Generalizing Everett’s Quantum Mechanics for Quantum Cosmology

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Quantum Cosmology

- If the universe is a quantum system, it has a quantum state.
- A theory of this state and calculations of its observable predictions are the objectives of quantum cosmology.
- Such a theory is a necessary part of any final theory. Otherwise there are no predictions.
Why Extrapolate Quantum Mechanics to Cosmology?

- The ever expanding domain of success of quantum theory on laboratory scales.
- The remarkable lack of alternative ideas. All current fundamental theories are quantum mechanical.
Quantum Mechanics Permits a Simple Fundamental Theory of the Universe’s Initial State.

- Were the laws **deterministic**, present complexity would have to be encoded in the fundamental initial condition.

- But in quantum mechanics, present complexity can arise from the **quantum accidents** of past history.
Example of a Current Question in Quantum Cosmology
T. Hertog, S.W. Hawking, J.H.

- Assume the no-boundary theory of the initial quantum state.
- Assume a matter field and a positive $\Lambda$.
- What is the probability that the universe behaved classically in the past and bounced at a minimum radius $R$?
What Quantum Cosmology Requires from Quantum Mechanics

- Probabilities for alternative coarse-grained histories of geometry and matter fields.
- Coarse-grained alternatives defined in four-dimensional, diffeomorphism invariant terms.
- Alternatives for the past as well as future history.
The Past in Cosmology and in Quantum Mechanics

- Reconstruction of the past in cosmology is essential to understand our present and simplify the prediction of the future.

- Decoherent histories quantum theory allows a coherent discussion of the past in quantum mechanics through probabilities for past histories conditioned on present data and the initial condition of the universe.
Cosmology is the Killer App for Everett Quantum Mechanics
Everett’s Quantum Mechanics

- The textbook quantum mechanics of measurements and observers has to be generalized to apply to cosmology.

- Everett’s key idea was to take quantum mechanics seriously for the universe.

- Understanding quantum mechanics for cosmology helps understand how it applies in the laboratory.
Everett’s ideas were extended and clarified by many.

The modern synthesis of decoherent histories quantum theory is adequate for the model cosmology of fields in a box when quantum gravity is neglected.

But we don’t live in a box, and quantum gravity is not negligible in cosmology.

A further generalization is needed.
Quantum Mechanics and Spacetime

Familiar quantum theory assumes a **fixed spacetime**:  

- To define the “$t$” in the Schroedinger equation:
  \[ i\hbar \frac{d}{dt} |\Psi\rangle = H |\Psi\rangle \]

- To define the spacelike surfaces on which the wave function is reduced on measurement or on which alternatives are defined in decoherent histories:
  \[ |\Psi\rangle \rightarrow P|\Psi\rangle/\|P|\Psi\rangle\| \]
  \[ |\Psi_\alpha\rangle = P_{\alpha_n}^{n}(t_n) \cdots P_{\alpha_1}^{1}(t_{\alpha_1})|\Psi\rangle \]

- But in quantum gravity spacetime geometry is fluctuating and without definite value so a generalization of these laws of evolution is needed.
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The Simplicity of Everett QM

• The conceptual simplicity of the Everett formulations provide a springboard for generalizations and extensions, because they are free from a fundamental dependence on complex physical phenomena such as measurements, observers, consciousness, etc.

• Measurements, observers, consciousness can be understood within quantum mechanics, but a detailed understanding is not necessary to understand quantum mechanics or its generalizations.
1. The sets of **fine-grained** histories.

2. The sets of **coarse grained** histories. (Generally partitions of the sets of fine-grained histories into classes \( \{\alpha\} \)).

3. A **decoherence functional** \( D \) defining the interference between coarse-grained histories and satisfying i) Hermiticity, ii) normalization, iii) positivity, and iv) the principle of superposition.

**Superposition Princ.** If \( \{\beta\} \) is a coarse graining of \( \{\alpha\} \):

\[
D(\beta', \beta) = \sum_{\alpha' \in \beta} \sum_{\alpha \in \beta} D(\alpha', \alpha)
\]
Generalized Quantum Theory (cont’d)

- **Decoherence:** \( D(\beta', \beta) \approx \delta_{\beta', \beta} p(\beta) \)

- The *probabilities* \( p(\beta) \) so defined are *consistent* as a consequence of decoherence.

\[
p(\beta) = \sum_{\alpha \in \beta} p(\alpha)
\]

- The decoherence functional of DH is one way of satisfying the axioms but not the only way.

\[
D(\beta', \beta) \equiv \langle \Psi_{\beta'} | \Psi_{\beta} \rangle
\]

\[
|\Psi_{\beta}\rangle = P_{\beta_n}^n (t_n) \cdots P_{\beta_1}^1 (t_1) |\Psi\rangle
\]

- Therein lies the possibility of generalization.
Key Idea about Histories:

Histories need not describe evolution in spacetime but can describe evolution of spacetime.
Fine Grained Histories of Spacetime

4d metrics with matter fields.

Simplicial geometries

Spin foams
Fine Grained Histories of Spacetime

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Coarse Graining

Every assertion that can be made about the universe corresponds to a partition of the fine-grained histories in the class where it is true and the class where it is false.

Example: Bounce Problem:

- A partition into the class C that are classical (to some approx.) and the class (NC) that are not.
- A partition of C into the class CB which bounce and the class CS which are singular.
Measure of Interference
(Schematic)

- **Branch State Vectors** (e.g. for classical bounce, CB)
  \[ |\Psi_{CB}\rangle = \int_{CB} \delta g \delta \phi \exp(iS[g, \phi]) |\Psi_{\text{no bound}}\rangle \]

- **Decoherence functional:**
  \[ D(\alpha', \alpha) \equiv \langle \Psi_{\alpha}' | \Psi_{\alpha} \rangle \]

- **Decoherence and probabilities:**
  \[ D(\alpha', \alpha) \approx \delta_{\alpha', \alpha} p(\alpha) \]
  \[ p(CB) = || |\Psi_{CB}\rangle||^2 \]

Ask speaker for details afterwards.
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What’s Real?

• Whether one set of histories is more real than other sets,

• Or whether one history in that set is real and the others are not,

• Or whether all the sets and all the histories are equally real.

Doesn’t seem to have much to do with the calculation of the probability that the universe bounces at a small radius or its interpretation.
A Fully Four-Dimensional Formulation

- Fine grained histories: 4d histories of spacetime geometry and matter fields.
- Coarse grainings: partitions of the fine grained histories into 4d diffeomorphism invariant classes.
- Measure of Interference: decoherence functional defined by 4d sums over histories.

Is there an equivalent 3+1 formulation in terms of the evolution of states on spacelike surfaces?
We derive states on spacelike surfaces, their inner products, and their unitary evolution \( i\frac{d\psi_A}{dt} = H\psi_A \)
Requirements for a 3+1 Formulation

• Fine-grained histories that are single valued in a time variable.
• Alternatives at a moment of time.

But, in quantum gravity:

• Histories of spacetime geometry are not single valued in any time variable.
• There are no diffeo invariant alternatives at a moment of time.

There is a 4-d formulation of quantum mechanics but not a 3+1 formulation.
Recovering States Approximately when Geometry is Approximately Classical

- For coarse-grainings defining geometry well above the Planck Scale and for particular initial conditions $|\Psi\rangle$
  the semiclassical approximation to the sum over geometries may be adequate:

$$|\Psi_\alpha\rangle \equiv \int_\alpha \delta g \delta \phi \exp(iS[g, \phi]) |\Psi\rangle \approx \int_\alpha \delta \phi \exp(iS[\hat{g}, \phi])) |\Psi\rangle$$

- This defines a quantum field theory on a background spacetime $\hat{g}$ which gives:
  - well defined time(s).
  - states on spacelike surfaces
  - alternatives at a moment of time
  - unitary evolution
In this way, familiar 3 +1 Hamiltonian quantum mechanics, emerges as an approximation in the late universe, appropriate to those initial conditions and coarse-grainings that imply classical spacetime there, in a more general 4d sum-over-histories formulation of quantum theory.
`Ideas that were accepted as fundamental, general, and inescapable that were subsequently seen to be consequent, special, and dispensable to reach a more general perspective. They arose from true physical facts, but ones which are special situations in a yet more general theory.'
Excess Baggage

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Old examples: earth centered solar system, absolute time, fixed Euclidean space, an exact second law, etc.
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- Determinism
- A central role for measurement and observers.
- A unique reality (one decoherent set).
- A distinguished role for time.
- Unitarily evolving states on spacelike surfaces.
Implications for Everett QM

- No wave function evolving unitarily in time.
- No branching at definite moments of time.

What’s Left:

- All the different decoherent sets of coarse-grained histories.
- The initial quantum state.
What’s Left of the “Measurement Problem”?

• When we have a quantum theory of the universe?
• When there are no states at a moment of time to reduce?
• When there are no alternatives at a moment of time?
Gravity is not an option in cosmology.

Most generally when including quantum gravity we should think of Everett 4-dimensionally.
Is Quantum Theory Excess Baggage?

- The founders of quantum theory thought indeterminacy and complementarity arose from the limitations of measurement. Why indeterminacy and complementarity in a closed system that is never measured?

- Why the principle of superposition in a theory where there is only one quantum state?
Whatever replaces quantum theory will be consistent with Everett’s idea of taking fundamental physics seriously for the universe.
The Main Points Again

- The simplicity of the Everett framework allows quantum mechanics to be generalized to include: *closed systems, histories, and quantum spacetime.*

- In quantum gravity there is likely to be no notion of a state evolving in time or branching at moments of time. Such notions may be just FAPP.
Soundbites for Discussion

- DH may supply an umbrella interpretation of quantum mechanics in which other interpretations (Bohm included) arise from restrictions of the sets of histories.

- Quantum mechanics can be formulated time neutrally with initial and final conditions. Time asymmetries may be only apparent arising from our closeness to the big bang.

- Ordinary language evolved over many thousands of years from our focus on the quasiclassical realm of everyday experience. Ordinary language must be qualified or reformed to discuss incompatible realms.
Soundbites for discussion (cont’d)

- A simple theory of the initial condition is unlikely to predict all of the present complexity of the universe. The interesting correlations that test the theory involve conditional probabilities that assume some part of our data. These include probabilities that are involved in anthropic reasoning.

- Quantum mechanics can be formulated as a classical probability distribution for histories that include negative probabilities with an extended interpretation of probabilities.