Chapter 6

The persistence puzzle

6.1 Motivation

At the beginning of the lectures we derived evidence using structural vector autoregressions that monetary policy shocks have a delayed yet persistent effect on output. All of the models we have examined so far (Lucas islands, cash in advance, nominal wage rigidity, sticky prices) have failed to replicate this key feature of the data. Whilst they are able to generate real-nominal interactions, the effect is not very persistent and output quickly returns to baseline. In the literature this is referred to as the "persistence puzzle". In this lecture we show how it is a general failing of monetary RBC models.

6.2 Key readings

The origin of the persistence puzzle is Chari, Kehoe and McGrattan (2000) "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?", *Econometrica*, 68(5), 1151-1180. This is a difficult paper to read. Our discussion is a simplified version of Section 5 of the paper.

6.3 Related reading

Chari, Kehoe and McGrattan (2000) "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?", *NBER working paper,* no. 7869

Ellison and Scott (2000) "Sticky Prices and Volatile Output", *Journal of Monetary Economics*, 46, 621-632

Taylor (1980) "Aggregate Dynamics and Staggered Contracts", *Journal of Political Economy*, 88, 1-24

Watson (1993) "Measures of Fit for Calibrated Models", *Journal of Political Economy*, 101, 1011-1041

6.4 Introduction

The basic intuition behind the persistence puzzle can be seen by considering the case with a static money demand equation, $y_t + p_t = m_t$, where y_t is output, p_t is the aggregate price level and m_t is the aggregate money supply. This equation enables us to see how money supply shocks are divided between output and prices - to have a large output effect it must be the case that prices do not respond too much to money supply shocks. However, prices do respond to costs in the form of the wage rate w_t. In other words, if prices are not to respond too much to money supply shocks then wages must not respond too much either.

To see the response of wages we assume a general form for the labour supply equation, $w_t = \gamma y_t$, where $\gamma > 0$ is the elasticity of the real wage with respect to output. According to this equation, wages will not react to the money supply shock as long as γ is small. We will see in what follows that in all our models γ is by definition large so there is too much pass-through of money supply shocks to wages and prices and not enough to output.

6.5 Staggered price setting and persistence

The paper by Chari, Kehoe and McGrattan (2000), CKM, formally analyses whether staggered price setting can solve the persistence problem. To demonstrate their argument we will assume that there are two types of firms: one changes its price in even periods and one changes its price in odd periods. We denote the price set by firms able to change their price in period t as x_t, so the aggregate price at time t is $p_t = (x_t + x_{t-1})/2$. This is potentially a strong propagation mechanism. Because of the staggered and overlapping price contracts, if the money supply shock affects the price set at time t, x_t, then this affects the aggregate prices p_t and p_{t+1}. However, the aggregate price p_{t+1} affects the price set at $t+1$, x_{t+1}, and so also the aggregate price p_{t+2} and so on. CKM refer to this as a "contract multiplier" which could pass the shock on through time.

CKM derive an equation (6.1) for the optimal price-setting behaviour of a firm setting its price at time t. Using microfoundations, they show that the price set depends on (i) the price x_{t-1} set by other firms in the previous period, (ii) the price x_{t+1} expected to be set by other firms in the next period, (iii) expected wages in the current period and (iv) expected wages in the next period. This formulation is very close to Taylor (1980).

$$x_t = \frac{1}{2}x_{t-1} + \frac{1}{2}E_{t-1}x_{t+1} + E_{t-1}w_t + E_{t-1}w_{t+1} \tag{6.1}$$

With a static money demand equation, $y_t + p_t = m_t$, a labour supply function of the form $w_t = \gamma y_t$, and the definition of aggregate prices $p_t = (x_t + x_{t-1})/2$, this can be re-written as equation (6.2).

$$E_{t-1}x_{t+1} - \left[\frac{2\left(1+\gamma\right)}{1-\gamma}\right]x_t + x_{t-1} + \left[\frac{2\gamma\left(1+\gamma\right)}{\left(1-\gamma\right)^2}\right]E_{t-1}\left(m_t + m_{t+1}\right) = 0 \tag{6.2}$$

This is a standard second order difference equation. We make the further simplifying assumption that the money supply is a random walk $E_{t-1}m_{t+j} = m_{t-1} \ \forall\ j$, so the solution to the equation is of the form

$x_t = ax_{t-1} + bm_{t-1}$. Comparing coefficients on x_{t-1} we obtain the quadratic equation (6.3) in a and select the stable solution (6.4).

$$a^2 - \left[\frac{2(1+\gamma)}{1-\gamma}\right] a + 1 = 0 \tag{6.3}$$

$$a = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}} \tag{6.4}$$

The full solution to the model is as follows:

$$\begin{aligned}
x_t &= ax_{t-1} + (1-a)m_{t-1} \\
p_t &= ap_{t-1} + \frac{1}{2}(1-a)(m_{t-1} - m_{t-2}) \\
y_t &= ay_{t-1} + (m_t - m_{t-1}) + \frac{1}{2}(1-a)(m_{t-1} - m_{t-2})
\end{aligned}$$

The degree of persistence in the model is directly related to the parameter a. If $a = 0$ ($\gamma = 1$), a shock Δ to the money supply at time t leads to an increase Δ in output at time t, an increase of $\Delta/2$ at time $t+1$ and no increase for all subsequent periods. There is no persistence beyond the length of price stickiness. In contrast, if $a = 1$ ($\gamma = 0$), a shock Δ to the money supply at time t has a permanent effect on output. The value of a depends directly on the elasticity of the real wage with respect to output by equation (6.4). To generate the require degree of persistence needs a value of γ which is not too small (in which case there is too much persistence) but not too large either (in which case there is very little persistence).

6.6 Labour supply in classical models.

The elasticity of the real wage with respect to output, γ, is not a free parameter in classical RBC models. Rather, the supply of labour is determined by the tastes of workers in competitive labour markets. In such models, workers deciding their labour supply typically maximise their utility function (6.5) subject to a budget constraint (6.6).

$$\begin{aligned}
u(c_t, l_t) &= \ln c_t + \psi \ln(1 - l_t) && (6.5) \\
c_t &= w_t l_t && (6.6)
\end{aligned}$$

c_t is consumption, l_t is hours worked and $\psi > 0$ is a parameter measuring the weight of leisure time in the utility function. The utility function (6.5) is popular because it is consistent with balanced growth in the economy. In the budget constraint (6.6), w_t is the wage rate. Assuming no capital in the economy so $c_t = y_t \forall t$, the first order condition can be written as equation (6.7).

$$w_t = (1+\psi)y_t \tag{6.7}$$

This is the aggregate supply curve of labour in a competitive labour market. Compared with the general form $w_t = \gamma y_t$, we see that $\gamma = 1 + \psi > 1$, where ψ is a true structural parameter. This illustrates why there is a "persistence puzzle" in the models we have analysed. All our models have competitive labour markets so $\gamma > 1$ and *by definition* the money supply shocks do not have a persistent effect on output. The wage rate and prices react too strongly to money supply shocks to have persistent output effects. The CKM result is that price stickiness alone cannot solve the persistence problem. It is worth noting that Taylor (1980) treats γ itself as a structural parameter and calibrates it at 0.05 to generate persistence. However, CKM derive this parameter from first principles to arrive at a much larger calibration of 1.22.

6.7 Extensions

The paper by Ellison and Scott (2000) looks at nominal price rigidity in a fully-specified dynamic general equilibrium model. They calibrate ψ to 1.56 on the basis of an 8 hour working day in competitive labour markets. The elasticity of the real wage with respect to output is then $\gamma = 2.56$ so there are persistence puzzles in the model simulations. The impulse response in Figure 6.1 from the model is much less than that estimated from the data using a structural vector autoregression.

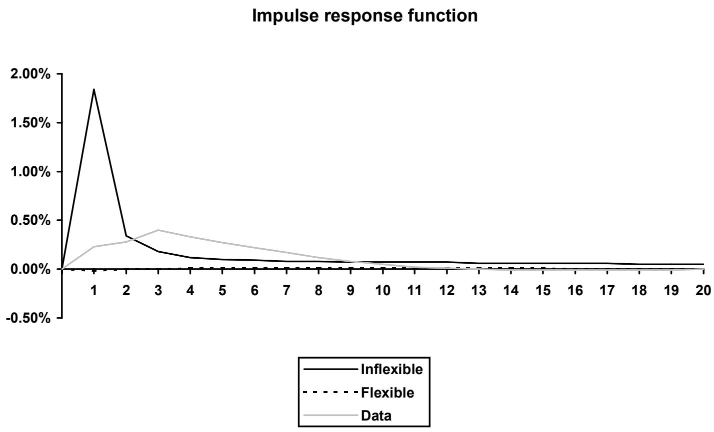

Figure 6.1: Impulse response function of monetary shocks on output

Ellison and Scott (2000) also identify other key failings of the model. Rather than rely on a qualitative assessment of stylised facts, they look at a quantitative measure of the goodness of fit of the model using the approach of Watson (1993). This measures how well the model fits the data in four dimensions: output, inflation, employment and consumption. The conclusion of the paper is that overall the presence of price rigidity *worsens* the ability of the model to match the data. A flexible price model actually performs better as a representation of the data than a sticky price model. Most of the deterioration in the performance is in the form of excessively high output volatility.

A second paper by Chari, Kehoe and McGrattan (2000) looks at the analogous problem of volatility and persistence of real exchange rates in an open economy. The conclusion is somewhat similar to the closed economy case - stickiness can only account for some properties of the data.