The Welfare Costs of Business Cycles in Robust Economies with Individual Consumption Risk

Martin Ellison  Thomas J Sargent
University of Oxford  New York University
Lucas (1987)
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- Calculate welfare gain in simple model

\[ \text{consumption} \]

\[ \text{time} \]
Lucas (1987)

- Calculate welfare gain in simple model
- $\approx 0.1\%$ of steady-state consumption
Increasing the welfare costs of business cycles

- Increase persistence of consumption shocks
- Increase risk aversion
- Introduce aggregate and idiosyncratic risk
- Introduce a preference for robustness
Increasing the welfare costs of business cycles

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- Introduce a preference for robustness
What we do

Introduce aggregate and idiosyncratic risk

Consumption subject to aggregate and idiosyncratic shocks

De Santis (2007)

Heterogeneous agent model

Introduce a preference for robustness

Tallarini (2000) uses Epstein-Zin preferences to separate risk aversion from intertemporal elasticity of substitution

High risk aversion needed for business cycles to be costly

Barillas, Hansen and Sargent (2009) reinterpret high risk aversion as preference for robustness

We look at combined effect of idiosyncratic risk and robustness
What we do

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Martin Ellison and Thomas J. Sargent
Robustness and Consumption Risk
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- We look at combined effect of idiosyncratic risk and robustness
A preference for robustness
A preference for robustness

- Agents fear model misspecification

\[ U_t = (1 - \beta) V(c_t) - \frac{1}{\sigma} \log E_t \exp(-\sigma\beta U_{t+1}) \]
A preference for robustness

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- Preference for robustness \( \sigma > 0 \)
A preference for robustness

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\[ U_t = (1 - \beta) V(c_t^i) - \frac{1}{\sigma} \log E_t \exp(-\sigma \beta U_{t+1}) \]

- Recursive multiplier preferences of Hansen and Sargent (2001)
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- \( \sigma \uparrow \rightarrow \) greater fear of misspecification
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- Recursive multiplier preferences of Hansen and Sargent (2001)
- Preference for robustness \( \sigma > 0 \)
- \( \sigma \uparrow \rightarrow \) greater fear of misspecification
- \( \sigma \rightarrow 0 \) means \( U_t = (1 - \beta) V(c_t^i) + \beta E_t U_{t+1} \)
Aggregate and idiosyncratic risk
Aggregate and idiosyncratic risk

Consumption process

\[
\begin{align*}
    c^i_t &= c_t + \delta^i_t \\
    \Delta c_{t+1} &= \sqrt{\epsilon} w_{1t+1} \\
    \Delta \delta^i_{t+1} &= \sqrt{\epsilon} w_{2t+1}
\end{align*}
\]
Aggregate and idiosyncratic risk

- Consumption process

\[ c^i_t = c_t + \delta^i_t \]
\[ \Delta c_{t+1} = \sqrt{\epsilon} w_{1t+1} \]
\[ \Delta \delta^i_{t+1} = \sqrt{\epsilon} w_{2t+1} \]

- \( w_{1t+1} \sim N(g - \tau_1^2/2, \tau_1^2) \) and \( w_{2t+1} \sim N(-\tau_2^2/2, \tau_2^2) \)
Aggregate and idiosyncratic risk

- **Consumption process**

  \[ c^i_t = c_t + \delta^i_t \]
  \[ \Delta c_{t+1} = \sqrt{\epsilon} w_{1t+1} \]
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- \( w_{1t+1} \sim N(g - \tau^2_1/2, \tau^2_1) \) and \( w_{2t+1} \sim N(-\tau^2_2/2, \tau^2_2) \)

- Result of aggregate and idiosyncratic shocks
Solution

\[ \text{Solution satisfies value function:} \]

\[ W(c_t) = (1 - \beta) V(c_{t+1}) + \sigma \log E_t \exp(\sigma \beta W(c_t)) \]

\[ c_{t+1} = c_t + p \epsilon (w_{1t} + 1 + w_{2t} + 1) \]


Solution of form \( W(c_t) = W_0(c_t) + h(c_t) \) satisfies value function for \( p \epsilon = 0 \)
Solution

Solution satisfies value function:

\[
W^E(c^i_t) = (1 - \beta) V(c^i_t) - \frac{1}{\sigma} \log E_t \exp(-\sigma \beta W^E(c^i_{t+1}))
\]

\[
c^i_{t+1} = c^i_t + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1})
\]
Solution

- Solution satisfies value function:

\[
W^E(c_t^i) = (1 - \beta) V(c_t^i) - \frac{1}{\sigma} \log E_t \exp(-\sigma \beta W^E(c_{t+1}^i))
\]

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c_{t+1}^i = c_t^i + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1})
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- Solution of form \( W^E(c^i_t) = W^0(c^i_t) + h(c^i_t) \)
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c^i_{t+1} = c^i_t + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1})
\]

- Solution of form \(W^E(c^i_t) = W^0(c^i_t) + h(c^i_t)\)
- \(W^0(c^i_t)\) satisfies value function for \(\sqrt{\epsilon} = 0\)
Approximate solution
Approximate solution

- No noise solution $W^0(c_t) = V(c_t)$
Approximate solution

- No noise solution: $W^0(c^i_t) = V(c^i_t)$
- Small noise solution:

\[
W^0(c^i_t) + h(c^i_t) = (1 - \beta) V(c^i_t) \\
- \frac{1}{\sigma} \log E_t \exp \left( -\sigma \beta W^E(W^0(c^i_{t+1}) + h(c^i_{t+1})) \right) \\
c^i_{t+1} = c^i_t + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1})
\]
Small noise expansion

\[ e^{\sigma \beta (W_0 (c_i t + 1) + h (c_i t + 1))} = e^{\sigma \beta (W_0 (c_i t) + h (c_i t))} \]

\[ 1 + \infty \sum_{n=1}^{\infty} \frac{\epsilon^n}{2^n \mu_0^n n!} (w_1 t + 1 + w_2 t + 1) n! \mu_0^n \]

\[ \kappa_n = \sigma \beta D_n (W_0 (c_i t) + h (c_i t)) \]

Take expectations, apply logarithm and expand
Small noise expansion

- Expansion for small $\sqrt{\epsilon}$:

\[
\begin{align*}
  e^{-\sigma \beta (W^0(c_{t+1}^i) + h(c_{t+1}^i))} &= e^{-\sigma \beta (W^0(c_t^i) + h(c_t^i))} \\
  \times &\left(1 + \sum_{n=1}^{\infty} \frac{\epsilon^{n/2} \mu_n'}{n!} (w_{1t+1} + w_{2t+1})^n \right) \\
  \mu_n' &= \kappa_n + \sum_{m=1}^{n-1} \binom{n-1}{m-1} \kappa_m \mu'_{n-m} \\
  \kappa_n &= -\sigma \beta D^n (W^0(c_t^i) + h(c_t^i))
\end{align*}
\]
Small noise expansion

- Expansion for small $\sqrt{\epsilon}$:

$$e^{-\sigma\beta(W^0(c^i_{t+1}) + h(c^i_{t+1}))} = e^{-\sigma\beta(W^0(c^i_t) + h(c^i_t))}$$

$$\times \left(1 + \sum_{n=1}^{\infty} \frac{\epsilon^{n/2} \mu'_n}{n!} (w_{1t+1} + w_{2t+1})^n\right)$$

$$\mu'_n = \kappa_n + \sum_{m=1}^{n-1} \binom{n-1}{m-1} \kappa_m \mu'_{n-m}$$

$$\kappa_n = -\sigma\beta D^n(W^0(c^i_t) + h(c^i_t))$$

- Take expectations, apply logarithm and expand
Undetermined coefficients

Solution satisfies:

\[
\beta_h(c_t) = \infty \sum_{k=1}^{\infty} (1+k) \sigma_n \sum_{n=1}^{\infty} \frac{\epsilon_n}{\mu_n} n! E_t(w_1 t + 1 + w_2 t + 1)
\]

Propose solution of form:

\[
h(c_t) = \infty \sum_{n=1}^{\infty} \epsilon_n \frac{h_n(c_t)}{2}
\]
Undetermined coefficients

Solution satisfies:

\[(1 - \beta) h(c^i_t) = -\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\sigma} \sum_{n=1}^{\infty} \frac{\epsilon^{n/2} \mu'_n}{n!} \mathbb{E}_t (w_{1t+1} + w_{2t+1})^n \]
Undetermined coefficients

- Solution satisfies:

\[(1 - \beta) h(c^i_t) = - \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\sigma} \left( \sum_{n=1}^{\infty} \frac{\epsilon^{n/2} \mu'_{n} E_t(w_{1t+1} + w_{2t+1})^n}{n!} \right)^k \]

- Propose solution of form:

\[h(c^i_t) = \sum_{n=1}^{\infty} \epsilon^{n/2} h^n(c^i_t)\]
Small noise solution

First order term:

\[ h_1(c_i t) = E_t \left( w_1 t + 1 + w_2 t + 1 \right) \beta \beta W_0(c_i t) \]

Second order term:

\[ h_2(c_i t) = E_t \left( w_1 t + 1 + w_2 t + 1 \right)^2 \sigma(\beta) \sigma \beta D^1 W_0(c_i t) + \left( E_t \left( w_1 t + 1 + w_2 t + 1 \right) \right)^2 \sigma(\beta) \sigma \beta D^2 W_0(c_i t) \]
Small noise solution

- First order term:

$$h^1(c^i_t) = \frac{E_t(w_{1t+1} + w_{2t+1})}{1 - \beta} \beta Dw^0(c^i_t)$$
Small noise solution

- First order term:

\[ h^1(c^i_t) = E_t(w_{1t+1} + w_{2t+1}) \frac{\beta DW^0(c^i_t)}{1 - \beta} \]

- Second order term:

\[
\begin{align*}
    h^2(c^i_t) &= E_t(w_{1t+1} + w_{2t+1}) \frac{\beta Dh^1(c^i_t)}{1 - \beta} \\
               &+ \frac{E_t(w_{1t+1} + w_{2t+1})^2}{2\sigma(1 - \beta)} \begin{pmatrix} 
    \sigma \beta D^2 W^0(c^i_t) \\
    - (\sigma \beta DW^0(c^i_t))^2 
    \end{pmatrix} \\
               &+ \frac{(E_t(w_{1t+1} + w_{2t+1}))^2}{2\sigma(1 - \beta)} (\sigma \beta DW^0(c^i_t))^2
\end{align*}
\]
Why second order?

\[ c_{i+1} = c_i + \epsilon \left( w_{1t+1} + w_{2t+1} \right) \]

First order in \( \epsilon \) has terms in \( E_t \left( w_{1t+1} + w_{2t+1} \right) \)

Second order has terms in \( E_t \left( w_{1t+1} + w_{2t+1} \right)^2 \)

Only second order approximation captures interaction between aggregate and idiosyncratic risk.
Why second order?

Consumption process \( c_{t+1}^i = c_t^i + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1}) \)

Only second order approximation captures interaction between aggregate and idiosyncratic risk.
Why second order?

- Consumption process $c^i_{t+1} = c^i_t + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1})$
- First order in $\epsilon$ has terms in $E_t(w_{1t+1} + w_{2t+1})^2 = \tau_1^2 + \tau_2^2 + t.i.risk$
Why second order?

- Consumption process: $c_{t+1}^i = c_t^i + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1})$
- First order in $\epsilon$ has terms in $E_t(w_{1t+1} + w_{2t+1})^2 = \tau_1^2 + \tau_2^2 + t.i.risk$
- Second order has terms in $E_t(w_{1t+1} + w_{2t+1})^4 = 3\tau_1^4 + 2\tau_1^2\tau_2^2 + 3\tau_2^4 + t.i.risk$
Why second order?

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Accuracy of solution
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- CRRA utility + no fear of misspecification solved analytically
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- Logarithmic utility + fear of misspecification solved analytically
Accuracy of solution

- CRRA utility + no fear of misspecification solved analytically

- Logarithmic utility + fear of misspecification solved analytically
  - First order approximation exact
Calibration

- Mean consumption growth: $g_{1.89}\%$
- SD of aggregate consumption shocks: $\rho_{1.9}\%$
- SD of idiosyncratic consumption shocks: $\rho_{10}\%$
- Discount factor: $\beta_{0.95}$
- Coefficient of relative risk aversion: $\gamma_{1.1,1.5}$
- Initial individual consumption: $c_{i0} \log(77.4)$
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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Calibrating the fear of misspecification

Andersen, Hansen and Sargent (2003)

Agent fears models that are statistically similar in finite samples

\[ P(\text{accepting incorrect model}) + P(\text{rejecting correct model}) \]

50%, 45%, 40%
Calibrating the fear of misspecification

- Detection error probability metric
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Role of risk aversion
Role of risk aversion

- Set $\sigma = 0$ so no fear of misspecification
Role of risk aversion

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- Welfare cost of aggregate business cycles in steady-state consumption equivalent

\[
\begin{array}{ll}
\gamma = 1 & \gamma = 1.5 \\
\text{Cost of business cycles} & 0.802\% & 1.262\% \\
= \text{Baseline cost} & = 0.802 & = 0.802 \\
+ \text{Contribution of idiosyncratic risk} & +0 & +0.460 \\
\end{array}
\]
Role of risk aversion

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- Cost independent of idiosyncratic risk if utility logarithmic
Add fear of misspecification
Add fear of misspecification

- Set $\gamma = 1.5$ so mildly risk averse

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- Interaction between robustness and idiosyncratic risk
Intuition
Intuition

- $f(c_{t+1})$ conditional density of aggregate and idiosyncratic consumption under approximating model
Intuition

- \( f(c_{t+1}) \) conditional density of aggregate and idiosyncratic consumption under approximating model
- \( \hat{f}(c_{t+1}) = m(c_{t+1})f(c_{t+1}) \) worst-case density to fear
Intuition

- $f(c_{t+1})$ conditional density of aggregate and idiosyncratic consumption under approximating model
- $\hat{f}(c_{t+1}) = m(c_{t+1})f(c_{t+1})$ worst-case density to fear
- $m_{t+1} \propto \exp(-\sigma \beta U_{t+1})$

<table>
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<td>$\bar{w}_{1t}$</td>
<td>0</td>
<td>$-0.532 \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{w}_{2t}$</td>
<td>0</td>
<td>$-6.315 \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td></td>
<td>(2.9%)$^2$</td>
<td>(2.90041%)$^2$</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>10%</td>
<td>10.0171%</td>
<td></td>
</tr>
<tr>
<td>$\rho_{w_1,w_2}$</td>
<td>0</td>
<td>$0.995 \times 10^{-3}$</td>
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Approximating and worst-case densities
Approximating and worst-case densities

\[ \sqrt{\in_1^2}, \sqrt{\in_1^1} \]
Alternative consumption process

\[
\begin{align*}
\text{idiosyncratic consumption process has cyclical job displacement risk:} \\
& w_2 t = \gamma = d_H \text{ with prob } \pi_H \\
& \text{with prob } \pi_H \\
\text{with prob } \pi_H \\
& \text{with prob } \pi_L \\
& \text{with prob } \pi_L \\
& \text{with prob } (1 - \pi_H) \\
\text{Compare with acyclical job displacement risk process:} \\
& \bar{w}_2 t = \bar{\gamma} = \bar{d}_H \text{ with prob } \bar{\pi}_H \\
& \text{with prob } \bar{\pi}_H \\
& \text{with prob } \bar{\pi}_L \\
& \text{with prob } \bar{\pi}_L \\
& \text{with prob } (1 - \bar{\pi}_H) \\
\end{align*}
\]

Krebs (2007)
Alternative consumption process

- Idiosyncratic consumption process has cyclical job displacement risk:

\[
w_{2t} = \begin{cases} 
-d_H & \text{with prob } \pi p_H \\
\frac{p_H d_H}{1-p_H} & \text{with prob } \pi (1 - p_H) \\
-d_L & \text{with prob } (1 - \pi) p_L \\
\frac{p_L d_L}{1-p_L} & \text{with prob } (1 - \pi)(1 - p_L)
\end{cases}.
\]

Compare with acyclical job displacement risk process:
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\[ \bar{w}_{2t} = \begin{cases} 
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- Krebs (2007)
Cost of business cycles
## Cost of business cycles

<table>
<thead>
<tr>
<th>detection error probability</th>
<th>50%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of business cycles</td>
<td>1.175%</td>
<td>2.083%</td>
</tr>
<tr>
<td>= Baseline cost</td>
<td>= 1.022</td>
<td>= 1.022</td>
</tr>
<tr>
<td>+ Contribution of cyclical displacement risk</td>
<td>+0.153</td>
<td>+0.153</td>
</tr>
<tr>
<td>+ Contribution of fear of model misspecification</td>
<td>+0</td>
<td>+0.615</td>
</tr>
<tr>
<td>+ Joint contribution</td>
<td>+0</td>
<td>+0.293</td>
</tr>
</tbody>
</table>
Worst case density
## Worst case density

<table>
<thead>
<tr>
<th>Detection Error Probability</th>
<th>50%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\pi}$</td>
<td>0.5</td>
<td>0.49970</td>
</tr>
<tr>
<td>$\hat{p}_H$</td>
<td>0.03</td>
<td>0.03002</td>
</tr>
<tr>
<td>$\hat{p}_L$</td>
<td>0.05</td>
<td>0.05953</td>
</tr>
</tbody>
</table>

\[
E(\hat{w}_{1t} \mid \text{displaced in expansion}) = 0 - 0.699 \times 10^{-3}
\]
\[
E(\hat{w}_{1t} \mid \text{displaced in contraction}) = 0 - 0.743 \times 10^{-3}
\]
\[
E(\hat{w}_{1t} \mid \neg \text{displaced in expansion}) = 0 - 0.666 \times 10^{-3}
\]
\[
E(\hat{w}_{1t} \mid \neg \text{displaced in contraction}) = 0 - 0.664 \times 10^{-3}
\]
Conclusions

Welfare cost of business cycles higher than previously thought

- Lower mean aggregate and idiosyncratic consumption growth
- Greater variance in aggregate and idiosyncratic consumption growth
- + correlation between aggregate and idiosyncratic consumption shocks

Stabilising business cycle should be an important priority

Next up - implications for market price of risk

Martin Ellison and Thomas J. Sargent

Robustness and Consumption Risk
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