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BME: Biomedical Ultrasonics

Examples Sheet I

Governing Equations

1. (a) By applying conservation of mass and momentum to a spherical shell of thickness *dr* centred at the origin, show that a spherically symmetric sound wave must satisfy the equation

$$\frac{r^2}{c^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial}{\partial r}\left(r^2 \frac{\partial\ p}{\partial r}\right),$$

where *p* represents the small amplitude pressure fluctuations, *r* the distance from the origin and *c* the speed of sound.

(b) Re-express this equation as a function of derivatives of the product (pr) only, and thus determine its general solution.

2. A pulsating sphere radiates spherical waves into air, where the frequency is 100 Hz and the intensity is 50 mW/m2 at a distance of 1 m from the centre of the sphere. The sphere has a nominal radius of 10cm.

   a) What is the acoustic power radiated?
   b) What is the surface velocity of the sphere?
   c) What is the pressure amplitude and velocity amplitude at distance of 50cm from the centre of the sphere?

Transmission, Reflection and Refraction

3. Consider plane waves normally incident on a hard boundary, such as an air-water interface. The transmission coefficient is T = 2. That means the transmitted pressure amplitude is twice as great as the incident pressure amplitude. How is this consistent with energy conservation?

4. For plane wave reflection from a fluid-fluid interface, it is observed that at normal incidence the pressure amplitude of the reflected wave is 1/2 that of the incident wave. As the angle of incidence is increased, the amplitude of the reflected wave first decreases to zero and then increases until at 30˚ the reflected wave is as strong as the incident wave. Find the density and sound speed in the second medium if the first medium is water.

5. Ultrasound may be used to facilitate bone healing. You want to maximize the transmission of sound from water into bone at normal incidence.
   a) What is the optimum characteristic impedance of the material to be placed between the water and the bone?
   b) What must be the density of, and the sound speed in, a layer of 1-mm thickness that will produce 100% transmission at 200 kHz?

   [you may assume the following properties

   WATER: $\rho \approx 998$ kg/m3 $c \approx 1481$ m/sec

   BONE: $\rho \approx 3{,}400$ kg/m3 $c \approx 4566$ m/sec]

Sources of Sound

6. A 1.48 MHz ultrasonic transducer with a radius a=10 mm is driven in continuous wave mode and radiates into water with sound speed 1480 m/s and density 1000 kg/m^3.
   a) What is the wavelength of the sound and Rayleigh distance of the transducer?
   b) Find the number of nulls on-axis (that is in the near field)
   c) How many nulls does the far-field directivity pattern have?
   d) The RMS pressure on axis at a range of 4 Rayleigh distances is 1 MPa. Find the velocity amplitude u_0 on the face of the piston.
   e) If the transducer is excited with the same amplitude u_0 as in part (d) but with only a 2-cycle pulse of 1.48 MHz then determine the intensity on-axis at a range of 49.5 mm.

7. An ultrasound array consisting of 32 elements with a spacing of 0.5 mm is used to generate a beam at 2 MHz and 4 MHz in tissue with a sound speed of 1540 m/s. Answer the following questions for both frequency components and comment on the differences between the two frequencies for each section:
   a) What is the angular width (in degrees) of the “primary lobe”, which is the major lobe centred on the acoustic axis (i.e. $\theta = 0^\circ$). Define this width as the angular span between the directions where the pressure drops to 50% of the on axis value.

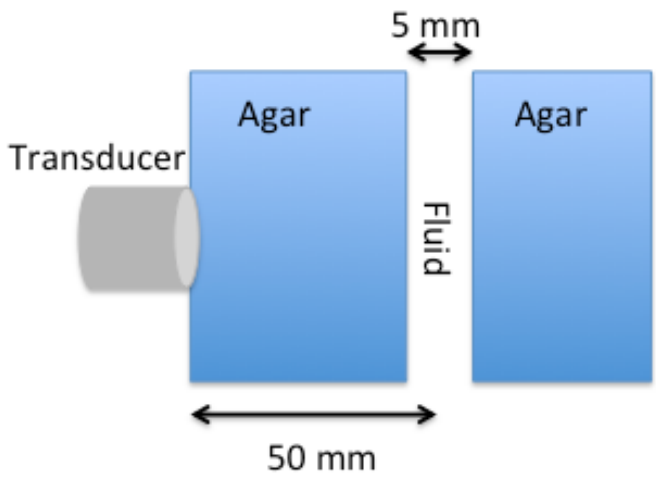
b) Estimate the relative pressure amplitude (relative to pmax, which is the pressure along the acoustic axis) of the 1st side lobe? How can the amplitude of the side-lobe be reduced?

c) At what angles are the grating-lobes for this array?

d) At what steering angle do the main lobe and grating-lobe result in the beams going in equal and opposite directions?

Experimental Case Study: Characterization of the acoustic properties of a fluid by ultrasound

8. An experimental apparatus for measuring the speed of sound, attenuation and characteristic impedance of an unknown fluid consists of a broadband, unfocussed ultrasound transducer with a centre frequency of 15 MHz, embedded in an agar gel, and driven in pulse-echo mode using a pulser-receiver. The outgoing pulse (1.5 cycles of a 15-MHz sine wave) propagates though the agar and is normally incident on a square flow vessel of cross-section 5mm x 5mm, through which a liquid can be made to flow. The vessel is cast inside the agar block and the distance from the face of the transducer to the centre of the vessel is 50 mm.



You may assume that the attenuation through both water and agar is negligible, and that their respective acoustic properties at 37 C are ρ_w = 1005 Kg m^{-3}, c_w = 1500.4 m/s, ρ_A = 1017 Kg m^{-3}, c_A = 1512.1 m/s

a) Assuming that the vessel is initially filled with water, sketch a time-amplitude diagram of the signal received by the transducer, indicating the timing of any reflections that you observe.

b) Describe a technique (give equations) for determining the speed of sound in another fluid given a known speed of sound through water.

c) Write down an expression for the pressure amplitude of the reflection received by the transducer from the near-wall (stating any assumptions clearly). Then devise a technique to measure the characteristic impedance of an unknown fluid, using the known properties of water and agar.

d) Write down an expression relating the pressure amplitude of the reflection received by the transducer from the **far-wall** for both water and an unknown (but attenuative) fluid?

e) Assuming that you have now been able to measure the characteristic impedance of blood, describe a technique for measuring ultrasound attenuation through the blood.

f) You have also obtained measurements for the attenuation through blood, which you have cast in units of Np/m. The attenuation coefficient expressed in those units gives the loss of pressure amplitude experienced by a plane wave when travelling through a length L of a particular medium, i.e.

$$p_2 = p_1 \exp(-\alpha_{Np/m} L).$$

You wish to compare your results to published values, which are given in units of dB/cm/MHz, where a dB is defined as $20\log_{10}(p_1 / p_2)$. Starting with the expression relating p_2 to p_1, determine the conversion factor between *Np/m* and *dB/cm*.

g) Explain why, even if you have performed your conversion correctly, your attenuation values may differ slightly from published ones. How could you resolve this issue?

[hint: think about the frequency content of the ultrasonic pulse]

Selected Answers

2. (a) 628 mW
   (b) 0.862 m/s
   (c) 12.9 Pa
   (d) 4.57 cm/s

4. density: 167 Kg/m^3 sound speed: 2,962 m/s

5. characteristic impedance: 4.79 kg/m^2s
   density: 5,987 kg/m^3 sound speed: 800 m/s

6. (a) 1mm and 100pi mm
   (b) 10
   (c) 19
   (d) 3.82 m/s
   (e) $4/\sqrt{3}$ MPa

7. (a) 3.33 degrees at 2 MHz
   (b) 13% at 2 MHz
   (c) No grating lobe at 2 MHz
   (d) 23 mm

8. (g) 1 Np/m = 0.2 $\log_{10} e$ dB/cm