Consumption and Labour Supply

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1 Overview

At the heart of modern macroeconomic models is the belief that business cycles should be explained by making explicit assumptions regarding the “deep” structural parameters of the economy: namely (i) tastes and preferences of agents (ii) production technology and (iii) market structure. In this lecture we focus on (i) and examine how consumption and labour supply are determined. The mechanisms we outline here form the heart of most neoclassical models, including Real Business Cycle models. As we shall see neoclassical models of consumption and particularly labour supply are at odds with the data in several respects. Nevertheless it represents a useful first step to analysing the labour market.

2 Key readings

3 Key concepts

Intertemporal Substitution, Euler Equations, Marginal Rate of Substitution

4 Tools for analysis

In this section we outline some simple summary statistics regarding the cyclical behaviour of consumption and labour supply. It provides an empirical reference point to help guide us in our subsequent analysis of the business cycle. In deriving summary statistics we face a problem - namely what does the business cycle component of the data look like. Observed macroeconomic data reflects many different components - an underlying trend, the business cycle component, seasonality, as well as purely random fluctuations - but business cycle theorists are interested in only one component. Effectively what is required is a filter, something which ignores everything other than the business cycle component.

![Figure 1: GNP and non-durable consumption in the US](image-url)
Figure 1 shows the logarithm of US GNP and non-durable consumption for 1947q1 to 2010q2. At present the figure is not very informative and it is difficult to see the interactions between output and consumption. To make things clearer there are an infinite number of different filters that can be used. Unfortunately because we can never know what the true business cycle component actually looks like we can never with complete certainty claim that one filter is better than another. Instead we have to choose a particular filter because it is plausible given certain beliefs about what a business cycle filter should do. Ideally, the cyclical component extracted from any dataset should not vary greatly between different filters. However, as we shall see this is not the case.

In the modern macroeconomics literature one filter has been used almost exclusively. This filter is based on an unpublished paper by Hodrick and Prescott “Postwar US business cycles: An empirical investigation”, Discussion Paper 451, Carnegie Mellon University, 1980. The idea behind this filter is that the data consists of a trend and a business cycle. Hodrick and Prescott’s starting point is that the trend must be a smooth time series - in other words, it does not make sense to think of a trend which fluctuates wildly from quarter to quarter. However, the trend (which we will denote by $\tau_t$) must follow the observed data ($y_t$) closely. They therefore infer the trend from the following minimisation problem:

$$\min_{\{\tau_t\}_{t=1}^T} \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2$$

The first part of the minimisation problem ensures that the trend component tracks the data fairly well. The constraint however prevents the change in the trend being too volatile. The larger the value of $\lambda$ the smoother the changes in the growth of the trend have to be. Hodrick and Prescott suggest a value of $\lambda = 1600$ for quarterly data. Although, as pointed out by many critics, the choice of $\lambda$ is likely to vary from variable to variable. Figure 2 shows recent US GNP and Hodrick-Prescott trends for different smoothing parameters.
Having solved this minimisation problem to arrive at an estimate of the trend, the cyclical component is defined as $y_t - \tau_t$. As mentioned above, the Hodrick-Prescott filter is now the industry standard approach towards detrending. However, there is widespread concern about its use.

(i) Business cycle facts are not invariant to the detrending filter used.

(ii) Other filters may be more optimal. A little bit of thought will reveal that if variables have different stochastic properties then a different detrending filter should be applied. Therefore we cannot expect any one technique to be optimal for all variables. The crucial thing is whether one particular technique is a better approximation across a wider range of variables than another.

(iii) The Hodrick-Prescott filter may produce spurious cycles. A well-known result in the econometrics literature is the spurious cycle result of Nelson and Kang “Spurious periodicity in inappropriately detrended time series” *Econometrica* 1981. They show that if a linear time trend is fitted to a series which follows a random walk then the detrended data will display spurious cycles. In other words, if the researcher mistakenly thinks the trend is deterministic then the cycles derived will be misspecified. Incorrect assumptions about the stochastic
behaviour of a variable similarly mean the HP filter will exaggerate the pattern of long term growth cycles at cyclical frequencies and depress the influence of cycles at other frequencies. The result is that the HP filter may exaggerate the importance of business cycles.

Even more striking, in the context of the Frisch-Slutsky paradigm, the HP filter can be dramatically misleading. Observed stylised facts about the business cycle reflect three factors: (i) an impulse (ii) a propagation mechanism and (iii) the data being detrended by the HP filter and the certain statistics reported. It can be shown that for a typical macroeconomic model (ii) is unnecessary - merely assuming a process for the shock and applying the HP filter will be enough to generate business cycle patterns even if they are not there in the model. In other words, so called “stylised facts” are nothing more than artifacts. This is why some call the HP filter the Hocus Pocus filter - it simply creates business cycles from nothing. However, some words of caution are necessary here. The reason why the HP filter goes wrong is that the researcher makes the wrong assumption about the trend behaviour of a series and so applies the wrong filter. It is not so much the HP filter that produces the wrong result but the misspecification of the trend by the researcher. Using another detrending filter aside from the HP does not remove the risk of misspecification.

These are clearly serious criticisms of simple stylised facts calculated from the HP filter. Some efforts have been made to put this literature on a more secure statistical footing. However, it still remains the case that many quantitative macroeconomic papers still attempt to justify themselves by using stylised facts constructed using the HP filter. Be warned.

5 Stylised facts

Figure 3 plots detrended real US GNP alongside non-durable consumption (all variables are in logarithms) for the period 1947q1-2010q2. Figure 3 shows a strong positive relationship. Interpreting detrended data should always be done with caution, but visual inspection suggests that consumption leads GDP by a quarter or two.
Figure 3: Detrended GNP and non-durable consumption

Table 1 gives a more complete description of the volatilities and cross correlations of consumption and labour market variables. We quote results from the US because most of the theoretical models we shall examine have been constructed with this data in mind. However, surprisingly the UK exhibits very similar properties as Table 1 except there is a more equal split between fluctuations in hours and employment. It should also be borne in mind that different data (for instance establishment rather than household data) can lead to quite different results.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sd%</th>
<th>Cross-correlation of output with:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>t-4</td>
</tr>
<tr>
<td>GNP</td>
<td>1.72</td>
<td>0.16</td>
</tr>
<tr>
<td>CND</td>
<td>0.86</td>
<td>0.40</td>
</tr>
<tr>
<td>CD</td>
<td>4.96</td>
<td>0.37</td>
</tr>
<tr>
<td>H</td>
<td>1.59</td>
<td>0.09</td>
</tr>
<tr>
<td>Ave H</td>
<td>0.63</td>
<td>0.16</td>
</tr>
<tr>
<td>L</td>
<td>1.14</td>
<td>0.04</td>
</tr>
<tr>
<td>GNP/L</td>
<td>0.90</td>
<td>0.14</td>
</tr>
<tr>
<td>Ave W</td>
<td>0.55</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Sd% denotes standard deviation. \( t - j \) denotes the correlation between GNP at time \( t \) and the variable denoted by the first column at time \( t - j \). CND stands for non-durable consumption, CD for durable consumption, H for total hours worked, Ave H is average hours worked per employee, L is employment, GNP/L is productivity, Ave W is average hourly wage based on national accounts. All employment data is based on household surveys.

There are six main stylised facts which emerge from Table 1:

(i) consumption is smoother than output
(ii) volatility in GNP is similar in magnitude to volatility in total hours
(iii) volatility in employment is greater than volatility in average hours. Therefore most labour market adjustments operate on the extensive rather than intensive margin
(iv) productivity is slightly procyclical
(v) wages are less variable than productivity
(vi) there is no correlation between wages and output (nor for that matter with employment)

In terms of the neoclassical model’s performance we will show that the model is relatively successful at explaining why consumption is smoother than output (at least for the US). Fact
(ii) shows how important labour market fluctuations are to the business cycle. The lectures on unemployment later in the course examine a number of models which try to account for (iii) but this represents a significant problem for the basic neoclassical model. Facts (iv)-(vi) are also very problematic for the neoclassical model which we now show.

6 The model

We assume the existence of a utility function $U(c_t, l_t)$ where $c_t$ is consumption and $l_t$ is leisure. Notice that utility depends only on current consumption and leisure - that is preferences are \textit{intertemporally separable}. This implies that previous choices of consumption and leisure do not influence marginal utility this period. Clearly previous values of $c_t$ and $l_t$ will influence the current choice of consumption and leisure through budget constraint effects but they do not directly influence the utility function. A number of recent studies have stressed the importance of not having intertemporally separable preferences.

Consumers have to make two decisions (i) how much to spend/save and (ii) how much to work. We assume that the number of hours available in a day is normalised to 1. Consumers receive an interest rate $R_t = (1 + r_t)$ on any savings and receive a wage rate $w_t$ for any hours worked. Both $R_t$ and $w_t$ are treated as beyond the consumer’s control and are also stochastic.

Assume the consumer wishes to maximise the present value of the discounted stream of utility. That is:

$$
\max_{\{c_{t+s}, l_{t+s}, a_{t+s+1}\}} E_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}, l_{t+s})
$$

s.t.

$$
a_{t+s+1} = R_{t+s} a_{t+s} + w_{t+s} (1 - l_{t+s}) - c_{t+s} \text{ for } \forall s \geq 0
$$

where $a_t$ denotes the consumer’s assets holdings. 1 is the time endowment per period so that $1 - l_t$ is time spent working and $\beta$ is the discount factor. The second constraint is a no-Ponzi condition that rules out consumption plans based on ever-increasing levels of debt. It serves as the transversality condition to uniquely pin down the optimal path for consumption. We will solve this problem in four different but complementary ways.
6.1 Direct substitution approach

We can substitute the budget constraint directly into the utility function so (1) becomes:

\[
\max \left\{ \sum_{s=0}^{\infty} \beta^s U(R_{t+s}a_{t+s} + w_{t+s}(1-l_{t+s}) - a_{t+s+1}, l_{t+s}) \right\}
\]

where the consumer maximises utility by choosing assets and leisure. Maximising w.r.t assets \( a_{t+1} \) in the next period we get the following Euler equation:

\[
E_t \left( \beta R_{t+1} \frac{U_{c,t+1}}{U_{c,t}} - 1 \right) = 0
\]

where \( U_{c,t} \) refers to the marginal utility of consumption at time \( t \). We shall refer to (3) as EC because this is the Euler equation which determines consumption. Notice that it is an intertemporal condition as it links the marginal utility of consumption in one period with that in another. Maximising (2) w.r.t. leisure \( l_t \) gives the following first order condition:

\[
U_{l,t} = w_t U_{c,t}
\]

This is an intratemporal condition linking the marginal utility of leisure and consumption in any one period and we shall refer to it as ILC. Equation (4) says that the marginal rate of substitution between leisure and consumption equals the real wage rate. Finally, if we substitute (4) into (3) we have the following equation which explains the intertemporal behaviour of labour supply:

\[
E_t \left( \beta R_{t+1} \frac{U_{l,t+1} w_t}{U_{l,t} w_{t+1}} - 1 \right) = 0
\]

We shall refer to this condition as EL, the Euler equation for labour supply.

6.2 Graphical approach

Consider two consecutive periods \( t \) and \( t+1 \) in the maximisation problem (1). From the utility function we can draw indifference curves in \((c_t, c_{t+1})\) space.
The slope of an indifference curve can be calculated by total differentiation of the utility function. Ignoring the expectations term for convenience, the (deterministic) slope of the indifference curve is given by

\[
0 = U_{c,t} dc + \beta U_{c,t+1} dc_{t+1} \\
\frac{dc_{t+1}}{dc_t} = -\frac{1}{\beta} \frac{U_{c,t}}{U_{c,t+1}}
\]

This is the marginal rate of substitution (MRS). We then add the budget constraint

\[
C_{t+1} = C_t + \frac{1}{R_{t+1}} dc_{t+1} + \frac{\omega_{t+1}}{R_{t+1}} (1 - l_{t+1}) - c_{t+1}
\]

with a slope given by iterating the budget constraints forward

\[
a_{t+1} = R_t a_t + w_l (1 - l_t) - c_t \\
a_{t+2} = R_{t+1} a_{t+1} + w_{t+1} (1 - l_{t+1}) - c_{t+1}
\]

to obtain

\[
\frac{a_{t+2} - w_{t+1} (1 - l_{t+1}) + c_{t+1}}{R_{t+1}} = R_t a_t + w_l (1 - l_t) - c_t
\]

from which it is clear that

\[
\frac{1}{R_{t+1}} dc_{t+1} = -dc_t \\
\frac{dc_{t+1}}{dc_t} = -R_{t+1}
\]

which is the marginal rate of transformation (MRT). At the point of tangency we have
and:

\[
\frac{1}{\beta} \frac{U_{c,t}}{U_{c,t+1}} = R_{t+1} \\
\left( \beta R_{t+1} \frac{U_{c,t+1}}{U_{c,t}} - 1 \right) = 0
\]

as required.

### 6.3 Value function approach

Taking the dynamic programming approach, we can write the value function as

\[
V(a_t) = \max_{a_{t+1}, l_t} \left[ u(R_t a_t + w_t (1 - l_t) - a_{t+1}, l_t) + \beta E_t V(a_{t+1}) \right]
\]

The first order condition with respect to assets \(a_{t+1}\) is

\[
U_{c,t} = \beta E_t V'(a_{t+1})
\]

As is usual in dynamic programming, we do not know the form of the value function \(V(a_t)\), but we do know its first derivative \(V'(a_t)\). Differentiating from the definition of the value function

\[
V'(a_t) = U_{c,t} R_t
\]

Rolling forward one period

\[
V'(a_{t+1}) = U_{c,t+1} R_{t+1}
\]

and substitute in the first order condition to obtain

\[
E_t \left( \beta R_{t+1} \frac{U_{c,t+1}}{U_{c,t}} - 1 \right) = 0
\]

as before. The first order condition with respect to leisure \(l_t\) is

\[
U_{l,t} = w_t U_{c,t}
\]

and the intratemporal labour supply equation is as before. In a moment we shall consider particular function forms for the utility function but from (3)-(5) we can already see some of the major implications of the neoclassical model for consumption and labour supply.
6.4 Lagrangean approach

The Lagrangean approach begins by defining:

\[ \mathcal{L} = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}, l_{t+s}) + \sum_{s=0}^{\infty} \lambda_{t+s} \beta^s (R_{t+s} a_{t+s} + w_{t+s} (1 - l_{t+s}) - c_{t+s} - a_{t+s+1}) \]

This is the present value formulation of the Lagrangean as the Lagrange multiplier \( \lambda_{t+s} \) is discounted by \( \beta^s \) back to its present value. It is equally valid to work with the current value Lagrangean and write the second term without discounting, i.e. \( \tilde{\lambda}_{t+s} = \lambda_{t+s} \beta \). They are mathematically equivalent but sometimes it is more convenient to work with one than the other. The first order conditions with respect to \( \lambda_t \), \( \lambda_{t+1} \) and \( a_{t+1} \) are

\begin{align*}
U_{c,t} &= \lambda_t \\
U_{c,t+1} &= \lambda_{t+1} \\
\lambda_{t+1} \beta R_{t+1} - \lambda_t &= 0
\end{align*}

which when combined imply

\[ E_t \left( \beta R_{t+1} \frac{U_{c,t+1}}{U_{c,t}} - 1 \right) = 0 \]

as required. The first order condition with respect to leisure gives the intratemporal Euler equation as before.

6.5 Consumption

Focusing on EC, the Euler equation for consumption, we can see that what determines the growth in the marginal utility of consumption (which as we shall see is closely linked to consumption) is the interest rate, \( R_t \). In our model we have assumed that the consumer can only invest in one asset, \( a_t \). However, equation (3) holds for any asset the consumer invests in so we should think of \( R_t \) more widely as the return on any asset e.g. equity, bond, cash etc. To see this more clearly assume that \( \lambda_{t+1} = \lambda_t \) and that \( \lambda_{t+1} \beta R_{t+1} - \lambda_t = 0 \). As a consequence they are not expecting their consumption to change between time periods. As a consequence they are not expecting their consumption to change at all. Similar reasoning suggests that if \( \beta R_{t+1} > 1 \) then \( E_t (U_{c,t+1}/U_{c,t}) < 1 \) which given that marginal utility is declining in consumption (i.e. \( u'' < 0 \)) implies that agents must be expecting consumption to increase. Similarly if \( \beta R_{t+1} < 1 \) then consumption is expected to fall. In all cases the only thing which determines consumption growth is the rate of return/interest rate and not...
income\(^1\). The rationale for the interest rate effect is as follows. If consumers know that savings this period are going to earn a high rate of return there is an incentive for them to save more by having lower consumption. For a given end of period consumption level, the lower is the level of initial consumption the faster is the growth rate.

### 6.6 Consumption-leisure choice

Turning to ILC we can deduce the following key propositions from standard assumptions on the utility function.

1. If \( w_t \) increases in a period, then this implies that \( U_{l,t} \) has to rise relative to \( U_{c,t} \). Remember that \( U_l \) is the marginal utility of leisure. For this to rise consumers have to spend less time as leisure and more time working. Therefore ILC says that when the wage rate rises then hours worked increases.

2. However, as emphasised by Barro and King (1984), if economic activity increases but wages do not rise (and from Table 1 we can see that there is no real correlation between wages and GDP) then ILC implies that both consumption and leisure have to increase together. In other words, assume something happens which increases the output in a particular period but doesn’t change the real wage rate (say, for instance an increase in real government purchases). The higher output will encourage people to consume more which implies that \( U_c \) must fall. But for ILC to still hold (given an unchanged real wage) it must be the case that \( U_l \) falls too, which requires the consumer working less and taking more leisure. In other words, absent any movement in real wages, because consumption and leisure are not inferior goods they must move together in the same direction. Yet empirically we observe (see Table 1) both employment and consumption being highly procyclical and so they move together (or alternatively consumption and leisure move in opposite directions). In a neoclassical framework this tells us that we can only reconcile the model with the data if we assume that the real wage moves upwards in booms and downwards in recessions. Therefore in neoclassical models any candidate for the cause of business cycles has to involve an increase in real wages. This is one reason why productivity shocks (as in the Real Business Cycle literature) are popular. But we can also see from Table 1 that the real wage is itself not procyclical. This is a fundamental problem for neoclassical models which they are still struggling to overcome.

\(^1\)We shall not be able to establish this point, but *unexpected* changes in consumption are caused by income.
There are basically two responses. The first is to assume that as well as productivity shocks which shift the labour demand curve there are also shocks which shift the labour supply curve. With both shocks occurring it is possible to reconcile the model with the data. Some suggested explanations for labour supply shifts are (i) simple taste shocks, i.e. I suddenly find that my hatred of work has increased so my marginal utility of leisure increases (ii) government expenditure shocks which under certain plausible assumptions can lower wealth and increase labour supply (iii) changes in labour taxes. The second response is to say that real wages are not measured correctly in the data. For instance, the implicit contract literature argues that firms remove risk from workers by smoothing wages over the cycle. As a result the marginal rate of productivity does not equal the quoted real wage rate and we can interpret Table 1 as saying anything about wages. It’s your decision whether you find any of these responses plausible.

6.7 Labour supply

Equation (5) says that the determinants of changes in the marginal utility of leisure are (i) the interest rate/rate of return (ii) the ratio of future and current wages. Let us first concentrate on the effect of wages by setting \( R_{t+1} = R \) and \( R\beta = 1 \). We shall return below to the influence of rates of return on labour supply. Under this assumption we have that:

\[
E_t \left( \frac{U_{l,t+1} w_t}{U_{l,t} w_{t+1}} \right) = 1
\]

Therefore, if wages are constant between the two periods then the agent does not expect the marginal utility of leisure to alter between the two periods and so they will supply the same amount of labour today and tomorrow. However, if \( w_{t+1} > w_t \) the marginal utility at time \( t \) must be less than marginal utility at time \( t+1 \)\(^2\). In other words, when the agent expects wages to rise tomorrow they take more leisure today because they expect to work harder tomorrow. In other words, consumers reallocate their leisure so that they work when wages are high and work less when wages are low. This is the intertemporal substitution hypothesis for labour supply. Notice that (6) is a theory about employment rather than unemployment.

An important implication of (6) is that consumers respond differently to permanent and temporary changes in wages. What matters in (6) is the relative real wage. If wages change

\(^2\)Strictly speaking this discussion is not strictly accurate. I am essentially treating \( E(XY) \) as equal to \( E(X)E(Y) \) which is only valid if \( X \) and \( Y \) are independent, which in this case they are clearly not. However, if not strictly accurate the intuition is fairly general and as we shall see holds for standard functional forms for the utility function.
permanently by $x\%$ then $w_{t+1}/w_t$ will not alter and labour supply does not change. However, if wages are expected to rise next period by $x\%$ but this rise is temporary then $w_{t+1}/w_t$ increases and so will next period’s labour supply. Therefore the consumer responds more to temporary wage increases than to permanent ones.

Our discussion of the ILC suggested that productivity shocks might explain business cycles in the context of a neoclassical model because explaining the positive co-movement between consumption and labour supply required shifts in the wage rate. Equation (6) says that if we are to explain sustained periods of high employment (as we observe in an upswing) then we require productivity shocks which are not permanent but which are persistent. That is we require upswings in the business cycle to be explained by an increasing sequence of positive productivity shocks so as to generate a rising real wage profile and rising employment. It is important these shocks are not permanent or else labour supply will not increase. Similarly, recessions have to be explained by a succession of larger negative productivity shocks.

Table 1 reveals little evidence of serially correlated fluctuations in wages. The model we have outlined so far consists of a labour supply curve which depends on relative wages between periods and a labour demand curve which moves around over the cycle, possibly because of productivity shocks. If this model is to explain the observed lack of movements in wages with the cyclical behaviour of employment it must be that the labour demand curve is moving along a very horizontal/elastic labour supply curve. In other words, only small changes in relative wages bring forth large movements in labour supply. That is agents are very willing to intertemporally substitute labour between periods.

Returning to (5) it is clear that according to the neoclassical model $R$ as well as relative wages determines labour supply. Assuming a constant relative real wage between time periods we can see that when $R$ is high the individual works harder this period and less in the next period. There are two reasons for this. Firstly, with a higher interest rate there is greater reward for saving and so by working hard an individual can have more to save. Secondly, the interest rate discounts future earnings. The higher the interest rate the greater this future discounting and the lower is the discounted expected wage next period.

\footnote{Using only minor amendments it is possible to avoid this dependence on serially correlated shocks to explain labour market fluctuations. The trick is to build persistence in to the model. Sargent Ch. XVIII gives an example of how to do this by incorporating costs of adjustment in the firm’s labour demand decision and habits into the utility function.}
7 Econometric evidence

As the above discussion has hopefully made clear there are several features of the neoclassical labour market which are not consistent with the “stylised facts” of Table 1. Below we shall concentrate on the empirical evidence concerning EC and EL. We do not have time for a lengthy summary but the following conclusions hold:

i) the precise implications of EC and EL are strongly rejected

ii) there is some evidence in favour of intertemporal substitution in both consumption and leisure. However in both case the estimated intertemporal effects are very small and not large enough to provide a model of the business cycle

iii) while the exact restrictions of EC are rejected the model scores some successes. Relative to EL, EC performs reasonably well. The rejections of the full EL model are many and convincing. Therefore the neoclassical model does not offer a good description of the labour market.

7.1 Consumption

The first paper to examine EC was Hall (1978). He focused on utility functions which were well approximated by quadratic utility functions (i.e. those that satisfied certainty equivalence) and assumed a constant interest rate which satisfies $\beta R = 1$. The result of this model is that consumption changes should be unpredictable. This paper sparked off one of the largest literatures in applied econometrics. Hall found that consumption growth was unpredicted by income growth but could be forecast by stock market prices. He interpreted this as a mild victory for the EC model. Subsequent work has been less kind to the model and has found that consumption growth does display a small but significant dependence on past income growth. However, the overall prediction that agents try and smooth consumption over the business cycle is partly correct (see Table 1).

A number of authors have also examined EC allowing for the interest rate to be variable. If you assume that $U(c, l) = (1/(1-\sigma))e^{1-\sigma} + v(l)$, so the utility function is additively separable between consumption and leisure then under certain further distributional assumptions EC can be written as

$$\Delta \ln c_{t+1} = \alpha + \frac{1}{\sigma} \ln R_{t+1} + \epsilon_{t+1} \quad(7)$$

The coefficient $1/\sigma$ is called the intertemporal elasticity of substitution (if the utility function is of the expected utility form $\sigma$ is the coefficient of relative risk aversion) and determines
how much consumption responds to changes in interest rates. There is widespread consensus that $1/\sigma$ is small, in the order of 0.1 to 0.2, in other words intertemporal effects exist but are not very strong for consumption.

Equation (7) implies that the cyclical pattern of consumption growth is the same as that for rates of return. This implication of (7) is strongly rejected in the data, with interest rates tending to be counter-cyclical, i.e. highest in recessions, and consumption growth pro-cyclical. In addition, as commented earlier, other variables aside from interest rates, most notably income growth, seem to affect consumption growth contrary to (7).

Therefore, while some features of EC are consistent with the data (i.e. consumption smoothing, some intertemporal effect) the exact implications are rejected. While consumption is smooth it is not smooth enough and the predictable part of consumption growth is not tied to interest rates.

7.2 Labour supply

As commented earlier, by and large EL performs poorly when tested. Firstly, on the subject of the degree to which agents are prepared to switch leisure between time periods in response to relative wage movements. Altonji Review of Economic Studies 1982 shows that estimates of the crucial intertemporal elasticity are small (i.e. the labour supply curve is not horizontal but instead nearly vertical). Estimates of the intertemporal elasticity of substitution of leisure for men are in the range 0-0.5, far too small to explain business cycles. Estimates for females are significantly higher, in the range 0 to 14 according to some studies but overall these studies offer no suggestions that the labour supply curve is elastic with respect to relative wages. The validity of this criticism has been accepted by neoclassical macroeconomist. In the later lectures on unemployment you will see some popular neoclassical models that guarantee a flat labour supply curve regardless of the steepness of individual’s labour supply curves. As commented earlier another approach is to deny that observed real wages correspond with the marginal product of labour and instead are the result of an insurance contract between firms and workers. However, the finding of low intertemporal elasticity of substitution for leisure is a serious blow for the neoclassical model of employment fluctuations.

A number of studies have also examined whether interest rate movements influence labour supply decisions. Once more this is an area where the econometrics literature is not favourable to the neoclassical model. The vast majority of studies find no significant interest rate effects on employment.
EL is also rejected because other variables are found to influence the determination of labour supply, aside from wages and interest rates. For instance Hall *Carnegie Rochester Conference Series* 1980 while finding some evidence in favour of intertemporal substitution of leisure also finds that money supply shocks predict future employment, contradicting EL. It is clear that the intertemporal substitution argument misses a good deal of what actually drives employment over the business cycle.

A useful reference for tests of EL, ILC and EC is Mankiw, Rotemberg and Summers *Quarterly Journal of Economics* 1985. They perform a variety of tests using a variety of datasets of all three implications and find no evidence in favour of the model.

8 Why does the model fail?

The key model failures are:

1. the procyclicality of both consumption and employment while real wages are acyclical
2. the zero correlation of real wages and employment fluctuations
3. low estimates of the key intertemporal elasticity of substitution for leisure
4. the absence of large transitory real wage shocks
5. the lack of interest rate effects on labour supply
6. the predictability of consumption growth

However, all the models we have outlined above have been very simple. We have often assumed that the utility function is separable between consumption and leisure intratemporally. We have also assumed that the utility function is intertemporally separable so that current utility depends only on current leisure and consumption. If we believe habits are an important aspect of people’s behaviour then what matters for current utility is not just current leisure but also leisure over the past. This has the potential to seriously improve the empirical performance of the model.

Mankiw, Rotemberg and Summers (1985) examine whether the utility function is intertemporally separable between consumption and leisure. They find they cannot reject the hypothesis of separability, but even in models which do not impose this separability EC, EL and ILC fail Eichenbaum, Hansen and Singleton *Quarterly Journal of Economics* 1988 examine
whether the assumption of intertemporal non-separability can explain rejection of the model. 
they find that while this improves the performance of the models it does not rescue the theory, 
which is still rejected.

The two most popular explanations for why the consumption model fails are (i) liquidity 
constraints, which prevent agents from borrowing so as to smooth consumption in bad income 
states (ii) precautionary saving, when agents worry about future consumption growth can 
depend on many other things other than rates of return. Campbell and Cochrane Journal of 
Political Economy 1999 argue that the rejection is due to habits.

But why does the neoclassical labour market model fail? This is a substantial agenda for 
the neoclassical macroeconomic paradigm. A good summary of the situation can be found in 
the textbook of Blanchard and Fisher (these are most definitely not neoclassical economists!) 
Lectures on Macroeconomic p. 346.

“it appears that the view of employment and output fluctuations as being roughly 
consistent with the representative agent-competitive market-technology shock model 
is difficult to hold. This class of model must be extended in some fundamental 
way if it to explain the basic characteristics of aggregate fluctuations in output and 
unemployment”

In the next lecture we shall focus on Real Business Cycles. These are models which add to 
this lecture by including capital in the model and serially correlated productivity shocks. As 
we shall see this model performs reasonably well when it comes to explaining consumption, 
savings and investment. However, not surprisingly given this lecture, it performs very badly 
at explaining the labour market. Later lectures in the course consider a number of attempts 
to remedy this modelling failure of the labour market within a neoclassical paradigm. As we 
shall see all these models find that the crucial step in getting better explanations of the labour 
market is to separate out the decision of whether to work or not from the decision of how many 
hours to work. The model we have looked at in this section draws no distinction between the 
number of people employed and the number of hours they work. It is widely believed that this 
is a crucial distinction for explaining business cycle movements in the labour market. Another 
approach is to try and explain the zero correlation between wages and employment by arguing 
that both labour supply and labour demand curve move over the business cycle. As a result 
employment moves without any fluctuations in real wages.