Monetary Models of the Business Cycle

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Solid lines with plus signs are VAR-based estimates
Money in Real Business Cycle Model

- Money earns no interest and provides no utility in RBC model
- No reason to hold money in equilibrium in RBC model
- Large literature on microfoundations giving rise to money
  - Search theoretic framework of Kiyotaki and Wright (1989)
- Shortcuts
  - Money in Utility function of Sidrauski (1967)
  - Cash in Advance of Clower (1967)
Money in utility function (MIU)

- Households receive utility from real money balances
- Captures various motives for holding money
  - Transactions demand
  - Precautionary demand
  - Speculative demand
- Motivation not explicitly specified or modelled
Household (nominal) problem in MIU model

\[
\max_{\{L_{t+s}, C_{t+s}, M_{t+s+1}, K_{t+s+1}\}} \quad E_t \sum_{s=0}^{\infty} \beta^s \left( \log C_{t+s} + \log \frac{M_{t+s+1}}{P_{t+s}} - \chi L_{t+s} \right)
\]

s.t.

\[
P_{t+s} K_{t+s+1} + M_{t+s+1} = R_{t+s} K_{t+s} + W_{t+s} L_{t+s} + M_{t+s} - P_{t+s} C_{t+s}
\]

\[
K_t, M_t \text{ given, } \lim_{T \to \infty} E_t \frac{K_{T+1}}{\tilde{R}_T} \geq 0
\]
Household (real) problem in MIU model

\[
\max_{\{L_{t+s}, C_{t+s}, m_{t+s+1}, K_{t+s+1}\}} \quad E_t \sum_{s=0}^{\infty} \beta^s \left( \log C_{t+s} + \log m_{t+s+1} - \chi L_{t+s} \right)
\]

s.t.

\[
K_{t+s+1} + m_{t+s+1} = r_{t+s} K_{t+s} + w_{t+s} L_{t+s} + \frac{m_{t+s}}{1 + \pi_{t+s}} - C_{t+s}
\]

\[
K_t, M_t \text{ given, } \lim_{T \to \infty} E_t \frac{K_{T+1}}{\tilde{R}_T} \geq 0, m_{t+s+1} = \frac{M_{t+s+1}}{P_{t+s}} - 1 + \pi_{t+s} = \frac{P_{t+s+1}}{P_{t+s-1}}
\]
First order conditions in MIU model

\( \frac{1}{C_t} = \beta E_t \left( \frac{1}{C_{t+1}} r_{t+1} \right) \) \hspace{1cm} (1)

\( \chi = \frac{w_t}{C_t} \) \hspace{1cm} (2)

\( \frac{1}{m_{t+1}} = \frac{1}{C_t} - \beta E_t \left( \frac{1}{C_{t+1}} \frac{1}{1 + \pi_{t+1}} \right) \) \hspace{1cm} (3)

\( r_t = \alpha K_t^{\alpha-1} (\theta_t L_t)^{1-\alpha} \) \hspace{1cm} (4)

\( w_t = (1 - \alpha) K_t^\alpha \theta_t (\theta_t L_t)^{-\alpha} \) \hspace{1cm} (5)
Response of real economy to technology shocks

- (1), (2), (4) and (5) same as in RBC model without money
- Equilibrium \( \{ r_t, w_t, C_t, K_t, L_t \} \) independent of money
- Responses to technology shocks independent of money
Money in equilibrium

Fisher equation defines $i_t$ as expected nominal return between $t$ and $t+1$

$$1 + i_t = r_{t+1}(1 + \pi_{t+1})$$

First order condition for real balances (3)

$$\frac{1}{m_{t+1}} = \frac{1}{C_t} - \beta E_t \left( \frac{1}{C_{t+1}} \frac{r_{t+1}}{1 + i_t} \right) = \frac{1}{C_t} \left( 1 - \frac{1}{1 + i_t} \right)$$

Equilibrium relationship between $m_{t+1}$ and $i_t$

$$m_{t+1} = C_t \left( \frac{1 + i_t}{i_t} \right)$$
Can specify policy in terms of $M_{t+1}$ or $i_t$

Choose $M_{t+1}$ and equilibrium determines $i_t$ or vice versa

In both cases money **neutral** wrt real variables $C_t$ and $r_{t+1}$

Stochastic process for $M_{t+1}$ or $i_t$ only affects nominal variables
Money supply as policy instrument

\[ M_t - \bar{M} = 0.95 (M_{t-1} - \bar{M}) + \varepsilon_t \]

\[ M_t \uparrow \rightarrow P_t \uparrow, m_t \uparrow, \pi_t \uparrow \rightarrow i_t \downarrow \]
Cash in advance model (CIA)

- Cash as a medium of exchange
- Household has to pay for consumption in cash in advance
- Nominal CIA constraint
  \[ M_{t+s} \geq P_{t+s}C_{t+s} \]
- Real CIA constraint
  \[ m_{t+s} \geq C_{t+s}(1 + \pi_{t+s}) \]
- Inflation acts as a tax on consumption → inflation tax
Household (real) problem in CIA model

\[
\max_{\{L_{t+s}, C_{t+s}, m_{t+s+1}, K_{t+s+1}\}} E_t \sum_{s=0}^{\infty} \beta^s (\log C_{t+s} - \chi L_{t+s})
\]

s.t.

\[m_{t+s} \geq C_{t+s}(1 + \pi_{t+s})\]

\[K_{t+s+1} + m_{t+s+1} = r_{t+s} K_{t+s} + w_{t+s} L_{t+s} + m_{t+s} - C_{t+s}\]

\[K_t, M_t \text{ given, } \lim_{T \to \infty} E_t \frac{K_{T+1}}{R_T} \geq 0, m_{t+s+1} = \frac{M_{t+s+1}}{P_{t+s}}, 1 + \pi_{t+s} = \frac{P_{t+s}}{P_{t+s-1}}\]
First order conditions in MIU model

\[ \frac{1}{C_t} = \lambda_t + \mu_t (1 + \pi_t) \] (6)

\[ \lambda_t = \beta \lambda_{t+1} r_{t+1} \] (7)

\[ \chi = \lambda_t w_t \] (8)

\[ \frac{1}{m_{t+1}} = \lambda_t - \beta E_t \left( \lambda_{t+1} \frac{1}{1 + \pi_{t+1}} \right) + \beta \mu_{t+1} \] (9)

\[ r_t = \alpha K_t^{\alpha - 1} (\theta_t L_t)^{1-\alpha} \] (10)

\[ w_t = (1 - \alpha) K_t^\alpha \theta_t (\theta_t L_t)^{-\alpha} \] (11)

\( \lambda_t \) and \( \mu_t \) are Lagrange multipliers on budget and CIA constraint.
Money supply as policy instrument

\[ M_t - \bar{M} = 0.95 (M_{t-1} - \bar{M}) + \varepsilon_t \]

\[ M_t \uparrow \rightarrow y_t \uparrow \]
Money non-neutrality in CIA model

- Monetary policy shocks affect real variables
- Inflation tax means household prefers to consume (untaxed) leisure
- 1% increase in $M_t \rightarrow \approx 0.04\%$ increase in $Y_t$
- c.f. 1% increase in $z_t \rightarrow \approx 1\%$ increase in $Y_t$
- CIA constraint creates only weak non-neutralities
The way forward

Chapters 2 and 3
Abstract from $K_t$ as it is weak propagation mechanism

Have government issue fixed quantity of one-period bonds

Allow households to buy and sell government bonds

Make prices sticky to amplify non-neutralities

Specify policy for $i_t$

$M_{t+1}$ determined endogenously by *ad hoc* demand for $m_t$

\[
\frac{M_{t+1}}{P_t} = Y_t i_t^{-\eta}
\]  
(G3.4)
Household problem with government bonds

\[
\max_{\{C_t, N_t, B_t\}} \quad \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)
\]

s.t.

\[
P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t
\]

\[
B_{t-1} \text{ given, } \lim_{T \to \infty} \mathbb{E}_t B_T \tilde{Q}_T \geq 0
\]

\[B_t \text{ are bond holdings, } T_t \text{ possible transfers}\]

\[Q_t = \frac{1}{1+i_t} < 1 \text{ is price in period } t \text{ of a bond that matures in period } t+1\]
Euler equation for consumption

Household sets $MRT = MRS$ as before

$$C_t^{-\sigma} = \beta E_t \left( C_{t+1}^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \right)$$

Log-linear approximation with $\log E_t (X_t) \approx E_t (\log X_t)$

$$-\sigma \log C_t = \log \beta - \sigma E_t \log C_{t+1} + \log (1 + i_t) - E_t \log (1 + \pi_{t+1})$$

Approximate with $\log (1 + i_t) \approx i_t$ and $\rho \equiv -\log \beta$

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \quad \text{(G3.3)}$$
Equilibrium output and output gap

- No capital so in equilibrium $c_t = y_t$
- Natural level of output $y^n_t$ if prices are flexible
- Natural level of employment $n^n_t$ if prices are flexible
- Natural rate of interest $r^n_t$ if prices are flexible
- $y^n_t, n^n_t, r^n_t$ determined by technology (and non-price rigidities)
- Output gap $\tilde{y}_t = y_t - y^n_t$
Dynamic IS equation (#1 of the 3 equations)

No capital

\[ y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \]

In terms of output gap \( \tilde{y}_t = y_t - y^n_t \)

\[ \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t) \quad \text{(G3.22)} \]

\[ r^n_t = \rho + \sigma E_t \Delta y^n_{t+1} \quad \text{(G3.23)} \]

where \( r^n_t \) is natural rate of interest \( (= r_t \) if prices are flexible so \( \tilde{y}_t = 0 \) \)
Euler equation for labour supply

Household optimises across goods and leisure

\[ \frac{N_t^\phi}{C_t^{-\sigma}} = \frac{W_t}{P_t} \]

Log-linear approximation

\[ w_t - p_t = \sigma c_t + \varphi n_t \quad (G3.2) \]

Real wage is increasing in equilibrium output

\[ w_t - p_t = \sigma y_t + \varphi n_t \]
Calvo (1983) price rigidity

Probability $1 - \theta$ firm can change price in a period
Probability $\theta$ firm cannot change price in a period
Proportion $1 - \theta$ of firms can change price in a period
Log-linear approximation of aggregate price

\[ p_t = (1 - \theta)p_t^* + \theta p_{t-1} \]
\[ \pi_t = (1 - \theta)(p_t^* - p_{t-1}) \]  \hspace{1cm} \text{(G3.7)}

where \( p_t^* \) is price set by firms able to change prices
Optimal price setting under price rigidity

Log-linear approximation of optimal price setting

\[ p_t^* - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \{ \mu + mc_{t+k|t} + (p_{t+k} - p_{t-1}) \} \]

- \( \mu \) is log of desired gross markup
- \( mc_{t+k|t} \) is (nominal) marginal cost in period \( t + k \) conditional on firm having last changed its price in period \( t \)
- \( p_t^* \) is a markup over current and expected (nominal) marginal costs
- \( \theta \uparrow \) or \( \beta \uparrow \) → more weight on future marginal costs
Optimal price setting under flexible prices

With flexible prices $\theta = 0$

$$p^*_t - p_{t-1} = \mu + mc_t + (p_t - p_{t-1})$$

Since prices are flexible $p^*_t = p_t$

$$mc = -\mu$$

Real marginal cost is constant
Marginal cost

Definition of real marginal cost with production function $Y_t = A_t N_t^{1-\alpha}$

$$MC_t = \frac{W_t}{P_t} \frac{dN_t}{dY_t} = \frac{W_t}{P_t} \frac{1}{(1 - \alpha) A_t N_t^{-\alpha}}$$

Log-linear approximation for individual firm

$$mc_{t+k|t} = (w_{t+k} - p_{t+k}) - a_{t+k} + \alpha n_{t+k|t} - \log (1 - \alpha)$$

Log-linear approximation of economy-wide average

$$mc_{t+k} = (w_{t+k} - p_{t+k}) - a_{t+k} + \alpha n_{t+k} - \log (1 - \alpha)$$
A useful expression for marginal cost

Relation between conditional and average marginal cost

\[ mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k|t} - y_{t+k}) \]

Imperfectly competitive firm faces downward-sloping demand schedule

\[ Y_{t+k|t} = \left( \frac{P^*_t}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \]  

Expression for marginal cost

\[ mc_{t+k|t} = mc_{t+k} - \frac{\alpha \varepsilon}{1 - \alpha} (p^*_t - p_{t+k}) \]
Putting it all together

Aggregate price dynamics

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}) \quad (G3.7)$$

Optimal price setting under price rigidity

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \mu + mc_{t+k|t} + (p_{t+k} - p_{t-1}) \right\} \quad (G3.11)$$

Marginal cost of individual firm

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\epsilon}{1 - \alpha} (p_t^* - p_{t+k}) \quad (G3.14)$$
New Keynesian Phillips Curve in terms of marginal cost

After a lot of tedious algebra (that you don’t need to reproduce)

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda (\mu + mc_t) \quad \text{(G3.16)} \]

\[ \lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \]

\( mc_t \uparrow \longrightarrow \) inflationary pressure \( \rightarrow \pi_t \uparrow \)
Marginal cost and the output gap

Marginal cost is constant under flexible prices

\[ mc = (w_t - p_t) - a_t + \alpha n_t - \log(1 - \alpha) = -\mu \]

\( w_t - p_t \) from household labour supply and \( n_t \) from production function

\[ mc = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t^n - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) = -\mu \quad \text{(G3.18)} \]

Economy-wide average marginal cost under price rigidity

\[ mc_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) \]

In terms of output gap \( \tilde{y}_t = y_t - y_t^n \)

\[ mc_t = -\mu + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \tilde{y}_t \quad \text{(G3.20)} \]
New Keynesian Phillips Curve (\#2 of the 3 equations)

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \]  \hspace{1cm} (G3.21)

\[ \kappa \equiv \lambda \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \]

1. \( E_t \pi_{t+1} > \pi_t \)
2. Prices expected to rise in \( t + 1 \)
3. Firm able to set price in \( t \) sets high price already in \( t \)
4. \( \tilde{y}_t < 0 \)
Policy Rule (#3 of the 3 equations)

\[ i_t = 1 + 1.5\pi_t + 0.5\tilde{y}_t \]

“Simple enough to put on the back of a business card!”
Taylor Rule 1987-1992

Figure 1A

Very good
Taylor Rule since 1993

Figure 1: The Original Taylor Rule, 1993-Present

Good in “normal times” 1993-2009
Poor in “abnormal times” since 2009
Taylor Rule today

Link to Federal Reserve Bank of Atlanta Taylor Rule Utility
3-equation model

\begin{align}
\tilde{y}_t &= E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \\
i_t &= \rho + \phi_\pi \pi_t + \phi_\gamma \tilde{y}_t
\end{align} (G3.21) (G3.22) (G3.25)
Monetary policy shocks in the 3-equation model

\[ i_t = \rho + \phi \pi_t + \phi_y \tilde{y}_t + \nu_t \]

\[ \nu_t = \rho \nu_{t-1} + \varepsilon_t^v \]

\( \varepsilon_t^v \) is monetary policy shock
Monetary policy shocks in the 3-equation model

Figure 3.1: Effects of a Monetary Policy Shock (Interest Rate Rule)

- Output gap
- Inflation
- Nominal rate
- Real rate
- Money growth
- \( \nu \)
Technology shocks in the 3-equation model

Process for technology

\[ a_t = \rho a_{t-1} + \varepsilon_t^a \quad \varepsilon_t^a \text{ is technology shock} \]

Marginal cost constant under flexible prices

\[ \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t^n - \frac{1 + \varphi}{1 - \alpha} a_t - \log (1 - \alpha) = -\mu \]

Technology shock affects \( y_t^n \) and \( r_t^n = \rho + \sigma E_t \Delta y_{t+1}^n \)
Technology shocks in the 3-equation model

Figure 3.2: Effects of a Technology Shock (Interest Rate Rule)

- Output gap
- Inflation
- Output
- Employment
- Nominal rate
- Real rate
- Money growth
- \( a \)
Other shocks in the 3-equation model

\[ \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t) + \varepsilon^d_t \]

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + u_t \]

\( \varepsilon^d_t \) is demand shock

\( u_t \) is cost-push shock
3-equation model of a small open economy

Gali and Monacelli (2005)

\[ \tilde{y}_t = \bar{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma_\alpha} (i_t - \bar{E}_t \pi_{H,t+1} - r^n_t) \]  \hspace{1cm} (G7.38)

\[ \pi_{H,t} = \beta \bar{E}_t \pi_{H,t+1} + \kappa_\alpha \tilde{y}_t \]  \hspace{1cm} (G7.37)

\[ i_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t \]  \hspace{1cm} (G7.40)
Alternative policies for the small open economy

Domestic inflation-based Taylor Rule (DITR)

$$i_t = \rho + \phi_\pi \pi_{H,t}$$

CPI inflation-based Taylor Rule (CITR)

$$i_t = \rho + \phi_\pi \pi_t$$

Exchange rate peg (PEG)

$$e_t = 0$$

where $e_t$ is nominal exchange rate
Figure 7.1 Impulse Responses to a Domestic Productivity Shock under Alternative Policy Rules