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R.C. GEARY'S CONTRIBUTIONS TO ECONOMIC THEORY*

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R.C. GEARY'S CONTRIBUTIONS TO ECONOMIC THEORY

Roy Geary was not an economist by training or inclination. His jousts with "literary" economists have been well documented and he seems to have had little time for pure theory either.[1] Of course he worked with economic statistics for most of his life and in his later years published extensively on applied economics topics. But he appears to have had little appreciation of the deductive as opposed to the inductive side of economics. So it is all the more remarkable that, in addition to his seminal work in statistics and econometrics, he made a number of important contributions to economic theory. In this paper, I review three of these contributions. My perspective is mainly analytic rather than biographical, so in each case I try to explain in simple but modern terms what Geary did and to place his work in the context of others' before and since.

1. The Stone-Geary Utility Function

First and foremost among Geary's contributions to economic theory is his derivation of the utility function which underlies the linear expenditure system. How this utility function later came to be labelled "Stone-Geary" is a fascinating question in the history of economic thought to which I will return. First, I want to summarise what Geary did and, since his two-page note was a comment on an earlier paper by Klein and Rubin (1947-48), I need to review that paper too.

Klein and Rubin's starting point was a problem which continues to concern economists and policy makers: since published consumer price indexes use base-period weights they fail to allow

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[1] See Fanning (1984, pages 151-155) and Kennedy (1993, pages 235-237).

for substitution between commodities by consumers. Nowadays there is much concern that this leads to an *over*estimate of the true rate of inflation (since consumers can escape part of the costs of higher prices by consuming less of those goods whose prices have risen relatively more). Klein was concerned with a different problem: in conditions of wartime rationing fixed-weight indexes lead to an *under*estimate of the true cost of living (since consumers substitute towards unrationed goods). In either case, the remedy in principle is to calculate a "true" cost of living index which takes account of consumer tastes, but since these are unknown the true index is unobservable.

To tackle this problem, Klein and Rubin began by showing how the true index can be calculated provided the consumer's demand functions are known and the Slutsky equation holds. However, they noted that without further restrictions this requires integrating a complicated partial differential equation. So, they proceeded to illustrate their method for what they called "a simple case," where the demand functions for n goods take the form:[2]

$$q_i \quad = \quad \sum_j \alpha_{ij} \frac{p_j}{p_i} \quad + \quad \beta_i \frac{z}{p_i} \qquad i=1, \ \ldots \ , \ n . \qquad \textbf{(1)}$$

In this almost casual fashion was the linear expenditure system, still one of the most widely-used specifications of consumer behaviour, introduced to economic theory.

The form of (1) is a natural way of specifying a general system of demand equations: the demand for each commodity is a linear function of the relative prices of all goods and of income, with the latter deflated by the price of the good in question. As written, there appear to be

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[2] q_i and p_i are the quantity and price of good i respectively; z is total expenditure or income (equal to $\Sigma_j \, p_j q_j$); and the α_{ij} and β_i are parameters.

$n(n+1)$ distinct parameters: n^2 α's and n β's. However, Klein and Rubin showed that, if the demand functions are to satisfy the budget constraint and Slutsky symmetry, the number of free parameters is much less than this: only $2n-1$. With the restrictions of rational choice imposed, the demand functions (1) can be written in the following much simpler form:

$$q_i \quad = \quad \gamma_i \quad + \quad \frac{\beta_i}{p_i} \left\{ z - \sum_j \gamma_j p_j \right\} \qquad i=1, \ \dots \ , \ n \ . \tag{2}$$

This form has only $2n-1$ independent parameters, n γ's and $n-1$ β's, since the income derivatives satisfy the restriction $\Sigma\beta_i = 1$. Klein and Rubin then proceeded to derive the true cost of living index corresponding to the demand functions (2) and to discuss how these might be estimated.

While Klein and Rubin's focus on price indexes led them to derive what would now be called the expenditure function corresponding to the linear expenditure system, they did not consider its implications for the form of the utility function. This was where Geary came in: he provided a simple derivation of the utility function which underlies the demand functions (2). Along any indifference curve, $du=0$ and so (since prices are proportional to marginal utilities) $\Sigma_i\, p_i\, dq_i = 0$. Substituting for p_i from (2), this becomes:

$$z' \sum_i \beta_i \frac{dq_i}{q_i - \gamma_i} \quad = \quad 0 \ . \tag{3}$$

Here $z' = z - \Sigma_j\, p_j\, \gamma_j$, which may be cancelled. What is left is a simple differential equation which may be integrated to give the utility function:

$$u \quad = \quad \prod_i \ (q_i - \gamma_i)^{\beta_i} \ . \tag{4}$$

Geary called this a hyperbolic form; alternatively, it could be described as a displaced or "translated" Cobb-Douglas defined with reference to an arbitrary origin. The standard Cobb-Douglas has all the γ's equal to zero and so implies that tastes are homothetic, with the β's giving both the marginal and average budget shares. By contrast, (4) starts from an arbitrary point defined by the γ's. All the income-consumption curves are straight lines through this point, but marginal and average budget shares differ and so the specification allows for non-homothetic tastes as well as a high degree of inter-commodity substitution. Figure 1 illustrates.

The rest is history. The linear expenditure system was empirically implemented by Stone (1954) who pioneered the estimation of systems of demand equations. He recognised the versatility of the system and developed procedures for estimating it which though primitive by modern standards were a major step forward in the 1950's. The utility function (4) came to be known as Stone-Geary[3] and soon emerged as one of the standard tools in empirical demand analysis. Certainly hundreds and perhaps thousands of papers have published estimates of the linear expenditure system as a model of expenditure allocation within a given period. Moreover, the relative tractability of the utility function has led to its application in a great variety of other contexts. Extensions to intertemporal choice were pioneered by Phlips (1972); it has been applied to modelling trade-union behaviour by Pencavel (1984); and even to the choice between armaments and consumer goods in defense economics by Hilton and Vu (1991). The versatility of the Stone-Geary form is shown by the fact (not often remarked upon) that it was independently developed by Nash (1953) as the solution of his axiomatic approach to bargaining theory. With two parties, each of whom has an outside option yielding a utility γ_i, $i=1,2$, and relative bargaining strengths (derived from their intertemporal elasticities of substitution)

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[3] I am not aware of the first use of this term.

measured by β_i, $i=1,2$, the outcome will be as if a welfare function of the form (4) were maximized, where the q_i should be reinterpreted as the utility levels of the two players.

Of course, in its original realm of application to systems of demand equations, the Stone-Geary utility function has important deficiencies. This is the downside of its desirable quality of a relatively parsimonious parameterization. Thus, while inferior goods are possible if some of the β's are negative, this violates the convexity to the origin of the indifference curves. Moreover, if convexity holds, then all goods are substitutes, so complementarity relationships are ruled out. Finally, since the utility function is additively separable, it shares a general property of additively separable demand systems which Deaton (1974) has called "Pigou's Law": to a reasonable approximation, the ratio of every own price elasticity of demand to the corresponding income elasticity is constant. Clearly we would prefer to test this restriction rather than have it imposed on the data.

Relaxing the assumption of additive separability while retaining some of the spirit of the linear expenditure system relates Geary's work to that of another leading Irish scholar, W.M. Gorman. Recall that Klein and Rubin derived the expenditure function underlying the linear expenditure system, which equals:[4]

$$e(p,u) \quad = \quad \sum_i p_i\gamma_i \; + \; u\,.\prod_i \;(p_i/\beta_i)^{\beta_i}\,. \qquad \textbf{(5)}$$

This is a linear function of utility where the intercept is linear in prices, giving the cost of the consumption bundle corresponding to the displaced origin, $q_i=\gamma_i$; and the slope is a Cobb-Douglas

[4] Strictly speaking, Klein and Rubin did not derive the expenditure function in full since they wrote (5) with the coefficient of the Cobb-Douglas term an undetermined constant of integration. Deriving it from the utility function shows that the constant must equal u.

term with β-deflated prices instead of quantities as arguments. The "Gorman polar form" can be thought of as a generalisation of this, where the expenditure function is linear in utility:

$$e(p,u) \quad = \quad a(p) \; + \; u\,.\,b(p)\,, \qquad \textbf{(6)}$$

with the only restriction that the functions *a(p)* and *b(p)* be linearly homogeneous in prices.[5] By choosing suitable functional forms for *a(p)* and *b(p)*, members of the Gorman polar form family can be derived which do not exhibit additive separability. Estimating these demand systems thus allows a direct test of the validity of the linear expenditure system, which not surprisingly is typically violated.[6] Even the Gorman polar form itself is quite restrictive. While it has the attraction that it allows consistent linear aggregation across quantities, more recent developments have followed the much weaker requirement, pioneered by Muellbauer (1975), of consistent linear aggregation across budget shares. Many of these models (such as those of Simmons (1980) and Blundell and Ray (1982 and 1984)) nest the LES although some of the most commonly used, notably the AIDS ("Almost Ideal Demand System") model of Deaton and Muellbauer (1980), do not.

So far, we seem to have an amazing success story for Geary: a two-page note stimulates an enormous literature making use of a functional form that continues to hold a central place in

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[5] See Gorman (1953 and 1961). Of course, I do not mean to imply that Gorman presented his work in the LES context. Indeed, I am not aware of any direct contact between Geary and Gorman.

[6] Pollak (1971) and Brown and Heien (1972) generalise the *b(p)* function to CES and nested CES forms respectively. The resulting demand systems exhibit additive separability but have budget shares which vary with prices. Nasse (1970) allows the *a(p)* function to take the Generalised Leontief form, so abandoning additive separability, while Blackorby, Boyce and Russell (1978) consider generalisations of the Gorman polar form which allow for more general forms of both *a(p)* and *b(p)* than (5).

consumer theory and that also has many applications in other fields. There is only one problem: Geary was not the first to derive the utility function (4) from the linear expenditure system (2)! That honour goes instead to Samuelson (1947-48), whose paper appeared in the same issue of the *Review of Economic Studies* as Klein and Rubin's, thus preceding Geary by two years. Why did Samuelson's name (perhaps along with Geary's) not attach itself to the utility function? Why was Samuelson's paper largely neglected and Geary's remembered? Indeed, why was Geary's published at all in its present form (with no reference to Samuelson)? These are intriguing questions on which I have little to say.[7] Instead I will confine my remarks to what can be gleaned from the published texts and consider the differences between the two papers.

The papers certainly differ in style. Samuelson's reads like a referee report on Klein and Rubin.[8] For example, he does not actually derive the utility function but merely sketches how

[7] It is noteworthy that the paper by Stone (1954) which pioneered the estimation of the linear expenditure system cites Samuelson but not Geary. By contrast, Stone (1987) in a biographical note on Geary reverses the honours. A counter-reaction is illustrated by Spencer (1983), who in deference to Samuelson's priority does not mention the Stone-Geary utility function in his summary of Geary's principal achievements.

[8] Klein (1985, pp. 14-15) confirms that Samuelson's paper originated as a referee's report. The introductory footnotes to the Klein-Rubin and Samuelson papers show that both originally formed part of the same Cowles Commission working paper. Klein's account of his contribution to this topic is worth quoting in full:

"[During my time at the Cowles Commission] I had an enduring interest in the economic theory of cost-of-living numbers, having been involved, while at MIT, in the dispute about formulas for wage indexation under wartime conditions of rationing. I was much impressed by work of N. Kaldor and J.L. Nicholson in Great Britain on the theory of measurement of the cost of living in such circumstances. I brought these two ideas together in formulating the linear expenditure system. I wrote down the marginal conditions for optimization of a consumer budget, where the demand functions were assumed to be linear in relative prices and real income. In discussions with Herman Rubin, encouraged by the collegial atmosphere of the Cowles Commission, where we regularly gathered in small groups in front of a blackboard to discuss common problems, I posed the problem of integrating the underlying differential equations. Herman, in a characteristic way, immediately saw the mathematical solution and we jointly had it can be done.[9] By contrast, Geary sets out the problem in a clear self-contained way, as we have seen. Geary's treatment of stochastic issues is also superior, as we would expect. Samuelson devotes four paragraphs to the question of how many empirical observations are needed to identify the parameters of (4) if that equation holds exactly; he then effectively concedes that his discussion is pointless since "All the above disregards the inevitable stochastical [sic] 'errors' present in any empirical situation." By contrast, Geary discusses the estimation problem in a short but insightful paragraph which mentions problems of measurement error and discusses whether prices or quantities should be viewed as the dependent variables.[10] Perhaps the real reason why Geary's paper was remembered more than Samuelson's may be that Geary was so much more positive about the prospects of implementing the theory empirically.[11]

None of these differences is of much consequence. There is however one other difference

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two formulas, the parametric expressions for the demand equations in the linear expenditure system and the expression for the 'true' cost-of-living index associated with this demand system.

"Paul Samuelson, who refereed our paper for publication, and R.C. Geary both saw the utility function that would produce the linear expenditure system, according to the economic theory of the consumer. This system was later discovered, independently, by Richard Stone."

[9] In my notation, all he says is: "By working with new variables $q_i' = q_i - \gamma_i$, we also quickly see [after rewriting the demand functions as (2)] that the ordinal utility or preference field must be of the form:

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\begin{aligned}
u &= F[\beta_1 \log q_1' + \ldots + \beta_n \log q_n'] \\
&= F[\beta_1 \log(q_1 - \gamma_1) + \ldots + \beta_n \log(q_n - \gamma_n)]
\end{aligned}
$$

where F is any function with $F' > 0$." Unlike Geary, Samuelson gives the logarithmic form of the utility function and stresses that it is ordinal rather than cardinal. The former difference is not quite so trivial as it might seem, as I suggest below.

[10] The estimation of the linear expenditure system remains a complicated problem, largely because (ironically) it is so non-linear. See Chipman and Tian (1989).

[11] I am grateful to John Aldrich for this suggestion and to Paul Samuelson for confirming its plausibility.

between the two papers which is of some theoretical interest. It concerns the signs of the γ's. As (2) is written (following modern conventions), the γ's are presumptively positive. This invites an interpretation which was first provided by Samuelson. The consumer can be thought of as first buying a necessary or subsistence set of goods, $\{\bar{q}_i=\gamma_i\}$; and then spending her surplus or "supernumerary" income, $z-\Sigma_j p_j \gamma_j$, in constant proportions (given by the β's) on the different commodities. As Samuelson noted, this interpretation provides an intuitive explanation for the expenditure function (5) and so for the true cost of living formula. Necessary expenditures contribute the linear term to the formula, while supernumerary income spent in constant proportions contributes the weighted geometric mean or Cobb-Douglas term. This interpretation can be applied to any member of the Gorman polar form. The *a(p)* and *b(p)* terms can be thought of as corresponding to the indifference curves of a "poor" and a "rich" consumer respectively; as utility rises the latter increasingly dominates the former.

However, a major problem with this elegant interpretation is that there is no guarantee in theory or practice that the γ's will be positive! Indeed, if the consumer's preferences are well-behaved for all nonnegative values of consumption, then the γ's cannot be positive. Provided all goods are normal, it is necessary for the actual consumption of each good to exceed the subsistence level if indifference curves are to be convex to the origin.[12] Hence, on economic

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[12] To see this, totally differentiate the utility function, setting $du = dq_k\ (k \neq i,j) = 0$, and rearrange to get the slope of an indifference curve:

$$\frac{dq_i}{dq_j} = -\frac{\beta_i}{\beta_j}\frac{q_j-\gamma_j}{q_i-\gamma_i}.$$

Now differentiate again to get the curvature of the indifference curve:

grounds, it may make more sense to follow Geary (as well as Klein and Rubin) and write the γ's as the negative of what I have written in (2) above.

How should we evaluate Geary's contribution in the light of these considerations? Cases of inappropriate or incomplete attribution abound in economics, as do cases of simultaneous discovery.[13] Even confining attention to functional forms which I have mentioned, it is worth recalling that the Cobb-Douglas function was first developed by Wicksell; and that the linear Engel curves property of the Gorman polar form originated with Antonelli (1886) and Nataf (1953), both of whom explicitly discussed its importance in the context of aggregation across consumers. Allowing for the interesting difference which I have noted between Geary and Samuelson in the interpretation of the γ coefficients, it seems reasonable that Geary get some credit. Perhaps the fairest nomenclature would be "the Samuelson-Geary utility function corresponding to the Klein-Rubin linear expenditure system whose estimation was pioneered by Stone". But there seems little point in trying to replace established practice with such a mouthful. Napoleon required of his generals only that they be lucky and even if it is only luck which attaches Geary's and Stone's names to the utility function we can surely rejoice in that.[14,15]

$$\frac{d^2 q_i}{dq_j^2} = -\frac{\beta_i}{\beta_j}\frac{1}{q_i-\gamma_i}\left[1+\frac{\beta_i}{\beta_j}\left(\frac{q_j-\gamma_j}{q_i-\gamma_i}\right)^2\right].$$

For well-behaved convex indifference curves, this must be negative. Hence, with both goods normal (β_i, $\beta_j > 0$), q_i must exceed γ_i.

[13] On the latter, see Niehans (1995).

[14] It is also perhaps fair that Geary gets more credit than he deserves in this case since he so conspicuously got less from Durbin and Watson who in their famous paper on testing for residual autocorrelation attributed a key result to von Neumann rather than to an earlier paper of Geary's. See Conniffe (1997) for further details.

2. International Comparisons of Real Income

While the Stone-Geary utility function is a household name, another of Geary's contributions is even more widely used though paradoxically less widely identified with him. This is his algorithm for calculating "world" prices and "true" exchange rates which correct for deviations from purchasing power parity between countries. These in turn lead to comparable measures of real output or income. Geary apparently developed his method as the outcome of advising the FAO in Rome on how to calculate an index of world agricultural output and published it as a three-page note in 1958.[16] It was later adopted by Irving Kravis and his co-workers at the University of Pennsylvania as the basic methodology underlying the International Comparison Project: a massive research project which has produced comparable data on GNP and its components for between 16 and 64 countries at five-yearly intervals since 1970. The ICP in turn forms the basis for the Penn World Table, an enormous data set giving comparable data on over 100 countries from 1950 onwards. These data are naturally of great intrinsic interest and have also made possible an explosion of international comparisons attempting to provide a quantitative basis for the causes of economic growth. Heston and Summers (1996) quote an anonymous claim that over twenty thousand regressions have been estimated using the Penn World Table![17]

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[15] Of course, the term "Stone-Geary" is not universal. A case in point is Casey (1973), drawn from a Ph.D. by an Irish student at Cambridge, where the linear expenditure system is attributed to Samuelson and Stone!

[16] Kawakatsu (1970, p. 173, footnote 3) cites an earlier version of Geary (1958) with the title "Note on national and international indices of agricultural output" which appeared as a mimeographed appendix to an FAO document circulated in June 1953. I am grateful to John Spencer for this reference.

[17] Kravis (1984) and Summers and Heston (1991) provide introductions to the ICP and the Penn World Table respectively. Further references on these and other aspects of this section may be found in Neary (1996a).

The problem of measuring real income can be illustrated in the two-good two-country case in Figure 2. Points J and K represent the output or consumption bundles of countries *j* and *k* respectively, with the slopes of the dashed lines through each point indicating the relative prices in each country. It is intuitively plausible that it would be desirable to compare the real incomes of the two countries expressed in a common set of prices but the problem is which set to choose. Measuring the real income of *j* relative to *k* along an arbitrary ray from the origin OQ, gives OA/OB if country *k*'s prices are used but OC/OD if country *j*'s prices are used. These differ greatly and the fact that OA/OB exceeds OC/OD reflects a standard bias known as the "Gerschenkron Effect": each country has a relatively higher real income if the *other* country's prices are used as reference. In practice some average of these Laspeyres and Paasche indexes is often used, such as their geometric mean, the Fisher "Ideal" index. However, multilateral extensions of such average methods do not exhibit a property known as "matrix consistency": that is, they do not give a consistent matrix of world consumption in constant prices, broken down by countries and commodities. It is precisely this gap which is met by the Geary method.

Geary's method can be illustrated for the special case of two goods and two countries in Figures 3 and 4, where points J and K have the same interpretation as in Figure 2. Rather than taking one country's prices as reference, he postulates the existence of "world" prices, corresponding to the slopes of the lines labelled π through J and K. Hence country *j*'s income at world prices must lie along the line aJ in Figure 3. To find out exactly where, assume that the price of good 2 is unity in both countries and on the (hypothetical) world market. Then we can measure values along the q_2 axis and in particular the price index for country *j* (the ratio of its output at world prices to its output at domestic prices) equals the ratio of Oa to Ob. This can be interpreted as country j's exchange rate corrected for purchasing power parity. With many goods ($i=1, ..., n$) and many countries ($h=1, ..., m$) the general formula for country h's exchange rate is:

$$\epsilon_h = \frac{\sum_i \pi_i q_{ih}}{\sum_i p_{ih} q_{ih}}, \qquad h = 1, \ldots m. \tag{7}$$

So far, this is just a standard matter of defining a price index for country j. The crucial next step is to require that, when valued at world prices, the worldwide consumption of each good equals its consumption valued at domestic prices converted at the true exchange rates:

$$\pi_i = \frac{\sum_h \epsilon_h p_{ih} q_{ih}}{\sum_h q_{ih}}, \qquad i = 1, \ldots n. \tag{8}$$

In the two-good two-country case, this takes a particularly simple form for the numeraire good 2 (since $\pi_2=p_{2j}=p_{2k}$):

$$q_{2j} + q_{2k} = \epsilon_j q_{2j} + \epsilon_k q_{2k}. \tag{9}$$

To locate $\varepsilon_j q_{2j}$ in the diagram, note that, by similar triangles, ε_j = Oa/Ob = OJ"/OJ = Lj/LJ. Hence, since LJ equals actual consumption of good 2 in j (q_{2j}), the vertical height of point J" (Lj) gives consumption of good 2 in j valued in world prices, $\varepsilon_j q_{2j}$. Projecting J"j rightwards to meet the line aJ therefore gives point J′ as the point at which country j's consumption of both goods is revalued at world prices.

Finally, matrix consistency is assured by requiring that, when this revaluation is repeated for country k, the two countries' total consumption bundles add up to equal actual world consumption, denoted by point W in Figure 4. Here, point J′ is repeated from Figure 3; point K′ is the corresponding point for country *k*; and the aggregation condition is satisfied as required. It may be thought surprising that the consumption levels revalued at world prices (given by points J′ and K′) are so different from the actual consumption levels in each country (given by points J and K). This follows from the normalisation assumed. With the price of good 2 equal to unity everywhere, the domestic price of good 1 in country *j* is considerably higher than that in country *k* (equal to the slopes of Bj and Dk respectively). Hence country *j*'s currency is highly overvalued. When measured at world rather than at domestic prices, country *j*'s consumption of good 1 is considerably greater, and its consumption of good 2 correspondingly less.

A recurring feature of Geary's contributions to economic theory is an ambiguity as to whether he should be given credit alone or with others! In the present case, the method I have credited to Geary is often referred to as the "Geary-Khamis" method. Khamis wrote extensively on this topic and seems to have had considerable input into the adoption of the method by the ICP. At a substantive level, he provided a set of necessary and sufficient conditions (using standard Frobenius techniques) for the method to yield non-negative world prices. He also advocated, notably in Khamis (1972), the use of both price and quantity indexes whose functional forms derive from (7) and (8). Whether these contributions merit a co-attribution of the method is debatable.

The Geary method is not the only one which has been proposed for measuring real incomes across countries. Indeed, many such measures have been proposed.[18] However, its main rival in practice is the so-called "EKS" method, developed by Gini in the 1930's and independently rediscovered by Eltetö and Koves (1964) and Szulc (1964). This amounts to a multilateral extension of the Fisher Ideal index number (discussed above), amended to ensure that relative real incomes are invariant to the choice of reference country and yield a transitive ranking across countries. This method is used by the OECD and Eurostat as the basis for their official publications.[19] It has also received considerable theoretical support from the writings of a number of theorists, most notably Erwin Diewert. He argues that the EKS index or its variants provide a good approximation to the true index for a general (specifically, a translog) specification of consumer preferences.

My own view, presented in Neary (1996a and 1996b), is that the claims for the theoretical superiority of the EKS method do not stand up. As I see it, there are two distinct questions which must be faced in choosing a desirable index for multilateral comparisons on economic theory grounds. First, what are we trying to measure? This question arises because, except in the special and highly unrealistic case of homothetic tastes, there are different candidates for the "ideal" or "true" index. Second, given a particular choice of true index, which of its nature is

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[18] For recent overviews, taking anti- and pro-Geary positions respectively, see Diewert (1996) and Neary (1996a). Mention should also be made of the methods of van Yzeren (1956) which were discussed by Geary. See the discussion in Balk (1989).

[19] Though this represents the outcome of a near-farcical compromise. These two organisations convened a conference in 1989, at which their expert advisors failed to agree on whether the Geary or EKS methods should be adopted. As a result, the OECD now publishes annually two complete tables of real income indexes for its member countries. However, "Eurostat requires that only one set of results be recognised as the official results of the [European] Community," so that based on the EKS method is released a year before that based on the Geary method. See OECD (1990).

unobservable, how can it best be approximated in practice?

One answer to the first question would be to use the true real income index due to Allen (1949). This compares the cost, at reference prices, of attaining each country's utility level. This is preferable to the Laspeyres and Paasche indexes in Figure 2 which compare the cost of purchasing each country's consumption bundle. The Allen index thus allows for substitution by consumers away from more expensive goods. However, its values depend on the reference price vector used. An obvious candidate is the price vector of one particular country. But why should one country be privileged in this way? The alternative I have proposed is to use an average set of prices which, like the Geary world prices, ensure matrix consistency. This leads to what I call the GAIA ("Geary-Allen International Accounts") system: a set of real income indexes which retain the desirable aggregation properties of the Geary indexes but, unlike them, are consistent with economic theory.

Of course, these GAIA indexes are unobservable, which leads to the second question: how best to approximate them? One approach would be to estimate a complete system of demand equations and use the estimated parameters to calculate the GAIA indexes. Neary and Gleeson (1997) show that this approach is feasible although it has not yet been adopted in large-scale practical applications. The alternative approach is to use one of the available observable multilateral index number formulae to approximate the GAIA indexes, which leads in turn to the question of which one does so best? In answer to this, Neary (1996b) argues that the (observable) Geary indexes provide a Laspeyres-type approximation to the (unobservable) true GAIA indexes and so can be justified as the appropriate ones to use in practice. It is also shown there that, when tastes are not homothetic, the EKS indexes only provide an approximation to an inconsistent set of multilateral indexes and so their theoretical basis is shaky.

While the jury is still out on these theoretical debates, the fact that the Geary method has been adopted by the most comprehensive and widely-used source of data on international real incomes seems likely to ensure it a significant place in the tool-kit of economists interested in international comparisons.

3. Real Income and the Terms of Trade

The final aspect of Geary's work which I want to discuss is his contribution to the debate on how to measure the gain in a country's real income arising from changes in its terms of trade. To understand the problem, consider the trade surplus *S*, defined as the difference between exports *X* and imports *M*, all measured in current prices:

$$S = X - M. \tag{10}$$

Assuming we have accurate price indexes for exports and imports, p_X and p_M, we can use them to deflate *X* and *M* to obtain exports and imports in constant prices, $x=X/p_X$ and $m=M/p_M$. The problem now is what price index *p* should be used to deflate the surplus to obtain a measure of the "real" surplus $s=S/p$. The choice of deflator in turn implies different values for the "trading gain" *t*, which is the residual term when (10) is expressed in real terms:

$$s = x - m + t. \tag{11}$$

This "trading gain" (the term appears to have been coined by Burge and Geary (1957)) is the empirical counterpart of the change in welfare or real income attributable to changes in the terms of trade.[20]

The simplest and probably most widely-used method of deflating *S*, due to Nicholson (1960), is to use the import price index. This gives a value for the trading gain of $t = x(p_X - p_M)/p_M$. However, there are many alternative methods. Geary's name is associated with two distinct proposals for resolving this problem. The first originated with an Australian statistician R.W. Burge but is usually referred to as the "Burge-Geary" system.[21] It is identical to the Nicholson method when the surplus is negative, but uses the export price index as deflator when the surplus is positive (so the trading gain becomes $t = m(p_X - p_M)/p_X$). This seems like a curious compromise but it has the rationale that it only measures the realised gain. It also has the advantage that it aggregates correctly when there are two countries (or one country and the rest of the world), in the sense that there is no net trading gain for the world as a whole.

The question of whether the deflation method chosen should be required to aggregate across countries appears to have been much discussed at the 1959 conference of the Association for Research in Income and Wealth held in Portoroz in the former Yugoslavia, whose proceedings on this topic were introduced by Geary (1961). Nicholson, who presented his paper at the

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[20] As Geary (1973, p. 237) states, "[The trading gain] is related to the more familiar *terms of trade*, [italics in original] but far more significant, in yielding, instead of an index number, an absolute value measuring the improvement (or disimprovement) of the nation consequent on trading price movements." So the change in the terms of trade is the exogenous shock and the trading gain is the resulting change in real income.

[21] The original reference to this system is an unpublished paper, Burge and Geary (1957). Geary (1961) refers to the method twice, first (p. 5) in terms which suggest that it is exclusively due to Burge and second (p. 7) as "the method ... advocated by Burge and Geary". Geary and Pratschke (1968, p. 33) attribute the method to Burge. I am grateful to John Spencer for invaluable assistance on this matter and also for tracking down a copy of Burge and Geary (1957) in the Bodleian.

conference though it was published elsewhere, argued that the net gain in income should not be zero in the two-country case. Rather, he claimed, "the gain (or loss) from changes in the terms of trade in the [national] product of the one country is necessarily equal to the loss (or gain) in the [national] income of the other country." However, it is not clear how this argument extends to the many-country context. In his introduction to the conference proceedings, Geary quoted criticisms of the Burge-Geary method along these lines by A.L. Gaathon. In response, and literally in what he called "A Footnote on the Trading Gain", Geary set out some axioms which a measure of the trading gain should satisfy and proposed yet another method which overcomes the Gaathon objection. This uses the arithmetic mean of the export and import deflators, $p=½(p_X+p_M)$, giving a value for the trading gain of:

$$t \quad = \quad (x+m)\left(\frac{p_X - p_M}{p_X + p_M}\right). \tag{12}$$

As Geary noted, this is robust to subdivisions or amalgamations of countries: the net trading gain of any set of countries is the same (and that of the world as a whole is zero) however they are grouped together.

The further development of this literature would take me too far afield. Neither of Geary's proposals appears to be widely used, though both are considered as candidates in most discussions of the topic.[22] Geary himself appears to have been relatively agnostic about which method should be used, arguing in Burge and Geary (1957) that any weighted average of p_X and

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[22] The Nicholson method is used in the Irish national accounts, where the issue is of considerable practical importance. As Geary noted, this was identical to the Burge-Geary method in the years when he was Director of the Central Statistics Office, though trade surpluses have been more common in recent years. See Leddin and Walsh (1995, p. 65).

p_M would suffice. As Spencer (1997) notes, this is exactly the recommendation of the most recent UN System of National Accounts. Characteristically, Geary was concerned with the quantitative significance of the choice between different measures and in Geary and Pratschke (1968, pp. 33-35) he presented evidence that they yielded very similar results. However, this was for Irish data in the relatively tranquil 1950's and 1960's. A more recent and much more comprehensive study by Silver and Mahdavy (1989), which compares ten different measures (including the Burge-Geary and Geary measures), shows that the choice of method can make a considerable difference in practice.

As for theoretical discussions of this question, they have not reached any consensus: Hamada and Iwata (1984) mention the Geary method favourably but Diewert and Morrison (1986) do not discuss it at all. Significantly, the only major empirical study I have found which makes use of the Geary method is that of Weale (1984). His paper estimates a world trade model, and was concerned to preserve the aggregation properties which only the Geary method guarantees. In a final irony, the Penn World Table itself uses a different method. Summers and Heston (1991, p. 344) warn users that the resulting numbers do not aggregate to the actual totals across countries, though without mentioning the Geary method which would avoid this difficulty!

4. Conclusion

In this paper, I have attempted a centenary review of Roy Geary's three principal contributions to economic theory proper, as opposed to statistical or econometric methods: the Stone-Geary utility function; the "Geary method" for international comparisons of real income corrected for deviations from purchasing power parity; and methods for calculating the change in real income arising from changes in the terms of trade. I have given simple geometric expositions of these pioneering contributions, discussed their origin and use, and tried to assess Geary's place in the development of each sub-field.

With the benefit of hindsight, it is clear that the common threads linking Geary's three contributions are linearity and aggregation. The Stone-Geary utility function is the most general utility function consistent with demand functions which are linear in income and prices. It is also a special case of the Gorman polar form, which in turn is the only demand system which allows exact linear aggregation over quantities consumed by different individuals. As for the Geary method of correcting real incomes for deviations from purchasing power parity, it is the only one which permits "matrix consistency": that is, which gives a consistent matrix of world output in constant prices, broken down by countries and commodities. Finally, Geary's suggestion for estimating the terms of trade gain of an open economy is the only one which ensures that the net gains to the world as a whole are zero.

This intellectual cohesiveness of his three contributions to economics is all the more remarkable since they were not part of a coherent research programme but rather represented occasional diversions from his statistical work and his administrative responsibilities. It would be fascinating to learn from someone who is familiar with Geary's statistical work in detail if similar formal themes can be identified in his contributions on different topics.

Finally, this review of Geary's work on economic theory raises the tantalising prospect of how much more work of this kind he might have done if he had found a university position. He famously failed to do so on at least one occasion, to the eternal disgrace of University College Dublin. (See Spencer (1997) for further details.) So he acquired no Ph.D. students, no

Festschrift, no "school" of disciples. Of course, in 1960 he became the first Director of the ERI (later ESRI), where he inspired (and overawed!) junior colleagues until his death in 1983. But economic theory is not for the elderly and Geary's work in his last years was mostly on applied economic problems. If the rush of symposia dealing with Geary and his writings (no less than four in the *Economic and Social Review* since 1976) may seem suspiciously like ancestor worship to a foreigner, it can perhaps be viewed more positively as a belated celebration of his best work and an ongoing reflection on its continuing relevance.

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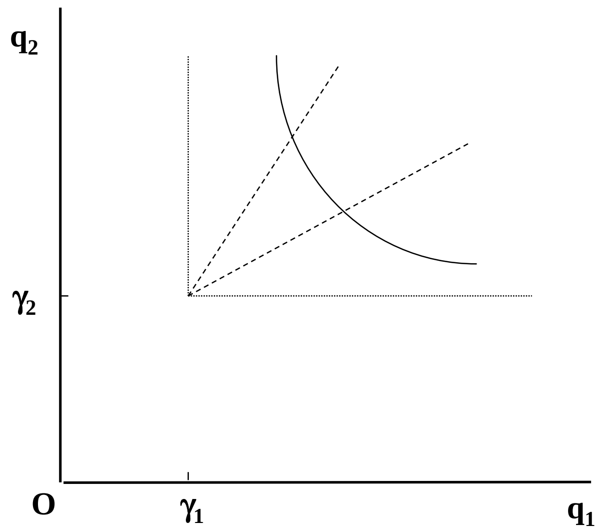

Figure 1: The Stone-Geary Utility Function

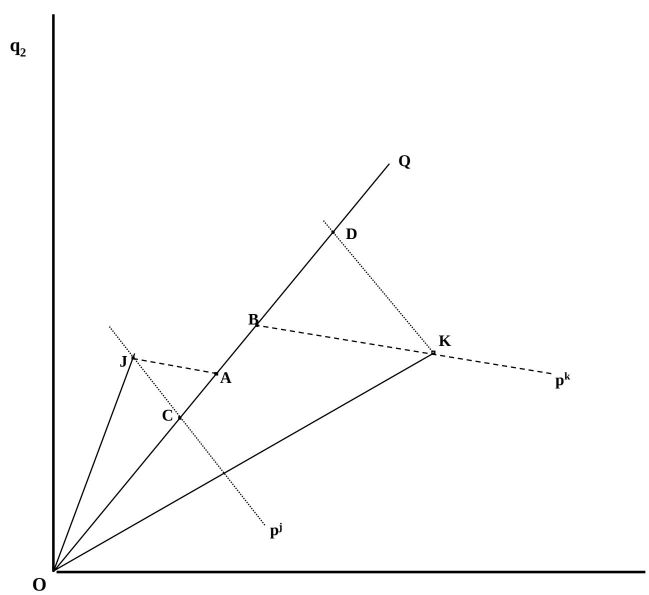

Figure 2: Measuring the Real Income of Country j Relative to Country k: Laspeyres = OA/OB; Paasche = OC/OD

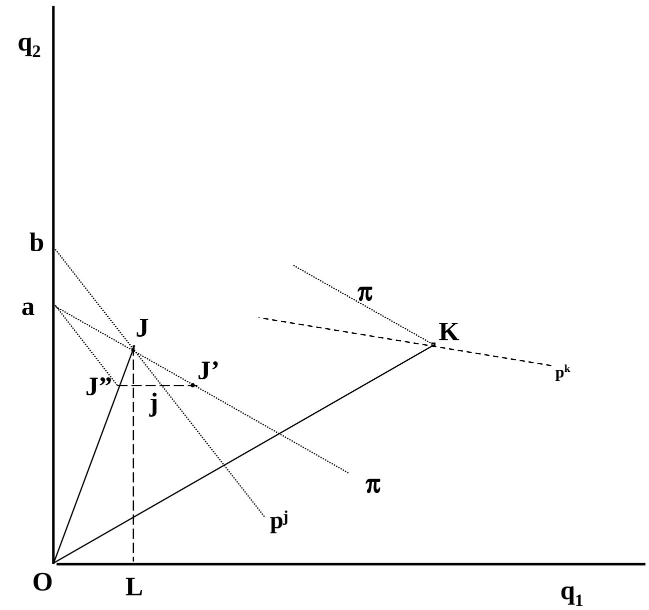

Figure 3: Calculating the Real Income of Country j at Geary World Prices

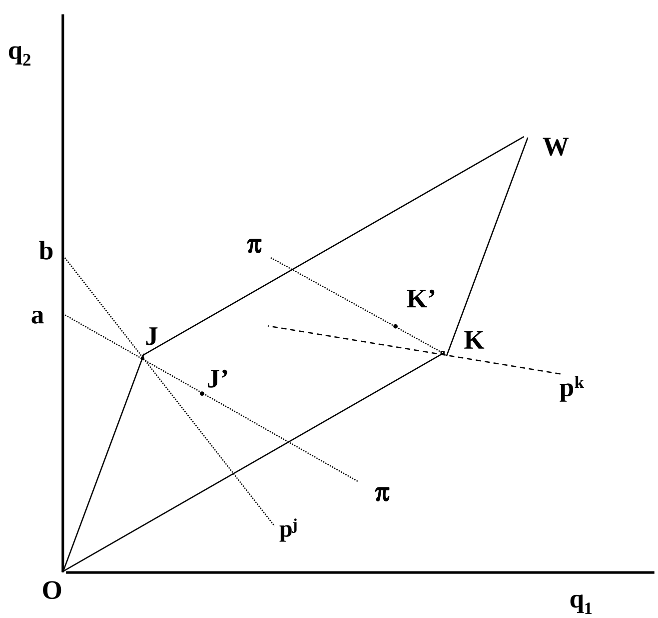

Figure 4: Calculating Geary World Prices and the Real Incomes of Countries j and k