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Competing Sales Channels

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Abstract

We study strategic interactions in a market where producers sell to consumers directly as well as via a competitive channel (CC) such as an online marketplace or price comparison website. We show how the size of the competitive channel can influence market outcomes. Equilibrium falls into one of two regimes: either the CC charges low commission and accommodates producers, or it charges high commission and faces strong competition from producers' direct sales channel. Seemingly pro-competitive developments that increase the number of prices consumers check can raise prices and reduce consumer surplus. We also use the model to study an active policy issue concerning which channels should be allowed to utilize data about consumers' past purchases. (*JEL* D43, D83, L11, M3)

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1 Introduction

Many kinds of goods and services can be purchased both directly from the seller and through a competitive sales channel (CC) where multiple sellers' offers can be considered simultaneously. Examples of CCs include price comparison websites (important in markets such as flights, hotels, telecommunications, and financial services) and online marketplaces such as Amazon Marketplace or eBay that sellers increasingly use as an additional channel.

The first instinct of many commentators has been to welcome the presence of competitive channels on the basis that they facilitate the comparison of prices and therefore strengthen competition between producers. But the operators of competitive sales channels are economic actors in their own right, whose incentives need not align with the interests of consumers.¹ Such concerns have led competition authorities in various jurisdictions to more closely scrutinize industries where CCs play a significant role.² This paper contributes to the ongoing policy debate by studying the implications of competition between (and within) competitive and direct sales channels. We recognize the strategic agency of the competitive channel and construct a model that allows us to examine when and how the direct sales channel (DC) serves to discipline a CC's behaviour and vice-versa.

The relative importance or size of direct and competitive channels varies across markets and over time. Indeed, technology has facilitated the emergence and rapid growth of new kinds of competitive channels (such as price comparison websites). Moreover, the operators of such channels often invest heavily in marketing leading many (such as Expedia in the US, CompareTheMarket in the UK, or Amazon Marketplace globally) to become household names in their own right. Unlike existing work, we introduce a model which allows both competitive and direct channels to vary in their market power, or "size". We then use our model to study the effects of variations in the competitive channel's size and show this to be an important determinant of market prices and consumer surplus. A growth in the competitive channel implies more consumers comparing prices, which is pro-competitive, but alters price competition in subtle ways that mean consumers can end up either better or worse-off overall. We also use the model to study so-called "non-resolicitation" contractual clauses that are used in markets with a competitive sales channel. These clauses dictate which channel will be allowed to utilize consumer data to solicit future sales and have been the subject of policy scrutiny. To our knowledge, we are the first to examine this practice.

¹For instance, recent work has highlighted various practices that might blunt or even eliminate the benefits of price comparison websites—such as price parity clauses (e.g., Edelman and Wright, 2015; Johnson, 2017) and high commission levels (e.g., Ronayne, 2015).

²For example, the UK's Competition and Markets Authority launched a comprehensive examination of so-called Digital Comparison Tools (i.e., price comparison websites, comparison shopping smartphone applications, and similar services) in 2017. See <https://www.gov.uk/cma-cases/digital-comparison-tools-market-study> for details.

To be more precise, the actors in our baseline model are two producers, a competitive sales channel, and a mass of consumers. Producers set prices: one for consumers who buy direct and, if they wish to, another for consumers who buy via the competitive channel. The competitive channel sets a commission fee that is paid by a producer each time it makes a sale through the CC. We adopt a clearinghouse framework à la Varian (1980). Some of the consumers are savvy “shoppers” who buy at the lowest price available anywhere in the market. The remainder of the consumers check only one sales channel and can be thought of as being influenced by brand marketing, being relatively uninformed or inactive, or as facing high search costs. For expositional convenience we term these consumers “loyals”. Loyals go to only one sales channel (either a producer’s direct channel or the competitive channel) and buy at the best price they see there, provided it is below their willingness to pay. The number of loyals a producer or CC has is a measure of its market power or “brand size”. We refer collectively to the masses of loyals and shoppers as the “market composition”.

We compute the equilibrium prices and fees for any given market composition. Equilibrium falls into one of two qualitatively-distinct regimes: low-fee and high-fee. In the case of a relatively weak CC (i.e., a CC with few loyals), the CC sets a *low fee* and hosts the lowest prices in the market. Shoppers buy through the competitive channel, while producers set a high direct price to exploit their loyal consumers. By contrast, a relatively strong CC demands a *high fee*, exploiting its position as bottleneck provider of access to its loyal consumers. To avoid paying the high fee, producers compete for shoppers via direct prices—undercutting the prices on the competitive channel. In this regime it is direct prices that are the lowest in the market.

Armed with this equilibrium characterization, we proceed to analyze the effects of market composition on equilibrium prices and consumer welfare. If the competitive channel becomes larger relative to the direct channel we observe two effects. The direct effect is that more consumers are subjected to the frictionless competitive channel where prices are easily compared, which tends to increase consumer surplus. But there is a countervailing effect that arises through the endogenous competitive responses of the two channels. That is, when a producer has fewer loyal consumers it becomes more focused on attracting price-sensitive shoppers, and therefore more willing to undercut the CC with its direct price. As the number of producer-loyal consumers decreases further, it eventually becomes so difficult for the CC to deter undercutting that it gives up and switches to the high-fee regime (where prices are higher) and focuses on selling to its own loyal consumers. This means that seemingly pro-competitive changes in market composition (such as uninformed consumers becoming informed or adopting a price comparison technology) that result in *more* consumers comparing *more* prices can *increase* prices and *reduce* consumer surplus.

The relative size of the competitive channel is also important in our analysis of non-resolicitation clauses (which determine whether the producer or the CC own the relation-

ship with the consumer). We find that these clauses can increase or decrease consumer surplus, depending on whether the competitive channel is relatively large or small.

1.1 Related literature and contribution

This paper builds on the established literature on price clearinghouses. In such models, a fraction of consumers only consider one firm’s price, while the remainder use a ‘price clearinghouse’ to see all firms’ prices, typically resulting in mixed-strategy pricing. Foundational papers in this literature include Varian (1980) and Rosenthal (1980); a survey can be found in Baye et al. (2006). Baye et al. (1992) extend Varian (1980) to allow non-uniform distributions of loyal consumers among producers. Unlike these early papers, we consider a market with a competitive channel that is a strategic actor in its own right.

In a symmetric setup with a monopolist intermediary, Baye and Morgan (2001) allow a more active role for the clearinghouse. They model a two-sided environment in which the clearinghouse sets fixed advertising and subscription fees in order to get both consumers and producers on board. We provide a new model with features that reflect empirical characteristics of many modern competitive channels: (i) CCs that charge commissions rather than lump-sum fees,³ (ii) producers that can set different prices for direct and CC sales and thus strategically interact with the CC, and (iii) some consumers only consider prices on the competitive channel. This allows us to study a new set of strategic considerations: whereas Baye and Morgan are mostly concerned with the platform’s problem of getting consumers and firms on board, the main trade-offs in our model revolve around simultaneous competition between and within the different sales channels available to firms.

A number of papers study competition within platform-based sales channels, but assume that these are the only sales channels available (e.g., Belleflamme and Peitz, 2010; Boik and Corts, 2016; Hagiu, 2009; Karle et al., 2017). Most recently, the platform literature has focused on issues such as price clauses in markets with differentiated products and platforms (e.g., Boik and Corts, 2016; Edelman and Wright, 2015; Johansen and Vergé, 2016; Johnson, 2017; Wang and Wright, 2016). In this paper, we directly model competition in prices, allow firms to sell through multiple channels, and study the impact of the market composition on equilibrium prices and consumer surplus. Another recent paper that allows competition between sales channels is Shen and Wright (2017), who ask why sellers don’t always undercut intermediaries to ensure sales take place direct. Their answer is that, when sellers set the fee paid to intermediaries, it is cheaper to attract consumers by offering a higher fee than it is by lowering price. In contrast, we study settings where fees are set by the intermediary (CC) and show how undercutting incentives depend on the market composition.

³Baye et al. (2011) allow intermediaries to choose between a fixed or contingent payment and find that it is optimal to set the fixed component equal to zero.

Our work is also conceptually related to a literature on the coexistence of retailers and direct sales by a manufacturer. Much of the focus in this literature has been on incentive conflicts that arise and the role that the direct channel can play in resolving these. For example, Chiang et al. (2003) and Arya et al. (2007) show how the manufacturer-retailer double marginalization problem can be alleviated by a direct channel, while Kumar and Ruan (2006) study the interaction of a manufacturer’s direct sales with a retailer’s incentives to promote its brand. Our work differs from this literature in a number of ways. Firstly, we focus on cases where the producer sets the price for both direct and intermediated sales. Secondly, in contrast to the above papers, we introduce a model that allows us to study the relative size of the two channels and the implications for competitive outcomes.

In an extension, we model firms’ endogenous investment in building consumer loyalty. In this respect, our work relates to a literature on advertising, particularly in a clearinghouse context (e.g., Baye and Morgan, 2009; Chioveanu, 2008). In contrast to these papers, we allow producers to set a direct price and to choose to advertise a price on the CC; for prices to be determined through the strategic interaction of producers and the CC; and for any firm (producer or CC) to engage in marketing activities to determine its market power.

2 Model description

Two producers sell a homogeneous good with zero production costs. Sales are made both directly and (possibly) through a competitive sales channel. Producers are indexed $i = 1, 2$, and the competitive channel, 0. There is a mass of consumers of measure $\mu + L$ who wish to buy one unit of a good and have reservation price $v > 0$. There are two types of consumer: shoppers and loyals. Shoppers are of mass $\mu > 0$. They are informed of all prices, buying at the lowest price available.⁴ Producers each have a mass of loyal consumers, $L_i > 0$, who shop directly i.e., on the producer’s website or at its physical store. Without loss of generality, we index the producers such that $L_1 \leq L_2$. The CC also has a mass of loyal consumers, $L_0 > 0$, who shop exclusively via the competitive channel.⁵

⁴If there is a tie for the lowest price where such prices are either all direct prices or all available through the CC, then shoppers choose one of these producers randomly. If there is a tie in the lowest price where some of these prices are direct and some are via the CC, the shopper completes the purchase directly through the producer’s site. The first tie-breaking rule is standard and allows us to nest the existing literature. The second is not arbitrary. In fact, it is the unique such tie-breaking rule for which there can be an equilibrium for any market composition. The proof can be found in Appendix W.1.

⁵In the context of price comparison websites (PCWs), a recent survey commissioned by the UK’s Competition and Markets Authority (Hanson et al., 2017) found that 58% of users knew which PCW they wanted to use and went straight there. Furthermore, 30% of PCW users consult only one PCW and no other source of price quotes and among the most commonly cited reasons for not shopping around is “brand loyalty”. Elsewhere, online marketplaces also appear to enjoy high degrees of loyalty. For example, web data firm Hitwise reports that in some categories as many as 77% of users conduct their whole search experience through Amazon.

The CC chooses a commission fee, $c \in \mathbb{R}_+$, which a producer must pay for each sale that takes place via the CC. Given c , producers choose a direct price, $p_i^d \in \mathbb{R}_+$, and a price to list on the CC, $p_i \in \mathbb{R}_+$.^{6,7} The solution concept is subgame-perfect Nash equilibrium. The timing of the game is as follows:

$t = 1$ CC sets commission fee c ,

$t = 2$ Producers observe c and set prices p_i^d and p_i ,

$t = 3$ Consumers shop.

We refer to $\mathbf{L} = (L_0, L_1, L_2, \mu)$ as the market composition.

3 Equilibrium

3.1 Pricing subgame

We begin by studying the best-responses of producers in stage $t = 2$ for a given choice of c by the competitive channel. In equilibrium, competition à la Bertrand in prices on the frictionless CC implies that producers make zero profits from any sales made there.

Lemma 1. *Fix $c \leq v$. In any equilibrium, the subgame starting at $t = 2$ must have $\min_i p_i = c$.*

Proofs are in the appendix. Lemma 1 means that every equilibrium of the subgame starting at $t = 2$ is pay-off equivalent to one with $p_1 = p_2 = c$, all else equal. We henceforth take $p_1 = p_2 = c$ as given and focus on each producer’s payoff-relevant choice of direct price.

Suppose that the lowest price in the market (at which the shoppers buy) can be found on the competitive channel. This implies that producers serve only loyal consumers through their direct channel and, therefore, that $p_i^d = v$ with corresponding profit vL_i . The best deviation for the producer would be to set p_i^d just low enough to induce shoppers to buy direct. But this requires the producer to undercut not only its rival’s direct price, but also the prices listed on the CC, i.e., to set $p_i^d \leq c$. The best such deviation yields

⁶Some platforms have sought to prevent discrimination of this kind through so-called ‘price-parity’ contractual clauses. Policymakers have become increasingly hostile to such agreements. For instance, Amazon dropped such clauses in the UK in 2013 under pressure from regulators, and in 2015 a general prohibition on such agreements was imposed in both France and Germany. This leaves firms free to set different fees across channels. For analyses of price-parity agreements, see, e.g., Boik and Corts (2016); De los Santos and Wildenbeest (2017); Edelman and Wright (2015); Hviid (2015); Johansen and Vergé (2016); Johnson (2017); Mantovani et al. (2017); Wang and Wright (2016).

⁷Producers might, in principle, choose not to list on the CC, which is formally equivalent to listing price $p_i > v$.

profit $c(L_i + \mu)$ and is, therefore, not profitable if

$$c \leq \underline{c}_i \equiv \frac{vL_i}{L_i + \mu}. \quad (1)$$

Producer 1 finds such a deviation profitable for lower levels of c relative to producer 2 because producer 1 has fewer loyals and therefore loses less from a reduction in its direct price. If condition (1) is satisfied for both producers then the unique equilibrium behaviour is for producers to set the monopoly price on their direct channel and allow the CC to serve the shoppers.

Lemma 2 (Low-fee regime). *Suppose $0 \leq c \leq \underline{c}_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$ and $p_1^d = p_2^d = v$. The resulting equilibrium profits are $\pi_0 = c(L_0 + \mu)$, $\pi_1 = vL_1$, $\pi_2 = vL_2$. When $0 \leq c < \underline{c}_1$, this equilibrium is unique.*

Now suppose $\underline{c}_1 < c < \underline{c}_2$. In this range, producer 1 finds it worthwhile to undercut the CC prices in order to attract shoppers: the resulting increase in demand more than compensates it for foregone monopoly rents on its few loyal consumers. Producer 2, on the other hand, earns more from monopoly pricing on its loyals and is unwilling to cut its price as low as c . This gives rise to a subgame equilibrium in which producers pursue asymmetric strategies with shoppers buying directly from producer 1:

Lemma 3 (Mid-fee regime). *Suppose $\underline{c}_1 \leq c \leq \underline{c}_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = c$, and $p_2^d = v$. The resulting equilibrium profits are $\pi_0 = cL_0$, $\pi_1 = c(L_1 + \mu)$, and $\pi_2 = vL_2$. When $\underline{c}_1 < c \leq \underline{c}_2$, this equilibrium is unique.*

For $c > \underline{c}_2$, both producers find it profitable to undercut the CC and fight for shoppers using direct prices. Because more than one producer now uses a single variable (direct price) to trade off competition for shoppers against sure sales to loyals, the equilibrium producer strategies are mixed. The equilibrium strategies, which also have to take the CC's fee into account, are given by Lemma 4.

Lemma 4 (High-fee regime). *Suppose $\underline{c}_2 < c \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, p_1^d and p_2^d mixed over supports $[\underline{p}, c]$ and $[\underline{p}, c] \cup v$ respectively via the strategies*

$$F_1(p) = \begin{cases} \frac{\mu p - L_2(v-p)}{\mu p} & \text{for } p \in [\underline{p}, c) \\ 1 & \text{for } p \geq c, \end{cases}$$

$$F_2(p) = \begin{cases} \frac{\mu p - L_2(v-p)}{\mu p} \frac{\mu + L_1}{\mu + L_2} & \text{for } p \in [\underline{p}, c) \\ \frac{\mu c - L_2(v-c)}{\mu c} \frac{\mu + L_1}{\mu + L_2} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

where $\underline{p} = \frac{vL_2}{L_2+\mu}$. The resulting profits are always $\pi_0 = cL_0$, $\pi_1 = \frac{vL_2(L_1+\mu)}{L_2+\mu}$, and $\pi_2 = vL_2$ for any $L_1 \leq L_2$. When $L_1 < L_2$ these equilibrium strategies are unique.⁸

3.2 Competitive channel fee-setting

Now we solve the game starting at $t = 1$. Accordingly, consider the incentives of the competitive channel when it chooses its fee level, c . For all levels of $c > 0$, the CC makes positive profit from the fees paid by producers for purchases made by its L_0 loyalists. In addition, when c is in the low-range of Lemma 2, producers do not offer a direct price that is competitive with the prices listed on the CC, so shoppers also buy through the competitive channel. For higher levels of c , at least one producer sets a direct price lower than all prices on the competitive channel, leaving the CC selling only to its loyalists. The CC therefore faces a trade off between i) a low-fee regime where it facilitates sales to shoppers and loyalists; and ii) a high-fee regime where it facilitates sales only to loyalists. By Lemmas 2–4, the highest c such that the competitive channel sells to shoppers and its loyalists is \underline{c}_1 , whereas the highest c such that it sells only to its loyalists is v . Hence the CC prefers i) to ii) iff:

$$vL_1 \frac{L_0 + \mu}{L_1 + \mu} \geq vL_0 \iff L_0 \leq L_1,$$

i.e., if the CC is small relative to the producers. This reflects the ways that the market composition affects the CC's profits. A higher L_1 increases \underline{c}_1 and hence profits for the CC in the low-price regime. On the other hand, a higher L_0 increases profits in both regimes, but increases profit by more in the high-fee regime precisely because that is where the marginal revenue is higher. Our model reveals that a comparison of brand-size is central to a competitive channel's trade-off. If the CC has a relatively strong brand, it chooses to set a high-fee and generate profit through sales to its loyalists. When it has a relatively weak brand, it sets a low-fee in order to capture shoppers. Proposition 1 formalizes these forces and states the equilibrium.

Proposition 1 (Equilibrium). *When $L_0 < L_1$, the low-fee equilibrium results: the competitive channel sets $c = \underline{c}_1$ and producers price in accordance with Lemma 2. When $L_0 > L_1$, the high-fee equilibrium results: the competitive channel sets $c = v$ and producers price in accordance with Lemma 4. When $L_0 = L_1$, both equilibria exist.*

When the CC has a relatively weak brand i.e., does not have many loyal customers relative to the producers, the low-fee regime results. Through its low fee, the CC disciplines the market, allowing shoppers and CC loyalists to reap the benefit of the fierce Bertrand competition that takes place there through a low equilibrium price. However, when the CC has a relatively strong brand, it chooses to set a high fee in equilibrium,

⁸See the Web Appendix for details on the case of $L_1 = L_2$, where equilibrium strategies can be slightly different.

preferring to exploit its loyal consumers rather than serve shoppers. Analysis by the UK’s Competition and Markets Authority accordingly finds that (in the insurance industry) an increase in the size of producers tends to be associated with lower CC commissions, while commissions tend to be higher for bigger CCs (CMA, 2017).

4 Effect of market composition

Proposition 1 suggests that market composition may be an important determinant of equilibrium market outcomes. But composition varies both across markets and across time.⁹ Given that competitive channels reduce search frictions and help consumers to compare prices, should a larger CC unambiguously be welcomed? For many, the obvious answer is yes. But our model suggests that caution is warranted.

To see why, let us begin by considering what happens to market prices when the CC grows by enough to switch equilibrium from the low-fee to the high-fee regime. When the high-fee regime occurs, the competitive force through producers’ direct prices (given by the mixed strategies of Lemma 4) is less than that exerted through producers’ prices listed on the competitive channel (where prices are subject to Bertrand competition). This yields the result stated in Corollary 1 that when a CC grows, *ceteris paribus*, such that the equilibrium switches from the low-fee to the high-fee regime, prices rise. Specifically, both the minimum and average prices in the market are higher in the high-fee regime.

Corollary 1 (Market Prices). *For $\mathbf{L} = (L_0, L_1, L_2, \mu)$ and $\mathbf{L}' = (L'_0, L_1, L_2, \mu)$ such that $L_0 < L_1 < L'_0$, both the minimum and average prices available in the market are higher under \mathbf{L}' with probability one.*

Next, we consider the effect of a growing CC on consumer surplus, derived in Lemma 5.

Lemma 5. *Equilibrium consumer surplus is*

$$CS = \begin{cases} v\mu \frac{L_0 + \mu}{L_1 + \mu} & \text{for } L_0 \leq L_1 \\ v\mu \frac{L_1 + \mu}{L_2 + \mu} & \text{for } L_0 \geq L_1. \end{cases} \quad (2)$$

Now consider the effect on consumer surplus of a change in market composition from $\mathbf{L} = (L_0, L_1, L_2, \mu)$ to some $\mathbf{L}' = (L'_0, L'_1, L'_2, \mu')$. To focus on understanding the effects of variations in the relative size of a competitive channel, it is useful to cast the differences

⁹For example, figures from the UK’s Competition and Markets Authority show that consumers are around twice as likely to rely on a price comparison site when buying motor insurance than when buying home broadband, while the largest PCW in the UK saw its sales increase five-fold between 2010 and 2016.

between the compositions \mathbf{L} and \mathbf{L}' in terms of an increase or decrease in the CC's size:

$$\begin{aligned} L'_1 &= L_1 - \Delta_1 \\ L'_2 &= L_2 - \Delta_2 \\ \mu' &= \mu - \Delta_\mu \\ L'_0 &= L_0 + \Delta_1 + \Delta_2 + \Delta_\mu + \tilde{\Delta} \end{aligned}$$

where the Δ s may be positive or negative. A competitive channel might grow because the market expands (higher $\tilde{\Delta}$), because it is adopted by consumers who were previously loyal to a producer (higher Δ_i), or because it is adopted by consumers who were previously shoppers (higher Δ_μ). The following proposition describes the effect of CC growth on consumer surplus.

Proposition 2. *Consumer surplus*

1. *is non-monotonic in Δ_1 and Δ_2 ;*
2. *is non-monotonic in $\tilde{\Delta}$ if $L_1 < L_2$; non-decreasing in $\tilde{\Delta}$ when $L_1 = L_2$;*
3. *is decreasing in Δ_μ .*

Parts 1 and 2 of Proposition 2 say that consumer surplus changes non-monotonically as the competitive channel grows. Conceptually, part 1 refers to the following exercise: suppose a mass of consumers loyal to a producer (i.e., those who see only one price) switch to using the competitive channel where competition is à la Bertrand. Although this corresponds to an increase in the number of consumers' comparing price quotes and might ordinarily be expected to intensify competition, the result can be a fall in consumer surplus. Similarly, part 2 of Proposition 2 says that a pure expansion in demand can also cause a fall in consumer surplus when the new consumers rely on the CC.

These results emerge because an increase in Δ_1 , Δ_2 , or $\tilde{\Delta}$ causes the competitive channel to become larger relative to the producers. This has three effects: (i) consumers compare more prices, (ii) the CC has more loyal consumers to exploit, and (iii) in the case of an increase in Δ_1 or Δ_2 , producers have fewer loyals and are more willing to fight for shoppers. The first and third effects, in isolation, tend to make the market more competitive. But the second and third effects together make the CC less interested in competing for shoppers, and more likely to set high fees. Proposition 2 establishes that these latter, anti-competitive effects eventually dominate, to the detriment of consumers.

Part 3 of Proposition 2 says that consumers who use solely the competitive channel are not as effective as shoppers when it comes to exerting competitive discipline on producers' pricing. Both CC loyals and shoppers cause fierce pricing within the CC, but shoppers also exert competitive pressure between channels. As a result, if shoppers become CC loyals then consumer surplus falls.

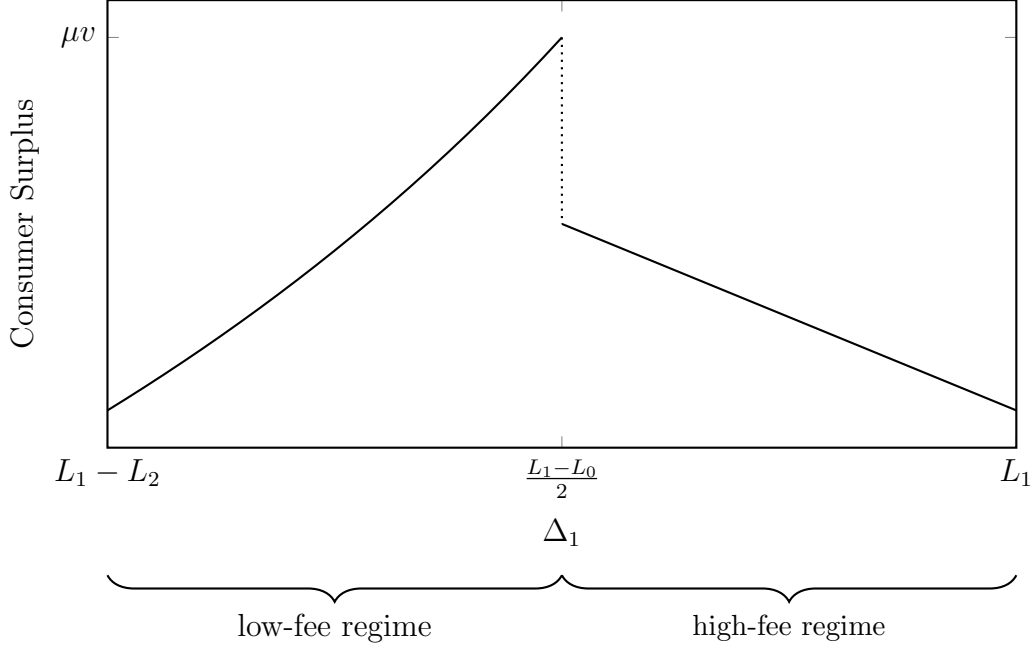


Figure 1: Effect of Δ_1 on consumer surplus, assuming $\Delta_2 = \Delta_\mu = \tilde{\Delta} = 0$.

As an example, point 1 of Proposition 2 is illustrated in Figure 1. The discontinuity arises because the CC endogenously chooses whether to fight for shoppers or exploit its loyalists. Once it reaches a sufficient size it switches from the former strategy to the latter, with different equilibrium pricing strategies arising as a consequence, taking the market from the low-fee regime, to the high-fee regime (as shown, once in the high-fee regime, consumer surplus is decreasing in Δ_1).

Exploiting this change in fee regime allows the following result, which says that more general changes in market composition where the competitive channel grows relative to a producer, can reduce consumer surplus.

Proposition 3. *For \mathbf{L} and \mathbf{L}' such that $L_1 - L_0 > 0 > L'_1 - L'_0$, there exists an $\epsilon > 0$ such that \mathbf{L} yields (weakly) higher consumer surplus than \mathbf{L}' if $\|\mathbf{L} - \mathbf{L}'\| < \epsilon$.*

Proposition 3 describes generally how consumer surplus behaves around the point where the equilibrium regime changes. It says that consumer surplus tends to fall following a change in market composition which takes the market out of the low-fee regime and places it in the high-fee regime. Proposition 3 also allows us to consider general redistributions of market power, beyond those considered in Proposition 2. One implication of the result is the following corollary:

Corollary 2. *For $\Delta_\mu \equiv -\Delta_1$, consumer surplus is non-monotonic in Δ_μ .*

In words, the corollary says that taking consumers who are captive to a single producer and turning them into fully-informed shoppers who compare all prices can reduce aggregate consumer surplus. This is because losing loyal consumers weakens the producer relative to the CC. This implies that competitive channels may affect the market

in ways that are anti-competitive even if their main effect were to provoke consumers to shop-around (a phenomenon often referred to as showrooming).¹⁰ Thus, optimistic assessments of competitive channels should be accompanied by the caveat that they can have harmful indirect effects on pricing incentives brought about by shifts in the balance of power between the channels in the market.

As well as overall consumer surplus, we can consider how changes in the market composition affect the distribution of surplus among consumers. To this end, it is useful to invoke Armstrong (2015)'s notions of search and rip-off externalities. Does the presence of informed shoppers serve to discipline firm pricing and thereby benefit loyal, or does the presence of shoppers induce firms to somehow exploit loyal to their detriment? Clearing-house models typically involve search externalities and, indeed, are used by Armstrong as the pro-typical example of how such externalities arise. But competition between channels can produce both search and rip-off externalities because of the subtle ways that changes in market composition affect pricing incentives.

To be more precise, fix \mathbf{L} and \mathbf{L}' such that the total mass of consumers is unchanged¹¹ and $\mu' > \mu$. We say that there is a *search externality* if all loyal consumers are weakly better-off under \mathbf{L}' than \mathbf{L} (with some strictly so). We say that there is a *rip-off externality* if \mathbf{L}' leaves shoppers better-off but makes at least one type of loyal consumer worse-off.¹²

Remark 1. *The model exhibits both search externalities and rip-off externalities.*

Search externalities arise for a similar reason as in previous clearinghouse models (e.g., Varian, 1980): price-sensitive shoppers increase the pay-off to firms that offer lower prices and thereby induce a reduction in equilibrium price offers; even loyal consumers benefit from these lower prices. This logic, though, is conditional upon remaining within the prevailing fee regime. If consumers become informed and thereby turn into shoppers, the result can be a switch between the low and high-fee regime. The low-fee regime allows producers to identify and extract all surplus from their loyal consumers, while the high-fee regime leads the competitive channel to exploit its own loyal. Thus, a change in fee regime (in either direction) generally results in some loyal consumers being harmed.

5 Non-resolicitation clauses

The issue of which party is allowed to utilize consumer data to solicit future sales is particularly relevant in markets where direct and competitive channels operate. In particular,

¹⁰See Shen and Wright (2017) for a model in which consumers can use an intermediary to learn about offers before switching to buy direct (in an environment where the commission paid to the CC is set by producers).

¹¹That is, $L_0 + L_1 + L_2 + \mu = L'_0 + L'_1 + L'_2 + \mu'$.

¹²Our definition departs a little from that used by Armstrong, where the surplus of loyal and shopper types tend to move in the same direction. But the spirit is the same: as more consumers become informed, there are externalities on others that could be either positive or negative.

“non-resolicitation” clauses have raised concerns because they manipulate the allocation of attention between a competitive sales channel and producers in a manner that some fear to be harmful. Our model can address such policy issues since it has an explicit role for the distribution of consumer attention.

Consumers often make a repeat purchase of the same or a similar product (e.g., insurance policies are often renewed annually). When a sale is conducted through the CC, both the chosen producer and the CC are party to the transaction, and either could potentially lay claim to data about the consumer. Data is valuable because it can be used in marketing aimed at steering repeat purchasers to buy through a particular sales channel. Non-re-solicitation clauses (NRCs) are contractual agreements between a CC and a producer that govern who is allowed to exploit consumers’ data by restricting the ability of one channel to communicate with consumers who previously bought through the CC.

The UK’s competition regulator has raised concerns about NRCs in insurance markets, worrying that a ban on CCs contacting consumers, which we refer to as “CC-binding” NRCs, may lead consumers to automatically renew directly with their existing provider at inflated prices.¹³ In contrast, online marketplaces like Amazon or eBay use conditions of sale to prevent sellers communicating directly with buyers which we classify as “DC-binding” NRCs.¹⁴

To model NRCs, we consider a stationary overlapping generations extension of our model where consumers each live for two periods. Consumers are ‘young’ in the first period of their life and ‘old’ in the second; a unit mass of young consumers enters the population in each period. The young generation are first-time buyers and behave (myopically) as in the baseline model according to the market composition \mathbf{L} . The old generation are inert/inactive and only respond to direct solicitations, buying at the lowest price offered to them. Old consumers who bought direct when young receive a renewal price \hat{p}_i^r from their previous supplier, i . Old consumers who bought via the competitive channel receive a set of offers determined by the NRC (if any) in place. We consider three cases. Firstly, under a *DC-binding NRC* only the competitive channel is able to contact them, so they are offered prices p_1 and p_2 . Under a *CC-binding NRC*, these old consumer receive only a personalized renewal quote from their existing supplier (p_i^r). Lastly, if *no NRCs* are in place (e.g., because they are banned) then both the CC and existing supplier are able to contact the consumer who therefore considers all three prices (p_1 , p_2 , and p_i^r).¹⁵

We look for stationary equilibria in which $(c, p_i, p_i^d, \hat{p}_i^r, p_i^r)$ are constant across periods.

¹³See paragraphs 65–69 of <https://assets.publishing.service.gov.uk/media/58d9310940f0b606e7000036/dcts-update-paper-appendices.pdf>, accessed 29 August 2017.

¹⁴For example, Amazon outlaws such practices as can be seen in their General Guidelines: <https://www.amazon.com/gp/help/customer/display.html?nodeId=200414320>, as does Ebay in their Policy Overview: <http://pages.ebay.com/help/policies/rfe-spam-ov.html>

¹⁵We have assumed that old consumers receive personalized renewal quotes from producers, but see the same CC prices as young consumers. This assumption is motivated by current practice in some price comparison markets where the use of NRCs has been subject to policy scrutiny.

Appendix C computes equilibrium behaviour in this model, with the results summarized as follows:

Lemma 6. *In each regime (CC-binding, DC-binding, no NRCs) there exists an equilibrium with a similar structure to the baseline case. In particular:*

1. *equilibrium is either a high-fee regime with firms mixing their direct prices and shoppers buying direct or a low-fee regime with high direct prices and shoppers buying via the CC;*
2. *there exists a cut-off value of L_0 below which the low fee regime emerges and above which the high fee regime emerges.*

Equilibrium has similar features to the baseline model but yields different pricing and fee-setting behavior. In particular, producers offer introductory discounts when they anticipate serving consumers at a profit in future periods, and the competitive channel takes lifetime competition with producers' direct channels into account when setting c . Building on this characterization of equilibrium we can now state the following result on the effects of NRCs for consumer surplus.

Proposition 4. *There exists an $\tilde{L} \in [\frac{L_1 - \mu}{2}, L_1]$ such that, compared with a policy intervention banning the use of NRCs,*

1. *a DC-binding NRC has no effect on consumer surplus;*
2. *a CC-binding NRC decreases consumer surplus if $L_0 < \tilde{L}$ and (weakly) increases consumer surplus if $L_0 > \tilde{L}$. If $L_1 < L_2$ then the increase is strict over the interval $L_0 \in (\tilde{L}, L_1)$.*

Figure 2 illustrates proposition 4. Under a CC-binding NRC, prices on the competitive channel are low because producers compete to attract young consumers (who will become captive when old). Since undercutting these low prices is costly, the CC can increase its fee without fear of being undercut and the CC-binding NRC makes the low-fee regime more attractive to the competitive channel. We thus observe two effects of an NRC: (i) within the low-fee regime, it increases CC fees (and hence prices), but (ii) it makes a low fee regime result for a wider range of parameters. A small CC (low L_0) implements a low-fee regime regardless of whether NRCs are permitted or not, so only the first effect survives and an NRC ban benefits consumers. If L_0 is larger, a ban on NRCs may cause a switch from the low- to the high-fee regime, which can harm consumers.

A DC-binding NRC reduces the lifetime value to producers of young consumers served via the competitive channel—and thus the c that they can be charged. But it also guarantees that the CC will serve consumers in both periods of their life instead of only the first. These two effects (lower fees, paid more often) exactly offset each other so that a ban on NRCs leaves both the CC's incentive to induce a low-fee regime and consumers' lifetime expenditure unchanged.

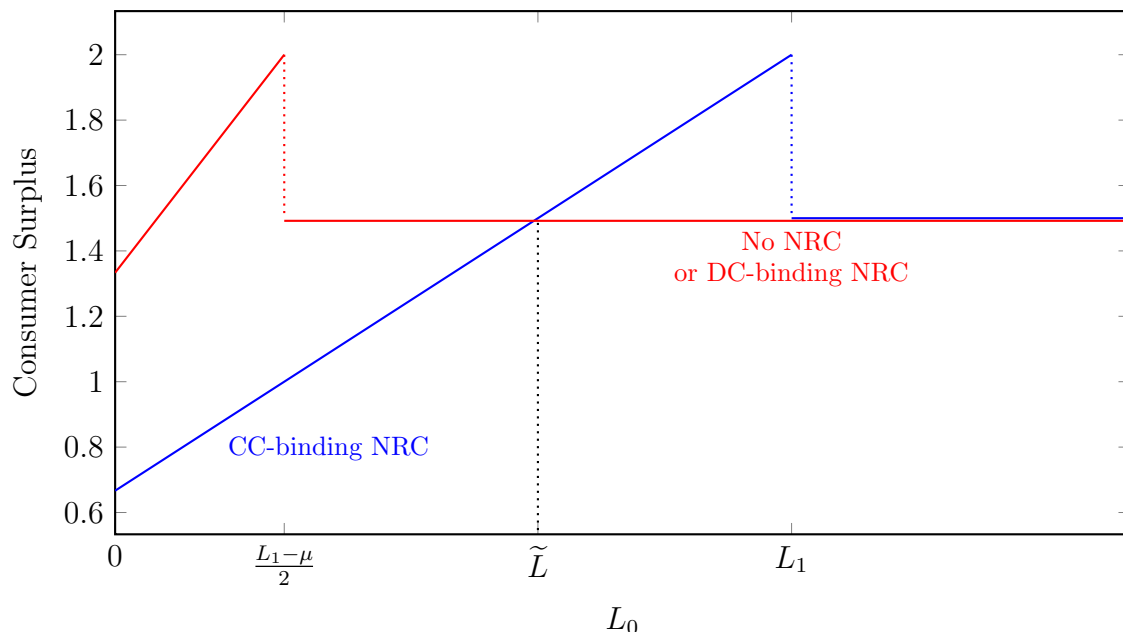


Figure 2: Consumer surplus under various NRCs where $v = \mu = 1$, $L_1 = 2$, $L_2 = 3$.

6 Extensions and Robustness

6.1 Endogenously-Determined Market Composition

In Section 4, we conducted comparative statics of the market composition. This allowed us both to make comparisons across markets with different compositions, and to consider the implications of shifts in power within a particular market. We found that consumer surplus can fall both when the competitive channel is larger and when consumers make more price comparisons. Both of these effects are due to the presence of the high-fee regime. In this section, we study the endogenization of market composition. This allows us to ask which regime we may expect to see in the long-run.

We add a pre-stage to the game where all firms (producers and the competitive channel) compete for market power using a costly technology. This can be interpreted as determination of the brand loyalty through advertising and marketing efforts. In reality, there is fierce competition between channels as reflected by high ad spends, not only by producers themselves, but also by competitive channels. In the UK, price comparison websites spent £123m in 2014-15¹⁶, with the biggest four entering the top 100 companies in the country by ad spend.¹⁷ Online giant Amazon's total global marketing spend was \$7.2bn in 2016 and it appears to have accumulated a position as the go to source for shopping searches.

To model this inter and intra-channel competition for market power formally, we

¹⁶Financial Times: <https://www.ft.com/content/8997ec7a-2f01-11e6-bf8d-26294ad519fc>

¹⁷This is Money: <http://www.thisismoney.co.uk/money/bills/article-2933401/Energy-price-comparison-sites-spend-110m-annoying-adverts.html>

add period $t = 0$ to the game. The firms $i = 0, 1, 2$ simultaneously choose their level of market power i.e., number of loyal consumers, L_i .¹⁸ However, the determination of market power is costly. The cost to player i of choosing L_i given the other players choose L_{-i} is $\psi_i(L_i|L_{-i})$ where $\psi_i : \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is twice continuously differentiable. We make the following additional assumptions on the cost function for $i, j = 0, 1, 2$:

1. Symmetry: $\psi_i = \psi_j \equiv \psi$
2. Costs nothing to do nothing: $\psi(0|\cdot) = 0$
3. Increasing: $\frac{\partial \psi(L_i|L_{-i})}{\partial L_i} = 0$ for $L_i = 0$; $\frac{\partial \psi}{\partial L_i} > 0$ for $L_i > 0$
4. Convex: $\frac{\partial^2 \psi(L_i|L_{-i})}{\partial L_i^2} > 0$
5. Inter-dependency: $\frac{\partial \psi(L_i|L_{-i})}{\partial L_j} = 0$ for $L_i = 0$; $\frac{\partial \psi(L_i|L_{-i})}{\partial L_j} > 0$ for $L_i > 0$

We consider only equilibria which feature pure-strategies at the $t = 0$ stage. Denote any cost function satisfying assumptions 1-5, ψ , and the set of all such functions Ψ . Let Ψ_L (Ψ_H) denote the set of all functions ψ under which a low-fee regime (high-fee regime) equilibrium exists. In general, for a given cost function ψ there may be many equilibria of the game. However, without the need to be more specific about the cost function in question, we report the following result.

Proposition 5. *There are more cost functions that lead to a high-fee regime than a low-fee regime in equilibrium: $|\Psi_H| \geq |\Psi_L|$.*

Proposition 5 says that there are more cost functions which allow the high-fee regime than there are those that allow the low-fee regime, in equilibrium. The proof shows that for any given cost function ψ , if there is a low-fee regime equilibrium, there is also a high-fee regime equilibrium. This follows because producers are more tempted to deviate from a market composition in the low-fee regime than one giving the high-fee regime. Given a composition yielding the low-fee regime i.e., when the competitive channel is small, deviating to a level of advertising (a choice of L_i) just lower than the CC prompts the high-fee regime instead. In the ensuing subgame, producers fight for shoppers with their direct price where the deviating producer fights the hardest and is rewarded with a high chance of selling to the shoppers. In contrast, for a composition yielding a high-fee regime, no such incentive exists because when the competitive channel is large, producers already serve the shoppers.

The result shows that in our framework, the high-fee regime is not an abstract possibility. Rather, the model predicts more situations where the competitive channel is large, and in equilibrium exploits their market power with high commissions.

¹⁸We hold μ fixed for this exercise. The results presented hold for any $\mu > 0$.

6.2 Percentage fees and positive marginal cost

In practice, competitive channels generally use per-sale fees. Sometimes these are in the form of flat commission rates, independent of the sale price; sometimes they are ad valorem, usually a percentage of the sale price; and sometimes there is a combination of the two.¹⁹ The baseline model we presented employed a flat fee, c , rather than an ad valorem commission. In this subsection we instead allow for arbitrary combinations of flat and ad valorem fees. We also allow for a positive marginal cost of production, $m \in [0, v]$ which producers incur for each unit sold.

Suppose a competitive channel can charge both a constant and a percentage commission, denoted $c \geq 0$ and $\tau \in [0, 1]$ respectively. It can be shown that $m + c > 0$ must hold in equilibrium and we focus on such situations to economize on the number of cases to be considered.²⁰

In our framework, the price set by producers on the competitive channel is a sufficient statistic to characterize the equilibrium. This price, in turn, is determined by c, τ . In other words, it does not matter what the exact level of c and τ are, what matters is the price on the competitive channel, p , that they generate. Indeed one p corresponds to many different (c, τ) pairs. Furthermore, in equilibrium the competitive channel's profits are not affected by (c, τ) other than through p , and hence the CC is indifferent between any of the (c, τ) pairs corresponding to that p . For a given p , producers also have the same equilibrium profit and consumers have the same surplus. Therefore, the analysis of the preceding sections is unchanged under the alternate assumption that commissions are a percentage of the sale price. For completeness, the equilibrium is stated in terms of p in the Appendix. Here, we make this equivalence between variables explicit.

Bertrand competition between producers on the competitive channel ensures that the equilibrium price there, p , gives producers zero profit through sales on the CC:

$$p(1 - \tau) - c - m = 0 \iff p = \frac{c + m}{1 - \tau}.$$

Suppose we would like to study equilibrium in the pricing subgame given arbitrary flat and percentage commissions (c, τ) . The induced CC prices are the same as those with alternative fee structure (c', τ') where $c' = \frac{c+m\tau}{1-\tau}$ and $\tau' = 0$. Moreover, because p is sufficient for equilibrium characterization, equilibrium under the two fee structures is equivalent and there is no loss from assuming $\tau = 0$.

¹⁹[Amazon](#) and [Ebay](#) make the majority of their third-party-generated revenue through percentage fees to sellers. Travel reservation websites such as [Booking.com](#) and [Expedia](#) [are understood](#) to charge their sellers 15-25% of the sale price. Price comparison sites in the UK [are reported](#) to charge flat fees per sale.

²⁰Briefly, if $m + c = 0$ then firms' net marginal cost is zero and Bertrand competition on the CC drives prices there to zero, implying the CC earns zero profit. The CC will therefore always choose a strictly positive c if $m = 0$.

6.3 Commission discrimination

In some markets, competitive channels do not charge different sellers different commission levels. For example, one can easily check seller fees on eBay and Amazon. However, in some of those markets e.g., hotel reservation sites, it is understood that there is a common base commission rate²¹ available to any hotel even though fees are set in private. Our framework allows firms to have different levels of market power. This suggests that a model should not constrain the competitive channel to charge the same commission level to all producers. However, here we show that this assumption is without loss of generality. We show that when the competitive channels can charge producers different commissions, it sets these to be equal in equilibrium.

Formally, allow the competitive channel to set two fee levels c_1, c_2 in $t = 1$ which are the commissions that producers 1 and 2 respectively pay upon a sale through the CC. In the Appendix we provide the analogous low, mid and high fee regimes that result as subgame equilibria in $t = 2$ given fee levels c_1, c_2 . We then show that when the competitive channel can set different fee levels for different producers it nevertheless sets $c_1 = c_2$.

To understand the result, suppose that instead $c_i < c_j$. Prices via the competitive channel are competed à la Bertrand down to c_j , but producer i is able to keep undercutting as $c_i < c_j$ and so secures all the sales through the CC at a fee of c_i . This means that the CC's income is solely determined by c_i but prices on the CC are set by c_j . The proof shows that the CC always wants to either increase c_i to receive more per-sale over the same number of sales, or to decrease c_j to lower CC prices and make undercutting more costly. Thus, when the CC can set discriminatory fee levels, it chooses not to.

7 Conclusion

We have studied an environment in which producers sell to consumers both directly and via a competitive sales channel. A competitive channel is a platform (such as a price comparison website or online marketplace) that lists price offers from multiple producers. A key question is whether the most price-sensitive consumers buy directly from the producer or via the competitive sales channel in equilibrium—a tension that causes equilibrium in our model to fall into one of two regimes. In the first regime, the competitive channel attracts the most sensitive consumers by demanding a low fee which results in it hosting the lowest prices on the market while producers instead focus on serving their captive consumers at high prices, through their direct channel. In the second regime the situation is reversed in the sense that fees (and hence prices) on the competitive channel are high while producers undercut it and compete with each other in their direct prices, serving the price-sensitive consumers. The more consumers rely on the competitive channel, the

²¹e.g., 15% on booking.com: <http://blog.directpay.online/booking-com>

more tempted the competitive channel is to increase its fee to exploit its position as a bottleneck gatekeeper. Moreover, as producers' captive audience shrinks, price-sensitive shoppers account for a growing share of their potential demand—increasing the temptation to undercut prices on the competitive channel and serve these consumers directly. Thus, the equilibrium regime changes with the relative size of the competitive channel and the producers.

These equilibrium dynamics have interesting implications for consumer welfare. Suppose consumers become less captive to individual firms and start comparing prices across firms more often (either by using the competitive channel, or by independently become more informed shoppers). The direct and immediate effect of more widespread price comparisons is to increase the intensity of competition between producers and cause them to lower their prices (i.e. competition within channels is strengthened). However, there is a countervailing effect, namely that the equilibrium regime can change in such a way that prices on the competitive channel increase, making it a less effective discipline on prices more generally (i.e., competition between channels is weakened). This countervailing effect means that consumer surplus can fall even as consumers become informed of a greater number of price offers. The key to understanding the result is that when the competitive channel is large enough relative to the producers' direct channels, equilibrium switches from one where price competition is most active within the competitive channel to one where it is most active within the direct channel. However, because competition is always more aggressive within the competitive channel such a switch can cause prices to rise and consumers to become worse off.

Within this framework, we analyse the effect of non-resolicitation clauses which are a feature of markets with both direct and competitive channels and have received attention from regulators. The clauses control whether the producer and/or competitive channel are able to contact past customers with follow-up marketing. Here, too, the relative size of the competitive channel is important for welfare evaluation. In particular, a large competitive channel that might otherwise provide little competitive discipline for the market can be induced to fight for price-sensitive consumers if it is denied access to data on purchase histories, which can raise consumer surplus. But such a contractual constraint helps producers to exploit inactive consumers, which reduces consumer surplus when the competitive channel is small.

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Appendix

A Baseline Model

A.1 Proof of Lemma 1

Lemma A1. *No firm lists any price strictly less than c on the PCW in equilibrium.*

Proof. Suppose there is a positive probability that prices strictly lower than c are listed on the PCW in equilibrium. There is then some price posted by some firm i , $p_i < c$, which has some positive probability, $\psi > 0$, of being the lowest posted on the PCW. When called upon to play p_i (and some arbitrary $p_i^d \geq 0$), firm i 's profit is,

- (i) $p_i^d L_i + \psi(p_i - c)L_0$ if shoppers do not buy from i ,
- (ii) $p_i^d(L_i + \mu) + \psi(p_i - c)L_0$ if shoppers buy from i directly, or
- (iii) $p_i^d L_i + \psi(p_i - c)(L_0 + \mu)$ if shoppers buy from i via the PCW.

A deviation to $p_i > v$ would cause profit in these three cases to change to (i) $p_i^d L_i$, (ii) $p_i^d(L_i + \mu)$, and (iii) either $p_i^d L_i$ or $p_i^d(\mu + L_i)$. In all cases the deviation strictly increases profit. \square

Denote the distribution of the lowest price listed on the PCW as G_{\min} . Denote the minimum and maximum of the support of G_{\min} by \underline{s} and \bar{s} respectively.

Lemma A2. *When $c < v$, $\underline{s} = c$*

Proof. By Lemma A1 we know $\underline{s} \geq c$. Now we show that $\underline{s} > c$ is not possible. Firstly, consider $\bar{s} = \underline{s} > c$, then at least one firm plays \underline{s} with probability one (but not all firms play \underline{s} with probability one, else one of them could profitably undercut). Denote firm i as one of the firms not playing \underline{s} with probability one and note that i could profitably decrease its PCW price to $p_i = \underline{s} - \epsilon$ in order to strictly increase its profit (in the case where $p_i^d = \underline{s}$ and there is a positive probability this is the cheapest direct price, then in addition consider firm i deviating to $p_i^d = \underline{s} - 2\epsilon$ to protect direct sales). Secondly, consider $\bar{s} > \underline{s} > c$. Then all firms must make positive profits via the PCW: if not, a firm, i , with zero profit could deviate to $p_i = \underline{s} - \epsilon$ (in the case where $p_i^d = \underline{s}$ and there is a positive probability this is the cheapest direct price, then in addition consider firm i deviating to $p_i^d = \bar{s} - 2\epsilon$). As firms make positive profits through the PCW and must be indifferent between all prices they play, all prices listed are no greater than v . Consider the highest price in the union of all the supports of all firms, \bar{p} , where $\bar{s} \leq \bar{p} \leq v$. If there is a positive probability of a tie at \bar{p} , any firm i would shift the associated probability mass to $\bar{p} - \epsilon$ (and wherever $p_i^d = \bar{p}$ and there is a positive probability this is the cheapest price, then in addition i deviates to $p_i^d = \bar{p} - 2\epsilon$). If there is a zero probability of a tie, any firm called upon to play a price at or approaching \bar{p} will make zero profit, a contradiction. \square

Lemma A3. *When $c < v$, $\bar{s} = c$*

Proof. From Lemma A2, $\underline{s} = c$. Therefore some firm i has c in its support and makes zero profit via the PCW when it plays $p_i = c$ (or vanishing profit as $p_i \searrow c$). If $\bar{s} > c$, then when i is called upon to play $p_i = c$ (or $p_i \searrow c$), it can instead play $\bar{s} - \epsilon$ for $\epsilon > 0$ small and net a strictly higher profit. \square

Lemma A4. *When $c < v$, at least two firms list $p = c$ with probability one.*

Proof. Lemmas A2 and A3 show $\bar{s} = \underline{s} = c$. This implies that there is at least one firm, say firm i , which lists c on the PCW with probability one. Suppose i was the only firm to do so. If so, i makes zero profit through the PCW, yet could profitably deviate to some slightly higher price at which there is a positive probability of having the lowest listed price. Such an increase in p_i cannot reduce demand through i 's direct sales channel, but would strictly increase expected profit through the PCW. \square

Lemma A5. *When $c = v$ forms part of an equilibrium, at least one firm lists $p = v$ with probability one.*

Proof. By Lemma A1, every firm sets $p_i \geq c$. When $c = v$, firms are indifferent between any $p \geq v$. However, suppose that there is no firm that plays $p = v$ with probability one. This implies PCW profit is lower than vL_0 . This can't be an equilibrium because the PCW could slightly reduce c to induce at least two firms to list $p = c$ with probability one (by Lemma A4) which increases PCW profit. \square

Lemma 1. *Fix $c \leq v$. In any equilibrium, the subgame starting at $t = 2$ must have $\min_i p_i = c$.*

Proof. Immediate from Lemmas A1-A5. \square

Lemma 1 means that every equilibrium of the subgame starting at $t = 2$ is pay-off equivalent to one with $p_1 = p_2 = c$, all else equal. We henceforth take $p_1 = p_2 = c$ as given and focus on each producer's payoff-relevant choice of direct price.

A.2 Proof of Lemmas 2–4

Lemma 2 (Low-fee regime). *Suppose $0 \leq c \leq \underline{c}_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$ and $p_1^d = p_2^d = v$. The resulting equilibrium profits are $\pi_0 = c(L_0 + \mu)$, $\pi_1 = vL_1$, $\pi_2 = vL_2$. When $0 \leq c < \underline{c}_1$, this equilibrium is unique.*

Proof. Firms earn zero profit from consumers served through the PCW in any equilibrium (Lemma 1). If a firm charges $p_i^d = v$ it makes total profit of $\pi_i = vL_i$. Prices $p_i^d \in (c, v)$ are dominated by v . The highest profit possible from setting any other value of p_i^d is from $p_i^d = c$ (with a zero probability of any other firm's direct price being tied or lower), which would generate $\pi_i' = c(L_i + \mu)$. Because $\pi_i' - \pi_i$ is decreasing in i , it suffices to check only firm 1 would not find this strategy more profitable,

$$\pi_1 \geq \pi_1' \iff c \leq \frac{vL_1}{L_1 + \mu}.$$

For $c < \underline{c}_1$, given that profit vL_i is the best a firm can ever do, uniqueness follows because a firm can always obtain such profit (only) by $p_i^d = v$. \square

Lemma 3 (Mid-fee regime). *Suppose $\underline{c}_1 \leq c \leq \underline{c}_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = c$, and $p_2^d = v$. The resulting equilibrium profits are $\pi_0 = cL_0$, $\pi_1 = c(L_1 + \mu)$, and $\pi_2 = vL_2$. When $\underline{c}_1 < c \leq \underline{c}_2$, this equilibrium is unique.*

Proof. Firms can guarantee themselves profit of vL_i by setting $p_i^d = v$; no greater profit can be earned by serving loyals. Attracting shoppers would require $p_i^d \leq c$, which is not profitable by definition when $c \leq \underline{c}_i$. Thus, in any equilibrium we must have $p_i^d = v$ for all $i \geq 2$. Given that its rivals all set $p_i^d = v$, firm 1 can choose between serving loyals and earning profit vL_1 or setting a price of c to attract the shoppers, resulting in profit of $c(L_1 + \mu)$. When $c \geq \underline{c}_1$, firm 1 prefers to attract the shoppers and must therefore set $p_i^d = c$ in equilibrium. \square

Lemma 4 (High-fee regime). *Suppose $\underline{c}_2 < c \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, p_1^d and p_2^d mixed over supports $[\underline{p}, c]$ and $[\underline{p}, c] \cup v$ respectively via the strategies*

$$F_1(p) = \begin{cases} \frac{\mu p - L_2(v-p)}{\mu p} & \text{for } p \in [\underline{p}, c) \\ 1 & \text{for } p \geq c, \end{cases}$$

$$F_2(p) = \begin{cases} \frac{\mu p - L_2(v-p)}{\mu p} \frac{\mu + L_1}{\mu + L_2} & \text{for } p \in [\underline{p}, c) \\ \frac{\mu c - L_2(v-c)}{\mu c} \frac{\mu + L_1}{\mu + L_2} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

where $\underline{p} = \frac{vL_2}{L_2 + \mu}$. The resulting profits are always $\pi_0 = cL_0$, $\pi_1 = \frac{vL_2(L_1 + \mu)}{L_2 + \mu}$, and $\pi_2 = vL_2$ for any $L_1 \leq L_2$. When $L_1 < L_2$ these equilibrium strategies are unique.²²

Proof. Here we show that the strategies given in Lemma 4 constitute an equilibrium when $L_1 < L_2$. For the special case of $L_1 = L_2$, see the Web Appendix. When firm 1 sets $p_1^d \in [\underline{p}, c]$, its profit is $\pi_1(p_1^d) = p_1^d \{L_1 + \mu [1 - F_2(p_1^d)]\}$. Firm 1's profits when $p_1^d = \underline{p} \equiv \frac{vL_2}{L_2 + \mu}$ are $\pi_1(\underline{p}) = \frac{vL_2}{L_2 + \mu} (L_1 + \mu)$. Setting $\pi_1(p_1^d) = \pi_1(\underline{p})$ and solving for F_2 yields

$$F_2(p) = \frac{\mu p - L_2(v-p)}{\mu p} \frac{\mu + L_1}{\mu + L_2}.$$

Similarly, when firm 2 sets $p_2^d \in [\underline{p}, c]$, its profit is $\pi_2(p_2^d) = p_2^d \{L_2 + \mu [1 - F_1(p_2^d)]\}$. Firm 2's profits when $p_2^d = v$ are $\pi_2(v) = vL_2$. Setting $\pi_2(p_2^d) = \pi_2(v)$ and solving for F_1 yields

$$F_1(p) = \frac{\mu p - L_2(v-p)}{\mu p}.$$

²²See the Web Appendix for details on the case of $L_1 = L_2$, where equilibrium strategies can be slightly different.

By construction, F_2 ensures 1 is indifferent over every $p_1^d \in [\underline{p}, c]$ and F_1 makes 2 indifferent over every $p_1^d \in [\underline{p}, c) \cup v$. We need to check that no firm can profit from a deviation outside of its support.

Deviations to $p_i^d > v$ yield zero profit and are not profitable. Neither firm can profit from a deviation to $p_i^d < \underline{p}$: this would result in the same demand as $p_i^d = \underline{p}$ but at a lower price. Any $p_i^d \in (c, v)$ is greater than the lowest price on the PCW and therefore only attracts loyal. The resulting profit, $p_i^d L_i$, is increasing in p_i^d so $p_i^d \in (c, v)$ is dominated by $p_i^d = v$. Since 1 has a mass point at c , $p_2^d = c$ induces a tie and yields strictly lower expected profit than does $p_2^d = c - \epsilon$ (for $\epsilon > 0$ small). Lastly, we check that firm 1 cannot profit from a deviation to $p_1^d = v$. We have $\pi_1(v) > \pi_1(\underline{p}) \iff vL_1 > \frac{vL_2}{L_2+\mu}(L_1 + \mu)$, which fails because $L_1 < L_2$. □

B Comparative Statics

Corollary 1 (Market Prices). *For $\mathbf{L} = (L_0, L_1, L_2, \mu)$ and $\mathbf{L}' = (L'_0, L_1, L_2, \mu)$ such that $L_0 < L_1 < L'_0$, both the minimum and average prices available in the market are higher under \mathbf{L}' with probability one.*

Proof. Under \mathbf{L} and \mathbf{L}' , the low-fee and high-fee regimes result respectively (Proposition 1). Under \mathbf{L} the minimum price on the market is $\frac{vL_1}{L_1+\mu}$ (Lemma 2). Under \mathbf{L}' the lower bound on the support of equilibrium prices is $\frac{vL_2}{L_2+\mu}$ (Lemma 4). Noting $\frac{vL_1}{L_1+\mu} < \frac{vL_2}{L_2+\mu}$ gives the result that the minimum price on the market is higher under \mathbf{L}' than \mathbf{L} with probability one. For average prices, it suffices to add that under \mathbf{L} there are two prices at $\frac{vL_1}{L_1+\mu}$ and two at v , whereas under \mathbf{L}' there are two prices in $[\frac{vL_2}{L_2+\mu}, v]$ and two at v . □

Lemma 5. *Equilibrium consumer surplus is*

$$CS = \begin{cases} v\mu \frac{L_0 + \mu}{L_1 + \mu} & \text{for } L_0 \leq L_1 \\ v\mu \frac{L_1 + \mu}{L_2 + \mu} & \text{for } L_0 \geq L_1. \end{cases} \quad (2)$$

Proof. The game is constant sum, hence consumer surplus is given by subtracting the sum of producer and CC profit (given in Lemma 2 and Lemma 4) from total surplus, $v(L_0 + L_1 + L_2 + \mu)$. The result follows. □

Proposition 2. *Consumer surplus*

1. *is non-monotonic in Δ_1 and Δ_2 ;*
2. *is non-monotonic in $\tilde{\Delta}$ if $L_1 < L_2$; non-decreasing in $\tilde{\Delta}$ when $L_1 = L_2$;*

3. is decreasing in Δ_μ .

Proof. Part 1. For the purposes of this proof, index the two producers i and j rather than 1 and 2 to distinguish between their identities and their relative sizes. For producer i and any L there exists some Δ_i^* such that $L'_0 = L'_i$.

- If $L'_i < L_j$: then consider values Δ_i close to Δ_i^* ,

$$CS = \begin{cases} v\mu \frac{L_0 + \Delta_i + \mu}{L_i - \Delta_i + \mu} & \text{for } \Delta_i < \Delta_i^*; \text{ increasing in } \Delta_i \\ v\mu \frac{L_i - \Delta_i + \mu}{L_j + \mu} & \text{for } \Delta_i > \Delta_i^*; \text{ decreasing in } \Delta_i \end{cases}$$

hence consumer surplus is non-monotonic in Δ_i .

- If $L'_i = L_j$: then consider values Δ_i close to Δ_i^* ,

$$CS = \begin{cases} v\mu \frac{L_0 + \Delta_i + \mu}{L_j + \mu} & \text{for } \Delta_i < \Delta_i^*; \text{ increasing in } \Delta_i \\ v\mu \frac{L_i - \Delta_i + \mu}{L_j + \mu} & \text{for } \Delta_i > \Delta_i^*; \text{ decreasing in } \Delta_i \end{cases}$$

hence consumer surplus is non-monotonic in Δ_i .

- If $L'_i > L_j$: then there is Δ_i^+ such that $L'_0 = L_j < L'_i$. Consider values Δ_i close to Δ_i^+ ,

$$CS = \begin{cases} v\mu \frac{L_0 + \Delta_i^+ + \mu}{L_j + \mu} = v\mu & \text{for } \Delta_i \nearrow \Delta_i^+ \\ v\mu \frac{L_j + \mu}{L_i - \Delta_i^+ + \mu} < v\mu & \text{for } \Delta_i \searrow \Delta_i^+ \end{cases}$$

hence consumer surplus is non-monotonic in Δ_i .

Part 2. Denote $\tilde{\Delta}^*$ the value of $\tilde{\Delta}$ such that $L'_0 = L_1$ and consider value of $\tilde{\Delta}$ close to $\tilde{\Delta}^*$. The jump discontinuity in consumer surplus is of size

$$\lim_{\tilde{\Delta} \searrow \tilde{\Delta}^*} CS - \lim_{\tilde{\Delta} \nearrow \tilde{\Delta}^*} CS = \frac{(L_1 - L_2)v\mu}{L_2 + \mu}$$

which is strictly negative for $L_1 < L_2$. When $L_1 = L_2$, consumer surplus is increasing in $\tilde{\Delta}$ up to $\tilde{\Delta}^*$, then remains constant thereafter.

Part 3. For Δ_μ such that $L'_0 < L_1$: from (2), consumer surplus is decreasing in Δ_μ . For Δ_μ such that $L'_0 > L_1$: from (2), consumer surplus is decreasing in Δ_μ . Denote the value of Δ_μ such that $L'_0 = L_1$ as Δ_μ^* and observe that the jump discontinuity is weakly negative:

$$\lim_{\Delta_\mu \searrow \Delta_\mu^*} CS - \lim_{\Delta_\mu \nearrow \Delta_\mu^*} CS = \frac{(L_1 - L_2)v\mu}{L_2 + \mu},$$

hence consumer surplus is decreasing in Δ_μ . \square

Proposition 3. *For \mathbf{L} and \mathbf{L}' such that $L_1 - L_0 > 0 > L'_1 - L'_0$, there exists an $\epsilon > 0$ such that \mathbf{L} yields (weakly) higher consumer surplus than \mathbf{L}' if $\|\mathbf{L} - \mathbf{L}'\| < \epsilon$.*

Proof. The difference in consumer surplus after a switch from \mathbf{L} to \mathbf{L}' is

$$\frac{v(\mu - \Delta_\mu)(L_1 - \Delta_1 + \mu - \Delta_\mu)}{L_2 - \Delta_2 + \mu - \Delta_\mu} - \frac{(L_0 + \mu)v\mu}{L_1 + \mu}. \quad (3)$$

We have

$$\|\mathbf{L} - \mathbf{L}'\| = \sqrt{(L_0 - L'_0)^2 + \Delta_1^2 + \Delta_2^2 + \Delta_\mu^2} < \epsilon,$$

which implies $|L_0 - L'_0|$, $|\Delta_1|$, $|\Delta_2|$, and $|\Delta_\mu|$ must all be small for ϵ small. Moreover, this observation along with $L_1 - L_0 > 0 > L'_1 - L'_0$ implies that $L_1 - L_0$ must approach zero with ϵ . Letting $L_0 \rightarrow L_1$ and $\Delta_1, \Delta_2, \Delta_\mu \rightarrow 0$, (3) becomes

$$\left[\frac{L_1 + \mu}{L_2 + \mu} - 1 \right] v\mu,$$

which is weakly negative. \square

Remark 1. *The model exhibits both search externalities and rip-off externalities.*

Proof. To see a search externality, suppose that $L_0 < L_1$ and consider a δ reduction in L_1 paired with a δ increase in μ (so that the fraction of shoppers in the population increases), where $\delta < L_1 - L_0$. The market is in the low-fee regime both before and after this change. Shoppers and CC-loyals pay \underline{c}_1 , which is decreasing in μ and increasing in L_1 ; they are therefore left better-off. Direct channel loyals pay v both before and after the change and are thus unaffected.

To see a rip-off externality, suppose that $L_0 = L_1 + \epsilon$ and consider a reallocation of 2ϵ consumers from L_0 to μ ($\epsilon > 0$ small). This induces a switch from the high to the low-fee regime and causes the surplus of shoppers to discontinuously increase (Corollary 1). But the price paid by DC loyals discontinuously increases to v , leaving such consumers worse-off. \square

C Policy

C.1 Characterization of equilibrium under NRCs

Begin with the observation that, because old consumers who bought direct when young are exposed only to \hat{p}_i^r , we must have $\hat{p}_i^r = v$ in any equilibrium.

C.1.1 Equilibrium under CC-binding NRC

If a CC-binding NRC is in place then old consumers receive solicitation messages only from their previous supplier, which can therefore price as a monopolist: $p_i^r = v$. A young consumer attracted via the CC is worth $p_i + p_i^r - c = p_i + v - c$ to firm i . In equilibrium, we must have $p_i = c - v$ (otherwise, a producer would undercut its rival to capture the CC consumers). To analyze pricing for young consumers, we proceed by looking for an equilibrium with a similar structure (low-, mid-, and high-fee regimes) to the baseline case.

A low-fee regime is such that shoppers buy through the CC and producers' direct prices serve only loyal young consumers. We must therefore have $p_i^d = v$. To check when this is equilibrium behavior, note that firm i 's per-period profit is $\pi_i^* = [vL_i + (c - v - c)(L_0 + \mu)/2] + [vL_i + v(L_0 + \mu)/2] = 2vL_i$, where the first square bracket is profit from young consumers and the second is profit from the old. The best deviation for i is to $p_i^d = p_i = c - v$, which results in young shoppers buying from it direct rather than through the CC. This yields profit $\pi_i = [(c - v)(L_i + \mu) + (c - v - c)L_0/2] + [v(L_i + \mu + L_0/2)] = c(L_i + \mu)$. We therefore find that the deviation is not profitable if

$$2vL_i \geq c(L_i + \mu) \iff c \leq \underline{c}_i \equiv \frac{2vL_i}{L_i + \mu}.$$

A mid-fee regime is such that firm 1 finds it worthwhile to undercut prices on the CC and serve shoppers directly, while firm 2 does not. The preceding analysis established that this will be the case if $\underline{c}_1 \leq c \leq \underline{c}_2$. Then $p_1^d = c - v$, $p_2^d = v$, $p_i = c - v$, $p_i^r = v$.

A high-fee regime is such that both firms undercut CC prices, which they wish to do if $c \geq \underline{c}_2$. The same Bertrand logic as above implies that $p_i = c - v$, provided $c \leq 2v$ (if $c > 2v$ then there is no way for firms to recoup the cost of sales through the CC so they will opt not to participate; this yields zero CC profit and can't be an equilibrium).

We start with the conjecture that, as in the baseline case, the equilibrium is such that producer 1 mixes over $[p, p_i]$ (with a mass point at p_i), while 2 mixes over $[p, p_i] \cup \{v\}$ (with a mass point at v), and verify that this is indeed consistent with equilibrium.

When playing $p_2^d = v$, firm 2's profit is $2vL_2$ (its direct prices attracts only loyal who buy in both periods of their life). An arbitrary price in $[p, p_i]$ yields profit $(p_2^d + v)L_2 + (1 - F_1(p_2^d))(p_2^d + v)\mu$, where F_1 is the distribution of 1's direct prices. Setting these two expressions for 2's profits equal and solving yields:

$$F_1(p) = 1 - \frac{(v - p)L_2}{(v + p)\mu}.$$

Additionally, $F(p) = 0$ implies $\underline{p} = v(L_2 - \mu)/(L_2 + \mu)$.

If producer 1 sets $p_1^d = \underline{p}$ then its profit is $(\underline{p} + v)(L_1 + \mu)$. This must be compared to profit from a generic $p_1^d \in [\underline{p}, p_i]$, which is $(p_1^d + v)L_1 + (1 - F_2(p_1^d))(p + v)\mu$. Solving for indifference yields

$$F_2(p) = 1 - \frac{(\underline{p} + v)(\mu + L_1) - (p + v)L_1}{(p + v)\mu}.$$

One can verify that these strategies along with $p_1 = p_2 = c - v$ and $p_1^r = p_2^r = v$ form a well-defined equilibrium. In particular, $\underline{p} \leq c - v$ (as required) whenever $c \geq \underline{c}_2$ and the distributions are increasing.

CC's decision and equilibrium payoffs Notice that in both the mid- and high-fee regimes the CC's profit is cL_0 so the optimal c is $c = 2v$. The low fee regime can be sustained if and only if $c \leq \underline{c}_1$ and yields CC profit of $c(L_0 + \mu)$; the optimal c is therefore \underline{c}_1 . Thus, the CC implements a low-fee regime with $c = \underline{c}_1$ if $\underline{c}_1(\mu + L_0) \geq 2vL_0 \iff L_1 \geq L_0$, and implements a high-fee regime with $c = 2v$ otherwise.

In the low-fee regime, the profits are $\pi_0 = 2vL_1(L_0 + \mu)/(L_1 + \mu)$, $\pi_1 = 2vL_1$, and $\pi_2 = 2vL_2$. Consumer surplus is $2(L_0 + L_1 + L_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 2v\mu(L_0 + \mu)/(L_1 + \mu)$.

In the high-fee regime, the profits are $\pi_0 = 2vL_0$, $\pi_1 = 2vL_2(L_1 + \mu)/(L_2 + \mu)$, and $\pi_2 = 2vL_2$. Consumer surplus is $2(L_0 + L_1 + L_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 2v\mu(L_1 + \mu)/(L_2 + \mu)$.

C.1.2 Equilibrium under DC-binding NRC

If a DC-binding NRC is in place producers can resolicit only old consumers who bought direct (for whom there is no competition) so they set $p_i^r = v$. Because producers have no possibility of serving old consumers who bought through the CC, the lifetime profit associated with a young consumer buying through the CC is $p_i - c$ and competition à la Bertrand implies $p_i = c$. We proceed by looking for an equilibrium with a similar structure (low-, mid-, and high-fee regimes) to the baseline case.

A low-fee regime is such that shoppers buy through the CC and producers' direct prices serve only loyal young consumers. We must therefore have $p_i^d = v$. To check when this equilibrium behavior, note that firm i 's per-period profit is $\pi_i^* = 2vL_i$. The best deviation for i is to $p_i^d = p_i = c$, which results in young shoppers buying from it direct rather than through the CC. This yields profit $\pi_i = c(L_i + \mu) + v(L_i + \mu)$. We therefore find that the deviation is not profitable if

$$c \leq \underline{c}_i \equiv \frac{v(L_i - \mu)}{L_i + \mu}.$$

A mid-fee regime is such that firm 1 finds it worthwhile to undercut prices on the CC and serve shoppers directly, while firm 2 does not. The preceding analysis established

that this will be the case if $\underline{c}_1 \leq c \leq \underline{c}_2$. We then have an equilibrium in which $p_i^d = c$, $p_2^d = v$, $p_i = c$, $p_i^r = v$.

A high-fee regime can be found in the same way as in the CC-binding NRC case (the sole difference being that now the maximum admissible c is v —because firms are prevented from directly selling to firms that bought through the CC when young). Indeed, one can verify that the expressions for equilibrium profit functions, and hence the equilibrium F_1 , F_2 , and \underline{p} are identical in the two cases.

CC's decision and equilibrium payoffs Notice that in both the mid- and high-fee regimes the CC's profit ($2cL_0$) is increasing so the optimal c is $c = v$. The low fee regime can be sustained if and only if $c \leq \underline{c}_1$ and yields CC profit of $2c(L_0 + \mu)$; the optimal c is therefore c_1 . Thus, the CC implements a low-fee regime with $c = \underline{c}_1$ if $2\underline{c}_1(\mu + L_0) \geq 2vL_0 \iff (L_1 - \mu)/2 \geq L_0$, and implements a high-fee regime with $c = v$ otherwise.

In the low-fee regime, the profits are $\pi_0 = 2v(L_0 + \mu)(L_1 - \mu)/(L_1 + \mu)$, $\pi_1 = 2vL_1$, and $\pi_2 = 2vL_2$. Consumer surplus is $2(L_0 + L_1 + L_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 4v\mu(L_0 + \mu)/(L_1 + \mu)$.

In the high-fee regime, the profits are $\pi_0 = 2vL_0$, $\pi_1 = [1 + (L_2 - \mu)/(L_2 + \mu)](L_1 + \mu)v$, and $\pi_2 = 2vL_2$. Consumer surplus is $2(L_0 + L_1 + L_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 2v\mu(L_1 + \mu)/(L_2 + \mu)$.

C.1.3 Equilibrium under no NRCs

In the absence of an NRC, old consumers who bought through the CC are exposed to offers from their previous supplier and the CC. If they buy through the CC again then their previous supplier earns at most $p_i - c$. But if they are offered $p_i^r \leq p_i$ they will buy direct yielding profit p_i^r . Thus, consumers buying through the CC are offered $p_i^r = p_i$ when old so that their lifetime value to a producer is $2p_i - c$. Bertrand competition on the CC is such that all profits are dissipated and $p_i = c/2$. Consumers who bought direct when young are offered $p_i^r = v$. We proceed by looking for an equilibrium with a similar structure (low-, mid-, and high-fee regimes) to the baseline case.

A low-fee regime is such that shoppers buy through the CC and producers' direct prices serve only loyal young consumers. We must therefore have $p_i^d = v$. To check when this is equilibrium behavior, note that firm i 's per-period profit is $\pi_i^* = 2vL_i$. The best deviation for i is to $p_i^d = p_i = c/2$, which results in young shoppers buying from it direct rather than through the CC. This yields profit $\pi_i = (v + c/2)(L_i + \mu)$. We therefore find that the deviation is not profitable if

$$c \leq \underline{c}_i \equiv \frac{2v(L_i - \mu)}{L_i + \mu}.$$

A mid-fee regime is such that firm 1 finds it worthwhile to undercut prices on the CC and serve young shoppers directly, while firm 2 does not. The preceding analysis established that this will be the case if $\underline{c}_1 \leq c \leq \underline{c}_2$.

A high-fee regime can be found in the same way as in the CC-binding NRC case. Indeed, one can verify that the expressions for equilibrium profit functions, and hence the equilibrium F_1 , F_2 , and \underline{p} are identical in the two cases.

CC's decision and equilibrium payoffs Notice that in both the mid- and high-fee regimes the CC's profit (cL_0) is increasing so the optimal c is $c = 2v$. The low fee regime can be sustained if and only if $c \leq \underline{c}_1$ and yields CC profit of $c(L_0 + \mu)$; the optimal c is therefore c_1 . Thus, the CC implements a low-fee regime with $c = \underline{c}_1$ if $\underline{c}_1(\mu + L_0) \geq 2vL_0 \iff (L_1 - \mu)/2 \geq L_0$, and implements a high-fee regime with $c = 2v$ otherwise.

In the low-fee regime, the profits are $\pi_0 = 2v(L_0 + \mu)(L_1 - \mu)/(L_1 + \mu)$, $\pi_1 = 2vL_1$, and $\pi_2 = 2vL_2$. Consumer surplus is $2(L_0 + L_1 + L_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 4v\mu(L_0 + \mu)/(L_1 + \mu)$.

In the high-fee regime, the profits are $\pi_0 = 2vL_0$, $\pi_1 = [1 + (L_2 - \mu)/(L_2 + \mu)](L_1 + \mu)v$, and $\pi_2 = 2vL_2$. Consumer surplus is $2(L_0 + L_1 + L_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 2v\mu(L_1 + \mu)/(L_2 + \mu)$.

C.2 Proof of Proposition 4

Part 1 of the proposition follows immediately from the above equilibrium characterization. To see part 2: Firstly, if $L_0 \leq (L_1 - \mu)/2$ there is a low-fee equilibrium under both CC-binding NRCs and a ban on NRCs, but the latter implies twice the consumer surplus of the former. If $(L_1 - \mu)/2 < L_0 \leq L_1$ then a CC-binding NRC supports a low-fee regime with consumer surplus equal to $2v\mu(L_0 + \mu)/(L_1 + \mu)$, while a ban on NRCs results in a high fee regime with consumer surplus of $2v\mu(L_1 + \mu)/(L_2 + \mu)$. Consumer surplus under a ban is constant in L_0 , while that under a CC-binding NRC is increasing. Moreover, as $L_0 \rightarrow L_1$, consumer surplus is (weakly) higher under a CC-binding NRC (strictly so if $L_1 < L_2$). Lastly, if $L_0 > L_1$ then both regimes yield the same consumer surplus.

D Extensions and Robustness

D.1 Endogenous Market Composition

This section provides the results allowing for an endogenous market composition $\mathbf{L} = (L_0, L_1, L_2)$ by allowing each firm to determine their stock of loyal consumers. We hold μ constant throughout where the proofs hold for any $\mu > 0$. Note that in the previous sections where \mathbf{L} was exogenous, we adopted the convention of naming the direct channels

such that $L_1 \leq L_2$, but now that order of the labels should be discarded: There are simply three firms, of any size, indexed $i = 0, 1, 2$ where 0 is a competitive channel and 1, 2 are direct channels. Formally, we add a stage, $t = 0$ to the game where each player $i = 0, 1, 2$ simultaneously chooses L_i subject to a C^2 cost function $\psi_i : \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$. The cost to player i of choosing L_i given the other players choose L_{-i} is written $\psi_i(L_i|L_{-i})$ where $L_{-i} = \mathbf{L} \setminus L_i$. We make the following additional assumptions on the cost function for $i, j = 0, 1, 2$:

1. Symmetry: $\psi_i = \psi_j \equiv \psi$
2. Costs nothing to do nothing: $\psi(0|\cdot) = 0$
3. Increasing: $\frac{\partial \psi(L_i|L_{-i})}{\partial L_i} = 0$ for $L_i = 0$; $\frac{\partial \psi}{\partial L_i} > 0$ for $L_i > 0$
4. Convex: $\frac{\partial^2 \psi(L_i|L_{-i})}{\partial L_i^2} > 0$
5. Inter-dependency: $\frac{\partial \psi(L_i|L_{-i})}{\partial L_j} = 0$ for $L_i = 0$; $\frac{\partial \psi(L_i|L_{-i})}{\partial L_j} > 0$ for $L_i > 0$

Players are assumed to have access to the same marketing tools, hence ψ is common across players (assumption 1). Assumptions 3 and 4 state respectively that determining market power is costly, and increasingly so. Assumption 5 states that it becomes more costly to generate the same market power when other players are marketing more.

Denote low-fee regime, LFR, and high-fee regime, HFR. We view this marketing stage of the game as reflecting long-run decisions and therefore restrict attention to pure-strategy equilibria. Denote a cost function satisfying assumptions 1-5 as ψ , and the set of all these functions Ψ . Let Ψ_L (Ψ_H) denote the set of all functions ψ under which a low-fee regime (high-fee regime) equilibrium exists.

Lemma D1. *In any equilibrium, $L_i^* > 0$ for all i .*

Proof. Suppose $L_i^* = 0$ for some i . Equilibrium profit for i is weakly positive and the marginal cost of L_i at $L_i = 0$ is zero. Therefore, if the marginal benefit of L_i at $L_i = 0$ is strictly positive, there is some strictly profitable deviation away from $L_i^* = 0$.

If $i = 0$, the marginal benefit at $L_i = 0$ is $\frac{d(vL_i)}{dL_i} = v > 0$ if the HFR results; and $v \frac{L_1^*}{L_1^* + \mu} > 0$ if the LFR results (note the LFR implies $L_1^* > 0$). If $i > 0$, then $\hat{L}_i > 0$ small produces the LFR with a marginal benefit of $v \frac{L_2^*}{L_2^* + \mu} > 0$ if $\hat{L}_i \leq L_j^*$ for all $j > 0$ or $v > 0$ otherwise. \square

Lemma D2. *No symmetric equilibrium exists.*

Proof. Suppose instead that $L_i = L^*$ for all i . By Lemma D1, $L^* > 0$. A deviation of any player to \hat{L}_i yields profit:

$$\hat{\pi}(\hat{L}_i) = \begin{cases} v \frac{L^*}{L^* + \mu} (\hat{L}_i + \mu) - \psi(\hat{L}_i|\mathbf{L}_{-i}^*) & \text{for } \hat{L}_i \leq L^* \\ v \hat{L}_i - \psi(\hat{L}_i|\mathbf{L}_{-i}^*) & \text{for } \hat{L}_i \geq L^*. \end{cases}$$

[Notice this is regardless of whether the LFR or HFR results in equilibrium.] The function $\hat{\pi}$ is continuous, but non-differentiable at $\hat{L}_i = L^*$. To see this, note that the left and right derivatives at $\hat{L}_i = L^*$ are $\partial_- \hat{\pi}(L^*) = v \frac{L^*}{L^* + \mu} - \frac{\partial \psi(L_i | \mathbf{L}_{-i})}{\partial L_i} \Big|_{\mathbf{L} = \mathbf{L}^*}$ and $\partial_+ \hat{\pi}(L^*) = v - \frac{\partial \psi(L_i | \mathbf{L}_{-i})}{\partial L_i} \Big|_{\mathbf{L} = \mathbf{L}^*}$ respectively, hence $\partial_- < \partial_+$. However, for there to be no strictly profitable deviation slightly below or above L^* we need $\partial_- \geq 0$ and $\partial_+ \leq 0$ which implies $\partial_- \geq \partial_+$, a contradiction. \square

Lemma D3. *If the LFR obtains in equilibrium, $L_0^* < L_1^*$. If the HFR obtains in equilibrium, $L_1^* < L_2^*$.*

Proof. If instead $L_0^* = L_1^*$, the profit from deviations of the CC, \hat{L}_0 is given by:

$$\hat{\pi}(\hat{L}_0) = \begin{cases} v \frac{L_1^*}{L_1^* + \mu} (\hat{L}_0 + \mu) - \psi(\hat{L}_0 | \mathbf{L}_{-0}^*) & \text{for } \hat{L}_0 \leq L_0^* \\ v \hat{L}_0 - \psi(\hat{L}_0 | \mathbf{L}_{-0}^*) & \text{for } \hat{L}_0 \geq L_0^*. \end{cases}$$

Following the proof of Lemma D2 shows that such an equilibrium implies a contradiction. Similarly, if $L_1^* = L_2^* \equiv L^* < L_0^*$, the profit from deviations by $i = 1, 2$, \hat{L}_i is given by:

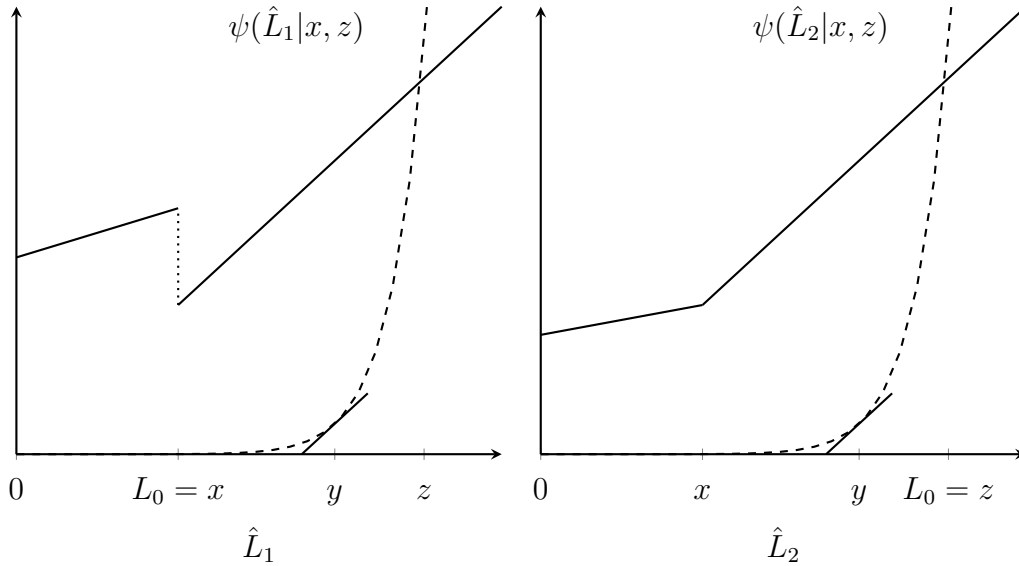
$$\hat{\pi}(\hat{L}_i) = \begin{cases} v \frac{L^*}{L^* + \mu} (\hat{L}_i + \mu) - \psi(\hat{L}_i | \mathbf{L}_{-i}^*) & \text{for } \hat{L}_i \leq L^* \\ v \hat{L}_i - \psi(\hat{L}_i | \mathbf{L}_{-i}^*) & \text{for } \hat{L}_i \geq L^*. \end{cases}$$

Following the proof of Lemma D2 shows that such an equilibrium implies a contradiction. \square

Proposition 5. *There are more cost functions that lead to a high-fee regime than a low-fee regime in equilibrium: $|\Psi_H| \geq |\Psi_L|$.*

Proof. We show that $\psi \in \Psi_L \implies \psi \in \Psi_H$ i.e., for any ψ that supports an equilibrium which gives the low-fee regime, then under the same cost function there is also an equilibrium which gives the high-fee regime.

Suppose $\psi \in \Psi_L$ and denote an equilibrium which gives the low-fee regime as $(L_0, L_1, L_2) = (x, y, z)$ where $x \leq y \leq z$. We show that this implies that there also exists an equilibrium where firms choose (z, x, y) which gives the high-fee regime. By Lemmas D3 and D1, $0 < x < y$ for (x, y, z) to be an equilibrium. Below on the left is an illustration of such a low-fee equilibrium (x, y, z) . On the right is an illustration of the corresponding high-fee equilibrium strategies $(L_0, L_1, L_2) = (z, x, y)$. The x-axis represents unilateral deviations of the player choosing y (producer in the left-hand panel and producer 2 in the right-hand panel). [To prevent clutter, the panels show only the revenue and cost curves of the players choosing y , but the procedure is unchanged if done instead for the players choosing x or z .]



Left panel: Deviation incentives for the smaller producer from the low-fee regime equilibrium (x, y, z) .
Right panel: Deviation incentives for the larger producer from the high-fee regime equilibrium (z, x, y) .

As (x, y, z) is an equilibrium, there are no profitable deviations for the players either (i) locally: the slope of the cost curve is equal to the slope of the revenue curve; or (ii) globally: there is no profitable deviation to, or slightly below the choice of the smallest player. It can now be seen that (z, x, y) is also an equilibrium. Because the same three values x, y, z are chosen (albeit by different players) in (x, y, z) and (z, x, y) , we show that (z, x, y) satisfies (i) and (ii), which implies it is an equilibrium. That it is a high-fee regime equilibrium is immediate because in (z, x, y) , $L_0 > L_1$. To see this for the player choosing y , notice that the figures in both panels are identical except for the revenue curve which is the same for deviations above x , but strictly higher in the left-hand panel for deviations below x . [Although only the deviations for the player at y are shown, it is also true that the revenue curve in the left panel is weakly above the revenue curve in the right panel for the players at x and z , hence the argument can be repeated for all players.] This implies immediately that there are no local deviations. It also shows that if a deviation to at or slightly below x is not profitable in the left panel, it is not profitable for the player in the right panel, which shows there are no global deviations. Hence if (x, y, z) is an equilibrium, then (z, x, y) is too. \square

D.2 Commission types and a marginal cost of production

Through its selection of fees (flat or ad valorem), the CC determines the equilibrium price available through it, p . As such, in this section it is convenient to write as if the CC is directly determining p . We state the equilibria in terms of p in the same order as in our presentation of the baseline model. In the baseline model, \underline{c}_i was the threshold fee level such that producer i would prefer to sell to shoppers and their loyal direct, but at the

CC price, and sell only to their loyal direct at the monopoly price. The corresponding expressions in terms of p , while allowing for a marginal cost of production, $m \in [0, v]$, are given below for $i = 1, 2$.

$$p_i = \frac{vL_i + m\mu}{L_i + \mu}$$

Lemma D4 (Low fee regime). *Suppose $0 \leq p \leq p_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$ and $p_1^d = p_2^d = v$. The resulting equilibrium profits are $\pi_0 = p(L_0 + \mu)$, $\pi_1 = (v - m)L_1$, $\pi_2 = (v - m)L_2$. When $0 \leq p < p_1$, this equilibrium is unique.*

Lemma D5 (Mid fee regime). *Suppose $p_1 \leq p \leq p_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$, $p_1^d = p$ and $p_2^d = v$. The resulting equilibrium profits are $\pi_0 = pL_0$, $\pi_1 = (p - m)(L_1 + \mu)$, and $\pi_2 = (v - m)L_2$. When $p_1 < p < p_2$, this equilibrium is unique.*

Lemma D6 (High fee regime). *Suppose $p_2 \leq p \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$, p_1^d and p_2^d mixed over supports $[\underline{p}, p]$ and $[\underline{p}, p] \cup v$ respectively via the strategies*

$$F_1(p^d) = \begin{cases} 1 - \frac{(v-p^d)L_2}{(p^d-m)\mu} & \text{for } p^d \in [\underline{p}, p) \\ 1 & \text{for } p^d \geq p, \end{cases}$$

$$F_2(p^d) = \begin{cases} 1 - \frac{(\underline{p}-m)\mu - (p^d-\underline{p})L_1}{(p^d-m)\mu} & \text{for } p^d \in [\underline{p}, p) \\ \frac{(\underline{p}-m)\mu - (p-\underline{p})L_1}{(p-m)\mu} & \text{for } p^d \in [p, v) \\ 1 & \text{for } p^d \geq v, \end{cases}$$

where $\underline{p} = p_2$. The resulting profits are $\pi_0 = pL_0$, $\pi_1 = \frac{(v-m)L_2(L_1+\mu)}{L_2+\mu}$, and $\pi_2 = (v - m)L_2$. When $p_2 < p \leq p$, this equilibrium is unique.

Proposition D1. *When $L_0 \leq L_1$, there is a low-fee equilibrium: the competitive channel sets $p = p_1$ and producers price in accordance with Lemma 2. When $L_0 \geq L_1$, there is a high-fee equilibrium: the competitive channel sets $p = v$ and producers price in accordance with Lemma 4.*

D.3 Commission discrimination

If the CC offers fees $c_i < c_j \leq v$ Bertrand competition on the CC implies that all sales via the CC go to firm i at a price of $p_i = c_j$.

Part (i): Suppose that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability 1 in the pricing sub-game and $c_i < c_j \leq v$. Then the CC's profit is $\pi_0 = c_i L_0$. There is a profitable deviation to $c_1 = c_2 = v$.

Part (ii): Suppose that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability 0 and $c_i < c_j \leq v$. We show that the CC can serve the same mass of consumers at a higher fee by reducing c_j . Indeed, firm i earns profit of $vL_i + (c_j - c_i)(L_0 + \mu)$ (assuming it sets $p_i^d = v$, which is optimal conditional on not undercutting the CC). The best deviation would be to $p_i^d = c_j$ to undercut the CC, yielding profit $c_j(L_i + \mu) + (c_j - c_i)L_0$. The deviation is not profitable if $c_i \leq \tilde{c}_i \equiv \frac{L_i(v - c_j)}{\mu}$. Similarly, firm j earns vL_j . A deviation to $p_j^d = c_j$ yields profit $c_j(L_j + \mu)$ and is not profitable if $c_j \leq \underline{c}_j$. Thus, the best that the CC can do, conditional on deterring undercutting, is to solve

$$\begin{aligned} & \max_{c_i, c_j} c_i(\mu + L_0) \\ \text{s.t. } & c_i \leq c_j, c_i \leq \tilde{c}_i, c_j \leq \underline{c}_j. \end{aligned}$$

At least one of $c_i \leq \tilde{c}_i$ and $c_j \leq \underline{c}_j$ must bind and both are made more slack by a reduction in c_j .

Part (iii) Lastly, if $c_i < c_j$ we show that it can't be the case that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability in $(0, 1)$. Indeed, this implies that both firms put positive mass on direct prices below c_j and positive mass on prices above. Direct prices in (c_j, v) never serve shoppers and are dominated by v . Standard arguments then imply that both firms must share a support, $[\underline{p}, c_j] \cup \{v\}$. To be indifferent between $p_j^d = v$ and $p_j^d = \underline{p}$, j must have $\underline{p}(L_j + \mu) = vL_j \iff \underline{p} = \underline{c}_j$, which does not depend on c_i or c_j . Similarly, i is indifferent if $\underline{p} = \frac{L_i v + \mu(c_j - c_i)}{L_i + \mu}$. For the two values of \underline{p} to coincide, we require $(L_j - L_i)v = (L_j + \mu)(c_j - c_i)$, which implies that $L_j > L_i$ (i.e., $j = 2$ and $i = 1$).

To be indifferent between \underline{p} and a $p_2^d < c_2$, 2 must have $\underline{p}(L_2 + \mu) + L_0(c_2 - c_1) = p_2^d L_2 + L_0(c_2 - c_1) + \mu p_2^d(1 - F_1(p_2^d))$, i.e.

$$F_1(p_2^d) = \frac{(L_2 + \mu)(p_2^d - \underline{p})}{p_2^d \mu} = \frac{(L_2 + \mu)(p_2^d - \underline{c}_2)}{p_2^d \mu}.$$

Similarly, 1 is indifferent if

$$F_2(p_1^d) = \frac{(L_1 + \mu)(p_1^d - \underline{p})}{p_1^d \mu} = \frac{(L_1 + \mu)(p_1^d - \underline{c}_2)}{p_1^d \mu}.$$

The CC's profit is $\pi_0 = c_1[1 - F_1(c_2)][1 - F_2(c_2)]\mu + c_1 L_0$. Observe that F_1 and F_2 do not depend on c_1 , so π_0 is linear in c_1 and maximized when $c_1 \uparrow c_2$. Letting $c_1 = c_2 = c$, it is easily verified that π_0 is convex and maximized at either $c = v$ (inducing firms to undercut with probability 1) or at $c = \underline{c}_1$ (inducing firms to undercut with probability zero). Profit when $c_1 \neq c_2$ must be strictly less so that the CC has a profitable deviation to the case considered in either part (i) or part (ii).

W Web Appendix

This Web Appendix is intended to house the more lengthy proofs of the paper and as supplementary material for readers who are especially interested in the details.

W.1 Uniqueness of tie-breaking rule

Proof of analog of Lemma 1

Suppose the lowest price on the market is p , set by the set of firms $T \neq \emptyset$ on the PCW and $D \neq \emptyset$ via direct prices. There are then $|T| + |D|$ sources of the lowest price on the market. In such a scenario, let $r_0 \in [0, 1]$ such that if:

- (1) $i \notin T, i \notin D$ the probability that i sells to a shopper is 0;
- (2) $i \in T, i \notin D$ the probability that i sells to a shopper is $\frac{r_0}{|T|}$;
- (3) $i \notin T, i \in D$ the probability that i sells to a shopper is $\frac{1-r_0}{|D|}$;
- (4) $i \in T, i \in D$ the probability that i sells to a shopper is $\frac{1-r_0}{|D|} + \frac{r_0}{|T|}$.

In the main text, we assumed that $r_0 = 0$ i.e., that shoppers buy through the direct channel with probability one where there is a tie in the lowest price across the direct and competitive channels. Here we show that this is the only such tie-breaking rule that gives an equilibrium of the game as a whole. To do so, suppose $r_0 \in (0, 1]$ and $c \leq v$.

Lemma W1 (Analog of Lemma A1). *No firm lists any price strictly less than c on the PCW in equilibrium.*

Proof. Suppose there is a positive probability that prices strictly lower than c are listed on the PCW in equilibrium. There is then some price posted by some firm i , $p_i < c$, which has some positive probability, $\psi > 0$, of being the lowest posted on the PCW. When called upon to play p_i (and some arbitrary $p_i^d \geq 0$), the only case not covered by Lemma A1 is when the tie-breaking rule affects profits. This occurs when p_i is both the lowest price on the market and ties with direct prices set by the set of firms $D \neq \emptyset$. But in this case, the analogous proof goes through: firm i 's profit is,

- (i) if $i \in D$, $p_i^d \left(L_i + \mu \frac{1-r_0}{|D|} \right) + \psi(p_i - c) \left(L_0 + \mu \frac{r_0}{|T|} \right)$,
- (ii) if $i \notin D$, $p_i^d L_i + \psi(p_i - c) \left(L_0 + \mu \frac{r_0}{|T|} \right)$.

A deviation to $p_i > v$ would cause profit in these cases to change to (i) $p_i^d \left(L_i + \mu \frac{1-r_0}{|D|} \right)$, and (ii) $p_i^d L_i$. In all cases the deviation strictly increases profit. \square

Lemma W2 (Analog of Lemma 1). *In any equilibrium, for $r_0 \in [0, 1]$ the subgame starting at $t = 2$ must have $\min_i p_i = c$.*

Proof. Lemmas A2-A5 are unchanged and Lemma 1 follows. \square

Our model requires a solution for any $\mathbf{L} \in \mathbb{R}_+^n$. Lemma W3 below shows that for $L_1 < L_2$, there is no equilibrium to the subgame starting in $t = 2$ for a particular range of c , and thus no subgame perfect equilibrium to the game as a whole.

Lemma W3 (Failure of the analog of Lemma 3). *When $r_0 > 0$ and $L_1 < L_2$, there is no equilibrium for $\underline{c}_1 < c < \underline{c}_2$.*

Proof. Given Lemma W2 we can try to establish the analogs for Lemmas 2-4 for $r_0 \in [0, 1]$. However, here we show that the proof of Lemma 3 fails for $r_0 > 0$. The problem is that firm 1 no longer wishes to set c . In fact, because $r_0 > 0$, firm 1 profits more from $p_1^d = c - \epsilon$ for $\epsilon > 0$ small, which avoids losing some profit through the tie-break sales that go through the PCW. But no such $p_1^d = c - \epsilon$ can be part of an equilibrium because if it were, $p_1^d = c - \frac{1}{2}\epsilon$ would constitute a profitable deviation. \square

W.2 Uniqueness of the high-fee regime subgame equilibrium

Here we show that the uniqueness claims in the equilibrium of the subgame denoted in Lemma 4 i.e., the producers' equilibrium responses following a fee level $c \in [\underline{c}_2, v]$. Proof that the equilibrium given is an equilibrium in the subgame is given in Appendix A. Given Lemma 1, we need to determine the equilibrium distribution of direct prices p_i^d for $i = 1, 2$, F_1, F_2 to complete the subgame's equilibrium characterization. We denote \underline{p}_i and \bar{p}_i as the min and max of the support of F_i . Firstly, we derive producer responses to $c \in (\underline{c}_2, v]$.

Lemma W4. $\bar{p}_1, \bar{p}_2 \leq v$.

Proof. Any $p_i^d > v$ results in no direct sales and $\pi_i = 0$, whereas $p_i^d = v$ results in direct sales to L_i , giving $\pi_i = vL_i$. \square

Lemma W5. $p_1^d, p_2^d \notin (c, v)$.

Proof. Any $p_i^d \in (c, v)$ results in sales to L_i only, such a price is therefore strictly dominated by v . \square

Lemma W6. $\underline{p}_1, \underline{p}_2 \geq \underline{c}_2$.

Proof. The best $p_i^d < \underline{c}_2$ can do for i is to sell to shoppers with probability one, netting $\pi_i = p_i^d(\mu + L_i)$, less than the vL_i made from setting $p_i^d = v$. \square

Lemma W7. $\min\{\underline{p}_1, \underline{p}_2\} \leq c$.

Proof. Suppose not. Then by Lemmas W4-W5, both producers charge pure strategies $p_i^d = v$. However, as $c > \underline{c}_2$, both producers have a profitable deviation of $p_i^d = c$ (selling to shoppers and L_i at c nets $c(\mu + L_i)$, selling only to L_i at v nets vL_i). \square

Lemma W8. $\underline{p}_1 = \underline{p}_2 \equiv \underline{p}$.

Proof. By Lemma W7, $\min\{\underline{p}_1, \underline{p}_2\} \leq c$. Suppose (without loss of generality) that $\underline{p}_1 < \underline{p}_2$. Case 1: $\underline{p}_1 < c$. Prices $p \in [\underline{p}_1, \underline{p}_2)$ are strictly worse than $p \nearrow \underline{p}_2$ for 1. Case 2: $\underline{p}_1 = c$. By Lemmas W4-W5, $p_2^d = v$, giving $\pi_2 = vL_2$. However, as $c > \underline{c}_2$, deviations to $p \nearrow \underline{p}_1$ are profitable for 2. \square

Lemma W9. $\underline{p} < c$.

Proof. By Lemmas W7-W8, $\underline{p} \leq c$. Now we rule out $\underline{p} = c$. Case 1: $\underline{p} = c$ and there is a zero probability of a tie at c . Then $c < v$ and both producers place all their mass on v by Lemmas W4-W5, $\pi_i = vL_i$ but deviating to $p_i^d \in (\underline{c}_2, c)$ nets $\pi_i = p_i^d(\mu + L_i) > vL_i$. Case 2: $\underline{p} = c$ and there is a positive probability of a tie at c . Both producers have an incentive to shift the mass placed on c to slightly below c (the arbitrary loss in price is compensated by the discrete gain in the probability of selling to shoppers). \square

Lemma W10. *Direct prices $[\underline{p}, c]$ are in the support of both producers.*

Proof. Suppose not. By Lemma W8 there is a common element to producers' supports. Denote $p < c$ highest common element to the supports such that $[p, p]$ is in both supports. Suppose without loss of generality that i then has a "hole" in their support of p_i^d , denoted by the interval $(p_l, p_h) \subseteq [p, c]$. This cannot be true in equilibrium because for prices close to p_l , j has a profitable deviation of $p_j^d \nearrow p_h$. \square

Lemma W11. *No producer has a mass point in $[\underline{p}, c)$.*

Proof. Suppose i had a mass point at $\hat{p} \in [\underline{p}, c)$. By Lemma W10, j 's support includes prices in small neighborhoods around \hat{p} , where j has a profitable deviation to relocate those p_j^d slightly above \hat{p} , to prices slightly below \hat{p} (the arbitrary loss in price is compensated by the discrete gain in the probability of selling to shoppers). \square

Lemma W12. *At most one producer has a mass point at c .*

Proof. If both did, then there is a positive probability of a tie at c . Hence, it would be profitable for either to shift the mass they place on c to slightly below c , (the arbitrary loss in price is compensated by the discrete gain in the probability of selling to shoppers). [Note that the point the mass is shifted to will not coincide with another mass point: if $\underline{p} = c$, because no prices below \underline{p} are charged; if $\underline{p} < c$, by Lemma W11.] \square

Lemma W13. *When $L_1 < L_2$, not both producers have mass points at v .*

Proof. Suppose so. Case 1: $c < v$. Then $\pi_1 = \underline{p}(\mu + L_1)$ (from Lemmas W10-W11) $= vL_1$ which gives $\underline{p} = \frac{vL_1}{\mu + L_1} \equiv \underline{c}_1 < \underline{c}_2$, ruled out by Lemma W6. Case 2: $c = v$. Lemma W12 applies. \square

Lemma W14. *When $c < v$, at least one producer has a mass point at v .*

Proof. Suppose neither producer has a mass point at v . By Lemma W12, at least one producer does not have a mass point at c , call this i . Note that for $j \neq i$, c is in their support by Lemma W10 and $\pi_j = cL_j$, but then j has a profitable deviation $p_j^d = v$, netting vL_j . \square

Lemma W15. *When $L_1 < L_2$ and $c = v$, exactly one producer has a mass point at v .*

Proof. By Lemma W13 it is not true that both have mass points at v . Now suppose neither producer has a mass point at v . Then producer $i = 1, 2$ makes $\pi_i = vL_i$, but we know $\pi_i = \underline{p}(\mu + L_i)$ (from Lemmas W10-W11) which gives $\underline{p} = \frac{vL_i}{\mu + L_i} \equiv \underline{c}_i$, which cannot be satisfied for both $i = 1, 2$ because $L_1 < L_2$. \square

Lemma W16. *When $L_1 < L_2$, producer 1 has no mass point at v , producer 2 does.*

Proof. When $c < v$, Lemmas W13 and W14 imply that exactly one producer has a mass at v . When $c = v$, Lemma W15 says the same. Next, we show that this producer is producer 2. Suppose instead it was 1, then $\pi_1 = vL_1$. However, 1 has a profitable deviation to $p_1^d = \underline{c}_2$ which generates $\pi_1 = \underline{c}_2(\mu + L_1)$ (the deviation wins shoppers with probability one by Lemmas W6 and W11), which is greater because $L_1 < L_2$. \square

Lemma W17. *When $L_1 < L_2$, the unique equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, p_1^d and p_2^d mixed over supports $[\underline{c}_2, c]$ and $[\underline{c}_2, c] \cup v$ respectively via the strategies*

$$F_1(p) = \begin{cases} \frac{\mu p - L_2(v-p)}{\mu p} & \text{for } p \in [\underline{c}_2, c) \\ 1 & \text{for } p \geq c, \end{cases}$$

$$F_2(p) = \begin{cases} \frac{\mu p - L_2(v-p)}{\mu p} \frac{\mu + L_1}{\mu + L_2} & \text{for } p \in [\underline{c}_2, c) \\ \frac{\mu c - L_2(v-c)}{\mu c} \frac{\mu + L_1}{\mu + L_2} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v. \end{cases}$$

The resulting profits are $\pi_0 = cL_0$, $\pi_1 = \frac{vL_2(L_1 + \mu)}{L_2 + \mu}$, and $\pi_2 = vL_2$.

Proof. In equilibrium, F_1 must be such that producer 2 is indifferent over all prices in their support. By Lemma W16, 2 has a mass at v , hence $\pi_2 = vL_2$. Therefore, F_1 must satisfy $\pi_2 = p_2^d L_2 + (1 - F_1(p_2^d))\mu p_2^d = vL_2$ for $p_2^d \in [\underline{p}, c]$. Solving gives F_1 over $[\underline{p}, c]$ as stated. By Lemmas W4, W5, W6, W11 and W16, the residual mass of $1 - F_1(c)$ must be located at c . Solving $F_1(\underline{p}) = 0$ gives $\underline{p} = \underline{c}_2$. Similarly, F_2 must keep 1 indifferent over all prices in their support. Solving $\pi_1 = \underline{p}(\mu + L_1) = p_1^d L_1 + (1 - F_2(p_1^d))\mu p_1^d$ gives F_2 over $[\underline{p}, c]$ as stated. With the addition of Lemma W12 (to W4, W5, W6, W11 and W16) the residual mass of $1 - F_2(c)$ must be located at v . Checks that the strategies described indeed constitute an equilibrium can be found in the proof of Lemma 4. \square

The following Lemmas correspond to the special case of $L_1 = L_2$, which as shown in Lemma D3, does not occur when market composition is endogenously determined. Therefore, we include these results mostly for completeness.

Lemma W18. *When $L_1 = L_2$ and $c = v$, the unique equilibrium of the subgame starting at $t = 2$ is as given in Lemma W17.*

Proof. One can follow the relevant part of the proof of Lemma W13 to see that not both producers have mass points at v . This means there is at least one producer who makes $\pi_i = vL_i$ (if neither have a mass at v , $\pi_i = vL_i$, $i = 1, 2$; if i does, then $\pi_i = vL_i$). This cannot be producer 1 because they have a profitable deviation to $p_1^d = \underline{c}_2$ which generates $\pi_1 = \underline{c}_2(\mu + L_1)$ (the deviation wins shoppers with probability one by Lemmas W6 and W11). Therefore, producer 2 makes $\pi_2 = vL_2$. To obtain the equilibrium strategies, one can now follow the proof Lemma W17 from “hence $\pi_2 = vL_2$ ”. \square

Lemma W19. *When $L_1 = L_2 \equiv L$ (so that $\underline{c}_1 = \underline{c}_2 \equiv \underline{c}$) and $c < v$, equilibria of the subgame starting at $t = 2$ have $p_i = p_j = c$, p_i^d and p_j^d mixed over supports $[\underline{c}, c] \cup v$ and $[\underline{c}, c] \cup v$ respectively for $i, j = 1, 2$ and $i \neq j$ via the strategies*

$$F_i(p) = \begin{cases} \frac{\mu p - L(v-p)}{\mu p} & \text{for } p \in [\underline{c}, c) \\ \alpha & \text{for } p \geq c \\ 1 & \text{for } p \geq v, \end{cases}$$

$$F_j(p) = \begin{cases} \frac{\mu p - L(v-p)}{\mu p} & \text{for } p \in [\underline{c}, c) \\ \frac{\mu c - L(v-c)}{\mu c} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

where $\alpha \in \left[\frac{\mu c - L(v-c)}{\mu c}, 1 \right]$. [Note in the case of $\alpha = 1$, v is not in the support of producer 1.] The resulting profits are $\pi_0 = cL_0$, $\pi_1 = \pi_2 = vL$.

Proof. By Lemma W14, at least one producer puts mass on v . This means that at least one makes $\pi_i = vL$ which gives F_j (and \underline{p}), over $[\underline{c}, c)$ as stated, in the same way as in proof of Lemma W17. Similarly, $\pi_j = \underline{p}(\mu + L)$ and again the proof of Lemma W17 can be followed to find F_i over $[\underline{c}, c)$ as stated. Both firms have residual mass $1 - F_i(c) = 1 - F_j(c)$ to be allocated. First, and again similar to Lemma W17, by Lemma W12 there can be only one producer with mass on c , hence by Lemmas W4, W5, W6 and W11, there must be one producer (denoted j here) with all this residual mass located at v . However, unlike Lemma W17, there is nothing dictate exactly how firm i should allocate their residual mass $(1 - \alpha)$ between c and v in equilibrium which leaves us with the strategies as stated above. Checks similar to those of Lemma 4 confirm these are equilibria. \square