Testing Wavefunction Collapse

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Abstract

The technique of measuring the wavefunction of a single system suggests a method for distinguishing between epistemological interpretations of quantum mechanics which postulate wavefunction collapse and ontological interpretations such as that of de Broglie and Bohm in which no collapse occurs.

In quantum mechanics it is convenient to divide interpretations of the wavefunction into the purely epistemolgical and the ontological. The first views the wavefunction as just a repository of statistical information on a physical system. The other sees the wavefunction primarily as an element of physical reality (while generally retaining the epistemological interpretation as a secondary property). A drawback of the epistemological interpretation is that it entails the hypothesis that the wavefunction ‘collapses’ at some stage in a measurement process, a notion that has engendered numerous paradoxes. It is known that there exist alternative ontological interpretations which dispense with the collapse hypothesis (one such consistent theory is described in ref. [1]) but as yet no empirical evidence has been forthcoming to decide between the two. This paper addresses that issue.

The measurement problem of quantum mechanics arises when one attempts to attribute definite outcomes to processes devoted to discovering some information on a quantum system [2]. The measurement of an operator $\hat{A}$ associated with a system of coordinate x is customarily modelled by an impulsive interaction generated by the Hamiltonian $H = f\hat{A}(-i\hbar\partial / \partial z)$ where z is the coordinate of the apparatus and f is a constant. At first the system and apparatus are non-interacting so the total initial state is $\Psi_0(x, z) = \psi_0(x)\phi_0(z)$ where $\psi_0(x) = \sum_a c_a\psi_a(x)$ is a superposition of eigenstates of $\hat{A}$, and $\phi_0(z)$ is the initial apparatus state (assumed to be a localized packet). The impulsive interaction acts as a beam splitter in configuration space generating a spectrum of macroscopically distinct apparatus states correlated with individual eigenfunctions. If the period of interaction is T we obtain

$$\Psi(x, z, T) = \sum_a c_a\psi_a(x, T)\phi_a(z, T) \tag{1}$$

where $\phi_a(z,T) = \phi_0(z - faT)$ represents a set of non-overlapping outgoing apparatus packets. In order to extract a definite result from this superposition in the epistemological interpretation, the hypothesis is invoked that the state (2) collapses into one of the summands, say the ath, with probability $|c_a|^2$:

$$\sum_{a'} c_{a'}\psi_{a'}(x, T)\phi_{a'}(z, T) \rightarrow \psi_a(x, T)\phi_a(z, T) \tag{2}$$

(after normalization). This transformation is not described by the evolutionary law of quantum mechanics (Schrödinger’s equation) and suggestions for how it might come about have ranged from the intervention of an observer who becomes aware of the outcome to modifications of the Schrödinger equation. But even if it is assumed that it does actually take place, the notion of collapse does not in itself solve the measurement problem. For to infer the outcome of the measurement the pointer of the apparatus must be assigned a location whose variation during the interaction can be unambiguously determined. In contrast, according to its usual interpretation the wavefunction attributed to the apparatus determines just the statistical frequency of measurement results. The wavefunction does not itself offer a description of an autonomous object. One may attempt to address this difficulty by invoking the feature of $\phi_0(z)$ that it is sharply peaked about a spacetime orbit, that is by making some kind of literal identification of the packet with the particle. Then one is tacitly shifting the interpretation of the wavefunction towards an ontological view, but not in a clearly consistent way - the eventual diffusion of the packet, or the possibility of splitting it into disjoint parts, means the 'particle' does not remain localized, for instance.

The other option is that the projection (2) does not take place. Rather, the correct wavefunction remains (1) so that all terms in the superposition continue to be finite but one is selected as representing the outcome of the measurement because it carries some special attribute. An example of a theory which solves the measurement problem in this way is due to de Broglie and Bohm [1]. Here the quantum state is defined by a set of point particles moving along spacetime tracks, as well as by the wavefunction. The primary role of the latter is to guide the particles according to a precise law of motion. In an ensemble of particle systems the probability density of presence in the initial state (from which the Born probability formula of observation follows) is $|\Psi_0(x,z)|^2$. Then in the measurement one of the outgoing summands is singled out because the de Broglie-Bohm system point (*x,z*) enters it. From the standpoint of the *particles* the transformation (2) does therefore in effect occur, even though the other ψ_as and ϕ_as are still finite (they will be called *empty*). Within this approach the entire measurement process may be treated by applying the usual linear, unitary Schrödinger equation, and the single concept of particle trajectory enables one to both avoid the collapse postulate and to solve the problem of the definiteness of the pointer (and object) position. Moreover, this ontological interpretation has the advantage of being unambiguous in application and of not assigning any special role to the consciousness of the observer. As far as we know it is free of paradoxes.

For historical rather than scientific reasons the de Broglie-Bohm proposal has been dismissed as an *ad hoc* metaphysical hypothesis and little consideration given to either its conceptual clarity and the insight it provides into the novelty of quantum mechanics, or to the possibility of subjecting it to an empirical test. Bohm [3] took the view that for the present it is not possible to distinguish the various interpretations because they are contrived to reproduce the same set of quantal predictions. This conviction may be unduly pessimistic because there exist what appear to be physically meaningful questions, such as the time taken for systems to move across spatial domains, for which the standard quantum formalism and hence the interpretations which regard it as complete do not give unambiguous answers, whereas the de Broglie-Bohm model does give clear results (in virtue of its particle law of motion). The ill-defined collapse hypothesis is an area where the de Broglie-Bohm theory may lead to alternative empirical predictions.

Previously suggested techniques for empirically investigating the empty wave concept have been found to be unsatisfactory [4]. The quite different method discussed here stems from a technique suggested recently for measuring the wavefunction of a single system [5-7] (this is not to be confused with the well known possibility of reconstructing the wavefunction from a statistical ensemble of measurements [8]). Aharonov and coworkers have shown how using suitably adapted interactions *described by quantum mechanics* (called 'protective' measurements) one can gain information about the wavefunction of an individual system without appreciably disturbing it. Adopting the positivistic attitude that (only) what is measurable is real, one may then infer from this procedure that the wavefunction is an element of physical reality. Let the initial moment of time be $t = T$, the wavefunction to be measured be $\alpha(x,T)$, and the initial wavefunction of the corresponding measuring apparatus be $\beta(y,T)$. Then in the protective interaction envisaged by Aharonov and coworkers the combined initial state $\Phi(x,y,T) = \alpha(x,T)\beta(y,T)$ has evolved at time t into:

$$\Phi(x,y,t) = \alpha(x,t)\beta(y,t)exp\left[-(i/\hbar)\int_T^t g(t)y\langle\hat{B}\rangle dt\right]. \quad (3)$$

Here $g(t)$ is a function characterising the interaction, $\alpha(x,t)$ and $\beta(y,t)$ are the wavefunctions obtained under free evolution of the two systems, and $\hat{B}$ is an operator associated with the system measured. It will be observed that this is still a product state in that the variables x and y have not become entangled. Because it is the expectation value $\langle\hat{B}\rangle$ of $\hat{B}$ in the initial state $\Phi(x,y,T)$ that appears in the exponent in (3), information on the state $\alpha(x,T)$ can be read off from the apparatus by measuring the change in its momentum. For example, we can choose $\hat{B} = |x_0\rangle\langle x_0|$ so that $\langle\hat{B}\rangle = |\alpha(x_0,T)|^2$ and the shift in momentum is proportional to the square of the wave amplitude at the point x_0. It is possible that the Hamiltonian needed to generate the protective interaction depends on the wavefunction being measured which would imply that we must first know the wavefunction before we can measure it. This difficulty does not impinge on the use we make of the protective technique below where the important issue is that the wavefunction of interest has a discernible influence on a non-reactive measuring device (we might prepare the wavefunction in advance by some state preparation procedure).

We apply this method to the wavefunction (1) in the case where the collapse (2) does not occur, that is, when (1) comprises the set of empty waves generated by the measurement interaction in addition to the one corresponding to the actual outcome. Our initial wavefunction is then the function (1); this is the wavefunction to be measured (so that we replace x in the previous paragraph by x and z). Let us suppose that the configuration point (x,z) of the de Broglie-Bohm model lies in the ath summand of $\Psi(x,z,T)$ and that we can determine this fact, and hence the location of corpuscle x, whenever we choose by observing z. Then as stated, in the de Broglie-Bohm theory the other summands are finite but empty from the moment the summands separate and remain so independently of whether or when we become aware of the location of x. Hence, if we apply the technique of Aharonov *et al.* we should be able to measure the finite, empty components of the total wavefunction and demonstrate empirically their reality. To this end, fix attention on the a'th component, $a' \neq a$, choose a point $(x_0,z_0) \in \psi_{a'}(x,T)\phi_{a'}(z,T) \neq 0$ (so that $\psi_{a''}(x_0,T)\phi_{a''}(z_0,T) = 0$ for all $a'' \neq a'$), and measure the operator $\hat{B} = |z_0\rangle|x_0\rangle\langle x_0|\langle z_0|$ according to the above method. Then from (3) we obtain

$$\Phi(x,y,z,t) = \left[\sum_a c_a \psi_a(x,t)\phi_a(z,t)\right]\beta(y,t) \times exp\left[-(i/\hbar)\int_T^t g(t)y|\psi_{a'}(x_0,T)\phi_{a'}(z_0,T)|^2\,dt\right] \tag{4}$$

To test whether in the first measurement the wavefunction has really collapsed simply requires observing the momentum of the detector y which in the ontological interpretation has been shifted by an amount depending on the finite amplitude of the empty wave $\psi_{a'}(x,T)\phi_{a'}(z,T)$. This may be achieved by a conventional momentum measurement which entails extending the configuration space to include the coordinates of the momentum-measuring apparatus.

Successful realisation of this experiment would provide justification for ontological interpretations of quantum mechanics according to which wavefunction collapse does not occur.

References

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