Chapter 4

Cash in advance model

4.1 Motivation

In this lecture we will look at ways of introducing money into a neoclassical model and how these methods can be developed in an effort to try and explain certain facts. As in previous lectures, we shall find that while we can develop models to improve our understanding of the business cycle we still remain some distance from a reliable model. While we introduce money in ways which break the neoclassical dichotomy, the models we look at are still partly neoclassical in nature. They are often referred to as monetary RBC models because they essentially keep the same structure of the real economy as RBC models but superimpose a monetary sector.

4.2 Key readings

Once again the origins of the modern literature rest with Lucas (1982) "Interest Rates and Currency Prices in a Two Country Model", *Journal of Monetary Economics*, and Lucas and Stokey (1987) "Money and Interest in a Cash in Advance Economy", *Econometrica*. However, I would recommend instead Svensson (1985) "Money and Asset Prices in a Cash in Advance Economy", *Journal of Political Economy*. Less theoretical papers (which are also easier to read) which focus on the business cycle implications of cash in advance are Cooley and Hansen (1989) "The inflation tax in a real business cycle model", *American Economic Review* and by the same authors "Money and the Business cycle" in Cooley (ed) "Frontiers of Business Cycle Research".

4.3 Introduction

The first problem with any neoclassical general equilibrium approach to business cycles when it comes to modelling monetary phenomena is how to explain why consumers need to hold money. Without a justification of why there is a demand for money it is obviously impossible to model the impact that variations in money supply will have on the economy. There are three broad approaches:

(i) Money in the Utility Function - if utility depends upon real money balances then money is like any other good and will be demanded by consumers. However, most people are reluctant to start with this assumption as it is rather *ad hoc*. It is a far better modelling strategy to try and point to a reason why money is held by consumers other than it is a direct source of utility. However, that said it is well known that under certain conditions there exists an equivalence between putting money in the utility function or specifying a transactions technology which involves money.

(ii) Cash in Advance Models - this is the route which has been most thoroughly explored in the literature. the assumption here is that before a consumer can buy goods they must pay for them in cash. Therefore money is demanded because it is the only means of purchasing some goods.

(iii) Transactions Cost (Shopping Time Technology) - in these models consumers have a choice (unlike in (ii)). They can obtain goods on credit or barter or they can purchase goods with cash. However, purchasing goods consumes resources and the more cash that an individual holds the lower these shopping costs are (e.g. they can avoid very costly bartering processes). By holding money, consumers lose any interest they would otherwise have gained on their savings but they economise on their transactions costs.

4.4 Cash in advance models

In what follows we shall focus entirely on cash in advance models. The basic cash in advance model is due to Lucas. Every period a consumer has to choose (a) their consumption (denoted c) (b) their money balances (denoted m) and (c) their savings (denoted a, assets). However, all consumption goods have to be paid for by cash so there is a constraint the consumer faces, $P_t c_t \leq m_t$. Assets deposited in the bank earn an interest rate $R > 0$ but no interest is earned on assets held in the form of money. Instead, money earns a rate of return equal to P_{t-1}/P_t, so if there is inflation money earns a negative return (it loses value). Consumers choose their consumption, assets and money balances once they observe the state of the world (i.e. after seeing what today's money supply growth is, what the value of the current productivity shock is, etc.) Because consumers earn interest on deposits but not on money they will always prefer to keep assets on deposit. Therefore they will hold only just enough cash to finance their consumption, e.g. $P_t c_t = m_t$. This has a rather unfortunate consequence that the velocity of money is constant. The velocity of money (V) is defined by the identity $MV = PY$, where M is the money supply, P is the price level and Y is the volume of transactions in the economy. Assuming no capital, the volume of transactions in this economy is just c, and because $m = Pc$ it must be that the velocity of money is always equal to 1. In reality, the velocity of money shows considerable variation and depends in particular on the interest rate. These are features which the basic cash in advance (CIA) model cannot account for.

Svensson (1985) proposes a simple amendment to Lucas' basic model. Like Lucas' article, Svensson's main concern is how to price assets when you have a cash in advance constraint. Svensson assumes that consumers have to choose how much cash to hold before they know the current state of the world (i.e. they are ignorant of the current money supply or productivity shock). As a result of this uncertainty the velocity of money is no longer constant. Agents will usually choose to hold $m > Pc$ for precautionary reasons. In a very good state of the world, agents know they would like their consumption to be high and they can only achieve this if they have high money balances. Therefore, agents tend to hold more money than they otherwise would need as a precaution in case they find themselves wanting to consume large amounts in a surprisingly good state of the world. The greater the uncertainty facing the consumer (e.g. the higher the probability of wanting to spend a lot on average) the larger these precautionary balances. However, the higher is the interest rate the lower the level of precautionary balances held by the consumer. Consumers have to trade the benefits of higher money balances (increased insurance against a good state of the world) against the costs (loss of interest). As a result the velocity of money becomes time-varying and depends on the interest rate.

4.5 Cash-credit models

Another version of the CIA model is the so called cash-credit model of Lucas and Stokey (1987). In this model agents gain utility from two goods, c_1 and c_2, where c_1 can only be purchased using cash but c_2 can be purchased on credit. The timing of the model is as follows. Agents observe the state of the world, decide on c_1 and c_2 and m, they then go and purchase cash goods paying for them with their money balances and also purchase credit goods, and then at the end of the period all credit bills are settled. This is another way of making the velocity of money variable. In this model, agents get utility from two goods, but on one good they have to pay cash and so lose R on any assets held in the form of cash. Therefore, when the interest rate is high they will tend to lower c_1 and increase c_2 to compensate, because they consume less of the cash good they also hold fewer money. Therefore the velocity of money $((c_1 + c_2)/m)$ varies positively with the interest rate - the higher the interest rate, the lower are money balances and the harder money has to work.

4.6 Simulation evidence

How well do these cash in advance models work in generating plausible patterns for money holdings in simulated economies? This is an issue examined in Hodrick, Kocherlakota and D. Lucas (1992) "The variability of velocity in cash in advance models", *Journal of Political Economy.* The authors choose a variety of values for certain key parameters, i.e. variability of money supply growth, persistence of money supply growth, risk aversion etc. They find that if they use the basic cash in advance model, whether it be that of Lucas or Svensson, they cannot generate under any plausible scenario enough variation in the velocity of money. For instance, over the last 100 years the velocity of money has had a standard deviation of around 4.5% but with the basic cash in advance model they cannot generate a standard deviation any greater than 0.09%. However, the cash-credit CIA model can generate more plausible numbers for the variability of velocity, with the numbers ranging between 0.6 % and 5.1%. However, to generate these more plausible numbers the authors have to assume very high levels of risk aversion. As a consequence, while they can explain observed volatility in the velocity of money they are unable to account for several other features of the data, such as the low level of actual interest rates. With the levels of risk aversion required to explain the velocity variability they need to have very large interest rates (20%) to explain consumption growth of around 2%, whereas in the data real interest rates tend to be around 3%. This is exactly the same type of failure others have identified as the equity premium puzzle. To account for one feature in the data we need to assume high risk aversion but in doing so we cannot explain why the risk-free rate of interest is so low. The core problem with these CIA models is that if interest rates are positive (i.e. consumers want to minimise cash holdings) and if consumption (or more precisely the marginal utility of consumption) is not very volatile (so that agents hold lower precautionary balances as they are less likely to have the need for very high levels of consumption) then because precautionary balances are very low, money is nearly equal to consumption of cash goods and so the velocity of money is low. Only by increasing risk aversion so that agents are concerned about even low levels of variability in consumption can you rescue the model. Not surprisingly, because the CIA models suffer from the same problems that generate the equity premium puzzle these basic CIA models are also not very successful in generating plausible asset price and interest rate data, an issue examined in Giovannini and Labadie (1991) "Asset Prices and Interest Rates in Cash in Advance Models", *Journal of Political Economy.*

4.7 Business cycle implications

The above suggests that CIA models have difficulty in explaining certain facts about nominal variables, but how do they fare in explaining business cycle fluctuations? This is investigated in Cooley and Hansen (1989) and also in their chapter in the volume edited by Cooley "Frontiers of Business Cycle Research". Before examining the outcome from simulations using these models it is important to analyse the propagation mechanism at work in the CIA model.

CIA models do not satisfy the conditions of the neoclassical dichotomy and so nominal variables such as money can have an impact on real variables such as consumption. The means by which this happens in a CIA model is through the "inflation tax". When inflation is high, the real value of money declines sharply so agents will hold less money. However, if cash is required to finance consumption purchases then high inflation will cause lower consumption and so a nominal variables (inflation/money supply growth) will affect a real variable (consumption).[1] Effectively, inflation acts as a tax on goods which require money to be purchased and as a subsidy on credit goods which do not require cash. Notice that what matters here is anticipated inflation and not unanticipated inflation. When agents choose their money holdings they do so on the basis of their expectations of inflation (which are assumed rational) and so unexpected inflation does not alter their decision.

Table 4.1 shows the results of simulating a CIA cash-credit model which has been calibrated for the US economy. An important feature of this simulation is that money growth is positively serially correlated, i.e. money growth tends to be above (below) average for several consecutive periods. Without this assumption CIA models tend to perform very poorly.

[1] Inflation acts as a distorting tax in this model. As a consequence, the fundamental welfare theorems do not hold and so the problem can no longer be analysed as the social planner's problem.

Variable	SD%	VariableCross-correlation of output with:							Correlation with M0 growth
		t-3	t-2	t-1	t	t+1	t+2	t+3	
Output	1.69	0.240	0.444	0.0692	1.000	0.692	0.444	0.240	-0.01
Consumption	0.53	0.397	0.488	0.582	0.676	0.378	0.153	-0.013	-0.60
Investment	5.90	0.169	0.381	0.643	0.975	0.699	0.473	0.282	0.16
Hours	0.35	0.145	0.362	0.637	0.987	0.715	0.492	0.305	-0.03
Prices	1.88	-0.135	-0.161	-0.190	-0.218	-0.126	-0.064	-0.021	0.43
Inflation	1.23	0.040	0.045	0.042	-0.138	-0.093	-0.065	-0.049	0.92
Nominal interest rate	0.58	0.010	0.011	-0.002	-0.014	0.003	0.011	0.007	0.72
ΔMoney	0.87	0.001	0.005	-0.000	-0.008	0.008	0.011	0.002	1.00
Velocity	1.40	0.141	0.351	0.613	0.948	0.691	0.477	0.295	0.27

Table 4.1: Simulated Cash-Credit RBC Model
(from Cooley and Hansen (ed) "Frontiers of Business Cycle Research")

The RBC model used in this simulation is essentially the basic Stochastic Growth model but with Hansen's assumption regarding indivisible labour markets. Comparison of the rows corresponding with the real variables in Table 4.1 with those from non-monetary RBC simulations suggest that little has altered except that consumption is slightly more volatile in Table 4.1. The effects of the inflation tax are immediately evident from the final column of Table 4.1, with consumption showing a strong negative correlation with money growth and investment a positive correlation, exactly what our intuition would expect. Unfortunately, comparing this with the earlier stylised facts we see that these correlations are not observed in the data. There are also a number of additional problems with Table 4.1. Firstly, money displays very little correlation with output either contemporaneously or in advance. The data suggest that high monetary growth is followed by higher output, which is not revealed in the simulations in Table 4.1. Further, the nominal interest rate displays no correlation with output at any leads or lags whereas the data shows strongly that high interest rates tend to be associated with lower output. Finally, the model is capable of generating a negative correlation between prices and output (because productivity shocks are the main source of fluctuations) but does not generate a positive correlation between inflation and output. In other words, adding a CIA model to as simple RBC structure cannot account either for the observed cyclical behaviour of nominal variables or the interaction between real and nominal variables. This suggests that in order to successfully account for the interaction between real and nominal variables in the data we need to introduce more sources of non-neutrality than just the inflation tax.

4.8 Exercise

Let the consumer's utility function be $U(c,l) = \alpha \ln c_{1t} + (1-\alpha)\ln c_{2t} - \gamma h_t$, where h denotes hours worked, c_1 is a good which can only be purchased with cash and c_2 denotes a good which can be purchased using credit. Households can hold two assets: money (m) or government bonds (b), the latter earn the return R. The household seeks to maximise utility subject to two constraints:

(i) a cash in advance constraint

$$p_t c_{1t} \leq m_t + (1 + R_{t-1})b_t - b_{t+1}$$

(ii) a resource constraint

$$c_{1t} + c_{2t} + \frac{m_{t+1}}{p_t} + \frac{b_{t+1}}{p_t} \leq w_t h_t + \frac{m_t}{p_t} + \frac{(1 + R_{t-1})b_t}{p_t}$$

(a) Write down an expression for the share of cash goods in total consumption as a function of the interest rate

(b) Write down an expression for the velocity of money. Is this a plausible model?

4.9 Solution

(a) The consumer maximises the present discounted value of utility subject to the two constraints, i.e.

$$\max E_0 \sum_{j=0}^{\infty} \beta^{t+j} \left[\begin{array}{l} \alpha \ln c_{1t+j} + (1-\alpha)\ln c_{2t+j} - \gamma h_{t+j} \\ +\lambda_{1t+j}(m_{t+j} + (1 + R_{t+j-1})b_{t+j} - b_{t+j+1} - p_{t+j}c_{1t+j}) \\ +\lambda_{2t+j}(w_{t+j}h_{t+j} + \frac{m_{t+j}}{p_{t+j}} + \frac{(1+R_{t+j-1})b_{t+j}}{p_{t+j}} - c_{1t+j} - c_{2t+j} - \frac{m_{t+j+1}}{p_{t+j}} - \frac{b_{t+j+1}}{p_{t+j}} \end{array} \right]$$

The first order conditions are:

$$w.r.t.\ c_1 \quad \tfrac{\alpha}{c_{1t}} - \lambda_{1t}p_t - \lambda_{2t} = 0 \tag{4.1}$$

$$w.r.t.\ c_2 \quad \tfrac{1-\alpha}{c_{2t}} - \lambda_{2t} = 0 \tag{4.2}$$

$$w.r.t.\ m \quad \lambda_{1t} + \tfrac{\lambda_{2t}}{p_t} - \tfrac{1}{\beta}\tfrac{\lambda_{2t-1}}{p_{t-1}} = 0 \tag{4.3}$$

$$w.r.t.\ b \quad \lambda_{1t}(1 + R_{t-1}) + \lambda_{2t}\tfrac{(1+R_{t-1})}{p_t} - \tfrac{1}{\beta}\lambda_{1t-1} - \tfrac{1}{\beta}\tfrac{\lambda_{2t-1}}{p_{t-1}} = 0 \tag{4.4}$$

$$w.r.t.\ h \quad \lambda_{2t}w_t = \gamma \tag{4.5}$$

When finding the share S_t of cash goods in total consumption we are interested in

$$S_t = \frac{c_{1t}}{c_{1t} + c_{2t}}$$

We can substitute out for consumption using (4.1) and (4.2).

$$\begin{aligned} S_t &= \left(\frac{\alpha}{\lambda_{1t}p_t + \lambda_{2t}}\right) \Big/ \left(\frac{\alpha}{\lambda_{1t}p_t + \lambda_{2t}} + \frac{1-\alpha}{\lambda_{2t}}\right) \\ &= \frac{\alpha\lambda_{2t}}{\alpha\lambda_{2t} + (1-\alpha)\lambda_{1t}p_t} \end{aligned}$$

Now substitute for λ_{1t} from (4.3) into (4.4).

$$\left(\frac{1}{\beta}\frac{\lambda_{2t-1}}{p_{t-1}} - \frac{\lambda_{2t}}{p_t}\right)(1 + R_{t-1}) + \lambda_{2t}\frac{(1 + R_{t-1})}{p_t} - \frac{1}{\beta}\lambda_{1t-1} - \frac{1}{\beta}\frac{\lambda_{2t-1}}{p_{t-1}} = 0$$

$$\lambda_{1t-1} = R_{t-1}\frac{\lambda_{2t-1}}{p_{t-1}}$$

And the share of cash goods is given by

$$S_t = \frac{\alpha}{1 + (1-\alpha)R_t}$$

and the share of cash goods is inversely related to the interest rate, $S_t = S_t(\overset{-}{R_t})$.

(b) Because there is an opportunity costs to holding money, the only money held will be to finance cash goods, i.e.

$$m_t = c_{1t}p_t$$

Total income at any time t is equal to total consumption spending on cash and credit goods.

$$y_t = p_t c_{1t} + p_t c_{2t}$$

Therefore, velocity of circulation is

$$v_t = \frac{y_t}{m_t} = \frac{p_t c_{1t} + p_t c_{2t}}{p_t c_{1t}} = \frac{1}{S_t}$$

$$v_t = \frac{1}{\alpha} + \frac{(1-\alpha)}{\alpha}R_t$$

This varies positively with the rate of interest, $v_t = v_t(\overset{+}{R_t})$, and so potentially the model is quite realistic.