Government Debt Management: the Long and the Short of It

E. Faraglia (U. of Cambridge and CEPR), A. Marcet (IAE, UAB and CEPR), R. Oikonomou (U.C. Louvain), A. Scott (LBS and CEPR)

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Angeletos (2002) studies optimal debt management in an economy with complete markets:

1. The optimal portfolio is to issue long term bonds and hold short term savings → **Government debt is long term**;

2. Positions are **several multiples of GDP** (Buera and Nicolini (2004));

3. Positions are **constant**.

Faraglia et al (2010) find also that modifying Angeletos’ framework generates **high volatility** of portfolios and reversal of the positions.

All these models assume that the government repurchases and reissues (r/r) the entire debt in every period.
Data: Share of Short Term Debt in the US
Data: Total Issuance

![Graph showing issuance over total debt over years](image-url)
The share of short term debt is sizeable: 43% on average but never below 20%;

Positions are not large multiples of GDP;

The shares of the different maturities are typically persistent and exhibit low volatility:
- First order autocorrelation of short bond is 0.94;
- Standard deviation is 0.078;

The portfolio shares are never zero or "negative";

Total issuance is smaller than 100% and 98% of the debt is redeemed at maturity.
Is the recommendation to issue only long term debt and engage in r/r operations robust to the introduction of reasonable market frictions?

We generalize Aiyagari et al. (2002) introducing an $N$ period zero coupon bonds and study two alternative environments:

- **"buyback"**: government always repurchases the outstanding debt in every period (with and w/o lending limits) $\Rightarrow$ common assumption in theory;
- **"no buyback"**: government never repurchases the outstanding debt (with and w/o lending limits) $\Rightarrow$ common assumption in practise;

We introduce calibrated costs of issuance and repurchase:
- Shadow costs calculation;
- Optimal buyback model;

Robustness: Introduction of coupon bonds, callable bonds, other maturities.
Summary of the Results

- The assumption of **no buyback** is essential to explain the coexistence of short and long debt/savings:
  - long bonds are still used for their fiscal insurance properties;
  - however imposing no buyback of the long bonds creates \( N \) period cycles in the tax schedules;
  - short bonds are necessary for the government to smooth the tax schedule.
- The assumption of no landing constraints helps to match the empirical facts.

- Introducing small **transaction costs** makes \( r/r \) too costly and **no buyback** arises endogenously.

- The results are robust to the assumption of different bonds (s.a. coupon bonds and callable bonds).
**Contribution**

- **Empirical contribution**: analysis of the buy back data for callable and non-callable bonds;

- **Theoretical contribution**: effects of the "no buyback" assumption for optimal fiscal policy models. With this assumption the Ramsey policy becomes a positive theory of debt management;

- **Methodological contribution**: new solution methods for portfolio and large state space problems with stochastic projection methods;
Model with Buyback

The Ramsey planner:

$$\max_{\{c_t, x_t, b_{1,t}, b_{N,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(x_t)]$$

subject to

$$g_t + b_{1,t-1} + p_{N-1,t} b_{N,t-1} = \tau_t (T - x_t) + p_{1,t} b_{1,t} + p_{N,t} b_{N,t}$$

$$c_t + g_t \leq T - x_t$$

$$\underline{M} \leq \beta^i b_{i,t} \leq \overline{M} \quad \text{for } i = 1, N$$

$$b_{1,-1}, b_{N,-1}, \ldots b_{N,-N} \text{ given}$$

- an exogenous and stochastic government spending process

$$p_{1,t} = \frac{\beta E_t\{u_{c,t+1}\}}{u_{c,t}}$$, $$p_{N,t} = \frac{\beta^N E_t\{u_{c,t+N}\}}{u_{c,t}}$$ and $$\tau_t = 1 - \frac{v_{x,t}}{u_{c,y}}$$. 
Model with Buyback

Off corners when $\xi_{L,t}^i = \xi_{H,t}^i = 0$ we get:

$$\lambda_t = \frac{E_t \{ \lambda_{t+1} u_{c,t+i} \}}{E_t \{ u_{c,t+i} \}}$$

that means that $\lambda_t$ is a risk adjusted random walk.

Following Marcet and Marimon (2014) and Aiyagari et al. (2002) the optimal solution has a recursive formulation where:

$$\begin{bmatrix}
    b_{N,t} \\
    b_{1,t} \\
    \lambda_t \\
    c_t
\end{bmatrix} = F \left( g_t, \lambda_{t-1}, \ldots, \lambda_{t-N}, b_{t-1}^N, \ldots, b_{t-N}^N \right)$$

In FMOS (2016) we show that the Lagrange multipliers are needed because they enforce in the appropriate continuation problem the promises of future taxes that affect interest rates.
We follow Marcet and Scott (2009):

- Annual horizon: $\beta = 0.95$;
- $u(c_t) + v(x_t) = \log(c_t) - \eta \frac{1}{x_t}$
- The process of government spending: $g_t = \rho g_{t-1} + (1 - \rho) \bar{g} + \epsilon_t$
- $\rho = 0.95$, $\bar{g} = 0.25\bar{y}$ and $\sigma_\epsilon = 1.44$

Debt constraints: $\underline{M} \leq \beta^i b_{i,t} \leq \bar{M}$

- Lending model: $+/ - 100\%$ of GDP for each $i$ (Faraglia et al. 2012)
- No lending model: $\underline{M} = 0$ (Chari and Kehoe (1999), Lustig et al. (2008), Faraglia et al (2013))
### Moments: Data and Model

<table>
<thead>
<tr>
<th>US DATA</th>
<th>BuyBack Lending</th>
<th>No Lending</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{ST}$</td>
<td>43%</td>
<td>4.10³%</td>
</tr>
<tr>
<td>$\sigma_{S_{ST}}$</td>
<td>7.8</td>
<td>3.10⁵</td>
</tr>
<tr>
<td>corr$(S_{ST, t}, S_{ST, t-1})$</td>
<td>0.94</td>
<td>0.47</td>
</tr>
<tr>
<td>corr$(\frac{B_{ST, t}}{GDP_t}, \frac{B_{LT, t}}{GDP_t})$</td>
<td>0.86</td>
<td>-0.01</td>
</tr>
<tr>
<td>%$S_t = 0$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>%$S_t \leq 0.1$</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Model: Average of 1000 samples of 60 periods
The No Buyback Assumption: Pros

We modify the budget constraint such that:

$$g_t + b_{1,t-1} + b_{N,t-N} = \tau_t (T - x_t) + p_{1,t} b_{1,t} + p_{N,t} b_{N,t}$$

after some algebra The intertemporal budget constraint is:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{u_{c,t+j}}{u_{c,t}} s_{t+j} = p_{N-1,t} b_{N,t-1} + p_{N-2,t} b_{N,t-2} + \ldots + b_{N,t-N} + b_{1,t-1}$$

where $s_t$ is the primary surplus.

$$\implies \text{Fiscal insurance motive is still present! However...}$$
The No Buyback Assumption: Cons

Assume only an $N$ period bond:

We modify the budget constraint such that:

$$g_t + b_{N,t-N} = \tau_t (T - x_t) + p_{N,t} b_{N,t}$$

Now the FOC shows that the random walk property no longer holds:

$$\lambda_t = \frac{E_t \left\{ u_{c,t+N} \lambda_{t+N} \right\}}{E_t \left\{ u_{c,t+N} \right\}}$$

Intuitively...

- If $g_t > g_{t+1}$ and $g_{t+1} = \bar{g}$ for $t + 1$ onwards then $\tau_t$ and $b_{N,t}$ will increase.
- I have to redeem the bond in $t + N$ by rising taxes and more debt...and so on
- There is an optimal $N$ period cycle in fiscal policy which violates tax smoothing.
Figure 7: Response of the Tax Schedule - No Buyback Model

Notes: The Figure plots the tax rate in a single bond economy without buyback. The solid line is a maturity of one year. The dashed line sets the maturity to three years and the crossed line to 10 years.
Model with Buyback and two Bonds

The government budget constraint

$$g_t + b_{1,t-1} + b_{N,t-N} = \tau_t (T - x_t) + p_{1,t} b_{1,t} + p_{N,t} b_{N,t}$$

Now the FOC become:

$$\lambda_t = \frac{E_t \{u_{c,t+N} \lambda_{t+N}\}}{E_t \{u_{c,t+N}\}}$$

$$\lambda_t = \frac{E_t \{u_{c,t+1} \lambda_{t+1}\}}{E_t \{u_{c,t+1}\}}$$

- Implications for debt management:
  - Long bonds have a hedging value
  - Short bonds are beneficial to smooth taxation
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<table>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lend.</td>
<td>No Lend.</td>
</tr>
<tr>
<td>$S_{ST}$</td>
<td>43%</td>
<td>4.10^3%</td>
<td>12%</td>
</tr>
<tr>
<td>$\sigma_{S_{ST}}$</td>
<td>7.8</td>
<td>3.10^5</td>
<td>13.0</td>
</tr>
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</tr>
<tr>
<td>$%S_t = 0$</td>
<td>0</td>
<td>-</td>
<td>13.1%</td>
</tr>
<tr>
<td>$%S_t \leq 0.1$</td>
<td>0</td>
<td>-</td>
<td>56.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>76%</td>
<td>48%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.10^3</td>
<td>8.1</td>
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<tr>
<td></td>
<td></td>
<td>0.42</td>
<td>0.92</td>
</tr>
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<td>0.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>0.01%</td>
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<tr>
<td></td>
<td></td>
<td>-</td>
<td>0.02%</td>
</tr>
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Model: Average of 1000 samples of 60 periods
No Buyback and No Lending
Optimal Buy Back and Transaction costs

\[ g_t + b_{1,t-1} + b_{N,t-N} - R_{t-N+1} + p_{N-1,t} R_t (1 + T^R (R_t)) = \tau_t (T - x_t) + \sum_{i \in \{S,N\}} p_{i,t} b_{i,t} (1 - T^i (b_{i,t})) \]

- Transaction costs are calibrated from the empirical evidence:
  - bid ask spreads and brokerage fees of bonds (0.038\%) and treasury bills (0.0099) (Amihud and Mendelson (1991))
  - auction effects on yields (3bp on the yield) (Lou, Yan and Zhang (2013))

Result: **The government does not want to buy back the debt:** similar results of NBB model.
## No Buyback Model: Moments

### Moments: Data and Model

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<td>0</td>
<td>-</td>
<td>0.01%</td>
</tr>
<tr>
<td>$% S_t \leq 0.1$</td>
<td>0</td>
<td>-</td>
<td>0.02%</td>
</tr>
</tbody>
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Model: Average of 1000 samples of 60 periods
Assuming Buyback does not match the empirical data.

Assuming No Buyback delivers very sharp predictions:

- Government debt is no longer (only) long term;
- The government finances deficits with both bonds;
- Introducing small transaction costs makes no buyback arise endogenously;
- Results are robust to different model specifications

The Ramsey policy becomes a positive theory of debt management.