Simple General Equilibrium Models
Notes for Oxford M.Phil. International Trade

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Plan of Lectures

1. The Heckscher-Ohlin Model: Introduction
2. Factor Price Equalization (FPE)
3. The Rybczynski Theorem
4. The Heckscher-Ohlin Theorem

N.B. This file is in beta; much of the material for this section of the course is in the PowerPoint files.
Plan of Lectures

1. The Heckscher-Ohlin Model: Introduction
   - Assumptions and Key Concepts

2. Factor Price Equalization (FPE)

3. The Rybczynski Theorem

4. The Heckscher-Ohlin Theorem
Assumptions:

- Perfect competition, constant returns to scale
- 2 countries: Home and Foreign (*)
- 2 factors: Capital $K$ and Labour $L$
- 2 sectors: $y_1$ and $y_2$; sometimes: $y_M$ (Manufacturing) and $y_F$ (Food)
- Complete intersectoral factor mobility

Key Concepts:

1. Factor Abundance (of countries):
   - Home more labour abundant: $k < k^*$. 

2. Factor Intensity (of sectors):
   - Facing the same factor prices, sector 1 is more capital intensive:
     - $K_1/L_1 > K_2/L_2$ or $k_1 > k_2$
   - Assume: No "factor-intensity reversals": One sector always more capital-intensive.
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Consider any industry:

Zero-profit condition implies that price equals unit costs: \( p = c(w, r) \)

- Unit cost function \( p = c(w, r) \): Same properties as expenditure function
- Given \( p \), this defines a locus in \((w, r)\) space
  - Convex to origin, because \( c \) is concave in \((w, r)\)
- Equilibrium must lie on this locus
  - Points above imply factor prices too high: losses \(\Rightarrow\) exit
  - Points below imply factor prices too low: super-normal profits \(\Rightarrow\) entry

So, equilibrium for both industries is where the loci meet

QED!!!
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Summary of Rybczynski Theorem

\[ \hat{L} > \hat{K} \implies \hat{X}_F > \hat{L} > \hat{K} > \hat{X}_M, \quad \hat{w} = \hat{r} = 0 \]

Economic mechanism?

- Increase in \( L \) puts downward pressure on wage \( w \)
- This encourages expansion of \( L \)-intensive sector, \( F \)
- \( L \)-intensive sector draws capital and labour from other sector
  - As diagram shows, \( F \)-sector ends up absorbing more labour than the economy’s extra endowment
- Since factor prices settle at their original level, both sectors end up with the same factor proportions as initially
- Expanding sector grows by more than the economy average
Rybczynski Theorem: Summing Up

- Theorem is very surprising: Economic growth occurs, yet output of one industry falls.
- Basic idea: Source of growth is asymmetric; effect on pattern of outputs is even more asymmetric: asymmetry is magnified.
- Real-world examples?
  - Black Death in 14th century Europe
  - Great Famine in Ireland, 1846-49
    - O’Rourke, AER 1994
  - Russian emigration to Israel in 1990s
    - Gandal-Hanson-Slaughter, EER, 2004
- Theoretical importance?
  - Essential building block for the Heckscher-Ohlin Theorem.
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4. The Heckscher-Ohlin Theorem
   - Theorem and Proof
   - The FPE Set
   - Empirical Evidence on Heckscher-Ohlin
Heckscher-Ohlin Theorem

Theorem: A country has a comparative advantage in the good that uses relatively intensively the country’s relatively abundant factor.

Proof:

- Rybczynski Theorem in Output Space:
  - Rise in labour endowment $\Rightarrow$ PPF shifts out, but in a biased way
  - “Rybczynski line” links points on old and new PPF with same slope
    - The theorem implies that it must be downward-sloping

- Now: Interpret Ryborscyznski Theorem in a Cross-Section:
  - Home is more capital-abundant; so at the same prices, it has:
    - Higher output of the capital intensive good $y_M$; and
    - Lower output of the labour-intensive good $y_F$
  - So: Home has a production bias towards $y_M$

- Final step: Homothetic tastes
  - Imply that relative demands are the same at home and abroad
  - So: Production bias translates into a trade bias
From Rybczynski to Heckscher-Ohlin
Next: Which divisions of world endowments are consistent with FPE?

Begin with an easier question: examine the Integrated Equilibrium

\[ k^W \Rightarrow p \Rightarrow w, r \Rightarrow a_{ij} \Rightarrow k_M, k_F \]

Trivially, any equi-proportionate division of factors is identical in aggregate to the Integrated Equilibrium

- Given constant returns to scale and homothetic tastes

Any other division \( \{k, k^*\} \) is consistent with the same world output if:

\[ k_M \geq k \geq k^W \geq k^* \geq k_F \]

So set \( ODO^*G \), where both countries produce with \( k_M \) and \( k_F \), is the desired one.
Constructing the FPE Set
Assume a given division of endowments $E$.

Where will consumption take place?

Equilibrium consumption point $C$ must be consistent with both:

1. Balance of trade equilibrium:
   
   $g(p, v) = e(p, u)$
   
   $\Rightarrow py_M + y_F = px_M + x_F$
   
   $\Rightarrow wL + rK = wL^c + rK^c$
   
   Home income is fixed at $wL + rK$, so this equation defines a budget constraint in factor space, passing through $E$ with slope $-w/r$.

2. Homothetic tastes: $\frac{K^c}{L^c} = \frac{K^W}{L^W}$

Trade triangle connects endowments $E$ and consumption $C$.

Next: For any endowment point in the FPE set, we can repeat this.

Iso-trade volume loci are straight lines parallel to the diagonal.

Empirical implications:

- Trade volume increases with the difference in factor endowments.
- Trade triangle is independent of differences in country size.
Trade in the FPE Set
Empirical Evidence on Heckscher-Ohlin

- Leontief (1951): The “Leontief Paradox”:
  - Applied input-output analysis to U.S. exports and imports.
  - Results: U.S. imports are relatively capital-intensive.
  - A paradox, since presumptively U.S. is relatively capital-abundant.
  - Leontief’s own explanation: U.S. workers more productive.

- Leamer et al. (1984 book; *AER* 1987 with Bowen and Sveikauskas):
  - Constructed endowment data: H-O rejected.

  - Tried to discriminate between H-O and monopolistic competition.
    - H-O: Larger market sucks in imports; MC: home-market effect encourages exports.
  - Results interesting though not conclusive.

  - Leontief was right!

  - Evidence in favour of $w^0.m^1 > 0$ from opening up of Japan in 1860s.