1 Overview

Real Business Cycle (RBC) analysis has been very controversial but also extremely influential. As is often the case with the neoclassical program it is important to discriminate between methodological innovations and economic theories. The RBC program instigated by Prescott has been controversial for three reasons (i) reliance on productivity shocks to explain the business cycle (ii) use of competitive equilibrium models which satisfy the conditions of the Fundamental Welfare Theorems implying business cycles are optimal (iii) the eschewing of econometrics in favour of calibration. Another key feature is the use of computer simulations to assess theoretical models. It is now more than 25 years since the seminal RBC paper of Kydland and Prescott. This paper seems to have had three long run impacts: i) a reassessment of the relative roles of supply and demand shocks in causing business cycles ii) widespread use of computer simulations to assess macroeconomic models iii) widespread use of non-econometric tools to assess the success of a theory. The RBC program is still a very active research area but current models are far more sophisticated in their market structure and while they still have an important role for productivity shocks, additional sources of uncertainty are allowed.

2 Key readings

The essential readings for this lecture are Chapter 5 of Romer and Chapter 2 of DeJong with Dave. The granddaddy of the RBC literature is Kydland and Prescott “Time to build and aggregate fluctuations” *Econometrica* 1982. However, it is a difficult read and a better reference is Prescott “Theory ahead of business cycle measurement” in *Carnegie Rochester Conference Series* 1986. This paper is also reproduced in Miller (ed) *The Rational Expectations*
Revolution 1994 as well as a very entertaining and insightful debate between Summers and Prescott, which is highly recommended. Campbell “Inspecting the mechanism: An analytical approach to the stochastic growth model” Journal of Monetary Economics 1994 is useful in using analytical expressions rather than computer simulations to illustrate the properties and failures of the RBC model. The best explanation of log-linearisation and eigenvalue-eigenvector decomposition in a macroeconomic context is “Production, Growth and Business Cycles” by King, Plosser and Rebelo in Computational Economics 2002. Our example will be a version of their basic neoclassical model.

3 Other reading


4 Key concepts

Productivity Shocks/Solow Residuals, Calibration, Hodrick-Prescott filter, Blanchard-Kahn decomposition

5 Empirical facts

Table 1 outlines some stylised facts of the US business cycle. We have looked at most of this Table before in the previous lecture, the only innovation is we now include investment expenditure. The most striking feature of Table 1 is how volatile investment is relative to GNP. Clearly investment is a significant contributor to business cycle volatility. As is to be expected investment is strongly procyclical.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sd%</th>
<th>Cross-correlation of output with:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>t-4</td>
</tr>
<tr>
<td>Y</td>
<td>1.72</td>
<td>0.16</td>
</tr>
<tr>
<td>C</td>
<td>0.86</td>
<td>0.40</td>
</tr>
<tr>
<td>I</td>
<td>8.24</td>
<td>0.19</td>
</tr>
<tr>
<td>H</td>
<td>1.59</td>
<td>0.09</td>
</tr>
<tr>
<td>Y/H</td>
<td>0.90</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Y is GDP, C is non-durable consumption, I is investment, H is total hours worked, Y/H is productivity. All calculations use only the cyclical component of each data. The first column quotes the standard deviation of each variable, the remaining columns show how each variable is correlated with GDP at time \( \tau \).

The RBC literature sets itself the task of trying to explain the observations in Table 1. In other words, the validity of a theory is assess by its ability to mimic the observed cyclical variability of numerous variables and the relative co-movements. We shall discuss whether this is a meaningful test of a theory later, for now we shall just post a warning. The RBC literature always focuses on the cyclical components of variables. To do this the data needs to be detrended and this involves making a decision about what the trend looks like. As we never observe the trend this is clearly controversial. The RBC literature, and many other people now, uses the Hodrick-Prescott filter. There is a wide literature which shows that the Hodrick-Prescott filter is probably seriously misleading. There is also a number of papers which show that (i) stylised facts such as are shown in Table 1 are often not robust to changes in detrending techniques (ii) different detrending techniques arrive at different results regarding the validity of different theoretical models.

The traditional view of macroeconomics has been that business cycle fluctuations, such as in Table 1, need to be explained by a different theory from that which explains economic growth or the trend. The starting point of RBC analysis is that this is incorrect. Instead they argue that the same model should be used to explain both the trend and cyclical nature of
the economy. Therefore they experiment to see if a slightly modified version of the stochastic growth model can explain the business cycle. This represents a significant move from previous models as (a) the model explains trend and cycle behaviour simultaneously (b) business cycles are caused by real rather than nominal phenomena.

To understand how radical and controversial the implications of RBC analysis are consider the following quote from Prescott (1986):

“Economic theory implies that, given the nature of the shocks to technology and people’s willingness to intertemporally and intratemporally substitute, the economy will display fluctuations like those the US economy displays ... In other words, theory predicts what is observed. Indeed, if the economy did not display the business cycle phenomenon, there would be a puzzle.”

The response to these claims has been equally vigorous, as the following quote from Summers comments on Prescott’s (1986) paper reveal:

“[RBC] theories deny propositions thought self-evident by many macroeconomists ... if these theories are correct, they imply that the macroeconomics developed in the wake of the Keynesian Revolution is well confined to the ashbin of history and they suggest that most of the work of contemporary macroeconomists is worth little more than that of those pursuing astrological science ... my view is that RBC models ... have nothing to do with the business cycle phenomenon observed in the US.”

As these reveal, by firmly grasping the full implications of standard neoclassical models the RBC literature has uncovered the faultlines of macroeconomics debate.

6 The basic model

Like all neoclassical models the starting point of RBC models is to specify the preferences and technology which characterise the model. The basic specification is:

\[
\begin{align*}
y_t &= A_t f(k_t, l_t) \\
k_{t+1} &= (1 - \delta)k_t + y_t - c_t \\
U(c_t, 1 - l_t)
\end{align*}
\]
where $A_t$ is a random productivity shock, $1 = T$ is the number of hours available in the period, $l_t$ is time spent working, $1 - l_t$ is time spent as leisure and $\delta$ is the depreciation rate. This model is essentially the Ramsey growth model except for (i) the random productivity shock (ii) consumers maximise utility by choosing consumption and leisure.

Basically, the RBC model as set out uses competitive markets, capital accumulation/consumption smoothing and intertemporal substitution as the propagation mechanism for business cycles. It uses as an impulse random productivity shocks, $A_t$.

In Lecture 1 we outlined the consumer’s first order conditions that arise from this model. Because this RBC model fulfils the conditions of the second welfare theorem it can be solved using the social planners problem and so prices are not involved. However, it will be useful, to complement Lecture 1, to outline how the firm responds to market prices in choosing how much capital and labour to select each period. It can easily be shown that maximising a firm’s profit leads to the following first order conditions:

$$\frac{\partial y}{\partial r} = r_t + \delta$$

$$\frac{\partial y}{\partial w} = w_t$$

so that the marginal product of capital is set equal to the real interest rate plus the depreciation rate and the marginal product of labour equals the real wage. These equations tie down investment and labour demand. Under standard assumptions on the production function we have that increases in $r_t$ reduce demand for capital and increases in $w_t$ reduce

1. every equilibrium of the competitive market is socially optimal
2. every socially optimal allocation can be supported by a competitive economy subject to an appropriate
distribution of resources.

The second welfare theorem implies that if we wish to study a competitive economy we do not need to consider each individual’s first order conditions and how this is then translated into an equilibrium sequence. Instead we can move straight to the first order conditions of the social planner. Even for the relatively simple economic model we have here this is a significant simplification. Solving economic models by appealing to the social planner’s problem makes it relatively easy to solve and analyse many models which would otherwise be intractable. It is partly for this reason that neoclassical models (which invariably satisfy the second welfare theorem) are better understood and articulated than Keynesian models. The latter involve many departures from the welfare theorems and as a result it is far harder to characterise the equilibrium properties of such models.
the demand for labour. Combining these with EC and EL for the consumer maximisation problem (see Lecture 1) gives a complete description of the workings of the economy.

Unfortunately for realistic assumptions on utility and production it is impossible to write down analytic solutions to the model. As a consequence models are analysed by using computer simulations. In other words, a laboratory model is set up and experimented with. However, to use computer simulations assumptions need to be made for key parameters such as the intertemporal elasticity of substitution, etc. This process of selecting parameter values for simulations and then using the output form simulations to evaluate the plausibility of a theoretical model is dubbed “calibration” and is a major methodological innovation of the RBC literature. Prescott, in a characteristically controversial manner, has argued that calibration should replace econometrics as the main tool of macroeconomics. While this particular debate is subsiding (with econometrics the victor) what is certainly the case is that calibration is rapidly becoming a standard means of evaluating the implications of theoretical models.

7 Calibration

The RBC view is to use observations from micro datasets and also from long run growth data to pin down the key parameters of the model, e.g. depreciation rates, capital share, intertemporal elasticities. In this way you are using non-business cycle studies to try and explain the business cycle.

One obviously important aspect of calibration is to arrive at a measure of $A_t$, the productivity shock. This is calculated in the following manner. Assume that $f(\cdot)$ the production function is of the Cobb-Douglas form so that:

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$$

$$\ln y_t - \alpha \ln k_t - (1-\alpha) \ln l_t = \ln A_t$$

Published data are available on $y, k$ and $l$ and assuming factors are paid their marginal products it can easily be shown that $\alpha$ is the share of capital income in output. Therefore it is possible to construct an estimate of $A_t$. Using US data gives:

$$\ln A_{t+1} = 0.95 \ln A_t + \varepsilon_{t+1}$$

$$\sigma_\varepsilon = 0.009$$

\footnote{McCallum shows that under strong assumptions an analytical solution is possible while Campbell uses approximations to solve the model. Both references are listed in the Key Readings section.}
so that productivity shocks (sometimes called the Solow residual) are highly persistent (some estimates suggest a value of 1 rather than 0.95). \( \sigma \) denotes the standard deviation of innovations to \( \ln A_t \) (these innovations are called the Solow residual) and obviously the Solow residual is very volatile. Very similar values hold for UK data as well. Therefore the RBC literature is trying to explain highly volatile and persistent business cycles by a high volatile and persistent impulse.

8 Bayesian estimation

The modern approach to calibration goes under the name of Bayesian estimation. The method argues that estimation should balance some prior information of the calibration type with actual data on the dynamics observed in the economy. For example, we may consider that the share of capital income in output \( \alpha \) is constant and close to 1/3, but we may be prepared to also entertain values a little bit away from 1/3 if that would help fit the data substantially. To capture the belief that \( \alpha \) should be close but not necessarily equal to some specific value, Bayesian estimation uses the idea of a prior. This is the distribution of \( \alpha \) that the econometrician has in mind before observing time series data from the economy. It could come from engineering studies or long run properties of the economy in just the same way as calibration sets some parameters. A typical prior distribution for \( \alpha \) might look something like the following.

\[
 f(\alpha)
\]

Note that the probability distribution \( f(\alpha) \) has the greatest mass at 1/3 - we say that the prior is centred on 1/3. We can change the standard deviation of the prior to reflect how confident we are about the distribution of \( \alpha \) before seeing the time series data. This is known as the tightness of the prior. The tighter the prior the more the econometrician believes they
already know the distribution of $\alpha$. In the limit when the prior is infinitely tight we get back to a pure calibration exercise.

How then to combine the prior with time series data? This is where the Bayesian bit comes in as we need to apply Bayes rule. If we denote the data as $y = \{y_t\}$ with distribution $f(y)$ then what we are interested in is $f(\alpha | y)$. A simple application of Bayes rule defines

$$f(\alpha | y) = \frac{f(\alpha \cap y)}{f(y)} = \frac{f(y | \alpha) f(\alpha)}{f(y)}$$

$f(\alpha | y)$ is known as the posterior, and from the equation above it is proportional to $f(y | \alpha) f(\alpha)$ where $f(y | \alpha)$ is the likelihood of the data. The Bayesian method selects $\hat{\alpha}$ as the value of $\alpha$ that maximises the posterior, i.e. the likelihood multiplied by the prior. If the prior is very tight then the maximum of the posterior will be close to that of the prior - a tight prior is dogmatic and does not “let the data speak” very much. In contrast, when the prior is diffuse the posterior is close to the likelihood so the data speaks a lot and estimation is close to classical maximum likelihood. An intermediate example is shown below, where data is given some weight so the posterior does differ from the prior to some extent.

9 Solving the model

The emphasis that RBC places on simulations has proved another source of methodological innovation. Much work has been spent developing fast and efficient numerical techniques with which to solve models. As commented earlier, RBC models are sufficiently complex that
they do not allow an analytical solution and so have to be solved numerically. As numerical
techniques develop it is possible to analyse more complicated models. By far the most common
approach to solving economic models is to use quadratic approximations. The first step here is
to work out the steady state of all variables. The model is then analysed in terms of deviations
from these steady state variables, that is you solve for how far each variable is away from its
steady state. The result is a series of linear equations linking all the endogenous variables, i.e.
output, consumption, with all the predetermined variables (the lagged endogenous variables
and current period shocks). Because the model is converted into linear form it is very easy
to solve and very fast. However, while this approach is computationally convenient it does
make some sacrifices. The result is only an approximation. The more linear is the original
model then the better the resulting approximation. However, many basic macromodels are
highly non-linear - particularly those with high risk aversion. In this case solutions based on
quadratic approximations would be misleading.

9.1 Approximation

By far the most common approach to solving decentralised economies is to take log-linear
approximations around the steady state and then solve the resulting linear expressions to
arrive at AR processes for the various endogenous variables (see King, Plosser and Rebelo
(2002) or DeJong with Dave Chapter 2). This approach has four main steps:

1. Calculate the steady state.

2. Derive analytical expressions for the approximation around the steady state.

3. Feed in the model parameter values.

4. Solve for the decision rules linking endogenous variables with predetermined and exoge-
nous variables.

The main reason why this approach is so common is it relative cheapness - the approxi-
mation leads to linear expressions for which there is a plentiful supply of cheap solution tech-
niques available. The main cost comes in deriving analytical expressions for the approximation,
whereas the actual computing time is reasonably trivial, which is a major gain compared to
all other solution techniques. Naturally, this computation cheapness comes at a cost. Firstly,
the model takes an approximation around the steady state. If the underlying model is fairly
log-linear then this approximation will be a good one. However, the more non log-linear the model the worse the approximation and the more misleading the resulting simulations will be. For many of the simple models that academics examine (such as the stochastic growth model with only one source of uncertainty) this is unlikely to be a problem. However, as the size of the model increases and as risk aversion and volatility become more important these log-linear approximations become increasingly unreliable. Secondly, this approach only works if it is possible to solve for the steady state. For some models, a unique steady state may not exist. In spite of these drawbacks, it would be fair to say that this approach is most prevalent in the literature.

9.2 First order conditions

To illustrate the technique of log-linearisation and eigenvalue-eigenvector decomposition, we assume the utility function has the form quasi-linear

$$U(c_t, 1 - l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi l_t$$

and solve the basic model in Section 6. The equilibrium in this economy is Pareto optimal so, by the second fundamental welfare theorem, the social planner’s solution and the decentralised equilibrium coincide. The social planner solves the following maximisation problem:

$$\max_{\{c_t, l_t\}} \sum_{s=0}^{\infty} \beta^s \left( \frac{c_{t+s}^{1-\sigma}}{1-\sigma} - \chi l_{t+s} \right)$$

s.t.

$$c_{t+s} + k_{t+s+1} = A_{t+s}k_{t+s}^{a}l_{t+s}^{1-a} + (1 - \delta)k_{t+s}$$

$$\ln A_{t+s+1} = \rho \ln A_{t+s} + \epsilon_{t+s+1}$$

The log of the stochastic term $A_{t+s}$ follows an AR(1) process with persistence parameter $\rho$. $\beta$ is the discount factor, $\delta$ is the depreciation rate and $\sigma$ is the coefficient of relative risk aversion. $\chi$ measures the disutility of working $l_{t+s}$. We want to solve this model, by which we mean we wish to calculate sequences for consumption, output, capital and labour which represent the equilibrium of the economy as it unfolds over time. The first order conditions for this model are:

$$c_t^{-\sigma} = E_t\beta \left[ c_{t+1}^{-\sigma} (\alpha A_{t+1}k_{t+1}^{a-1}l_{t+1}^{1-a} + 1 - \delta) \right]$$

$$\chi = c_t^{-\sigma} (1 - \alpha) A_t k_t^{a} l_t^{1-a}$$
9.3 Steady-state calculation

In steady state, consumption, output, capital and labour are all constant. The logarithm of the technology term $A_{t+s}$ is zero in steady state so $\bar{A}$ itself is unity. In terms of steady-state values $\bar{c}$, $\bar{k}$ and $\bar{l}$, the budget constraint and first order conditions are:

\[ \bar{c} + \bar{k} = \bar{k}^{1-\alpha} + (1 - \delta) \bar{k} \]
\[ 1 = \beta [\alpha \bar{k}^{\alpha-1} \bar{l}^{-\alpha} + 1 - \delta] \]
\[ \chi = \bar{c}^{-\sigma} (1 - \alpha) \bar{k}^\alpha \bar{l}^{-\alpha} \]

Solving for $\bar{c}$, $\bar{k}$ and $\bar{l}$, and adding $\bar{y}$ from the production function:

\[ \left( \frac{\bar{k}}{\bar{l}} \right) = \left( \frac{1 - (1 - \delta) \beta}{\alpha \beta} \right)^{\frac{1}{\alpha - 1}} \]
\[ \bar{c} = \left( \frac{1 - \alpha}{\chi} \right)^{\frac{1}{\sigma}} \left( \frac{\bar{k}}{\bar{l}} \right)^{\frac{1}{\alpha - 1}} \]
\[ \bar{k} = \bar{c} \left( \frac{\bar{k}}{\bar{l}} \right)^{\alpha - 1} \]
\[ \bar{l} = \bar{k} \left( \frac{\bar{k}}{\bar{l}} \right)^{-1} \]
\[ \bar{y} = \bar{k}^{\alpha \bar{l} - \alpha} \]

9.4 Log-linearisation

The budget constraint and first order conditions are both non-linear so we proceed with a log-linear approximation. The basic idea is to rewrite the equations in terms of variables that define how much a variable is deviating from its steady-state value. To aid exposition, we introduce the hat notation:

\[ \hat{x}_t = \frac{x_t - \bar{x}}{\bar{x}} \approx \ln x_t - \ln \bar{x} \]

In this case, rather than saying $x_t$ is 12 and $\bar{x}$ is 10, we refer to $\hat{x}_t$ as 0.2, meaning that it is 20% above its steady-state value. To transpose the Euler equation for consumption into hat notation, we first take logs:

\[ -\sigma \ln c_t = \ln \beta + E_t [ -\sigma \ln c_{t+1} + \ln(\alpha A_{t+1} k_{t+1}^{\alpha-1} l_{t+1}^{-\alpha} + 1 - \delta)] \]

Notice that already at this stage we have performed a trick by taking the expectations operator outside the logarithmic operator. In other words, we replace $\ln E(AB)$ with $E \ln A + E \ln B$
This is of course not strictly correct but is a necessary part of the approximation process. The left hand side and first two terms of the right hand side of the first order condition (1) will be easy to deal with. More problematic is the third term of the right hand side, which is a complex function of three variables, $A_t$, $k_t$ and $l_t$. To deal with this, we take a first order Taylor approximation of $\ln f(x, y, z)$ around $\ln f(\bar{x}, \bar{y}, \bar{z})$:

$$
\ln f(x, y, z) \approx \ln f(\bar{x}, \bar{y}, \bar{z}) + \frac{f_x(x, y, z)|_{x, y, z}}{f(\bar{x}, \bar{y}, \bar{z})}(x - \bar{x}) + \frac{f_y(x, y, z)|_{x, y, z}}{f(\bar{x}, \bar{y}, \bar{z})}(y - \bar{y}) + \frac{f_z(x, y, z)|_{x, y, z}}{f(\bar{x}, \bar{y}, \bar{z})}(z - \bar{z})
$$

Applying this to the third term in the right hand side of (1), we obtain:

$$
\ln(\alpha A_{t+1} k_t^{\alpha-1} l_t^{-\alpha} + 1 - \delta) \approx \ln(\alpha \bar{k}^{\alpha-1} \bar{l}^{-\alpha} + 1 - \delta) + \frac{\alpha \bar{k}^{\alpha-2} \bar{l}^{-\alpha}}{\alpha \bar{k}^{\alpha-1} \bar{l}^{-\alpha} + 1 - \delta} (A_{t+1} - \bar{A}) + \frac{\alpha(1 - \delta) \bar{k}^{\alpha-1} \bar{l}^{-\alpha}}{\alpha \bar{k}^{\alpha-1} \bar{l}^{-\alpha} + 1 - \delta} (l_{t+1} - \bar{l})
$$

The expression can be simplified by recognising in steady state, $\alpha \bar{k}^{\alpha-1} \bar{l}^{-\alpha} + 1 - \delta = \beta^{-1}$, $\alpha \bar{k}^{\alpha-1} \bar{l}^{-\alpha} = \beta^{-1} - (1 - \delta)$ and $\bar{A} = 1$.

$$
\ln(\alpha A_{t+1} k_t^{\alpha-1} l_t^{-\alpha} + 1 - \delta) \approx -\ln \beta + (1 - (1 - \delta)\beta) \hat{A}_{t+1} + (\alpha - 1)(1 - (1 - \delta)\beta) \hat{k}_{t+1} + (1 - \alpha)(1 - (1 - \delta)\beta) \hat{l}_{t+1}
$$

Notice again that this only holds to a first order approximation. There will inevitably be some loss of accuracy compared to the exact solution. Returning to condition (1), we write

$$
-\sigma \ln c_t = E_t \left[ -\sigma \ln c_{t+1} + (1 - (1 - \delta)\beta) \hat{A}_{t+1} + (\alpha - 1)(1 - (1 - \delta)\beta) \hat{k}_{t+1} + (1 - \alpha)(1 - (1 - \delta)\beta) \hat{l}_{t+1} \right]
$$

Adding $\sigma \ln \bar{c}$ to each side and writing $\ln c_t - \ln \bar{c} = \hat{c}_t$ gives the final form:

$$
-\sigma \hat{c}_t = E_t \left[ -\sigma \hat{c}_{t+1} + (1 - (1 - \delta)\beta) \hat{A}_{t+1} + (\alpha - 1)(1 - (1 - \delta)\beta) \hat{k}_{t+1} + (1 - \alpha)(1 - (1 - \delta)\beta) \hat{l}_{t+1} \right]
$$

A similar process can be used to log-linearise the second first order condition, the budget constraint and the law of motion for technology.

$$
\sigma \hat{c}_t = \hat{A}_t + \alpha \hat{k}_t - \alpha \hat{l}_t
$$

$$
\bar{c} \hat{c}_t + \bar{k} \hat{k}_{t+1} = (\alpha \bar{y} + (1 - \delta) \bar{k}) \hat{k}_t + (1 - \alpha) \bar{y} \hat{l}_t + \bar{y} \hat{A}_t
$$

$$
\hat{A}_{t+1} = \rho \hat{A}_t + \epsilon_{t+1}
$$
9.5 State space form

It is convenient to take expectations and express the four equations of the model (first order conditions, budget constraint and law of motion for technology) in matrix form.

\[
E_t \begin{bmatrix}
(1 - (1 - \delta)\beta) & (\alpha - 1)(1 - (1 - \delta)\beta) & (1 - \alpha)(1 - (1 - \delta)\beta) & -\sigma
\end{bmatrix} \begin{bmatrix}
\hat{A}_{t+1} \\
\hat{k}_{t+1} \\
\hat{l}_{t+1} \\
\hat{c}_{t+1}
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & -\sigma \\
\bar{y} & (\alpha\bar{y} + (1 - \delta)\bar{k}) & (1 - \alpha)\bar{y} & -\bar{c} \\
\rho & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\hat{A}_t \\
\bar{k}_t \\
\hat{l}_t \\
\bar{c}_t
\end{bmatrix}
\]

More succinctly, \( E_t Ax_{t+1} = Bx_t \). We will use state space forms in the rest of the lecture. What is required is to find the solution of \( E_t Ax_{t+1} = Bx_t \). Assuming \( B \) is invertible, we can pre-multiply each side of the equation by \( B^{-1} \) to obtain

\[
E_t CX_{t+1} = X_t
\]

where \( C = B^{-1}A \). Our technique does require that the matrix \( B \) is non-singular. However, other equally simple techniques such as Klein (2000), Sims (2000) and Söderlind (1999) exist for models where \( B \) is singular.

9.6 Eigenvalue-eigenvector decomposition

The technique suggested by Blanchard and Kahn solves the system \( E_t CX_{t+1} = X_t \) by decomposing the matrix into its eigenvalues and eigenvectors. Other techniques exist which do the job equally well, most notably the method of undetermined coefficients, which is the basis of Harald Uhlig’s toolkit for analysing nonlinear economic dynamic models “easily” (see http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm). The Blanchard-Kahn algorithm begins by partitioning the variables in \( X_t \) into predetermined and exogenous variables \( w_t \) and controls \( y_t \). In our model, \( w_t = (\hat{A}_t, \hat{k}_t) \) since technology is exogenous and the capital stock is predetermined. The control variables are consumption and labour supply so \( y_t = (\hat{l}_t, \hat{c}_t) \). With the variables partitioned, we have

\[
E_t C \begin{bmatrix}
w_{t+1} \\
y_{t+1}
\end{bmatrix} = \begin{bmatrix}
w_t \\
y_t
\end{bmatrix}
\]

(2)

13
The heart of the Blanchard-Kahn approach is the Jordan decomposition of the matrix \( C \). Under quite general conditions, \( C \) is diagonalisable and we can write

\[
C = P^{-1} \Lambda P
\]

In this Jordan canonical form, \( \Lambda \) is a diagonal matrix with the eigenvalues of \( C \) along its leading diagonal and zeros in the off-diagonal elements. \( P \) is the inverse matrix of the corresponding eigenvectors. In order to continue, we need the number of unstable eigenvalues (i.e., of modulus less than one) to be exactly equal to the number of controls. This is known as the Blanchard-Kahn condition. In a two-dimensional model (such as the Ramsey growth model) with one predetermined variable and one control, it is equivalent to requiring one stable root and one unstable root to guarantee saddle path stability in the phase diagram. If there are too many unstable roots than the system is explosive and we run into problems with the transversality conditions. If there are too few unstable roots then the system is super stable, which means there will be indeterminacy. Techniques do exist for handling models with indeterminacy, see “The Macroeconomics of Self-Fulfilling Prophecies” by Roger Farmer, MIT Press, 1993, but we restrict our attention here to models that satisfy the Blanchard-Kahn conditions.

We progress by partitioning the matrix of eigenvalues \( \Lambda \). \( \Lambda_1 \) contains the stable eigenvalues (of number equal to the number of predetermined and exogenous variables) and \( \Lambda_2 \) the unstable eigenvalues (of number equal to the number of controls). The matrix is similarly partitioned.

\[
\Lambda = \begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix}, \quad P = \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix}
\]

Using this partition and pre-multiplying each side by \( P \), equation (2) becomes

\[
E_t \begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix} \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix} \begin{bmatrix}
w_{t+1} \\
y_{t+1}
\end{bmatrix} = \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix} \begin{bmatrix}
w_t \\
y_t
\end{bmatrix}
\]

This is a cumbersome expression to work with so we prefer to solve a transformed problem, with

\[
\begin{bmatrix}
\tilde{w}_t \\
\tilde{y}_t
\end{bmatrix} = \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix} \begin{bmatrix}
w_t \\
y_t
\end{bmatrix}
\]

so that the equation to solve becomes

\[
E_t \begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix} \begin{bmatrix}
\tilde{w}_{t+1} \\
\tilde{y}_{t+1}
\end{bmatrix} = \begin{bmatrix}
\tilde{w}_t \\
\tilde{y}_t
\end{bmatrix}
\]
We will solve this equation for $\tilde{\varphi}$ and $E_t \tilde{\omega}_{t+1}$ and then work backwards to recover $y_t$ and $E_t w_{t+1}$. In the two dimensional case, the transformation rotates the phase diagram so that the stable eigenvector lies on the x-axis and the unstable eigenvector lies on the y-axis. The beauty of working with the transformed problem is that the two equations are now decoupled. In other words, we can write each time $t+1$ variable as solely a function of transformed predetermined variables, exogenous variables and transformed controls at time $t$.

\[
E_t \Lambda_1 \tilde{\omega}_{t+1} = \tilde{\omega}_t
\]
\[
E_t \Lambda_2 \tilde{\varphi}_{t+1} = \tilde{\varphi}_t
\]

The second equation shows the evolution of the transformed controls. Solving forward to time $t+j$ gives

\[
E_t \tilde{\varphi}_{t+j} = (\Lambda_2^{-1})^j \tilde{\varphi}_t
\]

Since $\Lambda_2$ contains the unstable eigenvalues (of modulus less than one), this is an explosive process. In this case, the only solution which satisfies the transversality conditions is $\tilde{\varphi}_t = 0 \quad \forall t$, in which case $E_t \tilde{\varphi}_{t+j} = 0$. The condition $\tilde{\varphi}_t = 0$ translates back into the original programme as

\[
0 = P_{21} w_t + P_{22} y_t
\]

We can therefore write the decision rule for the controls as

\[
y_t = -P_{22}^{-1} P_{21} w_t
\]

The reaction function defines the controls $y_t$ as a linear function of the predetermined and exogenous variables $w_t$. It is the equation of the saddle path. The linearity of the decision rule is a general feature of solution by log-linearisation.

To derive the evolution of the predetermined variables, we return to the first equation of the transformed problem:

\[
E_t \tilde{\omega}_{t+j} = (\Lambda_1^{-1})^j \tilde{\omega}_t
\]

In this case $\Lambda_1$ contains the stable eigenvalues (of modulus greater than one) and the system is stable. It already shows the expected evolution of the vector $\tilde{\omega}_t$. To return to the original problem, we recognise that

\[
\tilde{\omega}_t = P_{11} w_t + P_{12} y_t
\]
\[
= (P_{11} - P_{12} P_{22}^{-1} P_{21}) w_t
\]

15
Hence,
\[ E_t w_{t+1} = (P_{11} - P_{12}P_{22}^{-1}P_{21})^{-1} \Lambda_1^{-1}(P_{11} - P_{12}P_{22}^{-1}P_{21})w_t \]
and the evolution of the predetermined and exogenous variables is also linear.

9.7 Assessing the model

The standard way of assessing the validity of a model is to use econometrics to see whether the theory’s restrictions are consistent with data. However, Prescott argues that this is incorrect. His main argument is that almost by definition theoretical models are an abstraction of reality, in other words they will necessarily be rejected by data if tested econometrically. Instead RBC models should be assessed by seeing whether they “broadly” replicate the volatility and serial correlation properties of observed data. No metric is given of how “close” a model needs to be to the data before it is satisfactory. However, the RBC belief is that it is foolish to test the precise restrictions of an approximate theory. Instead one should just assess whether broad patterns in the data are replicated by the model. Prescott summarises this by saying that “a likelihood ratio test rejects too many good DSGE models”.

This emphasis on matching a handful of standard deviations and cross correlations is the subject of much criticism. The main criticisms are (i) why only these statistics? Is there anything optimal on focusing on this subset? (ii) how close a correspondence does there have to be between output and data to rate the model a success? In other words how do you perform inference. Both these criticisms of calibration focus on the lack of underlying statistical foundations. However, we should stress that they are criticisms of a particular approach to calibration. It is however possible to perform calibration in a more statistically rigorous fashion, see Del Negro and Schorfheide Journal of Monetary Economics 2008. Unfortunately this is seldom done.

10 Model results

Table 2 shows the results of simulating a simple stochastic growth model and then analysing the cyclical pattern implied by these simulations. To avoid undue dependence on a particular simulation results are normally quoted as an average over a large number of simulations.
Comparing the results with US data we can draw the following conclusions:

i) the stochastic growth model + productivity shocks appears capable of producing significant volatility in output, but not as much as is observed in the data.

ii) the model predicts only about half of the volatility in labour input as is observed in the data.

iii) hours and productivity move together in the simulations but not in the data.

iv) consumption is smoother in the simulations, but by too much compared with the data.

v) the relative volatility of output and investment is approximately correct.

In other words, the model seems to do reasonably well explaining investment, has a mixed record with consumption and performs poorly with respect to the labour market. Notice the model implies that all variables tend to move together. Given that the model assumes the existence of only one shock this fairly inevitable.

The basic point behind Kydland and Prescott “Time to build and aggregate fluctuations” *Econometrica* 1982 is to make some modifications to the stochastic growth model so that the model simulations match Table 1 better. They introduce three main modifications:

1. they specify the utility function so that current utility depends on all previous leisure decisions. that is the utility function is no longer intertemporally separable. Their specification is basically that what determines current utility is a weighted average of lifetime leisure rather than simply today’s leisure. As we remarked in Lecture 1 introducing non-time separable preferences is one way of improving the neoclassical approach to the labour market. The effect of making utility depend more on an average than the current
value is that today’s leisure has less of an impact on utility. As a result the consumer is more prepared to engage in intertemporal substitution. If the thing that matters is an average of previous leisure I am indifferent between working 0 hours this period and $H$ hours next period or simply working $H/2$ in both periods. In this way the model can produce more volatility in employment.

2. Kydland and Prescott assume “time to build” - or in other words, wine isn’t made in a day, there are gestation lags. Kydland and Prescott’s idea was that an investment project takes several periods between the commissioning of the project and its implementation. Further, during this period the project requires funding. The purpose of this assumption is to try and get additional persistence in fluctuations. In response to a favourable productivity shock firms invest today and over several periods so that investment remains higher for longer.

3. Kydland and Prescott assumed that inventories had a role in the production function. That is output was a function of capital, labour and inventories.

Table 3 lists the results of simulating the Kydland and Prescott economy. As can be seen the various amendments help the model to perform significantly better.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sd%</th>
<th>Cross-correlation of output with:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>t-1</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.79</td>
<td>0.60</td>
</tr>
<tr>
<td>$C$</td>
<td>0.45</td>
<td>0.47</td>
</tr>
<tr>
<td>$I$</td>
<td>5.49</td>
<td>0.52</td>
</tr>
<tr>
<td>$H$</td>
<td>1.23</td>
<td>0.52</td>
</tr>
<tr>
<td>$Y/H$</td>
<td>0.71</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Table 3: Kydland and Prescott Model Economy

However, for various reasons most subsequent work in the RBC literature has not utilised these three extensions. The time to build and inventory assumptions have been dropped because they essentially add very little in the way of volatility or persistence. This is because what matters for output is the capital stock and investment and inventories are small relative to the capital stock and so only exert a minor influence. By far the most important assumption of Kydland and Prescott was assuming the no-separability in the utility function. However,
this key assumption was poorly motivated and was very non-standard, further the weights which define the “average” measure of leisure were not defined by any independent study but simply assumed by the authors.

11 Empirical assessment


a) output growth is positively correlated with its own value over the last two quarters

b) there is evidence of significant temporary shocks which first cause a variable to increase sharply and the eventually returns to its old value. It is for this reason that shocks are called “temporary” - they must not be allowed to have a long term effect on output.

The Cogley and Nason paper is very illuminating but may be difficult to read for those of you who are not familiar with time series techniques. Basically they conclude that RBC models provide no propagation mechanism to productivity shocks. Basically RBC models are WYGIWYPI - what you get is what you put in. In other words, if you put into the neoclassical propagation mechanism a volatile and very persistent productivity shock then GNP fluctuations will also be very volatile and persistent\(^3\) But if you put in random productivity shocks then the model will provide basically random output fluctuations. Is this a problem? In other words, the neoclassical models reliance on capital accumulation as a propagation mechanism adds very little persistence.

If we could be sure that productivity shocks really were very volatile and persistent then the fact that the Kydland and Prescott model does not provide an propagation would not be a problem. The problem is there is very little independent evidence in favour of aggregate technology shocks: (i) if business cycles are caused by productivity shocks how do we explain recessions - technical regress? Is this plausible? (ii) different industries use different technologies. Why should all industries simultaneously experience a positive productivity shock? In other words, there may be no such thing as an aggregate productivity shock. (iii) are these shocks just oil price shifts? (iv) according to RBC theories the Solow residual should be exogenous. That is it should not be influenced by any other variable such as monetary policy,

\(^3\) Yet again, Cogley and Nason’s results point to the importance of modelling the labour market. They find that introducing costs of adjustment or time to build in the capital stock does not generate persistence. What matters for output is the stock of capital, not its flow. However, it is possible to generate more persistence in the model by altering the labour market specification as will be show later in the course.
government expenditure, etc. For the US there is strong evidence that this is not the case, that the Solow residual is predictable and further that it is predictable by demand variables. If this is the case then it cannot be interpreted as a pure productivity shock.

A further problem in assessing the empirical validity is the use of calibration. The big advantage of econometrics is that you can perform inference, in other words you can say with probability $x$ the data is consistent with the theory. However, as it is usually performed this is not the case for calibration. More recently some authors have shown how to do inference with calibrated models. Some authors have used Bayesian or GMM techniques, see Smets and Wouters “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area” Journal of European Economic Association 2003, while Watson “Measures of Fit For Calibrated Models” Journal of Political Economy 1993 provides a statistical way of summarising the explanatory ability of calibrated models. In all cases basic RBC models are rejected.

12 Conclusions

I. RBC versions of the stochastic growth model perform reasonably well (in a closed economy context) in explaining investment and to a lesser extent consumption. They perform very badly as far as the labour market is concerned.

II. The longer lasting impact of RBC models is not the importance of productivity shocks in business cycles (although they and related supply shocks are still given some role) but the use of calibration methods to understand model predictions and the use of simulations and associate numerical solution techniques as a way of developing theoretical models to mimic the real economy.

A A numerical example

To demonstrate the technique of eigenvalue-eigenvector decomposition in practice, we present a Matlab code to solve the stochastic growth model. We use the calibration $\beta = 0.95$, $\chi = 2$, $\alpha = 0.4$, $\sigma = 1$, and $\delta = 0.1$. The persistence parameter $\rho$ in the law of motion for technology is calibrated at 0.95, implying very high persistence. We begin by clearing the screen and defining the calibrated parameters.

```matlab
clear;clc;
beta=0.95;
```
\( \chi = 2; \)
\( \alpha = 0.4; \)
\( \sigma = 1; \)
\( \delta = 0.1; \)
\( \rho = 0.95; \)

Next, solve for steady state using

\[
\begin{align*}
\begin{pmatrix} \bar{k} \\ \bar{l} \end{pmatrix} &= \left( \frac{1 - (1 - \delta)\beta}{\alpha \beta} \right)^{\frac{1}{\alpha - 1}} \\
\bar{c} &= \left( \frac{1 - \alpha}{\chi} \right)^{\frac{1}{\beta}} \left( \frac{\bar{k}}{\bar{l}} \right)^{\frac{\alpha}{\beta}} \\
\bar{k} &= \bar{c} \left( \frac{\bar{k}}{\bar{l}} \right)^{\frac{\alpha - 1}{\beta}} - \delta \right)^{-1} \\
\bar{l} &= \bar{k} \left( \frac{\bar{k}}{\bar{l}} \right)^{\frac{\alpha - 1}{\beta}} \\
\bar{y} &= \bar{k}^\alpha \bar{l}^{1 - \alpha}
\end{align*}
\]

\[
k_{\text{bar}} = \left(1 - (1 - \delta)\beta\right) / \left(\alpha \beta\right) \right)^{1/(\alpha - 1)};
\]
\[
c_{\text{bar}} = \left(1 - \alpha\right) / \chi \right)^{1/\left(\sigma\right)} k_{\text{bar}} / \left(\alpha / \sigma\right);
\]
\[
k_{\text{bar}} = c_{\text{bar}} \left(1 / \left(\alpha / \sigma\right)\right) / \left(k_{\text{bar}} - \delta\right)^{-1};
\]
\[
l_{\text{bar}} = k_{\text{bar}} / \left(k_{\text{bar}} - \delta\right)^{-1};
\]
\[
y_{\text{bar}} = k_{\text{bar}}^\alpha l_{\text{bar}}^{1 - \alpha};
\]

The numerical values in our calibrated model are \( \bar{k} = 2.03, \bar{l} = 0.41, \bar{c} = 0.57 \) and \( \bar{y} = 0.77 \).

To write the model in state-space form, we define the matrices \( A \) and \( B \) in \( E_t A x_{t+1} = B x_t \).

\[
A = \text{zeros}(4,4);
\]
\[
B = \text{zeros}(4,4);
\]
\[
A(1,1) = 1 - (1 - \delta) \beta;
\]
\[
A(1,2) = (\alpha - 1) / \left(\alpha / \sigma\right);
\]
\[
A(1,3) = (1 - \alpha) / \left(\alpha / \sigma\right);
\]
\[
A(1,4) = -\sigma;
\]
\[ A(2,2) = \text{kbar}; \]
\[ A(3,1) = 1; \]
\[ B(1,4) = -\text{sigma}; \]
\[ B(2,1) = \text{ybar}; \]
\[ B(2,2) = \alpha \cdot \text{ybar} + (1 - \delta) \cdot \text{kbar}; \]
\[ B(2,3) = (1 - \alpha) \cdot \text{ybar}; \]
\[ B(2,4) = -\text{cbar}; \]
\[ B(3,1) = \rho; \]
\[ B(4,1) = 1; \]
\[ B(4,2) = \alpha; \]
\[ B(4,3) = -\alpha; \]
\[ B(4,4) = -\text{sigma}; \]

The numeric state-space form is

\[
E_t \begin{bmatrix} 0.145 & -0.087 & 0.087 & -1 \\ 0 & 2.0252 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{A}_{t+1} \\ \hat{k}_{t+1} \\ \hat{l}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} =
B \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0.773 & 2.132 & 0.464 & -0.570 \\ 0.95 & 0 & 0 & 0 \\ 1 & 0.4 & -0.4 & -1 \end{bmatrix} \begin{bmatrix} \hat{A}_t \\ \hat{k}_t \\ \hat{l}_t \\ \hat{c}_t \end{bmatrix}
\]

Inverting \( B \) and defining \( C = B^{-1} A \), we have

\[ C = \text{inv}(B) \cdot A; \]

\[
E_t \begin{bmatrix} 1.053 & 0 & 0 & 0 \\ -0.880 & 0.838 & -0.058 & 0.666 \\ 2.114 & 0.621 & 0.160 & -1.834 \\ -0.145 & 0.087 & -0.087 & 1 \end{bmatrix} \begin{bmatrix} \hat{A}_{t+1} \\ \hat{k}_{t+1} \\ \hat{l}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \hat{A}_t \\ \hat{k}_t \\ \hat{l}_t \\ \hat{c}_t \end{bmatrix}
\]
With the model in state-space form, we can perform the Jordan decomposition of $C$ into eigenvalues and eigenvectors. The eigenvalues are stored in the matrix $MU$, with corresponding normalised eigenvectors in the matrix $ve$.

$$[ve, MU] = eig(C);$$

The matrix $MU$ of eigenvalues has the following numerical values:

$$\Lambda = \begin{bmatrix} 1.053 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.218 & 0 \\ 0 & 0 & 0 & 0.78 \end{bmatrix}$$

In this case, we have two unstable eigenvalues (the 0.780 and 0) and two stable eigenvalues (the 1.218 and 1.053) so the Blanchard-Kahn condition is satisfied and we have saddle path stability. In general, before partitioning the $MU$ and $P$ matrices, we need to sort the eigenvalues so that the two stable eigenvalues are in the first two rows (corresponding to exogenous and predetermined variables) and the two unstable eigenvalues are in the last rows (corresponding to the controls). Even if the eigenvalues are already sorted and we have saddle path stability, it is good to utilise a more general algorithm which includes a sorting procedure and an eigenvalue stability test.

$$t = flipud(sortrows([diag(MU) ve']));$$

$$MU = diag(t(:, 1));$$

$$ve = t(:, 2:5);$$

$$P = inv(ve');$$

if $MU(2, 2) < 1$ \text{ or } $MU(3, 3) > 1$

$$error('No saddle path stability');$$

end

Partitioning $MU$ and $P$,

$$MU1 = MU(1:2, 1:2);$$

$$MU2 = MU(3:4, 3:4);$$

$$P11 = P(1:2, 1:2);$$
\[ P_{12} = P(1:2,3:4); \]
\[ P_{21} = P(3:4,1:2); \]
\[ P_{22} = P(3:4,3:4); \]

The model is now in standard Blanchard-Kahn form, i.e.

\[ \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w_t \\ y_t \end{bmatrix} \]

\[ \Lambda_1 = \begin{bmatrix} 1.218 & 0 \\ 0 & 1.053 \end{bmatrix} \]
\[ \Lambda_2 = \begin{bmatrix} 0.780 & 0 \\ 0 & 0 \end{bmatrix} \]
\[ P_{11} = \begin{bmatrix} -4.453 & 0.158 \\ 3.693 & 0 \end{bmatrix} \]
\[ P_{12} = \begin{bmatrix} -0.158 & 1.810 \\ 0 & 0 \end{bmatrix} \]
\[ P_{21} = \begin{bmatrix} -1.361 & -1.227 \\ -2.061 & -0.825 \end{bmatrix} \]
\[ P_{22} = \begin{bmatrix} -0.188 & 2.155 \\ 0.845 & 2.061 \end{bmatrix} \]

The decision rule is obtained from the formula \( y_t = -P_{22}^{-1}P_{21}w_t \) and the expected evolution of the predetermined variables from \( E_t w_{t+1} = (P_{11} - P_{12}P_{22}^{-1}P_{21})^{-1}(P_{11} - P_{12}P_{22}^{-1}P_{21})w_t \).

\[ \text{inv}(P22)*P21 \]
\[ \text{inv}(P11-P12*\text{inv}(P22)*P21)*\text{inv}(MU1)*(P11-P12*\text{inv}(P22)*P21) \]

The full solution is therefore

\[ \hat{l}_t = 0.756 \hat{A}_t - 0.347 \hat{k}_t \]
\[ \hat{c}_t = 0.698 \hat{A}_t + 0.539 \hat{k}_t \]
\[ E_t(\hat{A}_{t+1}) = 0.95 \hat{A}_t \]
\[ E_t(\hat{k}_{t+1}) = 0.358 \hat{A}_t + 0.821 \hat{k}_t \]
The decision rule for consumption shows that consumption is increasing in both the predetermined capital stock and technology, a parallel result to that obtained in the real business cycle model in the previous lecture. Consumption increases less than one-for-one with technology and the capital stock because of consumption smoothing.

B Dynare

It is a simple matter to simulate once the model has been solved into a recursive form. To obtain stylised facts comparable to those obtained from real data, we need to simulate the model in deviations from the steady state, calculate what this implies for the evolution of levels, and then apply the Hodrick-Prescott filter. This is not difficult but a little tedious. Fortunately, software now exists which automates the whole process. The best of these is Dynare, a free plug-in for Matlab that is available from http://www.dynare.org/. This user input is the first order conditions specified in a .mod file, which the software then proceeds to solve for steady-state, log-linearise and solve using a generalisation of the Blanchard-Kahn decomposition. One line commands then perform stochastic simulations, complete with Hodrick-Prescott filtering and calculation of stylised facts. It is also possible to calculate impulse response functions. The following .mod file shows how our real business cycle model would be specified and solved in Dynare:

```
% Basic RBC Model
%----------------------------------------
% 0. Housekeeping (close all graphic windows)
%----------------------------------------

close all;
%----------------------------------------
% 1. Defining variables
%----------------------------------------

var y c k l z;
```
\texttt{varexo e;}

\texttt{parameters beta chi delta alpha rho sigma;}

\%–––––––––––––––––––––-
\% 2. Calibration

\%–––––––––––––––––––––-
alpha = 0.4;
beta = 0.95;
delta = 0.1;
chi = 2;
rho = 0.95;
sigma = 1;

\%–––––––––––––––––––––-
\% 3. Model

\%–––––––––––––––––––––-
model;

c^{-sigma} = beta * c(+1)^{-sigma} * (alpha * \exp(z(+1)) * k^(alpha-1) * l(+1)^{(1-alpha)} + 1 - delta);
chi = c^{-sigma} * (1-alpha) * \exp(z) * k(-1)^{alpha} * l^{-alpha};
c+k = \exp(z) * k(-1)^{alpha} * l^{(1-alpha)} + (1-delta) * k(-1);
y = \exp(z) * k(-1)^{alpha} * l^{(1-alpha)};
z = rho * z(-1) + e;

end;

\%–––––––––––––––––––––-
\% 4. Computation

\% 26
The model file is run by typing `dynare rbc.mod` in the Matlab command window (once Dynare has been installed). The output in this case is:

`>> dynare rbc.mod`

**STEADY-STATE RESULTS:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>0.57025</td>
</tr>
<tr>
<td>k</td>
<td>2.02519</td>
</tr>
<tr>
<td>l</td>
<td>0.406542</td>
</tr>
<tr>
<td>y</td>
<td>0.772768</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
</tr>
</tbody>
</table>

**MODEL SUMMARY**
Number of variables: 5
Number of stochastic shocks: 1
Number of state variables: 2
Number of jumpers: 3
Number of static variables: 1

MATRIX OF COVARIANCE OF EXOGENOUS SHocks

Variables e

e 1.000000

Policy and transition functions

\[
\begin{align*}
y & \sim 0.772768 \ 2.025185 \ 0 \ 0.570250 \ 0.406542 \\
k(-1) & \sim 0.073109 \ 0.821355 \ 0 \ 0.151753 \ -0.069727 \\
z(-1) & \sim 1.067285 \ 0.689445 \ 0.950000 \ 0.377840 \ 0.292113 \\
e & \sim 1.123458 \ 0.725731 \ 1.000000 \ 0.397727 \ 0.307488 \\
\end{align*}
\]

THEORETICAL MOMENTS (HP filter, lambda = 1600)

VARIABLE MEAN STD. DEV. VARIANCE

\[
\begin{align*}
c & \sim 0.5702 \ 0.7580 \ 0.5745 \\
k & \sim 2.0252 \ 2.6611 \ 7.0814 \\
l & \sim 0.4065 \ 0.3800 \ 0.1444 \\
y & \sim 0.7728 \ 1.5416 \ 2.3764 \\
z & \sim 0.0000 \ 1.3034 \ 1.6990 \\
\end{align*}
\]

MATRIX OF CORRELATIONS (HP filter, lambda = 1600)

Variables c k l y z

c 1.000000 \ 0.9367 \ 0.5390 \ 0.9188 \ 0.8655

COEFFICIENTS OF AUTOCORRELATION (HP filter, lambda = 1600)

Order 1 2 3 4 5

c 0.8714 0.7062 0.5259 0.3459 0.1773
k 0.9425 0.8119 0.6404 0.4520 0.2643
l 0.6363 0.3568 0.1472 -0.0050 -0.1105
y 0.7475 0.5221 0.3263 0.1610 0.0256
z 0.7133 0.4711 0.2711 0.1098 -0.0163

>>