# Testing for Intertemporal Nonseparability<sup>\*</sup>

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**Abstract:** This paper presents a nonparametric analysis of intertemporal models of consumer choice that relax consumption independence. We compare the revealed preference conditions for the intertemporally nonseparable models of rational habit formation and rational anticipation. We show that these models are nonparametrically equivalent in the usual empirical setting.

JEL Classifications: D11, D12, D91.

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### 1. INTRODUCTION

The discounted utility model is the standard framework for thinking about dynamic consumer behaviour.<sup>1</sup> The model supposes that an agent's preferences over consumption profiles can be represented by  $\sum_t \beta^{t-1} u(x_t)$ , where u denotes a timeinvariant, cardinal, typically concave, instantaneous utility function defined over the period t consumption vector  $x_t$ , and where  $\beta$  is the discount factor defined as  $1/(1+\rho)$ , with  $\rho$  denoting the discount rate. A key feature of the discounted utility model is that it explicitly assumes time separability, or consumption independence. This embodies the assumption that an individual's preferences over consumption in any period are independent of consumption in any other period.

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<sup>&</sup>lt;sup>1</sup> For the origins of this literature, see Rae (1834), Böhm-Bawerk (1889), Fisher (1930), and Samuelson (1937).

That intertemporal separability is a strong assumption has of course long been recognised. Samuelson (1952) famously expressed the view that 'the amount of wine I drank yesterday and will drink tomorrow can be expected to have effects upon my today's indifference slope between wine and milk'. Koopmans (1960) argued that 'there is no clear reason why complementarity of goods could not extend over more than one time period'. Despite the manifest implausibility of this assumption, it remains popular, mainly because it greatly simplifies the analysis of intertemporal choice.

The two most obvious and straightforward approaches that incorporate intertemporal *nonseparability*, i.e., that allow preferences at a point in time to depend upon consumption choices at others, are rational habit formation and rational anticipation. Rae (1834) was perhaps the first to propose the idea that utility from current consumption can be affected by past consumption. The notion that knowledge of future consumption can affect present decision making goes back as far as Jevons (1871). In simple terms, having eaten an Italian meal last night may influence your trade-off today between Italian and Indian food, as may the knowledge that you are going out to an Italian restaurant tomorrow night. Both nonseparable approaches have delivered meaningful insights into consumer behaviour, and both are able to explain empirical consumption 'puzzles' where the time separable benchmark falls short.

Models representing habit formation have been taken up with some enthusiasm,<sup>2</sup> while models of anticipation have been slower to advance. Quiggin (1982) axiomatised a theory of anticipated utility<sup>3</sup>, which generalised the expected utility model in order to explain prominent behavioural anomalies, including the Allais paradox. Loewenstein (1987) proposed a formal model which assumes that an individual's instantaneous utility is equal to utility from current consumption plus some function of consumption in future periods. Incorporating future consumption in this way allows the consumer to have a preference for improvements over time and for suffering

<sup>&</sup>lt;sup>2</sup> Contributions include Becker and Murphy (1988) on the price-responsiveness of addictive behaviour, Meghir and Weber (1996) on intertemporal nonseparabilities and liquidity constraints, and Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) on asset-pricing anomalies, including the equity premium puzzle. Macroeconomists have appealed to habit formation to better explain movements in asset prices (Jermann, 1998; Boldrin, Christiano, and Fisher, 2001), to investigate the relationship between economic growth and savings (Carroll, Overland, and Weil, 2000), and to explain how aggregate spending responds to shocks (Fuhrer, 2000).

<sup>&</sup>lt;sup>3</sup> Quiggin's theory of anticipated utility is more commonly known as rank-dependent expected utility theory.

unpleasant outcomes quickly instead of delaying them. More recently, Caplin and Leahy (2001) have shown that anticipatory utility can explain the equity premium puzzle just as effectively as habit formation.

The idea that anticipation and habit formation are equally effective in explaining behaviour is at the core of this paper. The literature treats these models as though they are *distinct*. However, we show that this is not the case in general. We reconsider the relationship between models in which past and future consumption choices affect current preferences. We show that in the absence of specific parametric restrictions, these models are in fact observationally equivalent. That is to say that a finite data set containing consumption choices, spot prices, and interest rates can be rationalised by a model of rational habit formation if and only if it can be rationalised by a model of rational anticipation. We derive the empirical implications of these models in the nonparametric revealed preference tradition of Samuelson (1948), Houthakker (1950), Afriat (1967), Diewert (1973), and Varian (1982), and demonstrate an equivalence in the absence of any parametric assumptions.

The paper is organised as follows. Section 2 introduces the framework within which we investigate the observational content of intertemporal nonseparability. In particular, we highlight the differences between the perspectives of the agent and the observer when analysing intertemporal choice. Section 3 outlines the revealed preference conditions for models of rational habit formation and rational anticipation. Section 4 contains the main equivalence result of the paper. Section 5 provides some brief concluding remarks.

## 2. Framework

In order to isolate intertemporal nonseparability, we adhere to the principal assumptions of the benchmark discounted utility model—only consumption independence is relaxed. Note therefore that we continue to assume instantaneous preferences that are stable over some horizon,<sup>4</sup> perfect foresight, exponential discounting, and perfect liquidity.

We let  $x_t \in \mathbb{R}^K_+$  be a vector of consumption goods (where each good is indexed by  $k \in \kappa = \{1, \ldots, K\}$ ) purchased at corresponding spot prices  $p_t \in \mathbb{R}^K_{++}$ 

 $<sup>^{4}</sup>$  Whether the horizon is finite or infinite has no relevance in a revealed preference setting in which only a subset of periods is observed.

in period  $t \in \tau = \{1, \ldots, T\}$ , where  $\tau$  denotes the set of contiguous periods observed by the econometrician. In order to allow for leads and lags of consumption, we also make use of two augmented sets of periods. More specifically, we allow for  $N \in \{1, \ldots, T-1\}$  lags or leads, and we denote the augmented sets by  $\underline{\tau} = \{1 - N, \dots, T\}$  and  $\overline{\tau} = \{1, \dots, T + N\}$ . Discounted prices are given by  $\hat{p}_t \in \mathbb{R}_{++}^K$ .<sup>5</sup> Finally, we let  $B = \left\{ y_t \in \mathbb{R}_+^K \text{ for all } t \in \tau : \sum_{t \in \tau} \hat{p}_t \cdot y_t \le \sum_{t \in \tau} \hat{p}_t \cdot x_t \right\}$  denote the lifetime budget set. We assume that the econometrician observes a data set of discounted prices and consumption choices  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$ . Given these observables, we ask whether there are necessary and sufficient conditions which guarantee the existence of some instantaneous utility functions  $u: (\mathbb{R}^K_+)^{N+1} \to \mathbb{R}$  and  $v: (\mathbb{R}^K_+)^{N+1} \to \mathbb{R}$ , as well as a discount factor  $\beta \in (0, 1]$ , such that a consumer could have been solving either

$$\max_{\{x_t\}_{t\in\tau}} \sum_{t\in\overline{\tau}} \beta^{t-1} u(x_t, x_{t-1}, x_{t-2}, \dots, x_{t-N})$$
(1)

or

$$\max_{\{x_t\}_{t\in\tau}} \sum_{t\in\underline{\tau}} \beta^{t-1} v(x_t, x_{t+1}, x_{t+2}, \dots, x_{t+N})$$
(2)

subject to the lifetime budget constraint, where (1) corresponds to habit formation and (2) to anticipation. We also ask whether the utility functions u and v are necessarily distinct. We formalise this approach in the following section.

## **3.** Revealed Preference Analysis

#### 3.1 Rational Habit Formation

We begin with an examination of the revealed preference conditions for rational habit formation. Short memory habits are rationalisable in the following sense:

**Definition 1** The data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with a model of rational habit formation if there exist a non-satiated, concave, and differentiable<sup>6</sup> utility function  $u: (\mathbb{R}^K_+)^{N+1} \to \mathbb{R}$ , a discount factor  $\beta \in (0,1]$ , and unobserved consumption  $x_t = y_t \in \mathbb{R}$  $\mathbb{R}^{K}_{+}$  for each  $t \notin \tau$ , such that  $\sum_{t \in \overline{\tau}} \beta^{t-1} u(x_{t}, \dots, x_{t-N}) \geq \sum_{t \in \overline{\tau}} \beta^{t-1} u(y_{t}, \dots, y_{t-N})$  for all  $\{y_t\}_{t\in\tau}\in B$ .

<sup>&</sup>lt;sup>5</sup> Prices are discounted throughout according to  $\hat{p}_t = p_t / \prod_{s=1}^{s=t-1} (1+r_s)$  for all  $t \in \tau \setminus \{1\}$  and  $\hat{p}_1 = p_1$ , where  $r_t \ge 0$  denotes the rate of interest between period t and t+1 for all  $t \in \tau \setminus \{T\}$ . <sup>6</sup> Note that differentiability is without loss of generality throughout.

This definition simply states that a data set can be rationalised by habits if the observed consumption profile delivers weakly greater lifetime utility than any other consumption profile satisfying the lifetime budget constraint. We now establish the revealed preference conditions for this model.

#### **Lemma 1** The following statements are equivalent:

- 1. The data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with a model of rational habit formation.
- 2. There exist  $(u_t, \rho_t^0, \ldots, \rho_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \overline{\tau}$ ,  $x_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that

$$u_{t'} \le u_t + \begin{pmatrix} \rho_t^0 \\ \vdots \\ \rho_t^N \end{pmatrix} \cdot \begin{pmatrix} x_{t'} - x_t \\ \vdots \\ x_{t'-N} - x_{t-N} \end{pmatrix}$$
(H.1)

for all  $(t, t') \in \bar{\tau} \times \bar{\tau}$ ,

$$\beta^{t-1}\rho_{kt}^{0} + \dots + \beta^{t-1+N}\rho_{k(t+N)}^{N} = \hat{p}_{kt}$$
(H.2)

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} > 0$ , and

$$\beta^{t-1}\rho_{kt}^{0} + \dots + \beta^{t-1+N}\rho_{k(t+N)}^{N} \le \hat{p}_{kt}$$
(H.3)

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} = 0$ .

**Proof:** Necessity. Suppose that the data set  $\{(\hat{p}_t, x_t)\}_{t\in\tau}$  is consistent with a model of rational habit formation. Since u is non-satiated, with  $\tilde{x}_t = x_t$  for all  $t \notin \tau$ ,  $\{x_t\}_{t\in\tau}$ solves  $\max_{\{\tilde{x}_t\}_{t\in\tau}\in B}\sum_{t\in\tau}\beta^{t-1}u(\tilde{x}_t,\ldots,\tilde{x}_{t-N})$ , so that there exists  $\lambda > 0$  such that  $\beta^{t-1}\partial u(x_t,\ldots,x_{t-N})/\partial x_{kt}+\cdots+\beta^{t-1+N}\partial u(x_{t+N},\ldots,x_t)/\partial x_{kt} \leq \lambda \hat{p}_{kt}$  for all  $t\in\tau$ . Note that the inequality is binding for any  $(k,t)\in\kappa\times\tau$  with  $x_{kt}>0$ . Concavity of u implies that

$$u(x_{t'},\ldots,x_{t'-N}) \leq u(x_t,\ldots,x_{t-N}) + \begin{pmatrix} \partial u(x_t,\ldots,x_{t-N})/\partial x_t \\ \vdots \\ \partial u(x_t,\ldots,x_{t-N})/\partial x_{t-N} \end{pmatrix} \cdot \begin{pmatrix} x_{t'}-x_t \\ \vdots \\ x_{t'-N}-x_{t-N} \end{pmatrix}$$

for all  $(t, t') \in \overline{\tau} \times \overline{\tau}$ . Now let  $u(x_t, \dots, x_{t-N}) = \lambda u_t$  and  $\partial u(x_t, \dots, x_{t-N})/\partial x_t = \lambda \rho_t^0, \dots, \partial u(x_t, \dots, x_{t-N})/\partial x_{t-N} = \lambda \rho_t^N$  for all  $t \in \overline{\tau}$ .

Sufficiency. Suppose that there exist  $(u_t, \rho_t^0, \dots, \rho_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \overline{\tau}$ ,  $x_t \in \mathbb{R}^K_+$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that (H.1)–(H.3) are satisfied. Define  $u : (\mathbb{R}^K_+)^{N+1} \to \mathbb{R}$  as follows:

$$u(x^{0},\ldots,x^{N}) = \min_{t\in\overline{\tau}} \left( u_{t} + \begin{pmatrix} \rho_{t}^{0} \\ \vdots \\ \rho_{t}^{N} \end{pmatrix} \cdot \begin{pmatrix} x^{0} - x_{t} \\ \vdots \\ x^{N} - x_{t-N} \end{pmatrix} \right).$$

Notice that u is non-satiated,<sup>7</sup> concave, and differentiable. By the definition of u,  $u(x_t, \ldots, x_{t-N}) \leq u_t$  for all  $t \in \overline{\tau}$ . Since

$$u(x_t, \dots, x_{t-N}) = u_{t'} + \begin{pmatrix} \rho_{t'}^0 \\ \vdots \\ \rho_{t'}^N \end{pmatrix} \cdot \begin{pmatrix} x_t - x_{t'} \\ \vdots \\ x_{t-N} - x_{t'-N} \end{pmatrix} \le u_t$$

for some  $t' \in \overline{\tau}$ , it must be that  $u(x_t, \ldots, x_{t-N}) = u_t$  for all  $t \in \overline{\tau}$  in order to satisfy (H.1). Lastly, consider any  $\{y_t\}_{t\in\tau} \in B$  with  $y_t = x_t$  for any  $t \notin \tau$ . By the definition of u, it must be that

$$u(y_t, \dots, y_{t-N}) \le u_t + \begin{pmatrix} \rho_t^0 \\ \vdots \\ \rho_t^N \end{pmatrix} \cdot \begin{pmatrix} y_t - x_t \\ \vdots \\ y_{t-N} - x_{t-N} \end{pmatrix}$$

for all  $t \in \overline{\tau}$ , which implies that for some  $\beta \in (0, 1]$ ,

$$\sum_{t\in\overline{\tau}}\beta^{t-1}u(y_t,\ldots,y_{t-N}) \le \sum_{t\in\overline{\tau}}\beta^{t-1}u_t + \sum_{t\in\tau}\hat{p}_t\cdot(y_t-x_t)$$
(3)

$$\leq \sum_{t\in\overline{\tau}}\beta^{t-1}u_t\tag{4}$$

$$=\sum_{t\in\overline{\tau}}\beta^{t-1}u(x_t,\ldots,x_{t-N}).$$
(5)

Inequality (3) follows from (H.2) and (H.3), since  $\beta^{t-1}\rho_t^0 + \cdots + \beta^{t-1+N}\rho_{t+N}^N \leq \hat{p}_t$ for all  $t \in \tau$ ; inequality (4) follows since  $\sum_{t \in \tau} \hat{p}_t \cdot y_t \leq \sum_{t \in \tau} \hat{p}_t \cdot x_t$ ; and equality (5) follows since  $u(x_t, \ldots, x_{t-N}) = u_t$  for all  $t \in \overline{\tau}$ .  $\Box$ 

The restrictions in (H.1)-(H.3) exhaust the pure empirical implications of rational habit formation with N lags. In other words, if we observe a data set that satisfies

<sup>&</sup>lt;sup>7</sup> Non-satiation of u is given by the sign restrictions on  $(\rho_t^0, \ldots, \rho_t^N)$  for each  $t \in \overline{\tau}$  imposed by (H.2) and (H.3).

these conditions, then the observed consumption choices are *consistent* with a model of rational habit formation. The converse of this statement is also true, implying that data which do not satisfy the restrictions are *inconsistent*. Note that for each  $t \in \overline{\tau}$ , the parameters  $(\rho_t^0, \ldots, \rho_t^N)$  are not completely free to vary within  $(\mathbb{R}^K)^{N+1}$  due to the sign restrictions imposed by (H.2) and (H.3). However, as long as we observe some strictly positive consumption, some of these parameters must also be strictly positive, which guarantees non-satiation. Further note that rational habit formation contains the life-cycle model as a special case. To see this, let  $\rho_t^l = 0$  for all  $l \neq 0$  and  $t \in \overline{\tau}$ .<sup>8</sup> Notice that Lemma 1 has an equivalent cyclical monotonicity representation, which is first proven in Theorem 1 of Crawford (2010). However, the formulation presented here is much more computationally convenient. This is because cyclical monotonicity requires that we check *every* possible subset of the data—an enormous number of calculations even for a data set of moderate size—whereas the conditions in Lemma 1 can be implemented very efficiently using a simple grid or random search and standard linear programming techniques.

#### 3.2 Rational Anticipation

We now consider a model of rational anticipation, and we claim that the data are rationalisable by the theory in the following sense:

**Definition 2** The data set  $\{(\hat{p}_t, x_t)\}_{t\in\tau}$  is consistent with a model of rational anticipation if there exist a discount factor  $\beta \in (0, 1]$ , a non-satiated, concave, and differentiable utility function  $v : (\mathbb{R}^K_+)^{N+1} \to \mathbb{R}$ , and unobserved consumption  $x_t = y_t \in \mathbb{R}^K_+$ for each  $t \notin \tau$ , such that  $\sum_{t\in\tau} \beta^{t-1}v(x_t, \ldots, x_{t+N}) \ge \sum_{t\in\tau} \beta^{t-1}v(y_t, \ldots, y_{t+N})$  for all  $\{y_t\}_{t\in\tau} \in B$ .

As we saw earlier, this definition embodies the principle of revealed preference the data can be rationalised by rational anticipation if the observed consumption profile delivers weakly greater lifetime utility than any other consumption profile satisfying the lifetime budget constraint. We now establish the revealed preference conditions for this model.

### **Lemma 2** The following statements are equivalent:

 $<sup>^8</sup>$  If we further impose that  $\beta=1,$  the restrictions are equivalent to cyclical monotonicity in Browning (1989).

- 1. The data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with a model of rational anticipation.
- 2. There exist  $(v_t, \pi_t^0, \ldots, \pi_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \underline{\tau}$ ,  $x_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that

$$v_{t'} \le v_t + \begin{pmatrix} \pi_t^0 \\ \vdots \\ \pi_t^N \end{pmatrix} \cdot \begin{pmatrix} x_{t'} - x_t \\ \vdots \\ x_{t'+N} - x_{t+N} \end{pmatrix}$$
(A.1)

for all  $(t, t') \in \underline{\tau} \times \underline{\tau}$ ,

$$\beta^{t-1-N} \pi^{N}_{k(t-N)} + \dots + \beta^{t-1} \pi^{0}_{kt} = \hat{p}_{kt}$$
(A.2)

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} > 0$ , and

$$\beta^{t-1-N} \pi^{N}_{k(t-N)} + \dots + \beta^{t-1} \pi^{0}_{kt} \le \hat{p}_{kt}$$
(A.3)

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} = 0$ .

**Proof:** The proof of Lemma 2 is analogous to the earlier proof of Lemma 1. Necessity makes use of concavity in the instantaneous utility function v as well as standard optimality conditions for convex problems. Sufficiency constructs a piecewise linear utility function that rationalises the data, using the lower envelopes of the hyperplanes in (A.1).  $\Box$ 

As in Lemma 1, the restrictions in (A.1)–(A.3) exhaust the pure empirical implications of rational anticipation. Once again note that for each  $t \in \underline{\tau}$ , the parameters  $(\pi_t^0, \ldots, \pi_t^N)$  are not completely free to vary within  $(\mathbb{R}^K)^{N+1}$  due to the sign restrictions imposed by (A.2) and (A.3). Like habit formation, rational anticipation contains the life-cycle model as a special case. To see this, let  $\pi_t^l = 0$  for all  $l \neq 0$  and  $t \in \underline{\tau}$ .

## 4. Equivalence

The following proposition gives the main result of the paper.

**Proposition 1** The dataset  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with a model of rational habit formation if and only if it is consistent with a model of rational anticipation.

**Proof:** Necessity. Suppose that there exist  $(u_t, \rho_t^0, \ldots, \rho_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \overline{\tau}, x_t \in \mathbb{R}^K_+$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that (H.1)–(H.3) are satisfied. Define  $(\pi_t^0, \ldots, \pi_t^N) \in (\mathbb{R}^K)^{N+1}$  according to

$$\begin{pmatrix} \pi_t^0 \\ \vdots \\ \pi_t^N \end{pmatrix} = \beta^N \begin{pmatrix} \rho_{t+N}^N \\ \vdots \\ \rho_{t+N}^0 \end{pmatrix}$$

for all  $t \in \underline{\tau}$ , and  $v_t \in \mathbb{R}$  according to

$$v_t = \beta^N u_{t+N}$$

for all  $t \in \underline{\tau}$ , such that (A.1)–(A.3) are satisfied.

Sufficiency. Suppose that there exist  $(v_t, \pi_t^0, \ldots, \pi_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \underline{\tau}$ ,  $x_t \in \mathbb{R}^K_+$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that (A.1)–(A.3) are satisfied. Define  $(\rho_t^0, \ldots, \rho_t^N) \in (\mathbb{R}^K)^{N+1}$  according to

$$\begin{pmatrix} \rho_t^0 \\ \vdots \\ \rho_t^N \end{pmatrix} = (1/\beta^N) \begin{pmatrix} \pi_{t-N}^N \\ \vdots \\ \pi_{t-N}^0 \end{pmatrix}$$

for all  $t \in \overline{\tau}$ , and  $u_t \in \mathbb{R}$  according to

$$u_t = v_{t-N} / \beta^N$$

for all  $t \in \overline{\tau}$ , such that (H.1)–(H.3) are satisfied.  $\Box$ 

Within this particular class of intertemporal model (stable preferences, perfect foresight, exponential discounting, perfect liquidity), this nonparametric equivalence arises for several reasons: (1) we only observe a finite subset of the consumer's choices; (2) we only require non-satiation in the instantaneous utility functions; (3) unobserved consumption is the same in both models; and (4) we do not allow for a durable habit-forming or anticipatory good. As a result, intertemporal marginal rates of substitution between any two periods are observationally equivalent across models. In other words, given a finite data set, we cannot reject that they are the same.

It is easy to see that by imposing further structure on the problem, the equivalence no longer holds. For example, with stronger assumptions on the shapes of the utility functions u and v, we obtain further sign restrictions on  $(\rho_t^0, \ldots, \rho_t^N)$  and  $(\pi_t^0, \ldots, \pi_t^N)$  that can potentially differ. Furthermore, we could assume the observed subset contains the boundaries of the consumer's problem. A related modification would impose restrictions on unobserved consumption that vary across models. Lastly, if we treat habits/futures as durables (i.e., represent them by an unobservable stock variable which includes the entire history/future of consumption), as in Demuynck and Verriest (2013), then the observational equivalence no longer obtains.

## 5. Concluding Remarks

In the absence of parametric assumptions or sign restrictions, we have shown that data on prices, interest rates, and consumption profiles do not allow the econometrician to distinguish between the models of rational habit formation in (1) and rational anticipation in (2). This may go some way towards explaining why both models can provide rationalisations for the same behaviour. For example, Abel (1990) and Campbell and Cochrane (1999) address *inter alia* the equity premium puzzle via habits, while Caplin and Leahy (2001) adopt an anticipation approach. However, the same equivalence does not appear to hold when habits and anticipation models incorporate an unobservable stock variable, which suggests an interesting avenue for future work.

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