Cooperation and ESSs

Putting Humpty together again

Lecture Outline

- Introduction
- Iterated Prisoner's Dilemma (IPD)
- Is the IPD a good model for cooperation?

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Problem of sociality

- The Williams' revolution, as encapsulated in the *Selfish Gene*, says biological explanations must focus on individuals and individual advantage
- But many organisms are irreducibly social, and their biology reflects this
- Cooperation is one aspect of sociality that poses a challenge to individual selfishness

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Iterated Prisoner's Dilemma (IPD)

- Defined
- Biological Examples
- Some basic features
- History of study
- Some important results

IPD defined

- One-shot PD (Reward, Punishment, Temptation and Sucker). T>R>P>S, (R+R)>(S+T)
- Played many times. Usually a fixed probability, say w, of playing again.
- The game payoff is the sum of the separate payoffs, sometimes discounted.
- Kinds of strategies (AllD, AllC, TFT, TFTT)

C	S
T	P

He plays

		Cooperate	Defect
I play	Cooperate	I get C	I get S
	Defect	I get T	I get P

T>C, so I should defect if he cooperates

P>S, so I should defect if he defects

So, I should defect

Game equilibrium concepts

- A strategy is a Nash Equilibrium if it is a best reply to itself:
  For all y, $E(x,x) \geq E(y,x)$
- A strategy is an ESS if it is a Nash Equilibrium *and* for all equal best replies $y \neq x$, $E(x,y) > E(y,y)$
- This small crack widens to a chasm later!

Examples (see Dugatkin for more details)

- Egg-trading in polychaete worms (Sella)
- Reciprocal grooming (Hart and Hart)
- Predator Inspection (Milinski)

History of study

- Rapaport and Chammah 1965
- Other subjects, including economics, sociology, politics, mathematical game theory
- Biology (see Dugatkin)
  - Trivers, 1971
  - Axelrod, 1984 (see Ch 8 of Sigmund)
  - Axelrod and Hamilton, 1981

Some important results

- With finite maximum number of iterations
  - the only Population Nash Equilibrium is AllD
- In standard IPD,
  - AllD is always Nash
  - TFT is Nash when
    $w \geq \max\{ (T-R)/(R-S), (T-R)/(T-P) \}$
  - there are lots of Population Nash Equilibria
  - there is no ESS (Lorberbaum)

1. Suppose there is a last possible period T. Then in period T, the standard argument shows that the only solution is Defect.

2. Players in T-1 know that in T, all will defect. Hence in T-1, the standard argument again shows that the only solution is Defect.

3. and so on

4. Hence, with a maximum number of iterations, even as big as 10 to power 100, the only solution to the game is AllD.

5. Thus it is an essential part of the assumption of a fixed chance w of continuing that it sets no upper limit.

If everyone is playing AllD, then the best response is to play D each time. Hence, AllD is a *Nash Equilibrium*.

If everyone is playing AllC, then the best response is to play D each time. Hence, AllC is not a *Nash Equilibrium*.

I have taken this form of the argument from Maynard Smith (1982), Appendix K. Except that in step 3, he wrongly claims that TFT is an ESS when the inequality holds.

1. Suppose the population is playing TFT. Then only three classes of alternate strategies need be considered, according as they end up playing CCCCCCC…, DDDDDD…, or DCDCDCDC…

2. Their payoffs are $R/(1-w)$, $T+wP/(1-w)$, and $(T+wS)/(1-w^2)$.

3. Hence TFT is Nash when

$$w \geq \max\{ (T-R)/(R-S), (T-R)/(T-P) \}$$

4. But TFT is never an ESS in the full strategy set because AllC is an equal best response to TFT, but TFT does not do better against AllC than AllC does (in fact it does exactly the same)

Game equilibrium concepts

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Suppose a strategy plays AllD on even days, and TFT on odd days, with the understanding that the player plays whatever his opponent played on the previous odd day.

Then this strategy is a Nash equilibrium provided the TFT part is Nash, i.e.

$$w^2 \geq \max\{ (T-R)/(R-S), (T-R)/(T-P) \}$$

the same condition as before, except that now the probability of playing again refers to reaching one odd day from the previous odd day, hence w^2.

But we can take TFT and put any number of special "Defect Days" into them (provided they're not adjacent), and the result is also a Nash Equilibrium under the same conditions.

Some important results

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  - the only Population Nash Equilibrium is AllD
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Some basic features

- AllD and TFT can both be Nash…
- …and then which wins out depends on initial conditions
- TFT does well even though it never beats its opponent!
- TFT does not resist drift from AllC, which can then be invaded by AllD
- AllD is not inescapable either, though!

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A good model for cooperation? ‘Local’ points

- Discrete, single interactant, single iteration, no population structure, deterministic, no observation and learning, no punishment, simultaneous moves, no mistakes - all handled by variant models (see Dugatkin’s Table 3.1)
- Its a special model, and probably the literature is too concentrated on it
- Radical point about 0 payoffs.
- *But* the approach is important

A good model for cooperation? 'Global' points

- Humans aren't machines (but what about other animals?)
- Society and social interactions are emergent properties that cannot be reduced to an adding together of individuals' behaviour
- Super-confident game theorist vs Jeremiah-like humanist
- Intermediate - see what the approach can do

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References

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