Chapter 10

Monetary policy under uncertainty

10.1 Motivation

In recent times it has become increasingly common for central banks to acknowledge that the do not have perfect information about the structure of they economy they are attempting to control. There may be uncertainty surrounding the precise values of the key parameters of the model or, at a deeper level, there may be fundamental uncertainties regarding which is the correct model. In this lecture we concentrate on parameter uncertainty and discuss how the nature of monetary policy changes when such uncertainty is formally accounted for. The models we use will be static and quite stylised to highlight the precise mechanisms in action.

10.2 Key readings

Brainard (1967) "Uncertainty and the Effectiveness of Policy", *American Economic Review Papers and Proceedings*, 57, 411-425.

Sack (2000) "Does the fed act gradually? A VAR analysis", *Journal of Monetary Economics*, 46, 229-256.

10.3 Related reading

Shellekens (2000) "Caution and Conservatism in the Making of Monetary Policy", *European Central Bank Working Paper*, No. 25

Theil (1958) *Economic Forecasts and Policy*, North-Holland Amsterdam

Tinbergen (1952) *On the Theory of Economic Policy*, North-Holland Amsterdam

10.4 Certainty equivalence

If the uncertainty faced by the central bank takes a particularly simple form then the optimal policy of the central bank is to behave "as if" everything was known with certainty. This will typically be the case if the only source of uncertainty is an additive error term. Equation (10.1) describes a monetary transmission mechanism in which inflation π is determined by the interest rate i through the known coefficient b, where $b < 0$. u is an i.i.d. error term with mean zero and variance σ_u^2.

$$\pi = bi + u \tag{10.1}$$

The central bank is assumed to have a quadratic loss function (10.2), which penalises the deviation of inflation from a target level π^*.

$$\mathcal{L} = (\pi - \pi^*)^2 \tag{10.2}$$

The timing of the model is such that the central bank has to set the interest rate i before the error term u is revealed. In other words, the central bank does not know the true state of the world when it moves and so it has to set interest rates to minimise the expectation of the loss in (10.2). Substituting from (10.1) into (10.2) we can write

$$\begin{aligned} \mathcal{L}^e &= E\left[(bi + u - \pi^*)^2\right] \\ &= b^2 i^2 + E(u^2) + {\pi^*}^2 + 2biE(u) - 2bi\pi^* - 2\pi^* E(u) \end{aligned}$$

Note that the expectation operator only continues to apply to terms in u because b, i and π^* are all known by the central bank at the time the decision is taken. From the definition of u as a random error term, we also have $E(u^2) = \sigma_u^2$ and $E(u) = 0$ so the expected loss can be expressed as in (10.3).

$$\mathcal{L}^e = b^2 i^2 + \sigma_u^2 + {\pi^*}^2 - 2bi\pi^* \tag{10.3}$$

The central bank chooses i to minimise this expected loss and so derives the optimal policy under certainty equivalence (10.4).

$$i = \frac{\pi^*}{b} \tag{10.4}$$

This policy is completely independent of the uncertainty surrounding the error term u. It is as if the central bank has completely ignored the uncertainty and set policy such that the inflation target is met in expectation, i.e. $\pi^e = \pi^*$. In the literature this is known as the "certainty equivalence principle". When uncertainty is additive, as here in the case of a simple additive error term, the central bank can ignore the uncertainty and set policy as if everything was known with certainty. The result was first proposed by Theil (1958) and Tinbergen (1952). However, as we will see, the conditions under which it holds are quite restrictive and it is not really applicable to most real-world situations of interest.

10.5 Brainard conservatism

The paper by Brainard (1967) shows that certainty equivalence no longer holds for more complex specifications of uncertainty. More specifically, if there is uncertainty about the parameters of the model then the central bank should not behave as if the uncertainty does not exist, a result described some thirty years later by Alan Blinder as the "Brainard uncertainty principle". The key difference is that uncertainty about a parameter is multiplicative rather additive uncertainty: the more a policy is used the more that the uncertainty is multiplied into the system. To see how this changes the nature of the optimal policy consider the monetary transmission mechanism with parameter uncertainty (10.5).

$$\begin{aligned} \pi &= bi + u \\ b &\sim (\hat{b}, \sigma_b^2) \end{aligned} \tag{10.5}$$

The first part of equation (10.5) is identical to that in (10.1) in the discussion of certainty equivalence except now there is uncertainty about the parameter b. However, although the central bank does not know the precise value of b, it does know the distribution from which it is drawn, i.e. it knows its mean $\hat{b}$ and variance σ_b^2. There are many reasons why this might be a reasonable description of the central bank's knowledge of the monetary transmission mechanism. It could be that the central bank has poor information about how the transmission mechanism works, for example the current state of the ECB in Euroland. Alternatively, there may be fundamental uncertainties in the transmission of monetary policy which preclude ever being able to predict with certainty what the effect of interest rates on inflation is. The structure of the stylised economy (10.5) is shown in Figure 10.1. The central straight line shows the relationship $\pi = \hat{b}i$, which holds in expectation. The curved lines are confidence bands showing the range of inflation that is expected for given interest rates.

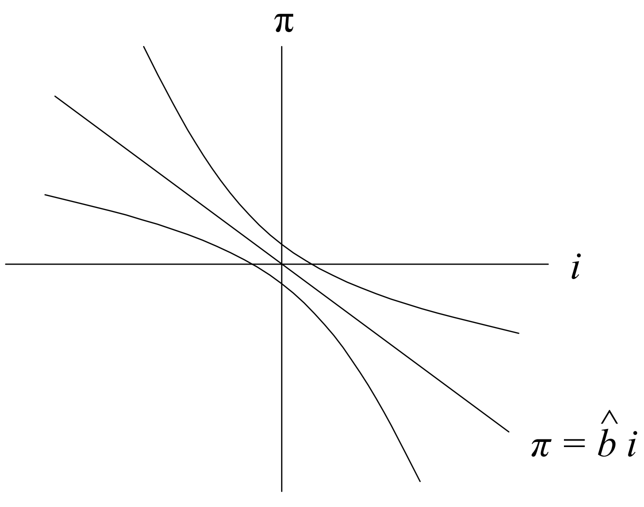

Figure 10.1: Uncertainty about the structural relationship.

Figure 10.1 shows how the parameter uncertainty is multiplicative. As interest rates are moved further away from zero it becomes increasingly more difficult to predict the level of inflation. Uncertainty would be minimised with a zero interest rate, at which the only uncertainty is additive due to the error term, but then expected inflation would be zero and not equal to target. Mathematically, the expected loss with parameter uncertainty is given by

$$\begin{aligned}
\mathcal{L}^e &= E\left[(bi + u - \pi^*)^2\right] \\
&= E(b^2)i^2 + E(u^2) + {\pi^*}^2 + 2E(bu)i - 2E(b)i\pi^* - 2\pi^* E(u)
\end{aligned}$$

Again, the definition of u as a random error term implies $E(u^2) = \sigma_u^2$ and $E(u) = 0$. The mean of b gives $E(b) = b$ and the variance of $\hat{b}$ can be written as $\sigma_u^2 = E(b - \hat{b})^2 = E(b^2) - E(\hat{b}^2)$, which gives an expression for $E(b^2)$. By making the further simplifying assumption that uncertainties about b and u are unrelated, in other words $E(bu) = 0$, the expected loss can be described by equation (10.6).

$$\mathcal{L}^e = \sigma_b^2 i^2 + \sigma_u^2 + (\hat{b}i - \pi^*)^2 \tag{10.6}$$

The optimal policy (10.7) is derived by differentiating (10.6) with respect to the interest rate i.

$$i = \frac{\hat{b}\pi^*}{\hat{b}^2 + \sigma_b^2} \tag{10.7}$$

This policy differs from the certainty equivalent policy (10.4) by the extra variance term σ_b^2 in the denominator. The presence of parameter uncertainty means that optimal policy is more cautious. $\left|i^{Brainard}\right| \leq \left|i^{Certainty.Equivalence}\right|$ implies that interest rates are closer to zero under the Brainard policy than under certainty equivalence. The reason is that additional caution reduces the amount of uncertainty that policy introduces into the system. At the extreme, as $\sigma_b^2 \to \infty$ and parameter uncertainty becomes infinite, the optimal policy is to do nothing and set interest rates to zero, $i \to 0$. In contrast, parameter uncertainty disappears as $\sigma_b^2 \to 0$ and the interest rate is set equal to that under certainty equivalence. This result is often referred to as Brainard conservatism - parameter uncertainty introduces a motive for caution in optimal policy. Such a policy means that the central bank does not expect to achieve its inflation target, i.e. $\pi^e \neq \pi^*$. The reason for this is that aiming to hit the target exactly involves large potential losses, especially if the parameter b turns out to be high and the monetary transmission mechanism is more potent than expected.

10.6 Is Brainard uncertainty empirically relevant?

Whether Brainard uncertainty is a useful concept to explain the behaviour of the Federal Reserve Board is the subject of a study by Sack (2000). He models the structure of the economy using a five dimensional vector autoregression in industrial production growth, unemployment, consumer price inflation, commodity price inflation (to control for price puzzles) and the federal funds rate. Structure is imposed by a recursive Choleski ordering in which the federal funds rate ordered last. Assuming the model is correctly identified, the first four equations in the model describe the structural form of the economy whilst the last equation is the estimated policy function of the fed.[1] The policy maker is assumed to minimise a present-value quadratic loss function (10.8)

$$\mathcal{L} = \frac{1}{2} E_t \left\{ \sum_{i=1}^{\infty} \beta^i \left[(\pi_{t+1} - \pi^*)^2 + \lambda_u (u_{t+1} - u^*)^2 \right] \right\} \tag{10.8}$$

where λ_u denotes the relative importance of the deviations of unemployment and inflation from their respective targets.

After estimating the model with OLS, Sack (2000) calculates a "certainty equivalent" policy rule by assuming that the point estimates from the VAR are the true values known with certainty. This rule will be linear in the past values of the variables in the system since it is a standard linear-quadratic control problem. The coefficients depend on preferences λ_u and the point estimates of the VAR coefficients. This rule is then compared to a "Brainard" policy rule which takes into account that the point estimates of the VAR are uncertain. The standard OLS errors of the parameter estimates are used as an indicator of the uncertainty surrounding each parameter. The new optimal rule is still a linear function of past values of variables in the system but now the coefficients of the rule depend on both the point estimates of the VAR and the variance-covariance matrix of the coefficient estimates.

Sack (2000) calculates the impulse response functions implied by the optimal rules with and without allowance for parameter uncertainty and compares these to those estimated purely from the data. He finds that the optimal policy rule taking parameter uncertainty into account is closer to the actual behaviour of the federal funds rate than an optimal policy disregarding parameter uncertainty. Figure 10.2 shows how the optimal policy with parameter uncertainty tracks the federal funds rate better, suggesting that caution induced by Brainard uncertainty is quantitatively important.

[1] Because this is not a true structural model based on microfoundations it is questionable whether the results are robust and not subject to Lucas critique problems.

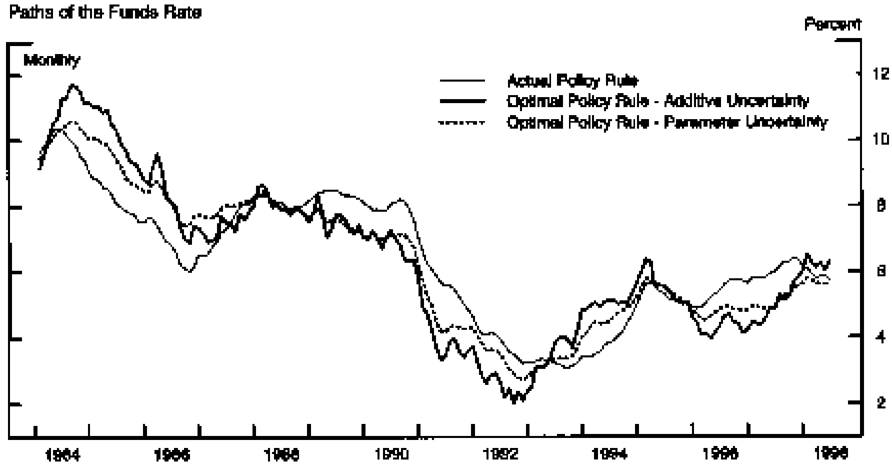

Figure 10.2: The interest rate under actual and optimal policies

Both of the optimal policy rules exhibit considerable persistence in interest rates. This is known as "interest rate smoothing" and implies that the fed should adjust interest rates gradually rather than aggressively. Table 10.1 shows simulation evidence for the persistence of interest rate changes under the two optimal policies and compares these to the actual estimated behaviour of the federal funds rate.

Federal funds rate	$\Delta i = -0.02 + 0.46\Delta i_{-1} + 0.01\Delta i_{-2}$
Optimal policy with parameter uncertainty	$\Delta i = -0.02 + 0.10\Delta i_{-1} + 0.22\Delta i_{-2}$
Optimal policy without parameter uncertainty	$\Delta i = -0.04 + 0.15\Delta i_{-1} + 0.09\Delta i_{-2}$

Table 10.1: Persistence structure of interest rates

The optimal rule with parameter uncertainty comes closest to matching the persistence observed in the federal funds rate, giving further support for the claim that Brainard uncertainty and caution are an important feature of the data and do help to explain the behaviour of the Federal Reserve Board. However, the degree of interest rate smoothing in the optimal rule with parameter uncertainty still falls short of that actually observed in the federal funds rate so there must be some additional explanation for the smoothness of interest rates.