Chapter 3

Lucas islands model

3.1 Motivation

The papers by Lucas (1972), (1973) and (1975) sought to explain the short term procyclicality of output and inflation (the "Phillips curve") while still maintaining the neoclassical assumption that in the long run money is a veil and does not affect output. The impulse in his model is money supply shocks and the propagation mechanism consists of (a) competitive markets (b) Rational Expectations (c) imperfect information, in the sense that agents have better information regarding the market they operate in than the economy as a whole.

These papers were enormously influential and generated a substantial literature investigating the relative impact of expected and unexpected money. However, the more long-lasting impact of this paper has been methodological. Firstly, it illustrates the power of Rational Expectations as a modelling technique. Secondly, it shows how to use simple general equilibrium models to arrive at structural models of economic fluctuations. Therefore, the best way to view these lectures is less as *the* model of the business cycle and more as a useful way of illustrating modelling techniques which are now standard in macroeconomics.

3.2 Key reading

Lucas (1973) "Some International Evidence on Output-Inflation Trade-offs", *American Economic Review*, 63, 326-334

3.3 Related reading

Lucas (1972) "Expectations and the neutrality of money", *Journal of Economic Theory*, 4, 103-24

Lucas (1975) "An Equilibrium Model of the Business Cycle", *Journal of Political Economy*, 83, 1113-1144

Barro (1978) "Unanticipated money growth, output and the price level", *Journal of Political Economy*, 86, 549-80

3.4 The model

There exist N islands. On each island is a producer who charges price $p_t(z)$, where z denotes a particular island. We shall denote the aggregate price as p_t, which is simply the average of $p(z)$ across all Z islands.

The main focus in the paper is on the supply side where the following "Lucas supply" function is assumed

$$y_t(z) = \gamma(p_t(z) - p_t) \tag{3.1}$$

so that a producer increases his output when his own price is greater than the aggregate price in the economy. Thus output is given by a standard upward sloping supply schedule. $y(z)$ should be interpreted as deviations in output from trend, so that when $y(z) = 0$ output is equal to its trend value. The innovation in this model is to assume imperfect information so that a producer on an island knows at time t his own price $p(z)$ but does not know the economy-wide price level p. Instead, they have to form a guess of p based on the information at their disposal. Let $I_t(z)$ denote the information available at time t to the producer in market z. Then $E(p_t\,|I_t(z)\,)$ denotes the expectation of the aggregate price p_t given the information available to the producer in island z. Because of imperfect competition, we have the incomplete information supply curve

$$y_t(z) = \gamma(p_t(z) - E(p_t\,|I_t(z)\,)) \tag{3.2}$$

However, the crucial question is how do agents form $E(p_t\,|I_t(z)\,)$? Without an expression for this guess of aggregate prices, we can do very little with our model.

Lucas' trick is to use Rational Expectations. There are two different but related interpretations of Rational Expectations. The first is a statistical one and implies that, when agents have to make a forecast, errors are unpredictable. In other words, agents use all the information at their disposal. The second interpretation is more economic and is due to John Muth. According to this definition, Rational Expectations is when agents use the economic model to form their price predictions.

Assuming rational expectations, it must be the case that $p_t = E(p_t\,|I_{t-1}\,) + \epsilon_t$ where ϵ is a forecast error which is on average zero and which cannot be predicted from I_{t-1}. We shall denote the variance of ϵ ($E(\epsilon^2)$) as σ^2.

Lucas also assumes that the price in each island, $p(z)$, differs only randomly from the aggregate price level, p. In other words, $p_t(z) = p_t + z_t$ where z is on average zero and has a variance equal to τ^2.

If the producer has perfect information about the aggregate price level then $y(z)$ would respond only to z, the relative price shock. However, due to imperfect information agents only observe the gap between $p(z)$ and their expectation of the aggregate price level, that is they observe the composite error $z + \epsilon$.

The producers problem is to decide how much of this composite error is due to mistakes in forecasting the aggregate price level (ϵ) and how much is the relative price shock (z) and to only alter output in response to the latter.

How does the producer decide how much of the composite shock is due to ϵ and how much is due to z? Technically this problem is called "signal extraction". The answer is to look at historical data. Our model only assumes that agents do not know the current value of ϵ, but they do observe historical data on z and ϵ. Running a regression of z on $(z+\epsilon)$ will provide a guess of what proportion of $(z+\epsilon)$ is due to z. Using standard OLS formulae we have

$$\theta = \frac{\tau^2}{\sigma^2 + \tau^2}, \text{ where } z_t = \theta(z_t + \epsilon_t) + u_t \tag{3.3}$$

Therefore an agents best guess of z is $\theta(z+\epsilon)$. By definition $p(z) = p + z$ so that an agent's best guess of the aggregate price level given that they observe $p_t(z)$ is

$$\begin{aligned} E(p_t\,|I_{t-1}(z), p_t(z)\,) &= p_t(z) - E(z_t\,|I_{t-1}(z), p_t(z)\,) \\ &= p_t(z) - \theta(p_t(z) - E(p_t\,|I_{t-1}\,)) \\ &= (1-\theta)p_t(z) + \theta E(p_t\,|I_{t-1}\,) \end{aligned} \tag{3.4}$$

Equation (3.4) gives an expression for $E(p_t\,|I_t(z)\,)$ which we can insert into (3.2) to get a supply curve. Aggregating across all islands gives an aggregate supply curve.

$$y_t = \gamma\theta(p_t - E(p_t\,|I_{t-1}\,)) \tag{3.5}$$

so that output only responds to unexpected aggregate price shocks. Note that by adding and subtracting p_{t-1} to the right-hand of (3.5) and re-arranging we have

$$(p_t - p_{t-1}) = \frac{1}{\gamma\theta}y_t + (E(p_t\,|I_{t-1}\,) - p_{t-1}) \tag{3.6}$$

so that inflation depends positively on output and expected inflation (y should be interpreted throughout this note as deviation from trend). Assuming a negative relationship between output and unemployment, (3.6) defines a Phillips curve. However, there only exists a short term trade-off between output and inflation. Due to the assumption of Rational Expectations, on average forecasts of inflation equal inflation so that for (3.6) to hold $y = 0$. Output only differs from its trend value due to unexpected forecast errors.

So far we have only discussed the supply side of the model, we now turn to the demand side. We assume that demand in each island, $y^d(z)$, depends on nominal money and the price level in each island, e.g. $y^d = m(z) - p(z)$ and that the money supply in each island equals the aggregate money supply plus a random error, e.g. $m(z) = m + \eta(z)$ where $\eta(z)$ has a zero mean and variance δ^2. Finally, we assume that the aggregate money supply evolves according to

$$m_t = m_{t-1} + \mu + \zeta_t \tag{3.7}$$

where μ is the expected growth in the money supply and ζ is the unexpected part of money growth with a zero mean and a variance equal to λ^2.

We now have expressions for demand and supply in each island. Equilibrium requires that supply equals demand in each market so

$$\gamma\theta(p_t(z) - E(p_t\,|I_{t-1})) = m_t(z) - p_t(z) \tag{3.8}$$

It is here that we use our assumption of Rational Expectations. Our supply functions all require $E(p_t\,|I_{t-1})$. Different assumptions regarding this term will lead to very different implications. According to Rational Expectations, we should use the model's predictions for p_t as the best possible forecast. To see how this works aggregate (3.8) over all markets to give

$$\gamma\theta(p_t - E(p_t\,|I_{t-1})) = m_t - p_t \tag{3.9}$$

The left-hand side of (3.9) depends on the forecast error made by agents in predicting the economy-wide price level. However, by the definition of Rational Expectations these forecasts must be unpredictable at $t-1$. If this is the case then the expected value of the left and right hand side of (3.9) at $t-1$ must equal zero. From this we can therefore deduce that $E(p_t\,|I_{t-1}) = E(m_t\,|I_{t-1})$ so that the forecast of next period's aggregate price is equal to the forecast of next period's money supply, which from (3.7) is $m_{t-1}+\mu$ (unanticipated money is forecast to be zero or else it wouldn't be unanticipated). Therefore we have used the implication of the model (3.8) to derive agent's expectations.

Using this price expectations term, we can now return to our equilibrium condition (3.8) and solve our model. Replacing $E(p_t\,|I_{t-1})$ with $m_{t-1}+\mu$ and rearranging we have

$$p_t(z) = m_{t-1} + \mu + \frac{\zeta_t + \eta_t(z)}{1+\theta\gamma} \tag{3.10}$$

$$p_t = m_{t-1} + \mu + \frac{\zeta_t}{1+\theta\gamma} \tag{3.11}$$

for the market clearing price in each market and for the economy as a whole respectively. Notice our assumption that agent's price expectations are rational. We assumed that $E(p_t\,|I_{t-1}) = m_{t-1} + \mu$ and having made this assumption we have derived (3.11) as our expression for prices and (3.11) implies $E(p_t\,|I_{t-1}) = m_{t-1} + \mu$. Therefore agent's expectations are consistent with the model, this is the essence of Rational Expectations.

We earlier assumed the existence of a forecasting error ϵ_t for the aggregate price level and a relative price shock for each island z_t. Using (3.10) and (3.11) we can now deduce that

$$\epsilon_t = \frac{\zeta_t}{1+\theta\gamma} \quad z_t = \frac{\eta_t(z)}{1+\theta\gamma} \tag{3.12}$$

and we can also write the crucial signal extraction parameter θ as $\delta^2/(\lambda^2+\delta^2)$, in other words it depends on the relative variability of aggregate and island specific money supply.

Inserting our price expectations (from (3.10) and (3.11)) into the aggregate and island specific supply curve gives

$$y_t(z) = \frac{\gamma\theta}{1+\gamma\theta}[\zeta_t + \eta_t(z)] \quad y_t = \frac{\gamma\theta}{1+\gamma\theta}\zeta_t \tag{3.13}$$

Equation (3.13) gives expressions for the equilibrium sequence of output in each island and in the economy as a whole. As is the case in Rational Expectations models, these equations are highly structural. Every parameter can be interpreted in terms of the basic parameters of the model.

3.5 Implications

i) anticipated money does not influence output, i.e. y_t is independent of μ. From (3.13) the only thing that matters is shocks to the money supply. Agents in each island will observe an increase in $p(z)$ but will be unsure whether this is a relative price shock or an inflation shock. Even though they respond rationally, they are partly fooled into increasing output in response to aggregate price shocks. However, they only respond to unanticipated shocks and so this cannot be exploited systematically by the monetary authorities.

ii) unanticipated money can influence the current value of y_t but not the average level of output. From (3.13) the average deviation of output from trend is 0. Therefore monetary policy cannot shift the so-called natural rate of output.

3.6 Empirical evidence

There exists some support for the key idea, namely that output responds more to unexpected money than it does to expected money although there is some evidence that in the short run both types of money influence output. The idea that money does not affect the long run level of output is widely believed.

However, even though there is this empirical evidence the Lucas model is not seen as a good model of the cycle. The key equation is the expression for aggregate output in (3.13). There are three drawbacks for this equation as a model of the business cycle:

i) ϵ_t is white noise whereas output fluctuations are serially correlated. In other words, the model does not generate any persistence in output. It has as an impulse white noise shocks and produces as output white noise fluctuations, This is a fairly standard failing of neoclassical models. Neoclassical propagation mechanisms do not appear to increase by much the persistence of shocks assumed as impulses to the model. Lucas (1975) tries to overcome this problem by introducing capital but with limited success.

ii) $\gamma\theta/(1+\gamma\theta)$ is less than one. In other words, the model not only does not increase the persistence of shocks assumed as the impulse to the model but it also serves to dampen down their effect. Money supply shocks influence output by less than one for one.

iii) ϵ is very small. Monthly innovations to the money supply are not very large and certainly not large enough given (ii) to generate observed business cycle movements. Further, given the welfare costs of business cycles it is relatively costless for the government every period to notify all agents what they have done with the aggregate money supply. In other words, the imperfect information could be easily rectified (although this raises important issues of credibility).

To see the failings of the model more clearly consider the following table taken from Cooley and Hansen "Money and the Business Cycle" in Cooley (ed.) "Frontiers of Business Cycle Research".

	SD%	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4
U.S. data	1.72	0.16	0.38	0.63	0.85	1.00	0.85	0.63	0.38	0.16
Simulations	0.30	-0.06	-0.06	-0.08	-0.09	1.00	-0.09	-0.08	-0.06	-0.06

Tables like this are now a standard way of examining the properties of economic models. The first row shows the information on the cyclical part of US data. The first column quotes the standard deviation of output growth. The second row shows the results of simulating a version of Lucas' model which has been "calibrated" using US data. The first column shows that the Lucas model fails to generate anything like the right amount of volatility in output fluctuations, as suggested by (ii) and (iii). In other words, the combination of monetary shocks and imperfect information cannot account for business cycle volatility. the remaining columns quote the autocorrelation coefficients of output at different lags. In other words, the $t-2$ column shows how the cyclical component of output is correlated with its own value two periods ago. The value of 0.63 for US data shows that there is a strong correlation. This reveals that cyclical output is positively serially correlated, that is that output growth tends to be high for several consecutive periods (booms) and then low for several consecutive periods (recessions). However, looking at the results from the simulated Lucas economy we see no sign of such serial correlation. Cyclical output seems uncorrelated with either its recent past or recent future. In other words, the Lucas island model does not generate business cycles of booms and recessions but occasional random high and low values.