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Efficient robust output feedback MPC

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Abstract: Research on robust model predictive control (MPC) has produced a plethora of results, most of which assume that the states are measurable. When not all states are measurable one must consider using output feedback. Earlier research on robust output feedback MPC either aims at simplified systems or is very computationally demanding. This paper exploits the convenient quasi-closed loop controller and separates the error dynamics and the nominal dynamics. The errors in the prediction steps are described in terms of polytopic sets with parallel edges but variable scalings and arbitrary complexity. In Mode 2, the error set is assumed invariant and thus a maximum admissible set is computed offline as a robust invariant terminal set. A quadratic programming problem is then solved online, just as done in the case of nominal MPC. The strategy enjoys guaranteed theoretical properties and can be applied to systems with multiplicative uncertainty, additive disturbances and measurement noise.

Key Words: output feedback, model predictive control, robust tube

1 Introduction

Model predictive control (MPC) provides an effective strategy that takes explicit account of constraints [19, 25]. The constraints can be imposed either on the input, state or output. In practice, system models describe plant dynamics only in an approximate way due to modeling errors, such as the multiplicative uncertainty, additive disturbances and measurement noise. Retaining robustness against these errors is one of the key research topics in the field of MPC. In the case when the states are measurable, Robust MPC has reached a considerable state of maturity [1, 10–12, 17, 20, 23, 26, 28]. To enlarge the applicability of robust MPC, the cases when not all the states are measurable and the output feedback is to be employed require further consideration.

For linear time invariant systems with additive disturbances, several formulations of output feedback MPC [9, 14, 21, 22] have been proposed. [16] considers a robust output feedback MPC strategy for systems with unstructured uncertainty. For systems with multiplicative uncertainty, which can be considered to be linear parameter varying (LPV) systems, the interplay of uncertainty, states and inputs, renders it difficult to characterize the evolution of the predicted errors and thus to ensure the recursive feasibility of the robust output feedback MPC. [15] exploits periodic invariance to assert stability, but the way in which invariant ellipsoids of the state estimate and the error are treated separately is conservative and the relevant online optimization involves a number of LMI's, which result in a heavy computational burden. In [4] and [24], the robust output feedback MPC strategy for quasi-LPV systems has been studied, where the term 'quasi-LPV' is used to denote that the model parameters are unknown in the future time but are known at the current instant. For systems with multiplicative uncertinaty, additive disturbances and measurement noise, a series of studies on how to design a dynamic output feedback have been conducted by Ding and his co-workers [4–6]. They make use of a dynamic output feedback controller and analyze various methods to handle the errors, e.g. through the use of polytopic or ellipsoidal sets, thereby providing guarantees for recursive feasibility and quadratic boundedness. They also show that there is no intrinsic difference between the feedback of the state estimator and the dynamic output feedback. However, their formulations introduce a number of LMI's to be used online and this restricts application to low dimensional problems with slow dynamics.

This paper designs a posterior state observer and exploits a convenient quasi-closed loop controller. The original system is then divided into the observer dynamics and the estimation error dynamics, and the observer state is further handled as the combination of a nominal state and an associated error. The predicted errors are steered into a series of polytopic sets with parallel *edges (bounding facets)* but variable scalings and arbitrary complexity, and these polytopic sets allow constraints to be invoked in one-step-ahead manner thereby generating a number of linear inequalities. In Mode 2, the error set is chosen to be invariant. Based on this setting, a maximum admissible set is computed offline as the robust invariant terminal set. The scalings of the error sets in the prediction steps constitute additional online decision variables, but their number depends linearly on the prediction horizon. Hence, the online optimization problem, which calls for the solution of a standard quadratic program, is of comparable computational complexity to that required by nominal MPC.

2 Problem Formulation

The system and constraints to be considered are given as:

$$x_{k+1} = A_k x_k + B_k u_k + w_k, \tag{1}$$

$$y_k = C x_k + D v_k, \tag{2}$$

$$F x_k + G u_k \le g, \tag{3}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$, $g \in \mathbb{R}^{n_g}$, $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $C \in \mathbb{R}^{n_y \times n_u}$, $D \in \mathbb{R}^{n_y \times n_v}$, $F \in \mathbb{R}^{n_g \times n_x}$, $G \in \mathbb{R}^{n_g \times n_u}$, w_k is the additive disturbances, $v_k \in \mathbb{R}^{n_v}$ represents the measurement noise. $[A_k\ B_k\ w_k\ v_k] \in Co\{[A^{(1)}\ B^{(1)}\ w^{(1)}\ v^{(1)}], \ldots, [A^{(L)}\ B^{(L)}\ w^{(L)}\ v^{(L)}]\}$. A posterior Luenberger state observer [3, 27] is designed as follows:

$$\hat{x}_{k+1} = A_0 \hat{x}_k + B_0 u_k + L_e[y_{k+1} - C(A_0 \hat{x}_k + B_0 u_k)], \tag{4}$$

where A_0, B_0 are the nominal (e.g. most likely or expected) values of A_k, B_k and L_e satisfies $\rho[(I-L_eC)A^{(j)}] < 1$ ($j = 1,\ldots,L$); this last condition arises from the requirement for the stability of the error dynamics (see (6) below). Use will be made of the quasi-closed loop control law

$$u_{k+i|k} = K\hat{x}_{k+i|k} + c_{k+i|k}, \quad c_{k+N+i|k} = 0,\ i \geq 0. \tag{5}$$

In (5), the first N control moves in Mode 1 are used to steer the state into the terminal set and optimize a predicted cost, whereas the control moves in the remainder of the predictions (Mode 2) are assumed to be $u = Kx$ where K is usually taken to be the unconstrained nominal LQ optimal gain matrix.

The error dynamics between the true state and the observer is obtained from (4), (1) and (5) as:

$$\begin{aligned} e_{k+1} =&(I-L_eC)A_k e_k + (I-L_eC)(\Delta_{A,k} + \Delta_{B,k}K)\hat{x}_k \\ &+ (I-L_eC)\Delta_{B,k}\, c_k + [I-L_eC,\ -L_eD]\varpi_k, \end{aligned} \tag{6}$$

where $\Delta_{A,k} = A_k - A_0$, $\Delta_{B,k} = B_k - B_0$, $\varpi_k^T = \begin{bmatrix} w_k^T & v_{k+1}^T \end{bmatrix}$. To analyze constraints (3) in the prediction horizon, as in [22], the nominal model of the original system is introduced:

$$z_{k+1} = \Phi_0 z_k + B_0 c_k, \quad \Phi_0 = A_0 + B_0 K. \tag{7}$$

Then the nominal error ε between the observer state $\hat{x}$ and the nominal state z is obtained:

$$\begin{aligned} \varepsilon_{k+1} =&(\Phi_0 + L_eC\Delta_{\Phi,k})\varepsilon_k + L_eC\Delta_{\Phi,k}\, z_k \\ &+ L_eC\Delta_{B,k}\, c_k + L_eCA_k e_k + [L_eC,\ L_eD]\varpi_k, \end{aligned} \tag{8}$$

where $\Delta_{\Phi,k} = \Delta_{A,k} + \Delta_{B,k}K$. Thus the dynamics for the lifted state space $\eta = [z^T\ \varepsilon^T\ e^T]^T$ can be formulated as:

$$\begin{aligned} &\eta_{i+1|k} = \Psi_{k+i}\eta_{i|k} + \tilde{B}_{k+i}c_{i|k} + \tilde{D}\varpi_{k+i}, \\ &\Psi_{k+i} = \begin{bmatrix} \Phi_0 & 0 & 0 \\ L_eC\Delta_{\Phi,i} & \Phi_0 + L_eC\Delta_{\Phi,i} & L_eCA_{k+i} \\ (I-L_eC)\Delta_{\Phi,i} & (I-L_eC)\Delta_{\Phi,i} & (I-L_eC)A_{k+i} \end{bmatrix}, \\ &\tilde{B}_{k+i} = \begin{bmatrix} B_0 \\ L_eC\Delta_{B,i} \\ (I-L_eC)\Delta_{B,i} \end{bmatrix},\ \tilde{D} = \begin{bmatrix} 0 & 0 \\ I-L_eC & -L_eD \\ L_eC & L_eD \end{bmatrix}. \end{aligned} \tag{9}$$

where $\Delta_{\Phi,i}$, $\Delta_{B,i}$ is the abbreviation of $\Delta_{\Phi,k+i}$, $\Delta_{B,k+i}$, and $[\Psi_{k+i},\ \tilde{B}_{k+i},\ \varpi_{k+i}] \in Co\{[\Psi^{(1)}\ \tilde{B}^{(1)}\ \varpi^{(1)}],\ \ldots,\ [\Psi^{(L)}\ \tilde{B}^{(L)}\ \varpi^{(L)}]\}$ with $\Psi^{(i)}\ \tilde{B}^{(i)}\ \varpi^{(i)}$ being defined (in an obvious way) by (9), (1) and (2). As is common practice we require that the state matrix of the overall dynamics of (9) is stable. Therefore, we make the following reasonable assumption.

Assumption 1. *Ψ_{k+i} is stable despite the uncertainty in $\Delta_{\Phi,i}$ and A_{k+i}.*

According to the formulation above, the true state can be expressed as

$$x_{i|k} = z_{i|k} + \varepsilon_{i|k} + e_{i|k} = [I,\ I,\ I]\eta_{i|k}. \tag{10}$$

Accordingly, constraint (3) can be rewritten as

$$(F+GK)z_{i|k} + (F+GK)\varepsilon_{i|k} + Fe_{i|k} + Gc_{i|k} \leq g. \tag{11}$$

3 Constraint Handling

One of the main difficulties in handling the constraint (11) arises from the need to characterize the errors (ε and e) in the prediction horizon and define a terminal set into which the lifted system state η will be steered at the end of Mode 1. Robust tubes that contain the state prediction errors at the predicted time steps are constructed in this paper and the edges of the tubes have fixed directions, but can be arbitrarily complex [7]. Assume that at the predicted step i the errors ε and e lie in separate polytopes

$$V_\varepsilon \varepsilon_i \leq \beta_{\varepsilon,i|k},\ \ V_e e_i \leq \beta_{e,i|k}, \tag{12}$$

with V_ε, V_e to be chosen offline and scalings β to be optimized online. The inequalities in (12) apply elementwise and can be reformulated as

$$V\zeta_i \leq \alpha_{i|k}, \quad \zeta_i = [\varepsilon_i^T\ \ e_i^T]^T, \quad V = diag(V_\varepsilon,\ V_e), \tag{13}$$

where $\alpha_{i|k} = [\beta_{\varepsilon,i|k}^T\ \ \beta_{e,i|k}^T]^T \in \mathbb{R}^{n_V}$ and $V \in \mathbb{R}^{n_V \times 2n_x}$. For ease of presentation, below we will not distinguish ε and e but instead we will make use of ζ in all possible cases. The definition of the polytopic tube in (13) is in fact a combination of n_V linear inequalities:

$$V_j\zeta \leq \sigma_j, \quad j = 1,\ldots,n_V, \tag{14}$$

where V_j is the jth row of V and σ_j is the jth component of $\alpha_{i|k}$. Each linear inequality defines an edge of the tube if it is active among the n_V simultaneous linear inequalities of (14). It is assumed in the paper that $\alpha_{0|0}$ is known and $V\zeta_{0|0} \leq \alpha_{0|0}$ represents an initial polytopic tube with the maximal number of edges, n_V. Since $\alpha_{i|k}$ is the online decision variable, some of the inequalities among (14) may become inactive and thus the number of the tube edges may decrease (less than n_V). However, each edge of the tube at the predicted time step $i|k$ can find its parallel one among the n_V edges of the initial tube defined by $V\zeta_{0|0} \leq \alpha_{0|0}$, since V_j ($j = 1,\ldots,n_V$) which gives the direction of the edge is chosen offline.

From (9), the following recursion equation is obtained:

$$V\zeta_{i+1} = V[\Psi_{\zeta,i}\zeta_i + \Psi_{\zeta z,i}\, z_i + \tilde{B}_{\zeta,i}\, c_i + \tilde{D}_\zeta\, \varpi_i] \leq \alpha_{i+1|k}, \tag{15}$$

where $[\Psi_{\zeta,i},\ \Psi_{\zeta z,i}]$, $\tilde{B}_{\zeta,i}$, $\tilde{D}_\zeta$ are conformal matrix partitioned from Ψ_i, $\tilde{B}_i$, $\tilde{D}$. Thus, the tube evolution can be handled through a one-step-ahead prediction scheme according to the following theorem.

Theorem 2. *For any ζ_i which lies in the polytope $\{V\zeta_i \leq \alpha_{i|k}\}$, the necessary and sufficient condition to meet $\{V\zeta_{i+1} \leq \alpha_{i+1|k}\}$ is that there exist matrices $H^{(j)}$ for $j = 1,\ldots,L$ with non-negative elements such that*

$$H^{(j)}V = V\Psi_\zeta^{(j)}, \tag{16a}$$

$$H^{(j)}\alpha_{i|k} + V(\Psi_{\zeta z}^{(j)}\, z_i + \tilde{B}_\zeta^{(j)}\, c_i + \tilde{D}_\zeta\, \varpi^{(j)}) \leq \alpha_{i+1|k}. \tag{16b}$$

Proof. It is known that for two linear sets, $S_1 = \{x : \Pi_1 x \leq \tau_1\}$, $S_2 = \{x : \Pi_2 x \leq \tau_2\}$, $S_1 \subseteq S_2$ if and only if there exists a matrix H with non-negative elements such that $H\Pi_1 = \Pi_2$ and $H\tau_1 \leq \tau_2$ (e.g. see the use of Farkas' Lemma in [2]). This result can be applied to (13) and (15) for each of the uncertainty vertices to prove the theorem. □

A similar treatment of the sets of (15) can be implemented to handle constraint (11) in the prediction horizon. The result is summarized without proof in the corollary below.

Corollary 3. *For any ζ_i which lies in the polytope $\{V\zeta_i \le \alpha_{i|k}\}$ for $i = 1, \ldots, N$, constraint* (11) *is satisfied if there exists a matrix H_c with non-negative elements such that*

$$H_c V = [F + GK,\ F], \tag{17a}$$
$$(F + GK)z_{i|k} + H_c\alpha_{i|k} + Gc_{i|k} \le g, \tag{17b}$$

To reduce the online computational burden, the matrices $H^{(j)}$, H_c in (16a) and (17a), which ideally should be optimized online, are designed offline. To provide a relaxation of the constraints (16b) and (17b), the maximum row sum of the absolute values of the H matrices are minimised through the offline calculation. Thus, a series of linear constraints in the prediction horizon, (16b) for tube evolutions and (17b) for constraint (11), have been obtained.

4 The Robust Invariant Terminal Set

We first assume that in Mode 2, the error set $\{V\zeta \le \bar{\alpha}\}$ is invariant, i.e. $\{V\zeta_{N+i} \le \bar{\alpha}\}$ leads to $\{V\zeta_{N+i+1} \le \bar{\alpha}\}$ for $i = 0, 1, \ldots, \infty$. Similar to (16b) in Theorem 2, a sufficient condition, which ensures the invariance of the error set, is as follows.

$$H^{(j)}\bar{\alpha} + V(\Psi_{\zeta z}^{(j)}\Phi_0^i z_N + \tilde{D}_\zeta\varpi^{(j)}) \le \bar{\alpha}, \quad i = 0, 1, \ldots, \infty. \tag{18}$$

Corollary 4. *If in Mode 2, $(F + GK)z_{N+i} + H_c\bar{\alpha} \le g$ and $\{V\zeta_{N+i} \le \bar{\alpha}\}$ are made to be satisfied for $i = 0, 1, \ldots, \infty$ under the dynamics $z_{N+i+1} = \Phi_0 z_{N+i}$, then constraint* (11) *will be satisfied for $N + i$ $(i = 0, 1, \ldots, \infty)$.*

Proof. This is a direct result from Corollary 3. □

Therefore, the terminal set for the parameters $(z_N,\ \bar{\alpha})$ is defined by

$$\hat{S}_\infty = \{(z_N,\ \bar{\alpha}):\ H^{(j)}\bar{\alpha} - \bar{\alpha} + V(\Psi_{\zeta z}^{(j)}\Phi_0^i z_N + \tilde{D}_\zeta\varpi^{(j)}) \le 0, \\ (F + GK)\Phi_0^i z_N + H_c\bar{\alpha} \le g, \quad i = 0, 1, \ldots, \infty\}. \tag{19}$$

Since Φ_0 is stable, one can represent the set $\hat{S}_\infty$ (which is defined in (19) by an infinite number of inequalities) using only a finite number, $n^* + 1$, of inequalities. These can be constructed using a straightforward extension of the approach for computing maximal admissible sets [8].

$$\hat{S}_{n^*} = \{(z_N,\ \bar{\alpha}):\ H^{(j)}\bar{\alpha} - \bar{\alpha} + V(\Psi_{\zeta z}^{(j)}\Phi_0^i z_N + \tilde{D}_\zeta\varpi^{(j)}) \le 0 \\ (F + GK)\Phi_0^i z_N + H_c\bar{\alpha} \le g, \quad i = 0, 1, \ldots, n^*\}. \tag{20}$$

The determination of n^* such that $\hat{S}_{n^*} = \hat{S}_\infty$ involves the offline solution of a set of linear programs to determine the smallest integer n^* such that $H^{(j)}\bar{\alpha} - \bar{\alpha} + V(\Psi_{\zeta z}^{(j)}\Phi_0^{n^*+1}z_N + \tilde{D}_\zeta\varpi^{(j)}) \le 0$ and $(F + GK)\Phi_0^{n^*+1}z_N + H_c\bar{\alpha} \le g$ for all $(z_N,\ \bar{\alpha})$ satisfying (20).

5 Robust Output Feedback MPC

The online objective is to minimise the nominal cost function, which is computed using the nominal system dynamics defined in (7):

$$J_k = \sum_{i=0}^{\infty}\{z_{k+i}^T Q z_{k+i} + (Kz_{k+i} + c_{k+i})^T R(Kz_{k+i} + c_{k+i})\}. \tag{21}$$

Using the lifted formulation of (9), J_k can be written as:

$$J_k = \sum_{i=0}^{\infty}\{[z_{k+i}^T,\ f_{k+i}^T]\tilde{Q}[z_{k+i}^T,\ f_{k+i}^T]^T\},$$
$$f_k = \begin{bmatrix} c_k \\ c_{k+1} \\ \vdots \\ c_{k+N-1}\end{bmatrix},\ \begin{bmatrix} z_{k+i+1} \\ f_{k+i+1}\end{bmatrix} = \begin{bmatrix}\Phi_0 & B_0E \\ 0 & M\end{bmatrix}\begin{bmatrix} z_{k+i} \\ f_{k+i}\end{bmatrix}, \tag{22}$$
$$M = \begin{bmatrix} 0 & I_{(N-1)n_u} \\ 0 & 0\end{bmatrix} \in \mathbb{R}^{Nn_u \times Nn_u},$$
$$f_{k+i} = M^i f_k, \quad c_{k+i} = Ef_{k+i},$$

with $\tilde{Q} = \begin{bmatrix} Q + K^TRK & K^TRE \\ E^TRK & E^TRE\end{bmatrix}$. Then the nominal cost takes the following form

$$J_k = [z_k^T,\ f_k^T]W_0[z_k^T,\ f_k^T]^T, \\ W_0 - \begin{bmatrix}\Phi_0 & B_0E \\ 0 & M\end{bmatrix}^T W_0\begin{bmatrix}\Phi_0 & B_0E \\ 0 & M\end{bmatrix} = \tilde{Q}. \tag{23}$$

In order to further enlarge the region of attraction and improve the performance, the initial nominal value $z_{0|k}$ can be seen as another decision variable [13], which must respect the constraint:

$$V_\varepsilon(\hat{x}_{0|k} - z_{0|k}) \le \beta_{\varepsilon,0|k}, \tag{24}$$

where $\hat{x}_{0|k}$ is the observer state at time k and $\beta_{\varepsilon,0|k}$ is an online decision variable. But at each time instant, the initial bound $\beta_{e,0|k}$ in (12) for the error $e_{0|k}$ is known, which should be updated with the latest estimation [1]:

$$\beta_{e,0|k} = \begin{cases}\beta_{e,0|0} = [0,\ I]\alpha_{0|0}, & k = 0, \\ \beta_{e,1|k-1} = [0,\ I]\alpha_{1|k-1}, & k \ge 1.\end{cases} \tag{25}$$

The overall robust output feedback algorithm can then be summarized as follows.

Algorithm 1.
Offline: Compute the paramater W_0 of the cost function J_k with (23)*; calculate $H^{(j)}$, H_c in* (16a) *and* (17a)*; find n^* in the terminal set $\hat{S}_{n^*}$ of* (20)*.*
Online: At each time step $k = 0, 1, \cdots$,

$$[z_{0|k},\ f_k] = \arg\min J_k, \quad \textit{subject to}$$
$$\begin{cases}\text{the error bound constraint (16b) } (i = 0, 1, \ldots, N-1), \\ \text{constraint for the predicted steps (17b) } (i = 1, \ldots, N-1), \\ \text{terminal constraint: } \alpha_{N|k} \le \bar{\alpha} \text{ and } (z_{N|k},\ \bar{\alpha}) \in \hat{S}_{n^*}, \\ \text{constraint on the initial nominal state (24).}\end{cases} \tag{26}$$

Then, using the first element, c_k, of the optimal value of f_k, implement $u_k = K\hat{x}_k + c_k$, update the error bound $\beta_{e,0|k}$ with (25) *and repeat the online step at the next time instant.*

[1] Therefore, at time k, $\beta_{\epsilon,0|k} = [I, 0]\alpha_{0|k}$ is an online optimization variable but $\beta_{e,0|k} = [0, I]\alpha_{0|k}$ is known.

Corollary 5. *With the formulation of the terminal set $\hat{S}_{n^*}$ defined in* (20)*, the recursive feasibility is guaranteed.*

Proof. In Mode 2, the formulation of the terminal set $\hat{S}_{n^*}$ guarantees that the error set $\{V\zeta \leq \bar{\alpha}\}$ is invariant and constraint (11) is satisfied for $N+i,\ i=0,1,\ldots,\infty$. Therefore if there is a set of optimal solutions at time k: $\{(\alpha_0,\alpha_1,\ldots,\alpha_N,\bar{\alpha}),\ (z_0,z_1,\ldots,z_N),\ (c_0,c_1,\ldots,c_{N-1})\}$, then at time $k+1$, one set of feasible solutions can be

$$\{(\alpha_1,\alpha_2,\ldots,\alpha_N,\bar{\alpha},\bar{\alpha}),\ (z_1,z_1,\ldots,z_N,\ \Phi_0 z_N),(c_1,c_2,\ldots,c_{N-1},0)\}.$$

□

Theorem 6. *Under Algorithm 1, the states will be steered into the vicinity of the origin, which implies stability.*

Proof. Due to the feasibility indicated in Corollary 5, the cost function J_k in (23) is a Lyapunov function [18] which gives that as $k\to\infty$, $z_k\to 0$, $c_k\to 0$. This yields that $x_k=\varepsilon_k+e_k=[I,\ I]\zeta_k\ (k\to\infty)$ will stay inside a terminal set, of which the vertices are defined by $\{V\zeta\leq\bar{\alpha}\}$, thus the stability is established. □

Remark 7. *The online optimization* (26) *is a standard quadratic programming problem, where the online decision variable is* $\{([I,0]\alpha_0,\alpha_1,\ldots,\alpha_N,\bar{\alpha}),\ z_0,\ (c_0,c_1,\ldots,c_{N-1})\}$. *It involves $n_V\frac{2N+3}{2}+n_x+n_uN$ scalar variables, where n_V reflects the dimension of V and represents the complexity of the polytopic tube edges. The larger n_V is, the tighter the bounds for the error estimation will be. Thus the choice of n_V will be a comprise between the accuracy of the error estimation and the number of online decision variables. However, in comparison with existing robust output feedback algorithms, the introduction of tubes into the prediction structure and the handling of tube evolutions via Farkas' Lemma, brings about a significant improvement in computational efficiency.*

6 Numerical Example

Consider a second-order system with constraint:

$$A^{(1)}=\begin{bmatrix}1.080 & 0.579\\ -0.805 & 0.123\end{bmatrix},\ A^{(2)}=\begin{bmatrix}1.142 & 0.579\\ -0.805 & 0.123\end{bmatrix},$$
$$B^{(1)}=\begin{bmatrix}-0.034\\ 0.731\end{bmatrix},\ B^{(2)}=\begin{bmatrix}-0.029\\ 0.855\end{bmatrix},\ C=[0.3,\ 0.5],$$
$$w^{(1)}=\begin{bmatrix}0.001\\ 0.002\end{bmatrix},\ w^{(2)}=\begin{bmatrix}-0.001\\ -0.002\end{bmatrix},$$
$$D=0.01,\quad v^{(1)}=1,\quad v^{(2)}=-1,$$
$$A_0=\frac{A^{(1)}+A^{(2)}}{2},\ B_0=\frac{B^{(1)}+B^{(2)}}{2},$$
$$F=\begin{bmatrix}0.2700 & 0.1447\\ 0 & 0\\ -0.2700 & -0.1447\\ 0 & 0\end{bmatrix},\ G=\begin{bmatrix}0\\ 0.25\\ 0\\ 0.25\end{bmatrix},\ g=\begin{bmatrix}1\\1\\1\\1\end{bmatrix},$$

with $Q=C^TC$ and $R=0.1$. The unconstrained nominal LQ optimal gain is $K=[0.1234,\ -0.3796]$, and $L_e=\left(\begin{smallmatrix}-1.4760\\ 2.6865\end{smallmatrix}\right)$ such that the eigenvalues of $(I-L_eC)A_0$ are 0.2 and 0.3. The initial tubes for ε and e are chosen to have 8 edges respectively, and $V_\varepsilon=V_e$ are matrices of 8×2 dimensions. On account of the fact that the α's are chosen online, it is possible that some of the inequalities that define tube edges will become inactive at some prediction instants and this will result in a decrease in the number of edges (tubes of lower complexity). The prediction horizon is chosen to be $N=4$.

The initial state estimate is $\hat{x}_0=(1,\ -8.3)$ and the initial error e_0 lies in an octahedron inscribed with a ball with the radius 0.2. Figure 1 shows the predicted tubes for time $k+i\ (i=1,2,3,4)$ at $k=0$. These predicted tubes grow very quickly, but as is designed in Section 4 and is evident from the figure, they converge to the vicinity of the origin. Figure 2 plots the closed loop tubes with the newest observer state $\hat{x}_k$ at each time instant k for one realization. These closed loop tubes grow as time k increases, and they converge to the maximum admissible set corresponding to the choice of the V. Moreover, at $k=1$, it is noted from Figure 2 that the closed loop tube (solid line) is much smaller than the predicted tube computed at $k=0$ (dashed line), which shows the key benefit brought by the receding horizon control. The closed loop trajectories, control inputs and system outputs for 100 random realizations of uncertainty for 10 steps are shown in Figure 3, 4 and 5 respectively.

7 Conclusion

A robust output feedback MPC strategy is proposed in the paper. It can handle systems with multiplicative uncertainty, additive disturbances and measurement noise. Through the introduction of tubes with parallel edges, variable scalings and arbitrary complexity into the prediction structure and the handling of tube evolutions via Farkas' Lemma, the online optimization is turned into a standard quadratic programming problem through the one-step-ahead prediction manner. Although the scalings of the error sets are additional online decision variables, the number of which grows linearly with the prediction horizon, N. Therefore, the developed algorithm not only guarantees the recursive feasibility and stability but also can be implemented very efficiently.

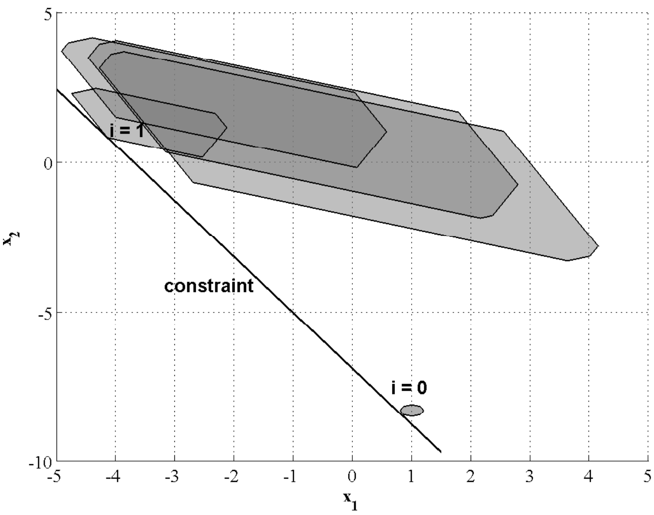

Fig. 1: Predicted tubes for $k+i\ (i\geq 0)$ at $k=0$

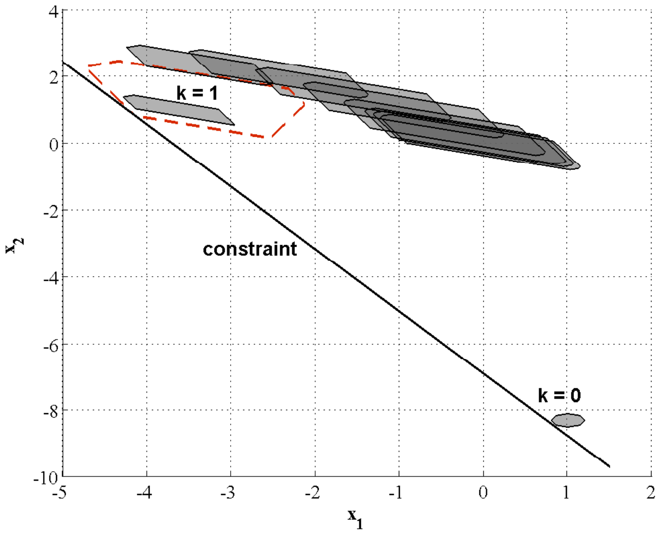

Fig. 2: Closed loop tubes using the newest observer state $\hat{x}_k$ for one realization

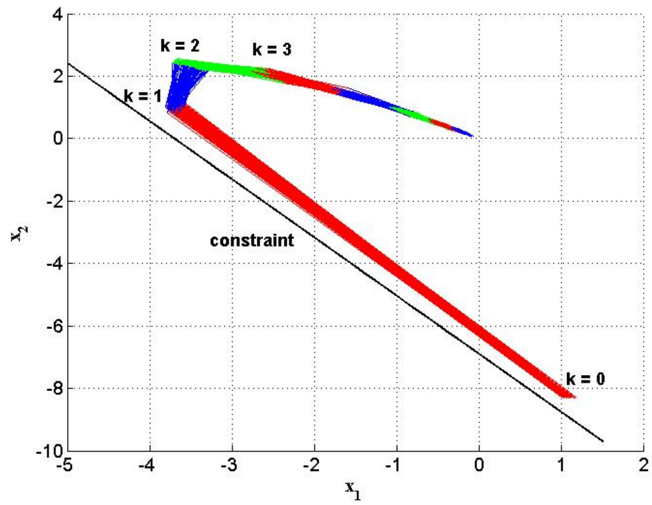

Fig. 3: Closed loop trajectories for 100 random realizations of uncertainty

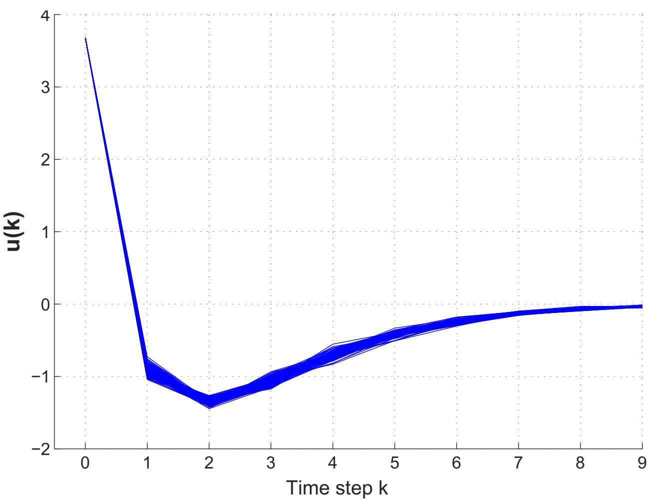

Fig. 4: Control inputs for 100 random realizations of uncertainty

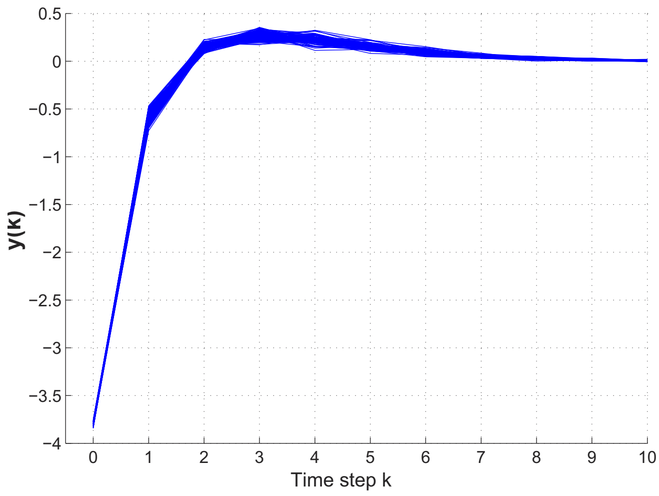

Fig. 5: System outputs for 100 random realizations of uncertainty

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