Elastic Behavior of Materials: Continuum Aspects

When a real material is studied at progressively diminishing scales, it is usually found to exhibit a variation in properties from point to point. This variation is called structure. The term is often used with reference to a certain scale, e.g., atomic structure, microstructure, etc. The mechanical response of material to loading depends on its structure, and a large section of materials science is devoted to the analysis of this dependence. However, some fundamental aspects of mechanical behavior can be understood very well if the material is considered to be structureless, a solid continuum.

Solids respond to applied external loads by developing internal forces. If an imaginary section through the solid is considered, the components of internal force acting on a unit elemental area are called stresses. Under the action of stresses solids deform, so that the distances between points change. However, provided the stresses are sufficiently small, the solid recovers its original shape and volume once the load is removed. This type of behavior is called *elastic*. Continuum elasticity considers the consequence of atomic interactions in solids, but disregards their nature. The atomic aspects of elastic deformation are considered in *Elastic Behavior of Materials: Physical Basis*. Non-elastic behavior is manifested in residual deformation persisting after load removal, and often gives rise to residual stress.

Under very low loads, deformation is found to be proportional to stress. The case of *linear continuum elasticity* is the main subject of this article. First, the foundations of stress and strain are laid out in the infinitesimal limit. Then simple forms of the elastic equations for isotropic bodies are introduced using Lamé's constants, as well as Young's modulus and Poisson's ratio.

1. *Deformation and Strain*

Each point in a solid continuum can be referred to by its initial position with respect to a certain coordinate system, $\mathbf{x} = (x_1, x_2, x_3)$. During deformation, the displacement of each point can be described by the vector $\mathbf{u} = (u_1, u_2, u_3)$. In Cartesian coordinates, the initial distance between any two closely positioned points is given by

$$\mathrm{d}l = \sqrt{(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2)}$$

After deformation it changes to

$$\mathrm{d}l' = \sqrt{(\mathrm{d}x_1 + \mathrm{d}u_1)^2 + (\mathrm{d}x_2 + \mathrm{d}u_2)^2 + (\mathrm{d}x_3 + \mathrm{d}u_3)^2}$$

The displacement differences $\mathrm{d}u_i$ can be written in terms of displacement gradients at point x_j (Fig. 1). Introducing the expression

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}\right),$$

the elemental length can be expressed as

$$\mathrm{d}l'^2 = \mathrm{d}l^2 + 2\varepsilon_{ij}\mathrm{d}x_i\mathrm{d}x_j + o(|\mathrm{d}\mathbf{x}|^2)$$

Here $o(|\mathrm{d}\mathbf{x}|^2)$ denotes the terms which decay more rapidly than $|\mathrm{d}\mathbf{x}|^2$ as $\mathrm{d}\mathbf{x}$ vanishes. The terms discarded here contain strain gradients, i.e., higher-order spatial derivatives of displacements. These terms can be taken into account in formulating a *linear* dependence of stress upon strain. They give rise to characteristic length scales related to the lattice structure of the crystal (see note in article *Elastic Behavior of Single Crystals: Anisotropy* and references therein). Following Cauchy, the last term in ε_{ij} is usually neglected for small deformations, so that

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{1}$$

The terms ε_{ij} defined according to Eqn. (1) are called strains, and form a symmetric second-rank tensor called the *strain tensor*. Strains can be separated into two groups. If $i = j$, then, e.g., $\varepsilon_{11} = \partial u_1/\partial x_1$, ε_{22}, ε_{33} are called *normal strains*, and describe elongation or contraction. Otherwise, $i \neq j$, and $\varepsilon_{12} = \frac{1}{2}(\partial u_1/\partial x_2 + \partial u_2/\partial x_1)$, ε_{23}, ε_{31} are called *shear strains*, and describe the change in the angle between two initially straight lines along the coordinate axes (Fig. 1).

By rotation, an alternative orthonormal system of axes can be found with respect to which shear strains
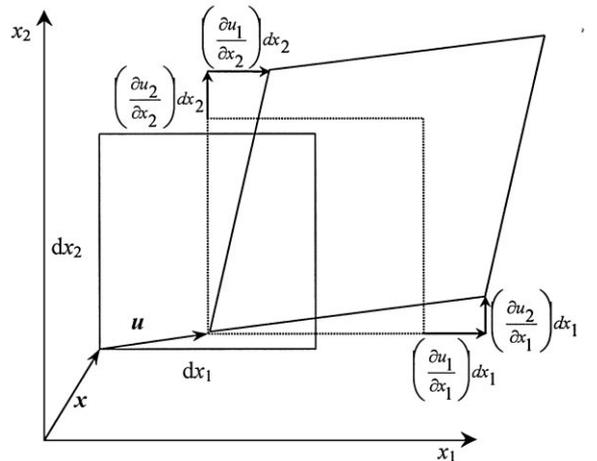

Figure 1
Illustration of strain.

vanish. The remaining normal strains with respect to these *principal axes* are called *principal strains*. A volume originally occupying an elemental cube with face normals along the principal axes deforms into a parallelepiped. The relative volume change (the *dilatation*) is described by the sum of normal strains, $\Delta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$. The sum of normal strains is invariant, i.e., remains the same with respect to any coordinate system.

For small deformations, uniform strains are adequately approximated by the ratios of elongations Δl_i to original gauge lengths l_j, e.g., $\varepsilon_{11} = \Delta l_1 / l_1$, etc., and are referred to as *engineering strains*. For larger distortions, *true* or *logarithmic strains* $\varepsilon_T = \ln(1 + \varepsilon)$ are often used.

2. Stress

A general element of area dA possesses orientation described by the unit normal $\mathbf{n} = (n_1, n_2, n_3)$, and transmits an internal force d$\mathbf{F} = (dF_1, dF_2, dF_3)$. The force has magnitude $dF = sdA$, and can be resolved into the normal σdA and shear τdA components (Fig. 2). In particular, for each rectangular element dA_i with normal in the direction Ox_i, internal force can be resolved into three Cartesian components written as $dF_j = \sigma_{ij} dA_i$. The terms σ_{ij} form a second rank tensor called the *stress tensor*. Components of torque exerted on an elemental volume dV by the surrounding solid can be written down as $(\sigma_{23} - \sigma_{32})dV$, $(\sigma_{31} - \sigma_{13})dV$, $(\sigma_{12} - \sigma_{21})dV$. The symmetry of the stress tensor, $\sigma_{ij} = \sigma_{ji}$, follows from the requirement that this expression must vanish in equilibrium. This assumption holds in the absence of body torque exerted by long range forces, e.g., by a magnetic field on magnetic crystals.

If $i = j$, the terms σ_{11}, σ_{22}, σ_{33} are called *normal stresses*, and can be classified further into tensile (positive) and compressive (negative). Otherwise, $i \neq j$, and the terms $\sigma_{23}, \sigma_{13}, \sigma_{12}$ are called *shear stresses*. By rotation an alternative orthonormal system of axes (the principal axes) can be found with respect to which shear stresses vanish, and an elemental cube is subjected to normal stresses only (called the *principal stresses*). The sum of normal stresses $\sigma_{11} + \sigma_{22} + \sigma_{33} = -3P$ is invariant, i.e., remains the same in any rotated system. It represents the *hydrostatic* component of the stress state, where P denotes pressure (assumed positive when compressive).

3. Strain Energy Density

The increment of strain energy density (i.e., elastic energy per unit volume) in a deforming solid is equal to the work done by the stresses to alter the strains, d$U = \sigma_{ij} d\varepsilon_{ij}$. The relationship between stresses and strains can therefore be expressed in terms of the strain energy density as

$$\sigma_{ij} = \frac{\partial U}{\partial \varepsilon_{ij}} \tag{2}$$

The energy density U may be expanded into series in terms of strains. If the initial state of the solid is assumed to be stress free, then for $\varepsilon_{ij} = 0$ so must $\sigma_{ij} = 0$. Hence, the linear terms in the expansion must vanish, and the expression for strain energy density should contain expressions quadratic in terms of strains,

$$U = U_0 + \sum_{ijkl} \tfrac{1}{2} c_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \tag{3}$$

Alongside *deformation* terms, the total internal energy of a solid contains *entropy* terms. These terms are neglected in the present analysis, although they may make a substantial contribution to the elastic effects in the case of configurational entropy of long molecules in polymeric solids (see *Elastic Behavior of Materials: Physical Basis*). For detailed discussion of the thermodynamics of elasticity, see *Elasticity: Thermodynamic Treatment*.

Equations (2) and (3) describe a linear relationship between stresses and strains,

$$\sigma_{ij} = \sum_{kl} c_{ijkl} \varepsilon_{kl} \tag{4}$$

Higher-order terms give rise to nonlinear stress–strain dependence. A mathematical treatment of nonlinear elastic deformations can be found in Ogden (1984).

Equation (4) states the generalized Hooke's law for a linear elastic solid continuum. The terms c_{ijkl} are
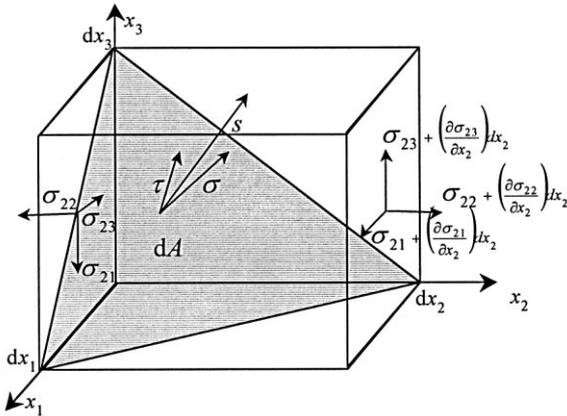

Figure 2
Illustration of stress.

called *stiffness coefficients*, and form a fourth rank tensor. For strains to be found in terms of stresses, the above expression may be inverted,

$$\varepsilon_{ij} = \sum_{kl} s_{ijkl} \sigma_{kl} \qquad (5)$$

The terms s_{ijkl} are called *compliance coefficients*, and form a fourth rank tensor equal to the inverse of tensor c_{ijkl}.

The tensorial property of ε_{ij}, c_{ijkl} (and σ_{ij}, s_{ijkl}) is manifested in their behavior under the rotation of a coordinate system. If a new Cartesian coordinate system is defined by the equation

$$x'_i = \sum_j t_{ij} x_j \qquad (6)$$

then the tensor components in the new system are related to the original ones by

$$\varepsilon'_{pq} = \sum_j t_{pi} t_{qj} \varepsilon_{ij}, \quad c'_{pqrs} = \sum_j t_{pi} t_{qj} t_{rk} t_{sl} c_{ijkl} \qquad (7)$$

4. Contracted Notation

Tensors ε_{ij} and σ_{ij} are symmetric with respect to the interchange of their indices, which must be reflected in the symmetry of c_{ijkl} and s_{ijkl}. Equation (3) shows further that c_{ijkl} and s_{ijkl} must remain unaltered if the first pair of indices is interchanged with the last. To take advantage of these symmetries, contracted notation can be introduced, giving six-component vectors for stress and strain, defined as follows:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix}, \quad \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix} = \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{pmatrix} \qquad (8)$$

The contraction of each pair of indices into one is carried out according to the rule:

$$11 \to 1, 22 \to 2, 33 \to 3, 23,32 \to 4, 31,13 \to$$
$$5, 12, 21 \to 6 \qquad (9)$$

The 6×6 stiffness matrix is introduced using the index transformation rule (9) as follows:

$$c_{\alpha\beta} = c_{ijkl}, \quad (\alpha,\beta = 1\ldots6, \; i,j,k,l = 1\ldots3) \qquad (10)$$

so that Equation (4) becomes

$$\sigma_\alpha = \sum_{\beta=1\ldots6} c_{\alpha\beta} \varepsilon_\beta \qquad (11)$$

It would be desirable to obtain an expression for the six-strains in terms of six-stresses and the compliance matrix in the same form as above,

$$\varepsilon_\alpha = \sum_{\beta=1\ldots6} s_{\alpha\beta} \sigma_\beta \qquad (12)$$

This, however, requires the introduction of factors 2 and 4 as follows:

$$s_{\alpha\beta} = s_{ijkl}, \text{ when } \textit{both } \alpha \text{ and } \beta \text{ are 1, 2, or 3;}$$
$$s_{\alpha\beta} = 2s_{ijkl}, \text{ when } \textit{either } \alpha \text{ or } \beta \text{ are 4, 5, or 6; } (13)$$
$$s_{\alpha\beta} = 4s_{ijkl}, \text{ when } \textit{both } \alpha \text{ and } \beta \text{ are 4, 5 or 6.}$$

In contracted notation, the number of stiffness or compliance components used to specify the elastic material properties is reduced from $3 \times 3 \times 3 \times 3 = 81$ to $6+5+4+3+2+1 = 21$ terms out of $6 \times 6 = 36$. This is the maximum number of independent elasticity coefficients needed to describe a general anisotropic continuum solid. Elastic symmetry characteristic of single crystal systems reduces this number further (*Elastic Behavior of Single Crystals: Anisotropy*). Many polycrystalline solids display directional dependence of elastic properties due to preferred orientation of their constituents. These systems can be described as anisotropic elastic continuua (*Elastic Behavior of Polycrystals*).

The lack of symmetry in the way six-vectors are introduced may be avoided by choosing an orthonormal dyadic base (Pedersen 1995), so that the fourth-rank unit tensor is represented by a unit 6×6 matrix. All off-diagonal stress and strain terms must be multiplied by $\sqrt{2}$, and the equivalent of Equation (13) (with $2 \to \sqrt{2}$ and $4 \to 2$) must be used for both c_{ijkl} and s_{ijkl}.

5. Elastic Isotropy

A practically important case concerns a body that develops the same strain independently of the direction in which stress is applied, and is called elastically isotropic. Although few single crystals approach this type of behavior, glasses and amorphous solids may be treated as macroscopically isotropic, as can polycrystals possessing no preferred orientation (*Elastic Behavior of Polycrystals*).

The strain energy density (Eqn. (3)) of isotropic solids must depend on two second-order scalars

composed of the strain components, and can be written as follows:

$$U = U_0 + \frac{\lambda}{2}\sum_i \varepsilon_{ii}^2 + \mu\sum_{i,j}\varepsilon_{ij}^2 \qquad (14)$$

Isotropic elasticity is fully described by two parameters λ and μ, known as Lamé's constants. Hooke's law for the isotropic body is obtained by differentiation according to Eqn. (2):

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\sum_i \varepsilon_{ii} \qquad (15)$$

Here Kronecker's delta symbol $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq 0$. Inverting Eqn. (15) gives

$$\varepsilon_{ij} = \frac{1}{2\mu}\left(\sigma_{ij} - \frac{\lambda}{(3\lambda+2\mu)}\delta_{ij}\sum_i \sigma_{ii}\right) \qquad (16)$$

6. Uniform Deformation

Simplest cases of deformation are observed when the strain tensor is uniform throughout the solid body.

6.1 Uniaxial Tension

A slender rod subjected to longitudinal tension develops uniform strains ε_{ij}, and uniform stresses σ_{ij}. Any section transmits axial stress alone, σ_{11}, while all other stress components are equal to zero. This state of stress is called *uniaxial tension*. From Equation (16), the axial elongation ε_{11} is found as

$$\varepsilon_{11} = \sigma_{11}\frac{(\lambda+\mu)}{\mu(3\lambda+2\mu)} = \frac{\sigma_{11}}{E},$$

$$E = \frac{\mu(3\lambda+2\mu)}{(\lambda+\mu)} \qquad (17)$$

The coefficient E is called Young's modulus. The ratio of transverse contraction experienced by the rod to axial elongation is called Poisson's ratio, and is given by

$$\nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = \frac{\lambda}{2(\lambda+\mu)} \qquad (18)$$

6.2 Equiaxial Compression

Consider a body subjected to hydrostatic pressure P, $\sigma_{ij} = -P\delta_{ij}$. The bulk modulus K of an isotropic body is found as the ratio of P to the relative volume

decrease, $\Delta = \Sigma\varepsilon_{ii}$. Summing up Equation (15) for $i = j = 1\ldots3$:

$$\frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33})}$$

$$= K = \lambda + \frac{2}{3}\mu = \frac{E}{3(1-2\nu)} \qquad (19)$$

6.3 Pure Shear

Consider uniform pure shear ε_{12} (all other strain components are zero). The only nonzero stress component is

$$\sigma_{12} = 2G\varepsilon_{12} \qquad (20)$$

The coefficient G is called the shear modulus, or modulus of rigidity,

$$G = \mu = \frac{E}{2(1+\nu)} \qquad (21)$$

6.4 General Uniform Strain

In the general case of isotropic material subjected to uniform deformation, the principal stresses and strains are related as follows:

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2+\sigma_3)] \qquad (22)$$

$$\sigma_1 = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_1 + \nu(\varepsilon_2+\varepsilon_3)] \qquad (23)$$

7. Thermoelasticity

Consider a body in a reference (undeformed) state at temperature T_0. Increasing the temperature by a small amount ΔT in the absence of external body force usually results in expansion. The strain energy density U must now contain strain tensor components not only as quadratic, but also as linear terms. For the isotropic body the only appropriate linear scalar combination is the sum Δ of normal strains ε_{ii}. For small ΔT, the additional term in Eqn. (14) may be assumed to be proportional to ΔT, i.e.,

$$U = U_0 - K\beta\Delta T\sum_i \varepsilon_{ii} + \frac{\lambda}{2}\sum_i \varepsilon_{ii}^2 + \mu\sum_{i,j}\varepsilon_{ij}^2 \qquad (14a)$$

and

$$\sigma_{ij} = -K\beta\Delta T\delta_{ij} + 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\sum_i \varepsilon_{ii} \qquad (15a)$$

In a body expanding without constraint no stresses arise and $\sigma_{ij} = 0$. It follows that ε_{ij} must be proportional to δ_{ij}. Taking into account Eqn. (19), it follows that $\Sigma \varepsilon_{ii} = \delta_{ij} \, \beta \Delta T$, where β represents the volumetric thermal expansion coefficient. Normal strain in any direction is given by $\varepsilon = \alpha \Delta T$, where α is the linear thermal expansion coefficient, and $\beta = 3\alpha$.

Anisotropy of thermal properties results in the orientation dependence of linear thermal expansion of crystals and polycrystals.

8. Plane Stress and Plane Strain

Plane problem of elasticity arises when deformation can be fully described in two-dimensional Cartesian coordinates. Two practically important cases are distinguished: plane stress and plane strain.

For plane stress, let x_3 be the direction normal to the surface of a thin flat plate loaded only along its edges. Then the out-of-plane stress components σ_{13}, σ_{23}, σ_{33} can be neglected, and values of the in-plane stresses σ_{11}, σ_{22}, σ_{12} can be averaged through the thickness. Strains due to the in-plane stresses can be found from Eqns. (20) and (22):

$$\varepsilon_{12} = \frac{2(1+v)}{E}\sigma_{12}$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - v\sigma_2]$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - v\sigma_1] \tag{24}$$

The only nonzero out-of-plane strain is found to be $\varepsilon_3 = -(v/E)(\sigma_1 + \sigma_2)$.

Plane strain conditions arise if displacements everywhere in a solid body are perpendicular to the axis Ox_3, and do not depend on the corresponding coordinate x_3. Then from Eqn. (1) strain components ε_{13}, ε_{23}, ε_{33} vanish, and from Eqns. (20) and (22) it follows that $\sigma_{13} = \sigma_{23} = 0$, and $\sigma_{33} = v(\sigma_{11} + \sigma_{22})$. It follows from Eqn. (22) that

$$\varepsilon_{12} = \frac{2(1+v)}{E}\sigma_{12}, \quad \varepsilon_1 = \frac{(1-v^2)}{E}\left[\sigma_1 - \frac{v}{1-v}\sigma_2\right]$$

$$\varepsilon_2 = \frac{(1-v^2)}{E}\left[\sigma_2 - \frac{v}{1-v}\sigma_1\right] \tag{25}$$

Note that Eqn. (25) can be given the form of Eqn. (24), if "plane strain elastic constants" are introduced:

$$E' = \frac{E}{(1-v^2)}, \quad v' = \frac{v}{1-v} \tag{26}$$

Table 1
Elastic properties of some engineering alloys.

Engineering alloys	Young's modulus (GPa)	Poisson's ratio
Aluminum	65–72	0.33–0.34
Copper	100–120	0.34–0.35
Magnesium	45	0.3–0.35
Nickel	200–220	0.31
Steels	200–215	0.27–0.29
Titanium	110–120	0.36
Zinc	105	0.35

This circumstance allows the use to the term *plane problem of elastiicty* to refer both to plane stress and plane strain.

9. Elastic Constants

In engineering practice, elastic properties of materials are often quoted in terms of Young's modulus and Poisson's ratio. Table 1 lists these parameters for common metallic alloys, Table 2 for ceramics and glasses, and Table 3 for polymers. Poisson's ratios of ceramics lie in the range 0.1–0.3, while for metallic alloys they are close to 0.3. Poisson's ratio of a polymer is not a constant, but shows strong dependence on stress, temperature, and time. Values generally lie in the range 0.3–0.5; those appearing in the table are given only as guidelines. Young's modulus shows a higher degree of variability between material classes, associated with the nature of atomic bonding (see *Elastic Behavior of Materials: Physical Basis*). Elastic properties of single crystals are discussed in article *Elastic Behavior of Single Crystals: Anisotropy*.

10. Equations of Equilibrium

In order for a solid body to remain in equilibrium, the balance of internal forces acting on any elemental volume within the body must be maintained. By considering a parallelepiped with edges along the coordinate axes (Fig. 2), the following equation is obtained:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0 \tag{27}$$

where F_i denotes the volume density of long-range forces, e.g., gravity. Deformation of a solid body is quasistatic if inertial forces in this equation can be neglected. When nonquasistatic deformations are considered, as in the case of elastic vibrations and waves (*Elastic Wave Propagation in Materials*), the right-hand side of this equation must contain the term $\rho\ddot{u}_i$.

If the elemental volume considered lies at the

Table 2
Elastic properties of some engineering ceramics and glasses.

Engineering ceramics and glasses	Young's modulus (GPa)	Poisson's ratio
Titanium diboride, TiB_2	540	0.11
Silicon carbide, SiC	400	0.19
Titanium carbide, TiC	440	0.19
Tungsten carbide, WC	670–710	0.24
Silicon nitride, Si_3N_4	110–325	0.22–0.27
Alumina, Al_2O_3	345–414	0.21–0.27
Beryllium oxide, BeO	300–317	0.26–0.34
Zirconia, ZrO_2	97–207	0.32–0.34
Fused silica	71	0.17
Soda-lime glass	69	0.24
Aluminosilicate glass	88	0.25
Borosilicate glass	63	0.20
High-lead glass	51	0.22

Table 3
Elastic properties of some engineering polymers.

Polymers	Young's modulus (GPa)	Poisson's ratio
Acrylics	2.4–3.1	0.33–0.39
Epoxys	2.6–3.1	0.33–0.37
Polystyrenes	3.1	0.33
Low-density polyethylene	0.1–0.3	0.45
High-density polyethylene	0.4–1.4	0.34
Polypropylene	0.5–1.9	0.36–0.40
PTFE	0.4–1.6	0.40–0.46
Polyurethanes	0.006–0.4	0.49

boundary, the internal stresses must be balanced by the surface tractions f_i, so that

$$\sigma_{ij}n_j = f_i \qquad (28)$$

where n_i denotes the components of the outward surface normal.

11. Fundamental Solutions

Mechanical behavior of real materials is strongly dependent on the presence within them of various defects, such as vacancies, dislocation lines and loops, inclusions, etc. These defects create self-equilibrated fields of stress and strain around them, which can be calculated on the basis of linear elasticity. The system of linear elastic equations introduced in the previous section admits fundamental singular solutions called *strain nuclei* (Mindlin and Chen 1950), which play a role similar to the concentrated and distributed charges in electrostatics.

An example of strain nucleus is given by an edge dislocation in an infinite solid. The boundary con-

ditions are given in terms of the displacement discontinuity $u_2(x_2+0) - u_2(x_2-0) = b$, prescribed over the half plane $x_2 = 0$, $x_1 > 0$. The displacement jump b is called the Burgers vector. The stress field around an edge dislocation is given by the following equations (Hull and Bacon 1984), sometimes also referred to as the Sneddon equations:

$$\sigma_{11} = -\frac{Gb}{2\pi(1-v)}\frac{(3x_1^2+x_2^2)x_2}{(x_1^2+x_2^2)^2},$$

$$\sigma_{22} = \frac{Gb}{2\pi(1-v)}\frac{(x_1^2-x_2^2)x_2}{(x_1^2+x_2^2)^2}, \quad \sigma_{33} = v(\sigma_{11}+\sigma_{22}),$$

$$\sigma_{12} = \frac{Gb}{2\pi(1-v)}\frac{(x_1^2-x_2^2)x_1}{(x_1^2+x_2^2)^2}, \quad \sigma_{23} = \sigma_{31} = 0 \quad (29)$$

The stresses are inversely proportional to the distance from the dislocation line in the plane (x_1, x_2). The strain energy associated with the edge dislocation may be obtained by integration in the form $W = \ln(\Lambda/c)Gb^2/4\pi(1-v)$, where the outer dislocation radius Λ and the dislocation core radius c must be

introduced to avoid divergence. Although continuum elastic description of defects such as dislocations breaks down in their immediate vicinity, it has been used successfully to model their interaction, as well as their effects on residual stress and deformation behavior.

See also: Elastic Behavior of Materials: Physical Basis; Elastic Behavior of Single Crystals: Anisotropy; Elastic Behavior of Polycrystals; Elasticity: Thermodynamic Treatment; Viscoelasticity/Anelasticity; Elastic Wave Propagation in Materials; Cosserat Media

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