it, which implies that iso-welfare loci are upward-sloping in this region and horizontal along the \( G(\cdot; \xi_0) = 0 \) locus, as stated in the text.

9.10 Mergers and Welfare

A small merger wave affects welfare as follows:

\[ \frac{dU}{d\tilde{z}} = U_{\tilde{z}} + U_W \frac{dW}{d\tilde{z}} = U_{\tilde{z}} - U_W L_W \frac{L_{\tilde{z}}}{L_W} = \frac{U_{\tilde{z}}L_W - U_W L_{\tilde{z}}}{L_W} \]  

Since the denominator \( L_W \) is negative, a small merger wave will raise welfare \( (dU/d\tilde{z} < 0) \) if and only if the numerator is positive. It is easily seen that this is equivalent to the case where the labour-market equilibrium locus is more steeply sloped than the iso-welfare locus, as illustrated in Fig. 6, which confirms the statement in the text. Combining the expressions for the individual terms from (44), (47), (50) and (53), the two slopes
can be written explicitly as follows:

\[
\frac{dW}{dz}_{LMEL} = \frac{1}{n+1} H (\tilde{z}) \frac{L_{\tilde{z}}}{L_W} = \frac{1}{n+1} \left[ \frac{1}{L_W} + \frac{r}{n+1} \right]
\]

\[
\frac{dW}{dz}_{U} = \frac{1}{U_W} \left[ \frac{2 \alpha \mu_1^S + n W \mu_2^S}{(n+1)^2} + \frac{n W (\mu_2^D + \gamma^2)}{(2n+1)^2} \right]
\]

where \( H (\tilde{z}) \) and \( J \) are defined in the text. Equations (55) and (56) imply, after eliminating common terms, that a small merger wave will raise welfare if and only if the following expression is positive:

\[
V (\tilde{z}) = H (\tilde{z}) \left[ \frac{2 \alpha \mu_1^S + n W \mu_2^S}{(n+1)^2} + \frac{n W (\mu_2^D + \gamma^2)}{(2n+1)^2} \right]
\]

\[
- \left[ \frac{3n + 2}{2(n+1)(2n+1)} \Theta (\tilde{z}) + W \alpha (\tilde{z}) \right] \left( \frac{\mu_2^S}{n+1} + \frac{J}{2n+1} \right)
\]

To explore the sign of \( V (\tilde{z}) \) it is helpful to consider separately its components corresponding to specialized and diversified sectors.

9.10.1 Effects on Specialized Sectors

Consider first the terms in \( V \) corresponding to sectors in which the home economy is specialized:

\[
V^S (\tilde{z}) = H (\tilde{z}) \left[ \frac{2 \alpha \mu_1^S + n W \mu_2^S}{(n+1)^2} + \frac{n W (\mu_2^D + \gamma^2)}{(2n+1)^2} \right]
\]

\[
- \left[ \frac{3n + 2}{2(n+1)(2n+1)} \Theta (\tilde{z}) + W \alpha (\tilde{z}) \right] \left( \frac{\mu_2^S}{n+1} + \frac{J}{2n+1} \right)
\]

Removing \( \alpha (\tilde{z}) \) from \( H (\tilde{z}) \) and collecting terms gives:

\[
V^S (\tilde{z}) = n \left[ \alpha (\tilde{z}) - \alpha (1 - \tilde{z}) \right] \left[ \frac{2 \alpha \mu_1^S + n W \mu_2^S}{(n+1)^2} + \alpha (\tilde{z}) \frac{2 \alpha \mu_1^S - W \mu_2^S}{(n+1)^2} \right] - \left[ \frac{3n + 2}{2(n+1)(2n+1)} \Theta (\tilde{z}) \mu_2^S \right]
\]

Next, multiply the second term by \( \frac{n(3n+2)}{2(2n+1)} \) (which equals one) and collect terms in \( 3n + 2 \):

\[
V^S (\tilde{z}) = \tilde{V}^S (\tilde{z}) + \frac{3n + 2}{2(n+1)^2(2n+1)} \left[ \alpha (\tilde{z}) \left( 2 \alpha \mu_1^S - W \mu_2^S \right) - \Theta (\tilde{z}) \mu_2^S \right]
\]

where:

\[
\tilde{V}^S (\tilde{z}) = n \left[ \alpha (\tilde{z}) - \alpha (1 - \tilde{z}) \right] \left[ \frac{2 \alpha \mu_1^S + n W \mu_2^S}{(n+1)^2} + \frac{n}{2(2n+1)} \alpha (\tilde{z}) \frac{2 \alpha \mu_1^S - W \mu_2^S}{(n+1)^2} \right]
\]

\( \tilde{V}^S (\tilde{z}) \) is unambiguously positive, recalling from (40) that \( 2 \alpha \mu_1^S - W \mu_2^S \) is proportional to total employment in the specialized sectors. That leaves only the term in brackets in (60) to be signed. Substituting for \( \Theta (\tilde{z}) \),
terms in $W\alpha(\tilde{z})\mu^S_2$ cancel leaving:

$$
\alpha(\tilde{z}) \left(2a\mu^S_1 - W\mu^S_2 \right) - \Theta(\tilde{z})\mu^S_2 = 2a \left\{ \alpha(\tilde{z})\mu^S_1 - \mu^S_2 \right\} + nW \left\{ \alpha(\tilde{z}) - \alpha(1-\tilde{z}) \right\}\mu^S_2
$$

(62)

This is unambiguously positive because $\alpha(\tilde{z})\mu^S_1 - \mu^S_2 = \int_{0}^{1-\tilde{z}} [\alpha(\tilde{z}) - \alpha(z)] \alpha(z) \, dz > 0$. Hence $V^S(\tilde{z})$ as a whole is positive for all $\tilde{z}$ in the range $\frac{1}{2} \leq \tilde{z} \leq \tilde{z}_0$.

### 9.10.2 Effects on Diversified Sectors

We first state some inequalities which will prove useful:

$$
\alpha(\tilde{z})\mu^D_1 > \mu^D_2 > \gamma^2 > \alpha(1-\tilde{z})\mu^D_1 \quad \text{and} \quad \alpha(\tilde{z})\gamma^2 > \alpha(1-\tilde{z})\mu^D_2
$$

(63)

These are easily checked; e.g., $\alpha(\tilde{z})\mu^D_1 - \mu^D_2 = \int_{1-\tilde{z}}^{\tilde{z}} [\alpha(\tilde{z}) - \alpha(z)] \alpha(z) \, dz > 0$.

Consider the terms in $V$ corresponding to sectors in which the home economy is diversified:

$$
V^D(\tilde{z}) \equiv H(\tilde{z}) \frac{2a\mu^D_1 + nW (\mu^D_2 + \gamma^2)}{(2n+1)^2} - \left[ \frac{3n+2}{2(n+1)(2n+1)} \Theta(\tilde{z}) + W\alpha(\tilde{z}) \right] \frac{J}{2n+1}
$$

(64)

Define $\Lambda^D \equiv 2a\mu^D_1 - WJ$; from (40) $\Lambda^D$ is proportional to the level of employment in the D-sectors. Substituting into the denominator of the first term in (64) gives:

$$
V^D(\tilde{z}) = H(\tilde{z}) \frac{\Lambda^D + (2n+1)W\mu^D_2}{(2n+1)^2} - \left[ \frac{3n+2}{2(n+1)(2n+1)} \Theta(\tilde{z}) + W\alpha(\tilde{z}) \right] \frac{J}{2n+1}
$$

(65)

$$
= \frac{W}{2n+1} \left[ H(\tilde{z})\mu^D_2 - \alpha(\tilde{z})J \right] + \frac{1}{(2n+1)^2} \left[ H(\tilde{z})\Lambda^D - \frac{3n+2}{2(n+1)} \Theta(\tilde{z})J \right]
$$

The first term is unambiguously positive, since $H(\tilde{z})\mu^D_2 - \alpha(\tilde{z})J = n [\alpha(\tilde{z})\gamma^2 - \alpha(1-\tilde{z})\mu^D_2]$, which is positive from (63). Note that this implies that $V^D(\tilde{z})$ evaluated at $\tilde{z} = \tilde{z}_0$ is definitely positive, since $\Theta(\tilde{z}_0) = 0$.

Next, we evaluate $V^D(\tilde{z})$ at $\tilde{z}_1$. From (18), we have at that point:

$$
2a = \frac{W}{\xi_1} [\alpha(\tilde{z}_1) - (1-\xi_1)\alpha(1-\tilde{z}_1)] = W \left[ \alpha(1-\tilde{z}_1) + \frac{1}{\xi_1} \delta(\tilde{z}_1) \right]
$$

(66)

where $\xi_1$ was defined in (5) and $\delta(\tilde{z}) \equiv \alpha(\tilde{z}) - \alpha(1-\tilde{z})$, the difference between home and foreign labour requirements in the threshold sector. Substituting for $2a$ into $\Lambda^D$ and $\Theta(\tilde{z})$ gives:

$$
\Lambda^D = W \left[ \alpha(1-\tilde{z}_1) + \frac{1}{\xi_1} \delta(\tilde{z}_1) \right] \mu^D_1 - WJ \quad \text{and} \quad \Theta(\tilde{z}_1) = W \left[ \frac{1}{\xi_1} - (n+1) \right] \delta(\tilde{z})
$$

(67)
Substituting these in turn into $V^D(\bar{z}_1)$, the term we wish to sign can be written as follows:

$$\frac{2n+1}{W} V^D(\bar{z}_1) = A + B - C \quad (68)$$

$$A \equiv H(\bar{z}_1)\mu_2^D - \alpha (\bar{z}_1) J \quad B \equiv \frac{1}{2n+1} \left[ \alpha (1 - \bar{z}_1) + \frac{1}{\xi_1} \delta (\bar{z}_1) \right] [H(\bar{z}_1)\mu_1^D - J]$$

$$C \equiv \Omega \delta (\bar{z}_1) J \quad \text{where:} \quad \Omega \equiv \frac{2n^2}{(n+1)(4n^2 - 4n - 1)}$$

It is easy to check that if this expression is positive, then $V^D(\bar{z})$ is also positive for any $\bar{z} \in [\bar{z}_1, \bar{z}_0]$. We turn next to two alternative routes to signing $V^D(\bar{z}_1)$.

9.10.3 Comparative Disadvantage on Average Versus at the Margin

A sufficient condition for $V^D(\bar{z}_1)$ to be negative is that these two measures of comparative disadvantage coincide. This occurs when $\mu_2^D$ attains its maximum value of $\alpha (\bar{z}) \mu_1^D$ and $\gamma^2$ attains its minimum value of $\alpha (1 - \bar{z}) \mu_1^D$. (Recalling (63), $\mu_2^D$ and $\gamma^2$ cannot actually attain these extreme values if, as we assume, $\alpha (z)$ is strictly increasing in $z$, but they can come arbitrarily close.) In this case $H(\bar{z}_1)\mu_1^D - J = 0$, so comparative disadvantage on average and at the margin are the same and $B$ is zero. In addition, $A$ reduces to $[H(\bar{z}_1)\mu_1^D - J] \alpha (\bar{z}_1)$ which is also zero. Hence $C$ is the only non-zero component of $V^D(\bar{z}_1)$, so the whole expression is negative.

To find a sufficient condition on the two measures of comparative disadvantage that ensures a positive value for $V^D(\bar{z}_1)$, rewrite the second and third terms as follows:

$$B - C = \frac{1}{2n+1} \alpha (1 - \bar{z}_1) + \frac{1}{2n+1} \delta (\bar{z}_1) [H(\bar{z}_1)\mu_1^D - \{1 + (2n+1) \xi_1 \Omega \} J] \quad (69)$$

Proposition 7 follows by inspection of the bracketed expression in the second term.

9.10.4 The Case of a Linear Cost Distribution

Suppose that the distribution of labour requirements across sectors is linear in $z$: $\alpha (z) = \alpha_0 + \beta z$. Routine calculations now show that the truncated moments are:

$$\mu_1^D = \left( \alpha_0 + \frac{1}{2} \beta \right) (2\bar{z} - 1) \quad (70)$$

$$\mu_2^D = \left[ \alpha_0 (\alpha_0 + \beta) + \frac{1}{3} \beta^2 (1 - \bar{z} + \bar{z}^2) \right] (2\bar{z} - 1)$$

$$\gamma^2 = \left[ \alpha_0 (\alpha_0 + \beta) + \frac{1}{6} \beta^2 (1 + 2\bar{z} - 2\bar{z}^2) \right] (2\bar{z} - 1)$$
and the measures of home comparative disadvantage are:

\[
H(\tilde{z}) = \alpha_0 + \beta \left[(2n + 1) \tilde{z} - n\right] \tag{71}
\]
\[
J = \left[\alpha_0 (\alpha_0 + \beta) + \frac{1}{6} \beta^2 \left(n + 2 - 2 (2n + 1) \tilde{z} (1 - \tilde{z})\right)\right] (2\tilde{z} - 1)
\]

Hence the first and second terms in the expression for \(V^D(\tilde{z}_1)\) become:

\[
A \equiv H(\tilde{z}_1) \mu_2^D - \alpha (\tilde{z}_1) J = n \left(\alpha_0 + \frac{1}{2} \beta\right) \left[\alpha_0 + \frac{1}{3} \beta (2 - \tilde{z}_1)\right] \beta (2\tilde{z}_1 - 1)^2 \tag{72}
\]
\[
C \equiv \Omega \left[\alpha (\tilde{z}_1) - \alpha (1 - \tilde{z}_1)\right] J = \Omega \left[\alpha_0^2 + \alpha_0 \beta + \frac{1}{6} \beta^2 \left(n + 2 - 2 (2n + 1) \tilde{z}_1 (1 - \tilde{z}_1)\right)\right] \beta (2\tilde{z} - 1)^2 \tag{73}
\]

Subtracting gives:

\[
A - C = \left[(n - \Omega) \alpha_0^2 + \left\{\frac{n}{2} + \frac{n}{3} (2 - \tilde{z}_1) - \Omega\right\} \alpha_0 \beta + \frac{1}{6} \left[n (2 - \tilde{z}_1) - \Omega \left(n + 2 - 2 (2n + 1) \tilde{z}_1 (1 - \tilde{z}_1)\right)\right] \beta^2\right] \beta (2\tilde{z}_1 - 1)^2 \tag{74}
\]

All three coefficients in the expression in square brackets are positive for \(n \geq 2\) and \(\tilde{z}_1 \geq \frac{1}{2}\), which proves Proposition 8.

Note finally that a linear distribution is not sufficient for \(B - C\) to be positive, so Propositions 7 and 8 are independent of each other. However, using more direct measures, comparative disadvantage at the margin is considerably greater than that on average when the distribution is linear. Instead of the difference between \(H(\tilde{z}) \mu_1^D\) and \(J\), consider the ratio of \(\mu_2^D - \gamma^2\) to \([\alpha (\tilde{z}) - \alpha (1 - \tilde{z})]\) \(\mu_1^D\). It can be checked that in the linear case \(\frac{\mu_2^D - \gamma^2}{[\alpha (\tilde{z}) - \alpha (1 - \tilde{z})] \mu_1^D} = \frac{\beta (2\tilde{z} - 1)}{6(\alpha_0 + \frac{1}{2} \beta)}\), which is less than or equal to \(\frac{1}{3}\).
References


Fig. 1: Equilibrium Production Patterns for a Given Cost Distribution

Fig. 2: Takeover Incentives
Fig. 3: Value of $\xi_0$ Minus $\xi_1$

Fig. 4: Simultaneous Determination of Wages and the Extensive Margin

$$G(W, \tilde{z}, \xi) = 0$$

$$L = L(W, \tilde{z})$$

$G(\cdot; \tilde{z}_0) = 0$

$G(\cdot; \tilde{z}_1) = 0$
Fig. 5: Wage Adjustments Dampen Cross-Border Merger Waves

Fig. 6: Mergers May Increase Welfare

\[ G(W, z; \xi) = 0 \]
\[ L = L(W, z; \xi) \]
\[ \bar{U} = U(W, z; \xi) \]
\[ G(A_0; \xi_0) = 0 \]
\[ G(A_1; \xi_1) = 0 \]
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**Fig. 7: Proof of Lemma 2 by Backward Induction**

(Fractions in boxes between rows \(s\) and \(s-1\) give the probability that a high-cost firm which declines an offer in stage \(s\) will face the takeover offer indicated by the corresponding arrow in stage \(s-1\).)