Chapter 7

IS-LM is back

7.1 Motivation

The IS-LM model originally developed by Hicks (1937) fell out of favour with macroeconomists as the Rational Expectations Revolution forced researchers to work harder on the microfoundations of their models. However, the recent trend has been to recast modern stochastic dynamic general equilibrium models in the IS-LM framework. In this lecture, we see how this is possible and re-examine the persistence problem analysed in the previous lecture.

7.2 Key readings

Goodfriend and King (1998) "The New Neoclassical Synthesis and the Role of Monetary Policy" *Federal Reserve Bank of Richmond Working Paper*, No. 98-5

Jeanne (1997) "Generating Real Persistent Effects of Monetary Shocks: How Much Nominal Rigidity Do We Really Need?", *NBER Working Paper*, No. 6258

7.3 Related reading

Calvo (1983) "Staggered Prices in a Utility-Maximising Framework", *Journal of Monetary Economics*, 12, 383-398

Clarida, R., J. Galí and M. Gertler, 1999, "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* 27, 1661-701

Mankiw (2000) "The Inexorable and Mysterious Trade-Off Between Inflation and Unemployment", *Economic Journal*, 111, C45-C61

Mankiw and Reis (2001) "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve", *NBER Working Paper*, No. 8290

7.4 Log-linearisation

To derive the dynamic IS-LM model we will follow the paper and notation of Jeanne (1997). To make the algebra tractable we will describe a log-linearised version of the model, i.e. all the key equations are log-linearised using the formula (7.1).

$$\ln(f(x_t)) \simeq \ln(f(x_0)) + \frac{f'(x_0)}{f(x_0)} x_0 \ln\left(\frac{x_t}{x_0}\right) \tag{7.1}$$

After applying the approximation, the solution of the model can always be written in the form $X_t = \beta X_{t-1}$. This equation describes the evolution of an $n \times 1$ vector of variables X_t, where X_t is covariance stationary and β is a matrix of coefficients.

7.5 The IS curve

The IS curve is derived from intertemporal maximisation of the representative agents, who maximises a utility function subject to a resource constraint. The agent equates the marginal utility of consumption in adjacent periods after allowing for the real rate of return on savings (the nominal interest rate minus the rate of inflation). In other words, there is an Euler equation for consumption in the form of

$$u'(c_t) = \beta E_t \left(\frac{1+i_t}{1+\pi_{t+1}} u'(c_{t+1})\right)$$

$u'(c_t)$ is the marginal utility of consumption c_t at time t. This equals the marginal utility of consumption at time $t+1$ after allowance for nominal interest i_t paid on savings and inflation π_{t+1}. To log-linearise this equation first take logarithms.

$$\ln u'(c_t) = \ln \beta + E_t \ln(1+i_t) - E_t \ln(1+\pi_{t+1}) + E_t \ln u'(c_{t+1})$$

Using the CRRA utility function, $u(c_t) = c_t^{1-\frac{1}{\sigma_u}}/(1-\frac{1}{\sigma_u})$, $\ln u'(c_t) = -\frac{1}{\sigma_u}\ln c_t$. The above equation also holds at steady-state c_0, i_0, π_0 so we can write

$$\begin{aligned} -\frac{1}{\sigma_u}(\ln c_t - \ln c_0) \quad = \quad & E_t \ln(1+i_t) - \ln(1+i_0) \\ & -E_t \ln(1+\pi_{t+1}) + \ln(1+\pi_0) \\ & -\frac{1}{\sigma_u}(E_t \ln c_{t+1} - \ln c_0) \end{aligned}$$

We introduce the hat notation $\hat{x}_t = (x_t - x_0)/x_0 \approx \ln x_t - \ln x_0$, where $\hat{x}_t$ is the percentage deviation from steady-state. Finally, assume no capital so $\hat{y}_t = \hat{c}_t$ and we have the dynamic IS curve (7.2).

$$\hat{y}_t = -\sigma_u[\hat{\imath}_t - E_t\hat{\pi}_{t+1}] + E_t\hat{y}_{t+1} \tag{7.2}$$

This is a dynamic relationship between output and the real interest rate, as was true statically in Hick's (1937) analysis. When the real interest rate is expected to be high their either current output is low or future output is high.

7.6 The LM curve

We now turn our attention to the monetary side of the model. Money non-neutralities are introduced by assuming that the representative consumer faces a cash-in-advance constraint.

$$p_t c_t = M_t$$

M_t is nominal money holdings. The log-linearisation is simply (7.3), where $\hat{m}_t$ is real money holdings, $\hat{y}_t = \hat{c}_t$ because of no capital, and the inequality is assumed to always bind.

$$\hat{y}_t = \hat{m}_t \tag{7.3}$$

Equation (7.3) is the dynamic LM curve, linking output to the money supply. Because of our assumption of cash-in-advance, the velocity of circulation of money is always constant at unity and interest rates do not have a separate effect. A more general specification such as a cash-credit model would lead to an additional interest rate term in the LM equation.

7.7 The AS curve

To complete the model we have to specify an equation for aggregate supply in the economy. The supply curve in Jeanne (1997) is an example of the New Keynesian Supply Curve made popular by Clarida, Galí and Gertler (1999). They begin with the Calvo (1983) approach to price stickiness in which a fraction $1-\phi$ of firms are allowed to change their price each period. Under such a scheme, the aggregate price level (in percentage deviations from steady state) follows a simple linear expression.

$$\hat{p}_t = (1-\phi)\hat{p}_{it} + \phi\hat{p}_{t-1}$$

$\hat{p}_{it}$ is the price set by the fraction $1-\phi$ of firms able to change their price in the current period. The remaining fraction ϕ retain the previous average price $\hat{p}_{t-1}$. Using standard assumptions, it is possible to derive from microfoundations the pricing behaviour of a firm able to set its price in period t.

$$\hat{p}_{it} = (1-\beta\phi)\hat{p}_t^* + \beta\phi E_t\hat{p}_{it+1}$$

This expression shows that the price the firm sets is a weighted average of what it would set purely looking at the current period ($\hat{p}_t^*$) and the price it anticipates setting if able to in the next period ($E_t\hat{p}_{it+1}$).

$\beta < 1$ is the subjective discount factor. Note that if $\phi = 0$ then there is perfect price flexibility, the probability of being able to change price each period is 1 and the firm sets its price according to short-run aims, $\hat{p}_{it} = \hat{p}_t^*$. In contrast, as $\phi \to 1$ and the degree of price stickiness increases, the firm pays less and less attention to current market conditions and focuses on the future expected price, $\hat{p}_{it} \simeq E_t \hat{p}_{it+1}$.

An expression for $\hat{p}_t^*$ can be obtained by considering the equilibrium that would prevail when the firm maximises its short-run profit. Typically, in such a situation relative price is set as a mark-up over marginal cost, where the magnitude of the mark-up depends on the degree of market power held by the firm. This relationship, $p_t^*/p_t = k \times mc_t$ when log-linearised becomes

$$\hat{p}_t^* = \hat{p}_t + \widehat{mc}_t$$

Since there is no capital in the model, marginal costs arise only from wages and, assuming log-linearity between wages and marginal costs, we have that $\widehat{mc}_t = \hat{w}_t$. To complete the aggregate supply curve we assume a general form for the labour supply function as in the previous lecture.

$$\hat{w}_t = \frac{1}{\alpha} \hat{y}_t$$

Note again that this is consistent with a competitive labour for a suitable choice of the α parameter. Combing the various behavioural relationships (the evolution of aggregate prices, optimal price setting, definition of short-run desired price, and the labour supply function) gives the New Keynesian aggregate supply curve (7.4).

$$\hat{y}_t = \frac{\alpha\phi}{(1-\phi)(1-\beta\phi)} (\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}) \tag{7.4}$$

The intuition behind this supply function has some appeal. If inflation is expected to increase then output falls: firms able to change their price in the current period set high prices (thereby choking demand) in anticipation of future high prices. Similarly, if inflation is expected to fall then firms set lower prices (stimulating demand) ready for the future. This supply function is part of what McCallum calls the nearest thing we have to a standard model in monetary economics. However, recently there has been some unease about certain features of the model, see Mankiw (2000) and Mankiw and Reis (2001).

7.8 The money supply

The money supply is assumed to be exogenous with growth rate $\mu_t = M_t/M_{t-1}$ following an AR(1) process $\mu_t - \bar{\mu} = \rho_m(\mu_t - \bar{\mu}) + \varepsilon_t$. A log-linearisation of the real money balances is (7.5).

$$\hat{\mu}_t = \hat{\pi}_t + \hat{m}_t - \hat{m}_{t-1} \tag{7.5}$$

7.9 The persistence puzzle revisited

Equations (7.2) - (7.5) describe an IS-LM-AS framework that is dynamic and derived from optimising behaviour of individual agents. To look at persistence we can solve the system of log-linear equations and find an expression for any endogenous variable in terms of exogenous variables. We have four equations in four unknowns ($\hat{y}_t, \hat{\imath}_t, \hat{\pi}_t$ and $\hat{m}_t$) and so have the solution

$$\hat{y}_{t-1} - \left[1 + \beta + \frac{(1-\phi)(1-\beta\phi)}{\alpha\phi}\right]\hat{y}_t + \beta E_t \hat{y}_{t+1} = \beta E_t \hat{\mu}_{t+1} - \hat{\mu}_t$$

This is a second order stochastic difference equation which satisfies standard assumptions. Using standard techniques, the solution of the equation may be written as

$$\hat{y}_t = \rho \hat{y}_{t-1} + \rho \frac{1 - \beta\rho_m}{1 - \beta\rho\rho_m}\hat{\mu}_t$$

where ρ is the stable root satisfying

$$1 - \left[1 + \beta + \frac{(1-\phi)(1-\beta\phi)}{\alpha\phi}\right]\rho + \beta\rho^2 = 0$$

Once again, ρ is the crucial parameter for the persistence of monetary policy shocks. Jeanne (1997) asks how much rigidity do we need for a certain level of persistence. He shows that both nominal price rigidity (ϕ) and real wage rigidity (α) increase the level of persistence, i.e. $\partial\rho/\partial\phi > 0$ and $\partial\rho/\partial\alpha > 0$. To make this clearer we can ask which combinations of nominal and real rigidity give rise to a certain level of persistence. This is equivalent to holding ρ constant in the quadratic equation and searching for suitable values of ϕ and α. Re-arranging the quadratic equation gives a simple expression for the level of wage rigidity needed for persistence ρ when nominal rigidity is ϕ.

$$\alpha = \frac{\rho(1-\phi)(1-\beta\phi)}{\phi(1-\rho)(1-\beta\rho)}$$

We can plot this curve in (ϕ, α) space to see which combinations of nominal and real rigidity give a desired level of persistence. The iso-persistence curves are shown in Figure 7.1.

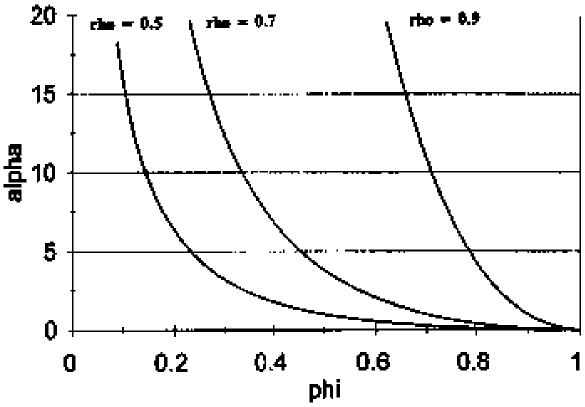

Figure 7.1: The iso-persistence curves in (ϕ,α) space

The figure reveals important non-linearities in the relationship. The curve are flat around $\phi = 1$ so a small increase in real rigidity is sufficient to offset a large amount of nominal rigidity. This suggests that it is not necessary to assume implausibly large amounts of wage stickiness to generate persistence. Jeanne (1997) suggests a calibration of $\phi = 0.5, \alpha = 3, \rho_m = 0.5$ as realistic. The impulse response function of output to a money supply shock for this calibration is shown in Figure 7.2. The response comes very close to that estimated from the data. Note, though that $\alpha = 3$ implies a considerable amount of wage rigidity compared with competitive labour markets.

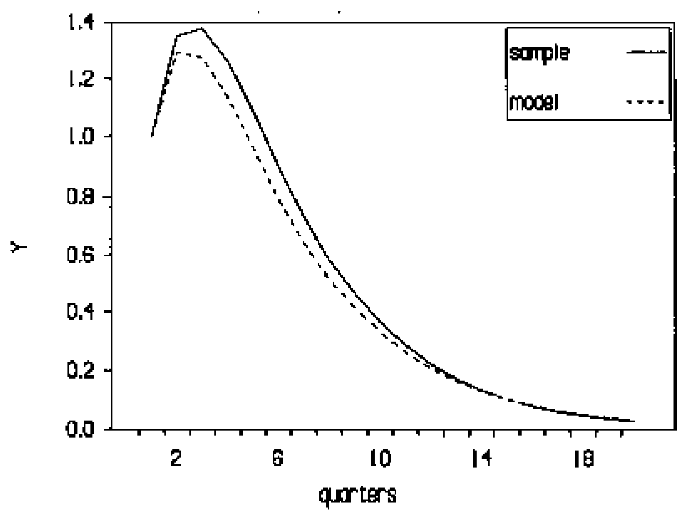

Figure 7.2: Impulse response functions