A Model of Biased Intermediation*

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Abstract

This paper studies situations in which some consumers rely on a potentially biased intermediary to choose among downstream firms. We introduce the notion that firms' and consumers' payoffs can be *congruent* or *conflicting*, and show that this has important implications for the effects of bias. Under congruence, the firm towards which the intermediary is biased invests more than its rival and consumers can be better-off than under no bias. Under conflict, bias hurts consumers and the favored firm charges higher prices. We study various oft-proposed policies for dealing with a biased intermediary and show that the efficacy of each intervention depends strongly on whether the environment exhibits congruence or conflict. We discuss how the model relates to recent issues in online markets.

Keywords: intermediary, quality, bias.
JEL Classification: D21, L15, L40.

1 Introduction

In many markets consumers rely on recommendations from an intermediary or other third party when choosing between competing products or services. For example, search engines provide a ranked list of the most relevant websites for a given query, physicians prescribe the best remedy for a given set of symptoms, and financial advisers help their clients to choose their investments. "Recommendations" can also take the form of the choice of the default provider of a complementary product, as when a web browser comes equipped with

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a default search engine. In these markets, consumers look for (and often expect) unbiased advice from intermediaries. Yet the intermediary's recommendation is often driven by financial incentives, such as commissions paid by the recommended firm or exclusivity contracts.[1] Another common arrangement is for the intermediary to vertically integrate into the downstream market and recommend its own products—thereby capturing the associated downstream revenue.[2]

The existence of these kinds of financial incentives has prompted numerous high-profile investigations by regulators and competition authorities that worry about potentially harmful effects. Google's promotion of its own subsidiary websites, the payment of commissions to financial advisers, paid endorsements on price comparison websites, and the provision of kickbacks to physicians by pharmaceutical firms are among the cases that have been (and continue to be) subject to scrutiny. Interventions are typically motivated by two main concerns: (i) will intermediaries systematically direct consumers towards inferior products? (ii) will downstream firms' behavior be affected in a way that harms consumers? Even though these concerns are common to the situations described above, a certain degree of heterogeneity remains as to the institutional or strategic environments. A key concern in the price comparison website industry, for example, has been the effect of paid endorsements on final consumer prices. Search engines and websites, by contrast, are mostly free to access so that attention has focused on innovation and investment decisions rather than on pricing. It is therefore doubtful that the analysis of a particular market could be transposed directly to another one.

In this paper we develop a model where an intermediary, who wields influence over some consumers, derives revenues through an endorsement contract with one firm. This stripped-down model of biased intermediation allows us to address questions (i) and (ii) while taking into account the heterogeneity discussed above in a parsimonious way. In particular, we introduce the notions of congruence and conflict, which reflect the relationship between utility and revenue (and encompass quality and price competition respectively). We show that both the effects of bias and the efficacy of various policies differ significantly between these two cases. The congruence-conflict dichotomy can therefore be a useful tool for practical policy design.

More specifically, we consider a market composed of an intermediary and two ex ante

[1]For example, the technology press reports that heavy bidding forced Google to pay \$300 million for the right to be the default search engine in Mozilla Firefox (All Things D, 2011) and \$1 billion for similar rights across Apple's suite of products (Recode, 2014). Other examples include doctors, who receive payments from pharmaceutical companies that are known to influence their prescription behavior (Engelberg, Parsons, and Tefft, 2013), and commissions paid by financial service providers to financial advisers.

[2]For example, Google is vertically integrated with a number of specialised websites (such as Google Maps and YouTube) and affords them favourable placement in its search results. Edelman and Lai (2013) provide evidence that this favouritism drives more traffic to these websites than they would enjoy without the upstream support.

identical downstream firms. Each firm takes an action (e.g. invests in quality, sets a price) that simultaneously determines the firm's per-consumer revenue and the utility it delivers to consumers. Consumers share the same preferences with respect to firms, but differ with respect to their information about the offers: informed consumers go directly to the best firm, whereas uninformed ones rely on the intermediary to choose among the two. Our main focus is on contracts where the intermediary sells its recommendation to one of the firms, thereby committing to direct all uninformed consumers to that firm (an arrangement we refer to as integration).

We study two different cases corresponding to different strategic environments. In the first a firm's action is a fixed-cost investment that increases both the utility that it delivers and the revenue that it derives from each consumer. We refer to this as the *congruent payoffs model*. A key application of this model is one where advertising-supported websites invest in the quality of their content, which directly benefits consumers and increases advertisers' willingness to pay.[3] This part of the paper is of particular interest, both because of its relevance to several ongoing cases in technology markets and because most models of intermediation focus on environments without congruence, leaving a gap in the literature.

In the second case, the firm's action increases revenue but decreases the utility delivered to consumers. We refer to this as the *conflicting payoffs model*. While markets where firms' main strategic decision is their price are the most natural application of this model, the framework also allows us to discuss investments in quality with variable costs.

For each of the congruent and conflicting payoffs models, we characterize the Nash equilibrium of the game with a biased intermediary and compare it to a benchmark where the intermediary is objective. In both cases equilibria are in mixed strategies but deliver starkly different messages. In the congruent payoffs model (Section 3), integration reduces the investment incentives of the non-integrated firm and can fully-foreclose competition—even when there is a positive share of informed consumers in the population. Although the intermediary is biased towards the integrated firm and recommends it even when it is inferior, this firm tends to invest more than its rival (in a stochastic sense) due to an increased scale. This strategic effect partially *offsets* the initial bias, so that the intermediary produces the right recommendation "on average". In some cases the scale effect is so strong as to make integration beneficial for consumers.

In the conflicting payoffs model (Section 4), the bias towards the integrated firm is *compounded* by a strategic effect that leads both firms to choose higher revenues (e.g. higher prices), with the integrated firm offering a lower utility to consumers on average. The bias induced by integration therefore always hurts consumers. Unlike in the congruence case, the profitability of integration does not stem from scale effects and foreclosure, but

[3] We provide a microfoundation for this reasoning below, as well as an example of a competition case where the court explicitly took this view of the market.

rather from a softening of competition.

We next examine a range of common policy interventions and show that the congruence-conflict dichotomy is also of utility here (Section 5). Mandatory disclosure of intermediary bias works well under conflicting payoffs, but not under congruent payoffs. Indeed in the latter case an intermediary that is ex ante biased tends to recommend the best firm, making it rational to follow the advice (even when bias is harmful in aggregate).

We also look at policies that directly regulate the recommendations: *neutrality* forces the intermediary to grant equal prominence to each firm, while *mandated access* forces it to run a "short term" auction between downstream firms. Under congruence, neutrality alleviates the risk of foreclosure but reduces the quality of the recommendation, whereas mandated access leaves scope for foreclosure but increases the quality of the recommendation. The net effect of these policies is thus ambiguous. Under conflicting payoffs, both neutrality and mandated access lead to lower prices. However mandated access in this case also reduces the quality of the recommendation, unlike neutrality.

We then consider competition policy tools. Under congruence, breaking up the integrated firm (divestiture) benefits consumers if and only if the intermediary's influence, measured by the number of uninformed consumers, is small. Fostering competition at the intermediary level is unlikely to be effective. Under conflict, both policies tend to increase consumer surplus.

In section 6 we consider several applications of the model, and in section 7 we discuss several assumptions.

1.1 Related literature

Intermediation and certification This paper contributes to the literature on intermediaries. First, several recent papers study intermediaries whose role is to help consumers choose among products, and focus on how commissions or other contracts affect both the quality of the recommendation and downstream firms' behavior. Inderst and Ottaviani (2012a) study the effects of mandatory disclosure of commissions, while Inderst and Ottaviani (2012b) focus on the intermediary's equilibrium compensation structure and associated policies. Unlike us, these papers also consider the intermediary's incentives to acquire information. Armstrong and Zhou (2011) look at several models where firms can become prominent, one of which involves the payment of commissions to an intermediary. de Cornière and Taylor (2014) study the determinants of search engine bias and its effect on websites' strategies. Using our terminology, these papers only consider environments with *conflicting payoffs*, and therefore ignore for instance the question of quality provision by the downstream firms. An important contribution of our work is to show how this kind of difference in strategic environment determines the implications of bias and the relevant policy prescription.

Second, a larger literature studies certification intermediaries, whose role is to disclose to consumers the quality of the products that are offered. In contrast with the papers mentioned above and with ours, this literature has mostly abstracted away from competition between downstream firms.[4] In the absence of commitment power by the intermediary,[5] an important concern is the threat of collusion between the intermediary and the firms it is supposed to certify. Several papers examine the role of reputational incentives as a disciplining device for the intermediary (Biglaiser, 1993; Strausz, 2005; Peyrache and Quesada, 2011; Durbin and Iyer, 2009). Rather than studying *conditions* under which collusion can or cannot occur when qualities are exogenous, we study the *effects* of collusion ("bias" in our model) on the equilibrium behavior of firms (e.g., choice of quality or price).[6] While our model is static and thus ill-suited to direct study of reputation effects in depth, we discuss how our model can shed new light on these effects in Section 7.

Some papers cover related themes in the context of intermediation on online markets. One aspect of the net neutrality debate concerns agreements between Internet Service Providers (ISP) and Content Providers (CP) whereby some CPs can ensure preferential treatment for themselves by paying the ISP (a "fast lane access"). Choi and Kim (2010) and Economides and Hermalin (2012), for instance, study how such agreements affect the ISP's incentives to invest. In contrast to this, we focus on *downstream* investment incentives.[7] Another series of papers study the effect of news aggregators on competition among content providers (Dellarocas, Katona, and Rand, 2013; Jeon and Nasr, 2012; Rutt, 2011). News aggregators help consumers identify quality content, and the above papers study how their presence affects content providers' incentives to invest in quality. Unlike the present paper, this literature has not investigated cases where the intermediary is biased towards a subset of content providers.

Bundling Formally, one could view our model as one on bundling between a product A (the intermediation service) and a product B (the downstream product), where some consumers (the uninformed) view A and B as perfect complements while others only care about B. Among other strategic reasons, bundling can be used to foreclose rivals (see, e.g., Whinston, 1990, Nalebuff, 2004, Carlton and Waldman, 2002 for models of price competition; Choi and Stefanadis, 2001, Choi, 2004 for models with R&D) or to soften competition (Carbajo, De Meza, and Seidmann, 1990, Chen, 1997), the two being somewhat incompatible (if bundling softens competition, thereby raising the entrant's post-entry payoffs, it cannot deter entry). Interestingly, the congruence/conflict dichotomy introduced by our model provides some guidance as to when one should expect either

[4]An exception is Buehler and Schuett (forthcoming).

[5]See Lizzeri (1999) and Albano and Lizzeri (2001) for models with perfect commitment power, respectively with exogenous and endogenous quality.

[6]Biglaiser and Friedman (1994) deal with reputational incentives in a setup with endogenous qualities.

[7]Choi and Kim (2010) also look at downstream investment, but assume that such investments do not benefit consumers, unlike in our setup.

effect to motivate bundling: foreclosure under congruent payoffs, softening with conflicting payoffs. Moreover, foreclosure may actually benefit consumers.

2 Baseline model

The market we consider is composed of two ex ante symmetric downstream firms, $i \in \{1, 2\}$, one intermediary, and a unit mass of consumers.

Investment and payoffs Each consumer can trade with one firm. When a consumer trades with firm i, he obtains a utility u_i and the firm's revenue is r_i. Both u_i and r_i are endogenous, and depend on an action taken by firm i. We model firm i's decision as the choice of a per-user revenue r_i, which it can achieve by paying $K(r_i)$. We assume that $K' \geq 0$, $K'' \geq 0$ and $K(0) = 0$. The resulting utility u_i can be written as $u(r_i)$.

In section 3, we can interpret r_i as firm i's quality, and assume that u is an increasing function of r_i. In section 4, we assume that increasing revenue comes at the expense of consumers (as would be the case if r_i is simply a price), so that u is decreasing in r_i. We refer to the $u'(r) > 0$ case as congruence, and that with $u'(r) < 0$ as conflict.

Information and intermediation Consumers have the same preferences regarding firms, but differ with respect to the information they have access to. *Informed* consumers are aware of the firms' offers, and always choose the one that delivers the highest utility. The mass of informed consumers is $1 - \mu$. The remaining μ consumers are *uninformed*; they can only access the product market through the intermediary, which directs them towards one of the two downstream firms. For the time being we assume that uninformed consumers do not behave strategically, and simply follow the recommendation. We come back to this issue in section 5.1.1.

An alternative interpretation of the model is that all consumers visit the intermediary, in which case μ captures its ability to steer consumers towards one of the firms, for instance by giving it more prominence. This interpretation is more relevant for the case of platforms that allow consumers to search: a share $1 - \mu$ of consumers examine all the options, while a share μ simply choose the most prominent firm.

We are interested in situations in which the intermediary has financial incentives to be biased in favor of one firm. A simple way to capture this is to assume that the intermediary runs an auction and commits to direct uninformed consumers towards the winning firm. We will compare this mechanism to a case in which the intermediary always recommends the best firm for consumers (the objective benchmark).

Timing and equilibrium The game proceeds as follows: At $t = 1$, the intermediary sells its recommendation through an auction between firms 1 and 2. At $t = 2$, firms choose

r_1 and r_2. At $t = 3$, informed consumers visit the firm providing the highest u_i, while uninformed consumers visit the firm recommended by the intermediary.

The solution concept is subgame perfect equilibrium. As will be apparent, the equilibrium sometimes involves mixed strategies at $t = 2$. We denote $F_i(r)$ the probability that firm i plays $r_i \leq r$.

Discussion Several of our assumptions warrant a discussion. First, in our setup, assuming that the intermediary runs an auction is formally equivalent to assuming that the intermediary becomes integrated with one of the firms and systematically recommends it: in the former case firms will bid the expected additional profit from being recommended, while in the latter this profit accrues directly to the intermediary at $t = 3$. We therefore refer to this regime as *integration* when we compare it to others. We postpone the discussion of the optimality of such a mechanism to section 7.

Second, our choice of timing (where the auction happens before the choices of r) is meant to capture situations in which the intermediary is engaged in a long-term relationship with one firm, such as under vertical integration. We discuss how reversing this timing changes the outcome when we consider the effects of a policy forcing divestiture.

Third, in our baseline model the intermediary has no incentives to cater to consumers' interests: their participation is inelastic (uninformed consumers cannot bypass the intermediary), and it cannot charge them for using its services. These simplifications allow us, in a first step, to focus on the consequence of biased recommendations rather than on conditions for bias to emerge, and in particular to contrast environments with congruence and conflict. In our policy analysis in section 5 we consider the implications of elastic demand and advice fees.

Our model also ignores realistic aspects such as product differentiation, cost asymmetries, or stochastic investment technologies. One benefit of our approach is that it allows us to derive clear analytical results in a transparent way. One feature of our simple framework is that it leads to mixed strategy equilibria in some cases. However, the main effects of vertical integration do not hinge on firms playing mixed strategies: models including differentiation or stochastic investment technologies deliver qualitatively similar results under pure strategies, albeit in a less tractable fashion.

3 Congruent payoffs model

In this section, we study situations in which investment by firms causes both their per-user revenue and consumers' utility to increase. Formally, we have $u'(r) > 0$. This assumption is useful in describing environments in which pricing is not the primary means through which firms seek to attract consumers. The main strategic implication is that a firm cannot sacrifice revenue to attract more consumers. Notice that the congruence between consumers' and firms' payoffs is not perfect, as firms alone bear the full cost of investment (here we assume that $K'(r_i) > 0$ and $K''(r_i) > 0$.). Slightly abusing language, we will refer to r_i as firm i's quality.

A key motivating example is the ongoing policy debate around the consequences of Google's alleged search bias, i.e. the preferential treatment granted to Google's affiliates (Youtube, Google Shopping, Google Maps, etc.). In this context, one can think of the two firms as websites that uninformed consumers can only access through a monopolistic search engine. The assumption that $u'(r) > 0$ is consistent with a situation where websites are vertically differentiated and purely advertising-supported.[8] Many other technology platform markets follow a similar template. For example, the firms might be free mobile phone apps, one of which is installed as default on a phone operating system (the "intermediary").[9] The uninformed consumers would be those who only consider the default application.

3.1 Benchmark: objective intermediary

We begin our analysis with the benchmark case in which the intermediary always recommends the firm with the highest quality.[10] In other words, here the intermediary's sole purpose is to inform otherwise uninformed consumers, with the result that all consumers choose the best firm. For future analysis, it will be useful to note that this case is formally equivalent to the integrated case with $\mu = 0$. Although an objective intermediary maximizes ex post welfare by directing consumers toward the best firm, it does not necessarily maximize ex ante welfare because investment incentives are ignored.

Assuming symmetric tie-breaking,[11] firm i's profit is

$$\pi_i = \begin{cases} r_i - K(r_i) \text{ if } r_i > r_j \\ r_i/2 - K(r_i) \text{ if } r_i = r_j \\ -K(r_i) \text{ if } r_i < r_j. \end{cases}$$

There is a unique equilibrium, which is as follows:

Lemma 1. *When the intermediary is objective, there is a unique equilibrium in the congruence model. Both firms set quality according to distribution $F(r) = K(r)/r$ on support $[0, \overline{r}]$, where $\overline{r}$ solves $K(r) = r$. Both firms earn zero (expected) profit in equilibrium.*

[8] For instance, suppose that a consumer who spends t units of time on a website of quality s obtains utility $v(s,t) - \kappa(t)$, where $v_1, v_2, v_{12} > 0$ and the opportunity cost of time, $\kappa(t)$, is increasing and convex. It is immediate that the optimal time spent on a website, $t^*(s)$, is increasing in s. The website displays one ad, which the consumer notices between t and $t + dt$ with probability λdt. If advertisers' willingness to pay to be noticed is a, then websites' per-visitor revenue is $r = \lambda a t^*(s)$. By the envelope theorem, $\partial u/\partial s = v_1(s, t^*(s))$. Both r and u are increasing in quality as required.

[9] The European Commission is currently conducting an investigation of such app bundling on Google's Android operating system.

[10] That is, there is no auction at $t = 1$.

[11] We make the symmetric tie-breaking assumption for brevity. Results are unaffected if ties are broken asymmetrically.

Omitted proofs are in the Appendix. The intuition for the presence of mixed strategies in equilibrium is analogous to all-pay auctions or contests, in which players incur the cost of effort (here $K(r)$) no matter the outcome.[12] A pure strategy equilibrium would necessarily involve one of the two firms (say 2) to exert no effort: $r_2 = 0$ and $r_1 > 0$. This would imply $\pi_2 = 0$. But this cannot be an equilibrium: if $\pi_1 > 0$ then 2 would have an incentive to choose $r_2 = r_1 + \epsilon$, and if $\pi_1 = 0$ then firm 1 could profitably deviate by choosing a lower r_1.

3.2 Equilibrium analysis of the integrated case

Let π_R be the expected gross profit of the firm that is recommended by the intermediary, and π_N that of the firm which is not recommended. Assuming that $\pi_R \geq \pi_N$, both firms are willing to pay up to $\pi_R - \pi_N$ to the intermediary at $t = 1$. The intermediary's profit is therefore $\pi_R - \pi_N$. We now study the subgame starting from $t = 2$ (i.e. the investment stage) assuming that firm 1 has won the auction and is therefore integrated with the intermediary.

At $t = 3$, the intermediary directs the μ uninformed consumers towards firm 1, while informed consumers visit the firm delivering the highest utility (in this case, the highest r_i). This implies that firm 1 has a guaranteed market share of μ. Profits can therefore be written as:

$$\pi_1 = \begin{cases} r_1 - K(r_1) \text{ if } r_1 > r_2 \\ r_1 [\mu + (1-\mu)/2] - K(r_1) \text{ if } r_1 = r_2 \\ r_1\mu - K(r_1) \text{ if } r_1 < r_2, \end{cases}$$

$$\pi_2 = \begin{cases} r_2(1-\mu) - K(r_2) \text{ if } r_2 > r_1 \\ r_2(1-\mu)/2 - K(r_2) \text{ if } r_2 = r_1 \\ -K(r_2) \text{ if } r_2 < r_1. \end{cases}$$

Let $\hat{r} \equiv \text{argmax}_r\{r - K(r)\}$ be the per-user revenue that would be chosen by a monopolist. We have the following result:

Lemma 2. *There exists a unique equilibrium of the sub-game in which the intermediary is integrated with firm 1.*

(i) If $\mu \geq 1 - \frac{K(\hat{r})}{\hat{r}}$, then $r_1 = \hat{r}$, $r_2 = 0$. Firm 1's (gross) profit π_R equals the monopoly profit. No consumer is matched with Firm 2 and $\pi_N = 0$.

(ii) If $\mu < 1 - \frac{K(\hat{r})}{\hat{r}}$, the equilibrium is in mixed strategies. Firm 1 chooses r_1 from a non-degenerate interval (given in the proof) $[\underline{r}, \overline{r}]$ with a mass point at $\underline{r}$. Firm 2

[12] Unlike a standard contest, here the value of the prize is increasing in the effort.

chooses $r_2 \in \{0\} \cup [\underline{r}, \overline{r}]$ with a mass point at 0. In the interior of the support, firms play according to

$$F_1(r) = \frac{K(r)}{(1-\mu)r}, \quad F_2(r) = \frac{K(r) + \mu(\overline{r} - r)}{(1-\mu)r}.$$

Firm 1 earns positive profit $\pi_R \in (0, \hat{\pi})$, while firm 2's profits are zero.

Intuitively, a pure strategy equilibrium must have $r_1 > 0$ and $r_2 = 0$. Indeed, given that firm 1 always serves the μ uninformed consumers, it always chooses $r_1 > 0$. Moreover having $r_2 > 0$ in a pure strategy equilibrium would mean that firm 2 is serving the informed consumers, so that $r_2 > r_1$. But if it is profitable for firm 2 to serve the informed consumers by investing $K(r_2)$, it must also be profitable for firm 1 to serve them by investing an extra $K(r_2) - K(r_1)$. If $r_2 = 0$ in a pure strategy equilibrium, it must be that $r_1 = \hat{r}$. The condition for this to be an equilibrium is that μ be large enough, so that firm 2 cannot profitably deviate to $r_2 = \hat{r} + \epsilon$.

If μ is below the threshold, the equilibrium is in mixed strategies. The transition between the mixed equilibrium and the pure equilibrium is continuous: $\underline{r}$ and $\overline{r}$ converge to $\hat{r}$, and the mass at 0 for firm 2 converges to 1, as μ increases to $1 - \frac{K(\hat{r})}{\hat{r}}$.

The next result describes the effects of integration.

Proposition 1. *With congruent payoffs, vertical integration:*

(i) leads firm 1 to invest more than firm 2, albeit only in a first-order stochastic sense when $\mu < 1 - \frac{K(\hat{r})}{\hat{r}}$;

(ii) leads firm 2 to invest less (in a FOSD sense) than it would under an objective intermediary;

(iii) leads to a rotation in F_1, and can increase or decrease $E[r_1]$ compared to the objective benchmark;

(iv) can increase or decrease consumers' expected utility compared to the objective benchmark.

Proposition 1 (i) reveals that under integration, the intermediary tends to recommend the right firm, but does not do so with probability 1.[13] Interestingly, it is because the intermediary recommends it that firm 1 tends to invest more than its rival, not the other way around.

Figure 1 illustrates the first three parts of the proposition. It is interesting to consider the intuition behind it. Fix a distribution, F_1, for r_1. Firm 2's profit is then

$$\pi_2(r_2) = (1-\mu)F_1(r_2)r_2 - K(r_2). \tag{1}$$

[13] If μ is small. When μ is large firm 1 is always better.

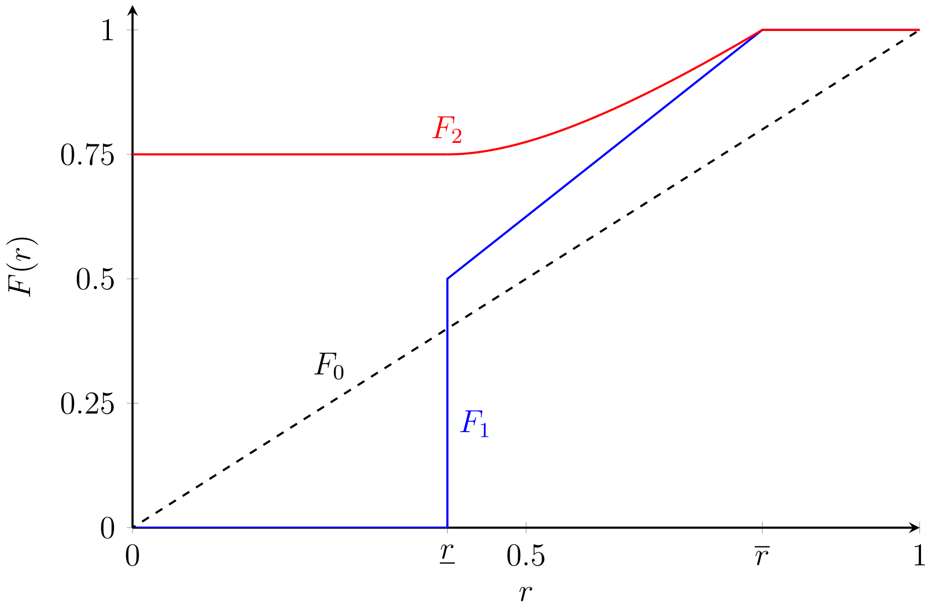


Figure 1: Equilibrium CDFs in the objective benchmark (F_0) and under vertical integration (F_1 and F_2) with $K(r) = r^2$ and $\mu = 1/5$.

Heuristically, the marginal return to an increase in r_2 under an objective intermediary (recalling that objectivity corresponds to $\mu = 0$) is

$$\left.\frac{\partial \pi_2}{\partial r_2}\right|_{\mu=0} = F_1(r_2) + F_1'(r_2)r_2 - K'(r_2).$$

There is a marginal effect (a higher quality increases expected demand) and an inframarginal effect (a higher quality implies more revenue per-consumer). The corresponding expression under vertical integration is

$$\frac{\partial \pi_2}{\partial r_2} = (1-\mu)F_1(r_2) + (1-\mu)F_1'(r_2)r_2 - K'(r_2).$$

We see that vertical integration affects the incentive to provide quality through two channels. The first, captured by $(1-\mu)F_1(r_2) < F_1(r_2)$, is a *scale effect*. Since costs are fixed, firms enjoy economies of scale in quality provision. By diverting consumers to firm 1, integration reduces firm 2's expected scale and thus its quality provision incentive. The second effect, reflected in $(1-\mu)F_1'(r_2)r_2 < F_1'(r_2)r_2$, is a *competition effect*. Under vertical integration, the uninformed consumers' demand is insensitive to quality so that the total mass of consumers over whom the firms compete is reduced and the incentive to compete in quality is diminished. Both the scale and competition effects point in the same direction, resulting in a (stochastically) lower equilibrium quality for firm 2.

Fixing the distribution of r_2 at F_2, the profit of firm 1 is

$$\pi_1(r_1) = (\mu + (1-\mu)F_2(r_1))\, r_1 - K(r_1). \tag{2}$$

Repeating the previous exercise, the marginal profit from quality provision under an objective intermediary is

$$\left.\frac{\partial \pi_1}{\partial r_1}\right|_{\mu=0} = F_2(r_1) + F_2'(r_1)r_1 - K'(r_1)\,,$$

whereas that under integration is

$$\frac{\partial \pi_1}{\partial r_1} = [\mu + (1-\mu)F_2(r_1)] + (1-\mu)F_2'(r_1)r_1 - K'(r_1)\,.$$

The competition effect experienced by firm 2 is also present here: like 2, firm 1 does not have to compete for consumers who will be directed to it anyway. However, the direction of the scale effect is now reversed ($\mu + (1-\mu)F_2(r_1) > F_2(r_1)$). By directing uninformed consumers to firm 1, the intermediary endows 1 with larger scale, which provides an impetus for 1 to increase its quality. Since the competition and scale effects work in opposite directions, the effect of vertical integration on firm 1's expected quality is, in general, ambiguous.

This ambiguity in firm 1's investment lies behind part (iv) of Proposition 1. Figure 2 compares the expected quality experienced by consumers under integration and under the objective benchmark for the constant-elasticity cost function $K(r) = r^\alpha$. The scale effect dominates the competition effect when α is small, in which case integration increases the expected quality.

4 Conflicting payoffs model

We now turn to markets with conflicting payoffs, where increases in revenues come at the expense of consumers ($u'(r) < 0$). The most natural instance of conflicting payoffs is when r_i is the price set by firm i and $u(r_i) = v - r_i$. Most existing models of intermediation consider variants of this setup and our discussion here will be framed in terms of pricing. But our model of conflict also applies to cases without pricing. For example, suppose firms compete in quality (along the lines of the previous section) but with costs borne on a per-consumer basis. Then it is easy to show that payoffs are in conflict and the below analysis applies.[14] A third way that conflict can arise is if firms make an up-front

[14]Suppose firm i offers quality q_i, which costs $k(q_i)$ to provide to each of its N_i consumers. Its profit is $N_i(q_i, q_j)[q_i - k(q_i)]$, where N_i is increasing in q_i. Firm i will never choose a q_i such that the average revenue, $r_i \equiv q_i - k(q_i)$, is increasing, because choosing $q_i + \epsilon$ would increase both the number of consumers and the revenue per-consumer. Therefore, over the relevant range, we have $u'(r) < 0$.

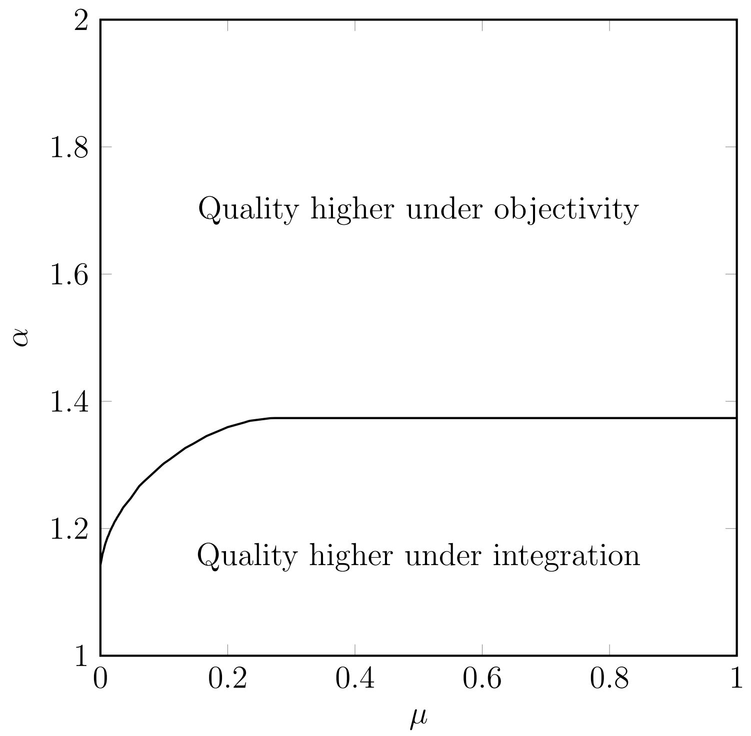


Figure 2: Comparison of expected transaction quality under objective benchmark and vertical integration when $K(r) = r^{\alpha}$.

investment in a technology that harms consumers. For example, a firm may invest in a tracking technology that compromises users' privacy in order to target advertisements. The technology, which requires an investment $K(r)$, allows firms to generate a per-consumer revenue of r. Provided that consumers dislike invasions of privacy, we would again have $u'(r) < 0$.

The purpose of this section is to study the effects of integration in such environments. To do so, we nest these possibilities in a model where firms choose a per-user revenue, r, at fixed cost $K(r)$ (with $K(r) = 0$ corresponding to the case of pure price competition). Informed consumers choose the firm with the *lowest* r, while uninformed consumers are directed to the integrated firm (firm 1) by the intermediary. Firms' payoffs are given by

$$\pi_1 = \begin{cases} r_1 - K(r_1) \text{ if } r_1 < r_2 \\ r_1 \left[\mu + (1-\mu)/2\right] - K(r_1) \text{ if } r_1 = r_2 \\ r_1 \mu - K(r_1) \text{ if } r_1 > r_2, \end{cases} \tag{3}$$

$$\pi_2 = \begin{cases} r_2(1-\mu) - K(r_2) \text{ if } r_2 < r_1 \\ r_2(1-\mu)/2 - K(r_2) \text{ if } r_2 = r_1 \\ -K(r_2) \text{ if } r_2 > r_1. \end{cases} \tag{4}$$

Lemma 3. *In the conflicting payoffs model, for any μ, there exists a unique equilibrium under integration.*

1. *Firm 1 mixes over $[\underline{r}, \widetilde{r}] \cup \{\overline{r}\}$, with a mass point at $\overline{r}$ ($\underline{r} \leq \widetilde{r} \leq \overline{r}$ are given in the proof). In the interior of this support, 1 plays according to*

$$F_1(r) = \frac{(r - \underline{r})(1-\mu) - K(r) + K(\underline{r})}{r(1-\mu)}.$$

2. *Firm 2 mixes over $[\underline{r}, \widetilde{r}]$ with a mass point at $\widetilde{r}$. In the interior of this support, 2 plays according to*

$$F_2(r) = \frac{r - \mu\overline{r} - K(r) + K(\overline{r})}{r(1-\mu)}.$$

As the cost function K becomes close to linear (a case that includes pure pricing with $K(r) = 0$), the model smoothly converges to an equilibrium in which both firms mix over a shared support as in Narasimhan (1988).

Example 1. *In the pure-pricing case ($K(r) = 0$ and $u(r_i) = v - r_i$), firms mix over $[\mu v, v]$. For any $r \in [\mu v, v)$, $F_1(r) = 1 - \frac{\mu v}{r}$; firm 1 plays $r_1 = v$ with probability μ. For any $r \in [\mu v, v]$, $F_2(r) = \frac{r - \mu v}{r(1-\mu)}$.*

As costs become sufficiently convex, the model converges to a pure strategy equilibrium, with firm 1 setting $r_1 = \overline{r}$ and catering to uninformed consumers, and firm 2 attracting informed consumers with $r_2 = \underline{r}$.

If the intermediary behaves objectively—always recommending the low r firm—then all consumers visit the firm choosing the lowest r. Firms facing an objective intermediary therefore compete in the manner of Bertrand, and the game has a symmetric equilibrium at $r_1 = r_2 = 0$.[15] From Lemma 3 the following proposition is immediate.

Proposition 2. *The equilibrium of Lemma 3 is such that*

1. *compared to the objective case ($\mu = 0$), vertical integration causes both firms to choose a higher r;*

2. *firm 1 chooses a (stochastically) higher r than does firm 2: $F_1(r) \leq F_2(r)$ for all r.*

The intuition is as follows: The presence of μ uninformed (captive) consumers makes firm 1's demand less elastic, causing it to increase its choice of r. The higher is r_1, the less competitive pressure is faced by firm 2, so firm 2 also increases its r (albeit to a lesser extent). In sum, integration reduces every consumer's expected surplus and leads the intermediary to recommend a firm that is (stochastically) worse than its rival. We thus observe a striking contrast with the congruent payoffs case considered above, where the effects of vertical integration were potentially positive for consumers and where the intermediary's advice tended to be good ex post.

[15] For a rigorous proof see the Proof of Lemma 3, where it can be seen that $\lim_{\mu \to 0} \overline{r} = 0$.

Discussion As the analysis of sections 3 and 4 reveals, contracts between the intermediary and a downstream firm are more likely to harm consumers when payoffs are conflicting than when they are congruent. In the former case the direct effect of bias is compounded by a strategic effect leading both firms—and in particular the integrated one—to extract more revenue from consumers. In the latter case the strategic effect partially offsets the direct effect, and may even overturn the results.

One interesting difference between the two environments concerns the channel through which integration increases the firms' profit. In the congruent payoffs model, integration allows firm 1 to partially or totally foreclose its rival and leverage increased scale. The logic is one of *exclusion*, whereby only the integrated firm profits from integration. By contrast, in the conflicting payoffs model integration is profitable because it *softens* competition and therefore benefits both firms.

By nesting both models in the same basic framework we are able to shed light on the forces responsible for driving this stark difference in predictions. In particular, the difference is not simply one between quality and price competition (for we have shown that the conflict model can be one of quality competition). Nor is the difference driven by the presence or absence of costs and scale economies as we have allowed for investment costs in both cases. Rather, the key distinction between the two models is the sign of $u'(r)$, suggesting that the notions of congruence ($u'(r) > 0$) and conflict ($u'(r) < 0$) are germane for understanding the effects of intermediary vertical integration. This becomes all the more apparent in the next section, where we find that the congruence-conflict distinction is also crucial for understanding the effects of various policy interventions.

5 Policy analysis

Various policies have been implemented or suggested to counteract the potentially undesirable effects of bias or vertical integration in the intermediary markets that we mention in the introduction. In this section we study some of these policies, and evaluate their effect on equilibrium bias and consumer surplus.

We classify the policies according to whether they primarily operate by regulating the intermediary's behavior (Section 5.1) or whether they rely more on competition policy tools to discipline the intermediary (Section 5.2). Our analysis shows that the distinction between the models with congruent and conflicting payoffs has important policy implications: what works in the conflict model (e.g. mandatory disclosure of bias) may not work with congruence, or vice versa (e.g. mandated access policies). In order to obtain further results, we sometimes restrict attention to quadratic investment cost in the congruence model (a case in which integration lowers surplus compared to an objective intermediary), or to the pure pricing case (i.e. $K(r) = 0$) in the model with conflicting payoffs.

For the sake of brevity, we omit (part of) the formal treatment of these policies in the text, and refer the interested reader to Appendix B.

5.1 Regulation or behavioral remedies

5.1.1 Mandatory disclosure of bias

In the baseline model, we have assumed that uninformed consumers always follow the intermediary's recommendation, irrespective of whether it is in their best interest to do so. This could result either from the intermediary only allowing consumers to visit firm 1, or from consumers' unawareness of the link between firm 1 and the intermediary. A first way to tackle the potential issues associated with integration would then be to (i) force the intermediary to allow consumers to reach both firms, and (ii) mandate the disclosure of the link to uninformed consumers.[16]

The effects of such a policy are described below:

Proposition 3. *1. In the model with congruent payoffs, the equilibrium of Lemma 2 survives the introduction of mandatory disclosure of bias.*

2. In the model with conflicting payoffs, the unique equilibrium under mandatory disclosure is such that uninformed consumers visit each firm with probability $1/2$*. Consumer surplus is higher than under unregulated integration.*

The proof of part 1 is straightforward and is omitted. With congruent payoffs, the integrated firm has a higher expected r than its rival (Proposition 1), so uninformed consumers optimally visit the integrated firm. The policy has no effect on equilibrium.[17]

With conflicting payoffs, this logic does not work. Indeed, in Proposition 2, the integrated firm provides a lower expected utility than its rival, so that disclosure of bias cannot leave the equilibrium unchanged. More generally, we show in Appendix B.1 that, if μ_1 uninformed consumers follow the recommendation and μ_2 go against it and choose firm 2, with $\mu_1 > \mu_2$, then firm 2 offers a higher utility on average. The unique equilibrium must therefore have $\mu_1 = \mu_2$. Regarding surplus, we show in Appendix B.1 that firms choose higher levels of r (in a first-order stochastic sense) as we increase the asymmetry $\mu_1 - \mu_2$, taking $\mu = \mu_1 + \mu_2$ as fixed.

[16]Without (i), condition (ii) would be inoperant as uninformed consumers would have no way of reaching firm 2. Formally, the game is as above except that uninformed consumers now form rational expectations about r_1 and r_2 and choose to either follow the intermediary's recommendation or two ignore it and visit firm 2 instead.

[17]There are two other equilibria: a formally equivalent one in which roles are reversed (all uninformed consumers go to firm 2), and one in which they ignore the recommendation and visit each firm with equal probability. This "babbling equilibrium" is outcome-equivalent to a neutrality policy, which we analyze below.

In the model with conflicting payoffs, the policy therefore alleviates the concerns related to integration—even though the outcome is still inferior for consumers to having an objective intermediary.

5.1.2 Charging consumers in the model with congruent payoffs

In order to reduce the (potential) problem of bias, one might look at ways to make the intermediary internalize consumer surplus. Such an approach has been used, for instance, in the the UK market for financial advice, where intermediaries' revenues must come from charges paid by consumers. We study such policies in the model with congruent payoffs.

We assume that consumers have access to both firms for free, but we allow the intermediary to charge consumers a fee in exchange for a recommendation, while maintaining mandatory disclosure of bias. When the intermediary can levy consumer charges, mandatory disclosure of bias can serve an additional role, by enabling the intermediary to commit *not* to be biased.

The timing is as follows: at $t = 1$ the intermediary chooses and publicly announces a bias, $\beta \in \{0, 1\}$, and transfer, $T \in \mathbb{R}_+$. $\beta = 1$ corresponds to a biased intermediary that unconditionally recommends its affiliated downstream firm, while $\beta = 0$ corresponds to an objective intermediary. At $t = 2$, firms simultaneously choose r_i. At $t = 3$, uninformed consumers decide whether to follow the intermediary's advice (paying a fee T), or bypass the intermediary and visit a firm at random. For a given β, the subgame starting at $t = 3$ is equivalent to the game studied in Section 3 with a mass $\beta\mu$ of uninformed consumers. The $T(\beta, \mu)$ that makes consumers indifferent between the intermediary's advice and the outside option[18] is

$$T(1, \mu) = E[u(r_1)|\mu \text{ uninformed}] - \frac{E[u(r_1) + u(r_2)|\mu \text{ uninformed}]}{2},$$

$$T(0, \mu) = E[\max\{u(r_1), u(r_2)\}|\text{no uninformed}] - \frac{E[u(r_1) + u(r_2)|\text{no uninformed}]}{2}.$$

We have the following:[19]

Proposition 4. *Suppose that payoffs are congruent with $u(r) = r$ and that the cost function is quadratic ($K(r) = r^2$). Then, for any $\mu > 1/5$, the intermediary can charge a higher consumer fee if it is biased (i.e. $T(1, \mu) > T(0, \mu)$).*

Proposition 4 says that (when μ is large enough) the intermediary can extract a higher fee from consumers if it is biased than if it commits to being objective.[20] This

[18] We assume that when an equilibrium with full participation exists consumers coordinate on it.

[19] The proof is given in Appendix B.1.

[20] This analysis only considers the profit that can be extracted from charging consumers. Bias also generates profits in the downstream market, meaning the intermediary will prefer to be biased for an even larger range of μ than identified in Proposition 4 (specifically, for any $\mu > 0.083$). The tendency for a

counter-intuitive result arises because bias reduces firm 2's quality relative to that of firm 1. This makes consumers' outside option of selecting a random firm less attractive relative to the intermediary's advice (which usually points to the best firm).

When the intermediary chooses $\beta = 1$, the outcome is formally equivalent to integration. The proposition therefore highlights that, in the congruence model, the efficacy of mandatory disclosure of bias, even when coupled with consumer fees, is limited.

5.1.3 Neutrality

The previous policies are mostly indirect: they do not forbid the intermediary from recommending its own firm. By contrast, we now turn to more direct approaches whereby the regulator can force the intermediary to change the way it recommends firms. Such approaches are regularly put forward in the policy debates.[21] The first, which we denote neutrality, forces the intermediary to grant equal prominence to the two downstream firms. Formally, both firms 1 and 2 receive $\frac{\mu}{2}$ uninformed consumers, while the informed consumers continue to choose the best firm.

In the model with congruent payoffs, profits are given by the following:

$$\pi_i(r_i, r_j) = \begin{cases} (1 - \mu + \frac{\mu}{2})r_i - K(r_i) & \text{if } r_i > r_j \\ \frac{\mu}{2} r_i - K(r_i) & \text{if } r_i < r_j. \end{cases} \tag{5}$$

In the model with conflicting payoffs, they are given by:

$$\pi_i(r_i, r_j) = \begin{cases} (1 - \mu + \frac{\mu}{2})r_i - K(r_i) & \text{if } r_i < r_j \\ \frac{\mu}{2} r_i - K(r_i) & \text{if } r_i > r_j. \end{cases} \tag{6}$$

The following lemma highlights an important property of the equilibrium distributions for a given μ_1 and μ_2 under both congruence and conflict.[22]

Lemma 4. *With both congruent and conflicting payoffs, $\mu_i \geq \mu_j$ implies that F_i first-order stochastically dominates F_j.*

Using this insight, as well as the derivation of equilibrium for $\mu_1 = \mu_2 = \mu/2$, which can be found in Appendix B.2, we have:

large intermediary to prefer bias is not special to the case of quadratic costs. One can check numerically, for instance, that a similar result holds for any $K(r) = r^\alpha$, $\alpha > 1$.

[21]For example, under the terms of a 2009 settlement, the European Commission required Microsoft to offer users of its Windows operating system a choice of web browsers from a list of competing alternatives. Since the ordering of the list was randomized, this amounts to equal treatment. See `https://blogs.technet.microsoft.com/microsoft_on_the_issues/2010/02/19/the-browser-choice-screen-for-europe-what-to-expect-when-to-expect-it/`, accessed 27 July 2016. Similar arrangements have been proposed by some opponents to Google. See for instance paragraphs 151-176 of the England and Wales High Court's StreetMap v. Google judgement (`http://www.bailii.org/ew/cases/EWHC/Ch/2016/253.html`).

[22]The proof is given in Appendix B.2 by Lemmas 5 (for congruence) and 7 (for conflict).

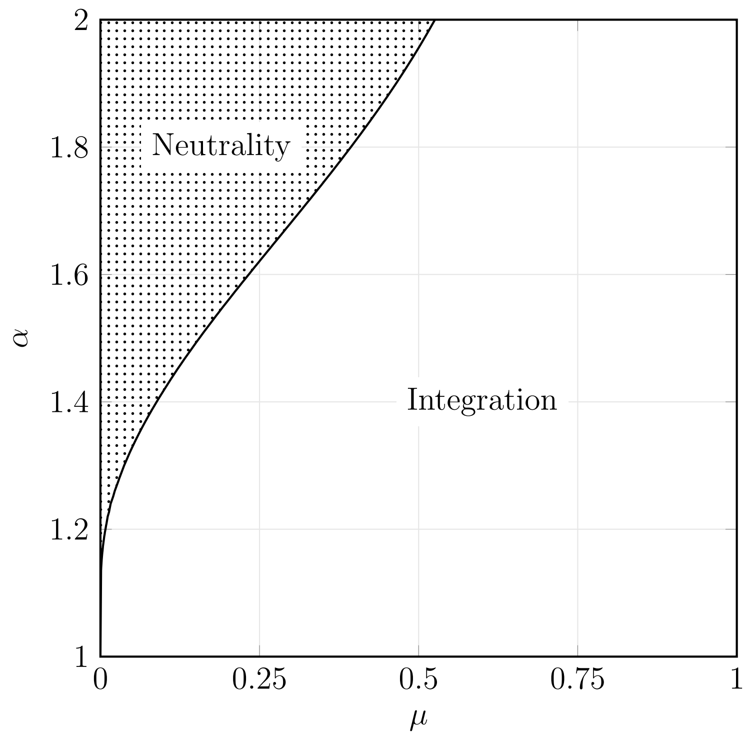


Figure 3: Suppose $K(r) = r^{\alpha}$ and payoffs are congruent. Then neutrality yields higher average transaction quality than integration in the shaded area.

Proposition 5. *Under intermediary neutrality, and compared to unregulated integration:*

- *In the congruence model, the intermediary recommends the right firm less often. Consumer surplus can increase or decrease.*
- *In the conflict model, the intermediary recommends the right firm more often. Consumer surplus increases.*

With $\mu_1 = \mu_2 = \frac{\mu}{2}$, the mixed-strategy equilibrium under neutrality is symmetric in both the congruence and the conflict models. Therefore, the recommendation is the right one with probability 1/2. But we know from the analysis of the integration regime that, in the model with congruent payoffs, the integrated firm (which is always recommended) delivers a higher utility the majority of the time, while the reverse is true in the model with conflicting payoffs. This explains the results on the recommendation quality.

Consumer surplus also depends on how the new regime affects firms' incentives. In the congruence model, neutrality eliminates foreclosure (firm 2 always invests a positive amount) but limits the extent to which firm 1 can enjoy economies of scale, which reduces its investment. The overall effect thus depends on the shape of the cost function, as illustrated in figure 3. In the conflict model, there is no ambiguity: both firms choose lower levels of revenues (in a stochastic sense), which benefits consumers.

Proposition 5 would also hold under a weaker regulation that would merely require a positive μ_2, without going as far as perfect symmetry.

5.1.4 Mandated access

In practice, a main challenge to the implementation of a neutrality regime consists in determining which downstream firm should benefit from the regulation. In particular, in a dynamic perspective, one would wish to avoid foreclosing future entry by innovators who would come after the beneficiaries of the regulation have been chosen. Instead of requiring the intermediary to recommend the non-integrated firm irrespective of its merits, an alternative and lighter approach is to force the intermediary to let non-integrated firms participate in an auction, the winner of which is recommended to uninformed consumers. This policy is similar in spirit to access arrangements that are often used when a monopolist controls an essential facility.

We model the mandated access policy using the following timing: at $t = 0$ the intermediary integrates with firm 1. At $t = 1$ firms simultaneously choose r_1 and r_2. At $t = 2$ firms observe both r_i's and bid in a first price auction for the right to be recommended.[23] At $t = 3$ informed consumers buy from the firm offering the greatest utility, while uninformed consumers buy from the firm recommended by the intermediary (i.e. the winner of the auction).

First, consider the stage 2 auction. For given r_1 and r_2, the unique equilibrium outcome in undominated strategies of the subgame is for firms to bid as follows: if $r_i > r_j$, firm j bids μr_j and firm i bids an amount arbitrarily close to, but larger than, μr_j. The firm with the largest r wins the auction under both congruent and conflicting payoffs.

With congruent payoffs, firms profits are given by

$$\pi_1(r_1, r_2) = \begin{cases} r_1 - K(r_1) & \text{if } r_1 > r_2 \\ \mu r_1 - K(r_1) & \text{if } r_1 < r_2 \end{cases} \tag{7}$$

and

$$\pi_2(r_1, r_2) = \begin{cases} (1-\mu) r_2 + \mu(r_2 - r_1) - K(r_2) & \text{if } r_2 > r_1 \\ -K(r_2) & \text{if } r_2 < r_1 \end{cases} \tag{8}$$

The key difference between the two is that firm 1 internalizes the revenue from the auction: when $r_1 < r_2$ it loses the auction for accessing the uninformed consumers, but the intermediary still receives μr_1. Firm 1's profit function is thus identical to the integrated case. For firm 2, the difference between mandated access and unregulated integration comes from the possibility of reaching uninformed consumers when $r_2 > r_1$ (albeit at a

[23] The first-price auction without reserve price is a convenient auction format to study in this complete information game. One property of this format is that, unlike e.g. a second-price auction, there are no incentives for firm 1 to "overbid" so as to increase the payment of firm 2. Additionaly, this mechanism allows firm 2 to capture the incremental value of its investment $(r_2 - r_1)$ when it wins.

cost of μr_1).

In the conflict model, focusing on the case of pure pricing (i.e. $K(r) = 0$), profits are

$$\pi_1(r_1, r_2) = \begin{cases} \mu r_1 & \text{if } r_1 > r_2 \\ r_1 & \text{if } r_1 < r_2 \end{cases} \tag{9}$$

and

$$\pi_2(r_1, r_2) = \begin{cases} (1-\mu) r_2 & \text{if } r_1 > r_2 \\ \mu(r_2 - r_1) & \text{if } r_1 < r_2. \end{cases} \tag{10}$$

We solve these two games in Appendix B.3. The main findings are summarized below:

Proposition 6. *Under mandated access, and compared to unregulated integration:*

- *In the congruence model, the intermediary always recommends the right firm. If $\mu \geq 1 - \frac{K(\hat{r})}{\hat{r}}$, foreclosure of firm 2 is the unique equilibrium and consumer surplus is unchanged. For smaller values of μ, consumer surplus can increase or decrease (see Figure 4).*
- *In the conflict model, the intermediary never recommends the right firm. With pure pricing ($K(r) = 0$), total consumer surplus is unchanged, but uninformed consumers are harmed by the regulation.*

The results regarding the quality of the recommendation are in stark contrast to Proposition 5, and show the impact of relying on a market mechanism to determine which firm gets recommended in a regulated regime. Under congruence, the market selects the right firm with probability 1 (a result reminiscent of Athey and Ellison, 2011 in the context of search engines). When payoffs are conflicting, the firm that is willing to bid more is also the one that extracts the most surplus from consumers, and a random recommendation (resulting from the neutrality policy) would be better for consumers.

In the model with congruent payoffs, when μ is large, mandated access results in the same outcome as integration: the price that needs to be paid to win the auction acts as an effective barrier to entry for firm 2. For smaller values of μ, the policy can increase investment incentives by enabling firm 2 to reach uninformed consumers (even though part of the surplus is extracted by the intermediary through the auction). It is thus in these situations that the policy is most likely to work.

With conflicting payoffs and $K = 0$ (pure pricing case), mandated access partially restores competition and leads firms to offer lower prices on average than unregulated integration, which benefits informed consumers. However, because uninformed consumers are now always directed towards the high-price firm, their surplus actually goes down. In practice, a natural requirement of policy interventions should be that they don't make uninformed consumers worse-off. Mandated access policies of the type we consider therefore appear somewhat inadequate.

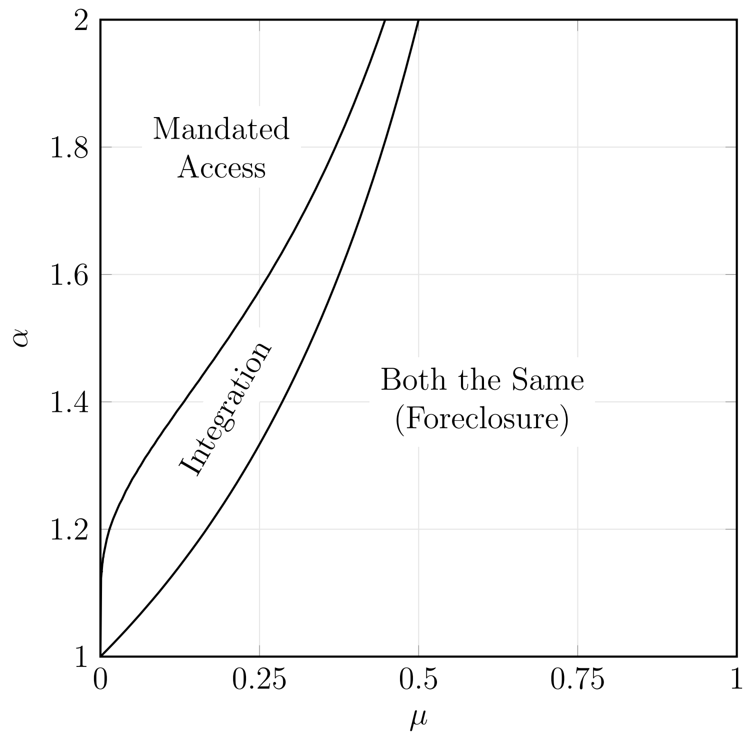


Figure 4: Suppose $K(r) = r^{\alpha}$ and payoffs are congruent. This plot shows whether mandated access or integration yield higher expected transaction quality.

5.2 Competition policy

We now turn our attention to a different set of policies, which we view as belonging to the competition policy rather than the regulatory domain. The first such policy (divestiture) consists in forcing the breakup of the integrated firm. We next study policies aimed at fostering competition at the intermediary level.

5.2.1 Divestiture

One way to restore a level downstream playing field is to require divestment of any interest in the downstream market by the intermediary.[24] We then assume that the intermediary relies on short-term contracts (or commissions) to generate revenues. More specifically, we use the same timing as for the mandated access policy to study the effects of divestiture: at $t = 1$ firms choose r_i, and at $t = 2$ they compete in a first price auction. The difference is that the intermediary's profit does not enter firm 1's objective function.

Because the subgame starting at $t = 2$ is identical to the case of mandated access, the results of Proposition 6 concerning the quality of the recommendation apply verbatim: divestiture leads to the right recommendation with probability 1 under congruence, and probability 0 under conflict.

[24]In this case long-term contracts between the intermediary and a downstream firm that replicate integration should also be prohibited.

Given divestiture, firms are ex ante symmetric and the profit function of firm i under congruence is

$$\pi_i(r_i, r_j) = \begin{cases} (1-\mu)r_i + \mu(r_i - r_j) - K(r_i) & \text{if } r_i > r_j \\ -K(r_i) & \text{if } r_i < r_j \end{cases} \tag{11}$$

If we restrict attention symmetric equilibria, we have the following:[25]

Proposition 7. *In the model with congruent payoffs there exists a unique symmetric equilibrium (in mixed strategies) under divestiture.*

- *The intermediary always recommends the right firm.*
- *If the cost function has a constant elasticity ($K(r) = r^\alpha$), there exists a pair $\{\mu', \mu''\}$ such that the average transaction quality is higher than under integration if $\mu < \mu'$, and lower than under integration if $\mu > \mu''$. If the cost function is quadratic then $\mu' = \mu'' = 1/2$.*

Under divestiture, each firm has the opportunity to reach uninformed consumers through the auction. The potential downside, compared to integration, is that the intermediary extracts a share of firm 1's profit, which reduces its investment incentives. When μ is large, this "hold-up" problem dominates, and divestiture reduces the average transaction quality. For smaller values of μ, divestiture increases quality. Figure 5 illustrates.

In the model with conflicting payoffs (and $K(r) = 0$), profits are given by

$$\pi_i(r_i, r_j) = \begin{cases} \mu(r_i - r_j) & \text{if } r_i > r_j \\ (1-\mu)r_i & \text{if } r_i < r_j \end{cases} \tag{12}$$

Proposition 8. *In the model with conflicting payoffs and pure pricing ($K(r) = 0$):*

- *There exists a unique symmetric equilibrium under divestiture (in mixed strategies).*
- *The intermediary never recommends the right firm.*
- *Consumer surplus is higher under divestiture than under integration.*

5.2.2 Entry by objective intermediary

A standard approach in competition policy is to take steps to ensure that an industry is adequately competitive and rely on the market to punish bad behavior. In this subsection and the next, we consider the effect of entry at the intermediary level, and study the potential of upstream competition to increase consumer surplus. We first envision a scenario in which initially all consumers are uninformed and an objective intermediary

[25] The formal treatment of this case appears in Appendix B.4.

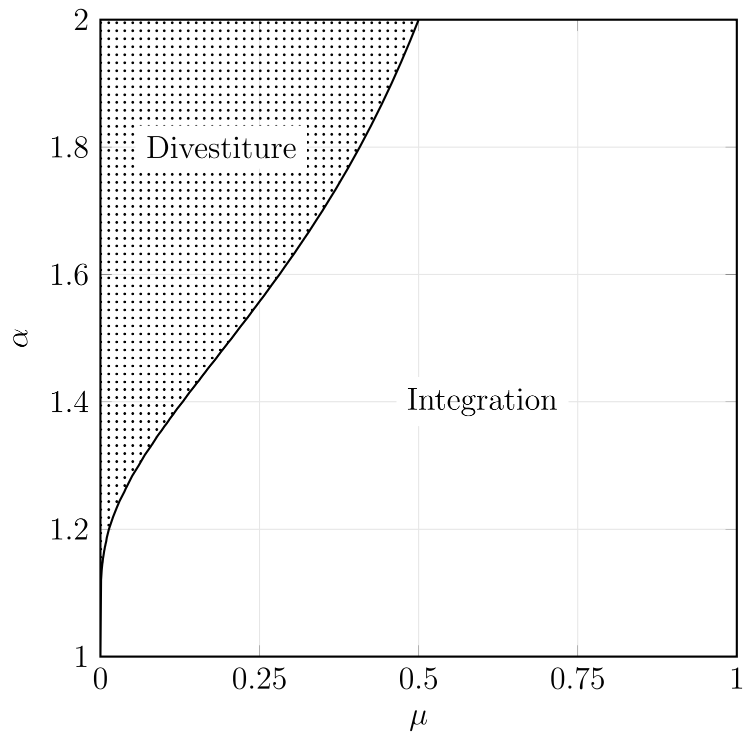


Figure 5: Suppose $K(r) = r^{\alpha}$ and payoffs are congruent. Then divestiture yields higher average transaction quality than integration in the shaded area.

enters the market. Uninformed consumers can switch to this new entrant (and effectively become informed) by incurring a cost s. We first provide a definition for the equilibrium of this game, and then focus on the case where s is arbitrarily small.

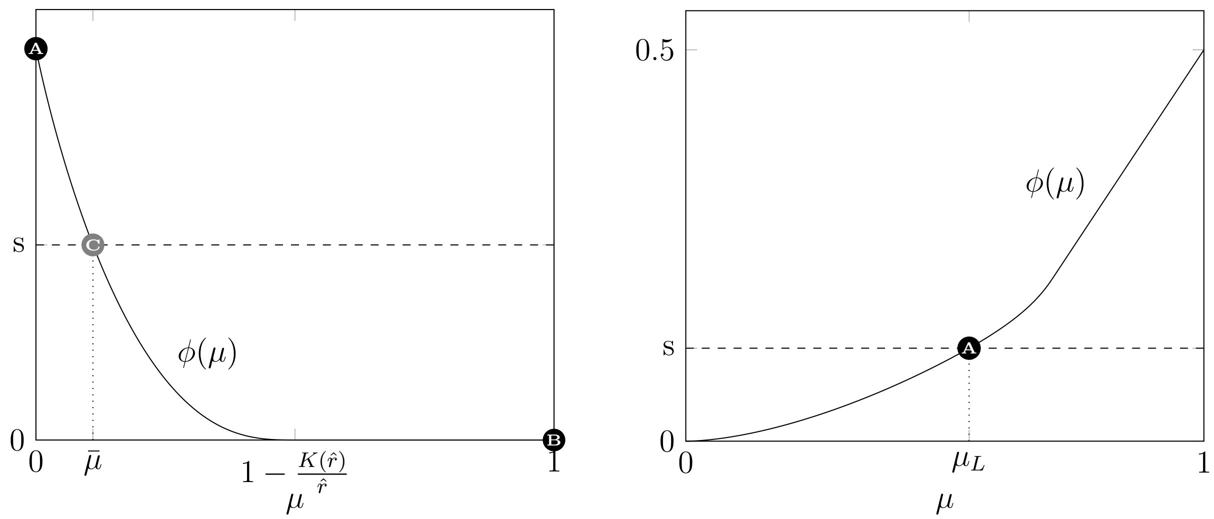
Let $\phi(\mu)$ be the extra utility brought by the objective intermediary compared to the integrated one when μ consumers remain with the incumbent. Thus, $\phi(\mu) \equiv E[\max\{u(r_1), u(r_2)\}] - E(u(r_1))$, where r_i is distributed according to the equilibrium strategy F_i for a given μ. If a consumer expects a mass μ of consumers to choose the incumbent, he will stay with the incumbent when $\phi(\mu) < s$, and switch to the entrant when $\phi(\mu) > s$. If $\phi(\mu) = s$, he can either stay or switch.

Definition 1. *An equilibrium of the game with competition between an integrated and an objective intermediary is a triple $\{\mu, F_1, F_2\}$ such that (i) given strategies F_1 and F_2 played by downstream firms, consumers behave optimally with respect to their switching decision; and (ii) F_1 and F_2 are equilibrium strategies of the game with μ uninformed consumers.*[26] *An equilibrium $\{\mu, F_1, F_2\}$ is stable if there exists $\bar{\epsilon} > 0$ such that, for every $\epsilon \in [-\bar{\epsilon}, \bar{\epsilon}]$, best-reply dynamics starting from a perturbation $\mu' = \mu + \epsilon$ converge to $\{\mu, F_1, F_2\}$.*[27]

To illustrate and build some intuition, Figures 6a and 6b show equilibria under

[26] Such strategies are given by Lemmas 2 and 3.

[27] I.e. firms best-reply to μ' by playing $\{F_1', F_2'\}$, then consumers best-reply to $\{F_1', F_2'\}$ which leads to μ'', and so on.



(a) Congruence. (b) Conflict.

Figure 6: Gain, $\phi(\mu)$, to switching when μ consumers use the incumbent (assuming $K(r) = r^2$).

congruent and conflicting payoffs respectively, both for $K(r) = r^2$. For instance, in the congruent case, there are three equilibria: everybody switches (A), nobody switches (B), and an interior equilibrium where consumers are indifferent (C). Equilibrium C is unstable because slightly reducing μ results in $\phi(\mu) > s$, causing yet more consumers to switch to the entrant.

We now focus on the most favorable scenario for the entrant, where $s = 0$. In that case, $\mu = 0$ is always a stable equilibrium. We look at whether there are other equilibria. We have the following:

Proposition 9. *With an objective entrant and arbitrarily small switching costs:*

- *With congruent payoffs, there is a stable equilibrium in which all consumers remain with the biased incumbent ($\mu = 1$).*
- *With conflicting payoffs, the unique stable equilibrium is such that all consumers switch to the objective entrant ($\mu = 0$).*

When payoffs are congruent, if all other consumers remain with the incumbent, firms' equilibrium play implies $r_1 = \hat{r}$ and $r_2 = 0$. Switching to the entrant therefore does not improve a consumer's utility, which shows that $\mu = 1$ is an equilibrium. It is stable because the threshold for μ ($\mu = 1 - K(\hat{r})/\hat{r}$) above which $(\hat{r}, 0)$ ensues is strictly less than 1.

When payoffs are conflicting, any interior μ leads to $\phi(\mu) > 0$. Indeed, the recommended firm tends to set a higher price than its rival (Corollary 2 (2)). Thus, even though $\mu = 1$ can be an equilibrium,[28] it cannot be stable.

[28]If $K(r) = 0$ an equilibrium for $\mu = 1$ is such that both firms charge the monopoly price and thus $\phi(1) = 0$.

The effects of competition at the intermediary level are very different depending on whether payoffs are congruent or conflicting. Proposition 9 suggests that competition is more likely to work in the latter case.

5.2.3 Competition between biased intermediaries

Another way to approach the issue is to study competition among two intermediaries, each integrated with (and recommending) a different downstream firm. Let μ_i be the mass of uniformed consumers that choose to consult the intermediary integrated with firm i (with $\mu_1 + \mu_2 = \mu$). Write F_i for the distribution of r chosen by intermediary i's integrated downstream firm. We define equilibrium as follows:

Definition 2. *An equilibrium of the game with two intermediaries is a 4-tuple* $\{\mu_1, \mu_2, F_1, F_2\}$ *such that (i) given* F_1 *and* F_2*,* $\mu_i \geq 0$ *only if* $E[u(r_i)] \geq E[u(r_j)]$*; and (ii) given* μ_1 *and* μ_2*,* F_1 *and* F_2 *are mutual best-responses. Stability is defined analogously to Definition 1.*

We have the following:

Proposition 10. *For the game with two integrated intermediaries:*

1. *With congruent payoffs, any stable equilibrium is such that all uninformed consumers use the same intermediary:* $\mu_1 = \mu$ *or* $\mu_2 = \mu$*. The outcome is thus the same as under a monopolistic intermediary.*

2. *With conflicting payoffs, the unique equilibrium is such that* $\mu_1 = \mu_2 = \frac{\mu}{2}$*. Consumer surplus is greater than with a monopoly.*

We only provide a sketch of the proof here. By Lemma 4, if payoffs are congruent, having a larger market share gives a firm more incentives to invest than its rival. Therefore $\mu_i \in (0, \mu/2)$ cannot be part of an equilibrium because consumers who use i would be better-off switching to j. When $\mu_1 = \mu_2 = \mu/2$, the firms both face the same incentives so that consumers are indifferent and this *does* constitute an equilibrium. However, this symmetric equilibrium is not stable: a perturbation to $\mu_1 = \mu/2 + \epsilon$ would provide firm 1 with more incentives to invest, prompting all uninformed consumers to switch towards 1.

Under conflicting payoffs, the logic is reversed: any configuration with $\mu_1 \neq \mu_2$ cannot be an equilibrium, because the firm with the largest number of uninformed consumers tends to charge the highest price. At $\mu_1 = \mu_2$, uninformed consumers are indifferent, and a small perturbation converges back to this symmetric configuration. The result on consumer surplus follows the same logic as under neutrality (see Lemma 7 in the appendix).

Together, the last two subsections indicate that upstream competition has the potential to alleviate concerns related to integration in the model with conflicting payoffs, but not so much when payoffs are congruent.

5.3 Discussion

Looking across the various policy options described above, a number of patterns emerge. Here, we distill these into some lessons for policy.

Lesson 1: policies that help consumers to circumvent a biased intermediary are effective under conflict, but not under congruence. Several of the policies considered above work by creating an opportunity for consumers to bypass the intermediary. For example, the presence of upstream competition allows consumers to switch to an alternative source of advice. Similarly, mandatory disclosure alerts consumers to the fact that there may be better alternatives to the firm recommended by the intermediary.

When payoffs are congruent, an integrated intermediary tends to recommend the firm that is best for consumers (Proposition 1). This means that consumers have little incentive to bypass the intermediary's advice—even if doing so is almost costless. The efficacy of policies that facilitate bypass of a biased intermediary (but rely on consumers to be proactive) is therefore significantly blunted. Under conflict, on the other hand, integration tends to result in bad advice (Proposition 2). This gives individual consumers a unilateral gain to bypassing the intermediary's advice and policies that remove barriers to such bypass are highly effective at disciplining the intermediary.

Lesson 2: market-based access policies eliminate bias under congruence, but exacerbate it under conflict. If an integrated intermediary cannot be relied upon to provide objective advice then perhaps a market mechanism can be used to promote the best firms. This is the reasoning behind policies such as mandated access and divestiture. When there is a competitive market to be recommended, the firm that is recommended will be the one with the greatest willingness to pay for access to a consumer. Under congruence this is also the firm that the consumer would most like to be matched with, so the market does a good job of eliminating ex post bias. Under conflict, on the other hand, the firm that most values access to consumers is the one that captures the biggest share of the surplus, so a market for access will tend to promote firms that leave little surplus for consumers. Athey and Ellison (2011) have shown that a market for search ads will tend to promote the most relevant ads to the top, while Armstrong and Zhou (2011) find that high-priced firms will tend to buy their way to prominence. In the parlance of our framework, the environments studied in these two papers respectively exhibit congruence and conflict.

Lesson 3: environments with congruence involve a trade-off between upstream objectivity and downstream incentives. Policies that facilitate circumvention of the intermediary have little bite under congruence. This forces us to consider more direct interventions in the market. One concern of regulators, supported by Lemma 2, might be the risk of foreclosure under integration. A policy that directly addresses this issue is neutrality. However, this policy comes with the downside that it degrades the quality of the intermediary's advice. Alternatively, policies that involve the creation of a market for access (such as divestiture or mandated access) ensure that the quality of advice is maximized, but do not reliably improve the state of downstream competition. Foreclosure may persist under such policies and a market for access can dilute the scale efficiencies of integration to such an extent that overall quality decreases.[29] Thus, there is often a trade-off between minimizing the level of bias on the one hand, and delivering effective downstream investment incentives on the other. A policy maker should consider whether integration already represents a good balance of these considerations and, if not, which of the two concerns is more pressing.

6 Applications

6.1 Search engine bias

The investigation of search engine bias has been, and continues to be one of the most significant competition issues of recent times. Google (the market leader in the US, most European countries, and much of the rest of the world) has repeatedly been accused of abusing its apparently dominant position by favoring its own subsidiary websites in the presentation of search results.[30] Some authorities, such as the Federal Trade Commission, have taken a fairly wide-ranging approach to the issue of search bias, whereas other investigations have focused on particular facets of the debate. This second approach is particularly interesting for us because (a) Google is active in dozens of different markets,[31] and (b) our analysis suggests that the specific market context is of great importance in determining the effects of bias.

One recent case, which was decided by the UK's High Court of Justice in February 2016, was Streetmap.EU Ltd v Google Inc. Streetmap alleged that, by prominently featuring Google maps in its search results, Google was abusing its dominant position in search to the detriment of competition in the online mapping market. In his judgment, Mr Justice Roth observes that maps "are free to users [and have] the feature that the more attractive the online product is to users, the greater the advertising revenue that will be earned." In other words, the court viewed this as a market exhibiting congruence. A key factor in the decision in Google's favor was the finding that Google Maps was a "pioneer" in developing many technologies that later became industry standards, where "there was significant evidence suggesting that Streetmap was deficient or lagging behind as regards many of these functional developments." This is consistent with our results, which show that the integrated firm tends to invest significantly more than its rival under congruence. Interestingly, the court appears to have considered the question of whether Google's success

[29] Indeed, we have seen that even if a regulator could directly force the intermediary to be objective, the result might be a reduction in quality.

[30] See Edelman (2014) for a description of the issue and a case against Google.

[31] Maps (Google Maps), video sharing (YouTube), and price comparison (Google Shopping) to name a few.

was due to a quality advantage *or* due to bias. Our analysis show that, in fact, the two may be directly related because ex ante bias results in asymmetric incentives to invest, which may give the misleading impression of success purely "on the merits". Part of the judgment also considered (and dismissed) a proposed remedy that would see Google displaying competitors' maps as well as its own. One criticism of this solution was that it might lead to a deterioration in the quality of Google's upstream advice because the featured rival maps would often be of lesser quality than Google's own offering. Our analysis of the neutrality policy under congruence suggests that this concern was well-founded, even after accounting for downstream strategic effects.

Another important case is the European Commission's ongoing investigation of Google. Initially, much like the FTC's investigation, this started out as a fairly broad inspection of the issue of search bias. During this phase, various policy options were mooted. Some members of the Commission were in favor of breaking up Google (divestiture). At one stage, it appeared as though the Commission and Google would reach a settlement that would see Google operate an auction, displaying winning firms on equal terms with its own websites (mandated access). However, this settlement was rejected by the EC in light of objections from Google's competitors. A key factor in this decision were complaints that the settlement would "lock in discrimination and raise rivals' costs instead of solving the problem of Google's anti-competitive practices."[32] The rivals' preferred approach seems to have been one of neutrality—unsurprising in light of our result that the unintegrated firm earns higher profits under such a policy.

Our analysis suggests that these policies have various strengths and weaknesses, depending on the downstream market in question. A better approach to policy is to consider each market in turn. By the end of 2015, the European Commission had narrowed its investigation of search bias to the comparison shopping sector, arguing that Google was "abusing a dominant position [...] by systematically favouring its own comparison shopping product in its general search results pages".[33] This particular market appears to be a better fit for our model of conflict: online shopping search engines' main strategic choice is arguably the commission or fee they charge to merchants whose products they list. A higher fee translates into a higher marginal cost for merchants, which should be passed through to consumers in the form of a higher price. Given our finding that environments with conflict tend to be those where bias is most harmful, the EC's focus on this particular market appears to be well-justified. As of July 2016, the Commission is yet to conclude its investigation, but our analysis suggests that the adverse effects of bias are considerably weakened by the presence of upstream competition (See Proposition 10). Thus, the extent to which the likes of Amazon and eBay are Google's upstream competitors takes on a

[32]See http://www.fairsearch.org/fairsearch-europe-initial-view-googles-proposed-commitments-are-worse-than-nothing/, accessed December 2015. Fair Search is an advocacy website that lobbies on behalf of Google's rivals.

[33]See `http://europa.eu/rapid/press-release_MEMO-15-4781_en.htm`, Accessed May 2016

special importance. If one views Amazon as an integrated firm consisting of an upstream search engine and a downstream product database, with Google Search/Google shopping being a competing integrated unit then our analysis of the effects of upstream competition suggests that Amazon's presence in the market will go a long way to disciplining Google. If, on the other hand, it is determined that Google is not competitively constrained with respect to product search then it may be appropriate to verify that consumers have ample opportunity to bypass Google's advice.

6.2 Android operating system and applications

In April 2016, the European Commission sent Google a statement of objections regarding what it considers to be an abuse of dominant position by Google on the market for mobile applications. The Commission argues that contractual restrictions imposed on phone manufacturers, whereby Google ties the licensing of its app-store for Android (Play Store) to the pre-installment on the devices of other Google applications, such as Google Search and Google Chrome, adversely affect competition on respectively the general search and the mobile browser markets.[34] According to the EC, it is important for manufacturers to to pre-install the Play Store, as this will give their customers access to a large number of applications for the Android OS. By tying it to other, less essential applications, Google would be able to extend its dominant position and exclude rivals.

Putting aside legal arguments, our model can shed light on some of the economic forces at play. To see how it fits the situation, suppose that all phone manufacturers pre-install the Google application (firm 1). A mass μ of consumers stick with the default option, while $1-\mu$ consumers are "savvy" and switch to a better application if one is available. We argue that the relevant model here is the one with congruent payoffs: the applications e.g., search engine) are purely ad-financed, and increasing their quality presumably increases usage rate, and therefore advertising revenue.

Our analysis (Lemma 2(i)) supports the concerns over foreclosure, especially if we assume that a large majority of consumers use the default version. Moreover, if returns to scale are not too large, integration may impede the level of investment and innovation. Assuming that integration is indeed found to be harmful, one must still wonder whether removing the restrictions imposed on manufacturers would improve the situation. Here we can think of two scenarios. In both scenarios manufacturers would all install Google play (the essential application), but would be free to choose the other default applications.

Suppose, first, that competition between phone manufacturers is so intense that a small advantage over the quality of a default application can induce a large shift in handset demand. One would the expect manufacturers to install the best application for free (i.e. without requiring payment from the app developper), or even to offer payments in order to

[34]Another aspect of the case is the use of exclusive contracts forbidding manufacturers who obtain a license for Google apps to sell devices that run on modified versions of the open-source Android OS.

obtain a license for the applications. The resulting configuration would then be formally equivalent to our objective benchmark, which, in the case of moderate returns to scale, would increase the quality enjoyed by consumers (see Figure 2).

The second scenario is one where competition between manufacturers is weaker, and consumers do not take into account the quality of the browser when choosing which phone to buy. One would then expect manufacturers to try to extract rents from the application developers, for instance by running an auction to be the default browser,[35] leading to a situation resembling the case of divestiture that we studied in section 5.2.1. There, we showed that the expected quality would be lower for high values of μ. In such a case, the prohibition of tying would actually harm consumers (see, e.g., Figure 5).

Of course, this discussion neglects the effects of the contractual restrictions on manufacturers' pricing incentives. For instance, absent restrictions, and assuming that manufacturers would be able to charge developpers for their application to become the default, one could expect that the price of phones would go down, as the marginal cost of selling a device would be lower. Extending our model to incorporate such pricing decisions seems like a worthy research avenue.

6.3 Price comparison websites

Our model can also shed light on the industry of price-comparison websites (PCWs). In our framework a PCW is an intermediary that operates in an environment with conflicting payoffs. The "informed " consumers are those who use the PCW and look at all the offers, whereas the "uninformed" ones are those who only look at the most prominent offer. Our model thus differs from Baye and Morgan (2001), where all the consumers who use the PCW become informed.

In this industry, the presence of vertical integration (between PCW and sellers)[36] and of commissions paid by sellers, often in return for favorable positioning, has led to regulatory scrutiny. For instance, OfGem, the UK energy regulator, provides PCWs with an accreditation provided they prominently list the energy companies from which they receive commission on sales, as well as make it clear that they earn commission on certain tariffs. They must also display all the available deals as a default setting. In this sector, OfGem estimates that 40% of consumers use PCWs to choose their provider.[37] In France, recent legislation[38] requires PCWs across all sectors to be more transparent regarding

[35]For instance, Google paid Apple $1bn in 2014 in exchange for having its search bar installed on the iPhone (see (Bloomberg Technology, 2016)). On the browser market, competition between search engines to be the default option can also generate large revenues: Google was paying Mozilla around $300m a year up to 2014, when Mozilla signed a 5-year deal with Yahoo! ((Ars Technica, 2014))

[36]For instance, the insurance PCW Confused.com is part of the Admiral group, a motor insurance company.

[37]See `https://www.ofgem.gov.uk/information-consumers/domestic-consumers/switching-your-energy-supplier/confidence-code`, accessed 18 July 2016.

[38]`https://www.legifrance.gouv.fr/eli/decret/2016/4/22/EINC1517258D/jo`, accessed 18 July

(i) the criteria they use to rank the offers they display, (ii) the existence of contracts (or capitalistic links) with firms, and (iii) the final price paid by the consumer and other "essential characteristics" of the offers.

Our analysis lends support to these interventions, as they are likely to increase consumer surplus. In particular, even though the disclosure of bias tends to reduce prices (see Proposition 3), it is only an imperfect substitute to increased transparency over prices (which can be seen as a decrease in μ).

Another example stems from Google's price comparison service, which allows merchants to bid to be prominently featured on the main Google results page (in addition to appearing in the Google Shopping price comparison listings). The auction arrangement resembles our model of divestiture which, under conflicting payoffs, predicts that the intermediary will tend to recommend high-priced products (and that prices will be higher than under objectivity). Consistent with this prediction, Google's critics have argued "merchants who have paid the most to be put in front of customers in the Google Product Listing Ads have every incentive to try to make up for it by charging higher prices."[39] A study by the Financial Times found that five out of every six items promoted by Google were more expensive than alternatives that were not promoted (with an average premium of 34%).[40] Google discloses the paid nature of these listings with a label alerting users to 'sponsored' content. This disclosure should go some way to offsetting the harms of bias, although some evidence suggests that consumers do not fully understand the meaning of such labels (e.g., Edelman and Gilchrist, 2012).

7 Discussion

7.1 On the optimality of integration

The contract we focus on in the baseline model, whereby the intermediary commits ex ante to send all uninformed consumers to one firm, captures the "long term" aspect of integration: firms know which one is going to be favored, and choose their actions accordingly. While natural, one might also ask whether this type of arrangement is optimal from the intermediary's perspective.

If the intermediary could sign any contract with both firms, specifying r_1 and r_2, it could achieve the maximal profit $\hat{\pi}$ irrespective of μ.[41] Such contracts would however most

2016.

[39]See https://next.ft.com/content/a004c830-552d-11e3-a321-00144feabdc0, accessed 1st August 2016.

[40]https://next.ft.com/content/8c1f2e90-5501-11e3-86bc-00144feabdc0, accessed 1st August 2016.

[41]One way to do it: ask 1 to choose $r_1 = \hat{r}$, and 2 to choose $r_2 = 0$ (under congruence) and $r_2 = \hat{r} + \epsilon$ (under conflict with pure pricing), which amounts to asking 2 to stay out. Require 1 to pay $\hat{\pi}$, and 2 to pay 0. If one firm refuses the contract, implement objective outcome. In equilibrium both firms accept.

likely be deemed anticompetitive, as they essentially amount to collusion.

Instead of defining the set of "non-collusive" contracts, a challenging task left for future research, we focus on some natural alternatives to integration. First, we compare it to the regulatory regimes of mandated access and divestiture of section 5. Second, we study the broader class of "partial integration", of which integration is a particular case, and which works as follows: : At $t = 0$ the intermediary makes a take-it-or-leave-it offer to one firm (say, firm 1): it commits to send $a \leq \mu$ consumers in exchange for a payment T_a . At $t = 1$ firms choose r_1 and r_2. At $t = 2$ the intermediary observes r_1 and r_2 and makes a take-it-or-leave-it offer to one of the firms, whereby it commits to sending $b = \mu - a$ uninformed consumers in exchange for a payment T_b. The case with $a = \mu$ corresponds to "full" integration or long-term contracting, while $b = \mu$ can be interpreted as a business model entirely based on short term commissions.

Proposition 11. *Under both congruence and conflict,*[42] *the intermediary's profit is larger under integration than under partial integration, mandated access, and divestiture.*

While not exhaustive, this exercise justifies our focus on integration in that it leads to larger profits for the intermediary in our setup. Of course in practice we sometimes observe variations of these contracts (Google's sponsored links for instance), which suggests that some considerations that we haven't modelled here are at play. We leave a thorough treatment of this issue to future research, but below we discuss the robustness of our results with respect to some assumptions.

7.2 Other assumptions

In our model firms' products are homogeneous up to any vertical differentiation. This assumption was made primarily to allow for a clean analysis, but is also a good approximation for many of the leading applications of our model. For example, in many technology markets the relevant downstream products are services such as mapping applications or price comparison websites. These are pure information services for which horizontal differentiation is likely to be relatively less important. Nevertheless, we can obtain results qualitatively similar to those above using a model of differentiated product competition.

Suppose that two firms are located at opposite ends of a Hotelling line and choose r. Consumers are uniformly distributed along the line. The $1 - \mu$ informed consumers go directly to the firm that maximizes their utility. A fraction μ of consumers are uninformed and observe neither the rs nor their match with the firms. The intermediary is able to direct these uninformed consumers to either firm. If the intermediary is constrained to be objective then both firms have identical incentives in choosing r and each consumer visits their best matching (closest) firm.[43] Under vertical integration a familiar pattern of

[42] With $K(r) = 0$ under conflict.

[43] Equilibrium with horizontal differentiation is in pure strategies.

distortions emerges. With congruence the integrated firm has a greater incentive to invest than its rival and it is rational for uninformed consumers to follow a biased intermediary's advice. Some consumers are steered away from their best (horizontal) match but this mis-matching is offset by the fact that the promoted firm becomes relatively more attractive for all consumers. As in our baseline model, the net effect of bias can be to leave consumers either better- or worse-off. Under conflict, the integrated firm will be that with a higher r.

Another assumption we make is that, in the model with congruent payoffs, firms compete using a deterministic investment technology. If firms are competing to innovate at the technological frontier then the outcome of (R&D) investment will have a stochastic component. With random investment outcomes, as in the baseline congruence model, realized qualities are random variables (now thanks to the stochastic innovation technology) so that an integrated intermediary sometimes directs consumers to the inferior firm. On average, though, the intermediary's advice is good because the integrated firm has a higher marginal incentive to invest than its rival. The intuition closely parallels that in Section 3.

A third simplification of our model has been to abstract away from dynamic considerations of intermediary reputation. However, the above policy analysis offers some insight into the effects that reputation should have: In Section 5.1.1 we force the intermediary to disclose details of its bias to consumers. Such disclosure amounts to an extremely strong reputation mechanism by which consumers learn the true level of bias immediately. Much like disclosure of bias, a reputation for bias will be very damaging under conflict because consumers expect to receive bad advice and will therefore seek ways to bypass the intermediary. Under congruence, on the other hand, a biased intermediary recommends the best firm on average and consumers will continue to follow its advice in spite of a reputation for bias. Indeed, Section 5.1.2 implies that the intermediary will often choose to be biased even when it completely internalises the value of its advice to consumers and faces a perfectly effective reputation mechanism.

We motivated our analysis of congruence with reference to environments in which firms offer free products and invest fixed costs in their quality. Meanwhile, a leading application of the conflict model is to cases where firms sell undifferentiated products and compete in price. One might wonder, what happens in a world where firms both invest to generate quality *and* compete in prices? We can study such an environment within our framework, letting utility be the difference between a firm's quality and its price. When we do so we find that, under fairly mild conditions, congruence emerges endogenously: the high revenue (i.e. high price) firm will also be that which offers the greatest quality-price differential. Moreover, as in the baseline congruence model, the integrated firm will usually be the one that offers consumers the highest utility.[44] This has important ramifications for policy. For example, just like the baseline congruence case, consumers have little incentive to bypass the intermediary because its advice tends to be good. In contrast,

[44]We obtain this result assuming quadratic costs. Details are available on request.

cases with undifferentiated price competition (exhibiting conflict) result in bad advice and a strong incentive to circumvent the intermediary. Thus, policies facilitating bypass of the intermediary are likely to be less effective under price competition with fixed investment in quality than under undifferentiated price competition. This serves to emphasize the value of the dichotomy at the centre of our analysis: to understand the implications of bias, the relevant distinction is not the mode of competition (e.g., price versus quality), but rather its implications for payoffs (congruence versus conflict).

8 Conclusion

In 2015 the European Commission launched a public consultation in recognition of the need for a unified strategy to regulate "the platform economy". A key challenge in building such a strategy is to come up with a coherent way of thinking about information intermediaries that operate in different institutional and strategic contexts. For example, the main concerns in the debates around search engine bias appear, on the face of it, to be very different to those for financial adviser's commission. The purpose of this paper has been to devise a model that can help determine how policy should respond to this heterogeneity. Using our parsimonious model, we distill regulators' most common concerns into worries about distortion of the intermediary's advice and worries about distortion of competition between downstream firms. We show that the severity of these concerns depends crucially on the relationship between firms' and consumers' payoffs.

We first consider environments with *congruence*—where strategies that increase firms' revenues also increase consumers' utility (such as when technology firms sink fixed costs to improve the quality of free services). We find that integration and similar vertical arrangements typically do not lead to bad advice because the market mechanism governing the provision of quality is such that the recommended firm tends to be the best one. Downstream investment *is* distorted, but the overall effect is ambiguous, with consumers potentially better-off under vertical integration than when the intermediary is independent and completely objective.

We contrast this case with environments exhibiting *conflict*, where higher revenues are obtained by extracting more surplus at consumers' expense (for example, when firms compete mostly in prices). In such environments, integration leads to bad advice and to higher prices so consumers are unambiguously harmed.

As well as establishing the importance of congruence and conflict in determining the effects of integration, we study various policy responses that are often mooted. Again, we find that the efficacy of a policy strongly depends on whether payoffs are congruent or in conflict. Policies that work by alerting consumers to bias or by giving consumers the opportunity to circumvent a biased intermediary are effective under conflict, where bias tends to reduce the quality of advice. However, the same policies have little bite under congruence because integration distorts downstream competition in such a way that consumers have little to gain by bypassing the intermediary. Another commonly discussed set of policies involve creating markets for access to the intermediary by allowing firms to bid to be recommended. Such markets have the perverse effect of entrenching bias under conflict because the firms with the most to gain from being recommended are those that are worst for consumers. Under congruence, the opposite is true: markets for access eliminate ex post bias but may diminish investment incentives.

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A Omitted Proofs From Sections 3–4

Proof of Lemma 1. In a supplementary appendix we establish that any equilibrium must involve mixed strategies[45] on (symmetric) support $[0,\overline{r}]$, with both firms making zero profits. Here we establish that the unique strategies that satisfy these conditions are those given in the statement of the lemma. We have

$$\pi_i(r_i) = \left[\Pr(r_j < r_i) + \frac{\Pr(r_j = r_i)}{2}\right] r_i - K(r_i) = 0.$$

For this to hold at every $r_i \in [0,\overline{r}]$, it must be that $\Pr(r_j = r_i) = 0$ (by continuity). Writing $F_j(r_i) = \Pr(r_j \le r_i)$: $F_j(r_i)\, r_i - K(r_i) = 0 \implies F(r) = K(r)/r$. To find $\overline{r}$, set $F(r) = 1$ and solve. ∎

Proof of Lemma 2 Part (i). Let us first show that $r_1 = \hat{r}$, $r_2 = 0$ is an equilibrium of the subgame when $\mu \ge 1 - [K(\hat{r})/\hat{r}]$. Firm 1 achieves the monopoly profit, so it can obviously not do better. The only way for firm 2 to attract some consumers is to choose $r_2 > \widehat{r}$, in which case the informed consumers would choose it over firm 1. The best such deviation is $r_2 = \widehat{r} + \epsilon$ (with ϵ close to zero). Indeed, the function $r \mapsto (1-\mu)r - K(r)$ is concave and its maximum is attained for $r < \widehat{r}$ (as $\widehat{r}$ maximizes $r \mapsto r - K(r)$). Firm 2's maximal deviation payoff is thus $(1-\mu)\widehat{r} - K(\widehat{r})$, which is non positive because $\mu \ge 1 - [K(\widehat{r})/\widehat{r}]$.

To show uniqueness, note first that any $r_2 > \hat{r}$ yields negative profit and is strictly dominated for firm 2. If firm 2 chooses a pure or mixed strategy involving only values of r_2 in $(0,\hat{r})$, firm 1's best response is to choose $r_1 = \hat{r}$. Firm 2 then makes a negative profit, which cannot be an equilibrium. If firm 2 plays $\hat{r}$ with positive probability, firm 1's best response is to play $r_1 = \hat{r} + \epsilon$, with ϵ a positive number arbitrarily close to zero. Firm 2 again gets a negative payoff which can't be part of an equilibrium. ∎

Proof of Lemma 2 Part (ii). We now turn our attention to the case in which $\mu < 1 - [K(\hat{r})/\hat{r}]$. We begin by establishing that there exists an equilibrium that satisfies the statement of the Lemma. More specifically, suppose the following four statements are true: (a) $\overline{r}$ solves $(1-\mu)\overline{r} - K(\overline{r}) = 0$; (b) $\underline{r}$ solves $\mu\overline{r} = \underline{r}K'(\underline{r}) - K(\underline{r})$; (c) firm 1 mixes over $[\underline{r},\overline{r}]$, with a mass point at $\underline{r}$; and (d) firm 2 mixes over $\{0\} \cup [\underline{r},\overline{r}]$, with a mass point at 0.

For firm 2 to be indifferent over $r_2 \in \{0\} \cup [\underline{r},\overline{r}]$, we must have

$$\underbrace{F_1(r_2)(1-\mu)r_2 - K(r_2)}_{\text{Profit from } r_2 \in (\underline{r},\overline{r})} = \underbrace{0}_{\text{Profit from } r_2 = 0}.$$

[45] The intuition is fairly standard: Each firm would like to set its r slightly above that of its rival to capture all consumers (à la Bertrand). Once $r_i - K(r_i) = 0$, however, one of the duopolists must be making negative profits and prefer to set $r = 0$. At this point the rival firm would prefer to deviate to some lower, but positive r and the cycle starts again.

Solving yields

$$F_1(r) = \frac{K(r)}{(1-\mu)r}.$$

Similarly, for firm 1 to be indifferent over any $r_1 \in [\underline{r}, \overline{r}]$ we must have

$$\underbrace{[\mu + F_2(r_1)(1-\mu)]r_1 - K(r_1)}_{\text{Profit from } r_1 \in [\underline{r}, \overline{r}]} = \underbrace{\overline{r} - K(\overline{r})}_{\text{Profit from } r_1 = \overline{r}} = \mu\overline{r}.$$

where the last equality comes from (a). Solving yields

$$F_2(r) = \frac{K(r) + \mu(\overline{r} - r)}{(1-\mu)r} \tag{13}$$

for $r \in [\underline{r}, \overline{r}]$. The mass point at zero in F_2 is found as

$$F_2(0) \equiv F_2(\underline{r}) = \frac{K(\underline{r}) + \mu(\overline{r} - \underline{r})}{(1-\mu)\underline{r}} = \frac{K'(\underline{r}) - \mu}{1-\mu}, \tag{14}$$

where the last equality follows from statement (b) above.

To establish that F_1, F_2 form an equilibrium, we need to rule-out any profitable deviation. Any $r_1 \geq \overline{r}$ yields profit $r_1 - K(r_1)$. A deviation to $r_1 > \overline{r}$ is therefore not profitable because $\overline{r} > \hat{r}$ (this follows from the definition of $\overline{r}$ and the fact that $\mu < 1 - [K(\hat{r})/\hat{r}]$) implying that $r_1 - K(r_1)$ is decreasing. Any $r_1 \in (0, \underline{r}]$ yields profit $[\mu + (1-\mu)F_2(0)]r_1 - K(r_1) = K'(\underline{r})r_1 - K(r_1)$, which is increasing in r_1. Thus, deviations to $r_1 \in (0, \underline{r})$ are not profitable. A deviation to $r_1 = 0$ would imply zero profit for firm 1 and can therefore also be ruled-out.

Any deviation by 2 to $r_2 \in (0, \underline{r})$ yields profit $-K(r_2)$ and is not profitable. Since $(1-\mu)r - K(r)$ is concave and $(1-\mu)\overline{r} - K(\overline{r}) = 0$, a deviation to $r_2 > \overline{r}$ must yield negative profits and be non-profitable.

So far we have shown that there is a unique equilibrium pair of distributions that satisfy conditions (a)–(d). That this equilibrium is unique is implied by establishing that every equilibrium must satisfy statements (a)–(d). We demonstrate that this is the case in Online Appendix D.3. ■

Proof of Proposition 1 part (i). For all $r \in [\underline{r}, \overline{r})$, the equilibrium distributions can be rewritten

$$F_2(r) = \frac{K(r)}{(1-\mu)r} - \frac{\mu r - \overline{r} + K(\overline{r})}{(1-\mu)r} = F_1(r) - \frac{\mu(r - \overline{r})}{(1-\mu)r}$$

(where the first equality stems from the fact that $\overline{r}$ is characterized by $(1-\mu)\overline{r} - K(\overline{r}) = 0$). It follows that

$$F_2(r) - F_1(r) = \frac{\mu(\overline{r} - r)}{(1-\mu)r} > 0$$

for any $r \in [\underline{r}, \overline{r})$. The proof is completed by noting that, for $r < \underline{r}$, $F_1(r) = 0 \leq F_2(r)$. ■

Proof of Proposition 1 Part (ii). From the proof of Lemma 2 Part (ii), note that the benchmark case yields identical quality distributions to the integrated case when $\mu \to 0$. It therefore suffices to show that $\partial F_2(r)/\partial\mu > 0$ for all $r < \overline{r}$. $F_2(0)$ is given in (14). In general, a function $F(x)/G(x)$ is increasing in x if and only if $F'(x)/F(x) > G'(x)/G(x)$. $F_2(0)$ is therefore increasing in μ if $[K''(\underline{r}) - 1] \,/\, [K'(\underline{r}) - \mu] > -1/(1-\mu)$. We know the numerator on the left hand side is larger than that on the right since $K''(r) > 0$. We know the denominator on the right is larger than that on the left since $\underline{r} < \widehat{r}$ implies $K'(\underline{r}) < K'(\widehat{r}) = 1$.

It thus remains to show that $\partial F_2(r)/\partial\mu > 0$ for all $r \in (0, \overline{r})$. $F_2(r)$ is given by (13). The denominator is decreasing in μ so it suffices to show that the numerator is increasing.

We have

$$\frac{d\left(\mu(\overline{r} - r)\right)}{d\mu} = \overline{r} - r + \mu\frac{d\overline{r}}{d\mu}. \tag{15}$$

Totally differentiating the expression characterizing $\overline{r}$, $(1-\mu)\overline{r} = K(\overline{r})$, yields

$$d\overline{r}(1-\mu) - \overline{r}d\mu = K'(\overline{r})d\overline{r} \implies \frac{d\overline{r}}{d\mu} = \frac{\overline{r}}{(1-\mu) - K'(\overline{r})}.$$

Substituting this into (15) yields

$$\frac{d\left(\mu(\overline{r} - r)\right)}{d\mu} = \overline{r}\left[1 + \frac{\mu}{(1-\mu) - K'(\overline{r})}\right] - r = \overline{r}\underbrace{\frac{1 - K'(\overline{r})}{(1-\mu) - K'(\overline{r})}}_{>1} - r.$$

Since $\overline{r} \geq r$ for all $r \in (\underline{r}, \overline{r}]$ this is positive. ∎

Proof of Proposition 1 Part (iii). To show that an increase in μ causes a rotation in F_1 suppose μ is increased to $\mu' > \mu$ with corresponding $S(F_1) = [\underline{r}', \overline{r}']$. We show that the following are true (a) $\underline{r}' \geq \underline{r}$; (b) $\overline{r}' \leq \overline{r}$; (c) $F_1(r)$ decreases for any $r < \underline{r}'$ and increases for any $r \geq \underline{r}'$.

Proof of (a) and (b): From the proof of Lemma 2 Part (ii), note that we can write $K'(\underline{r}) = \mu + (1-\mu)F_2(0)$. We therefore know that $\underline{r}$ is increasing in μ if and only if $\mu + (1-\mu)F_2(0)$ is increasing in μ. That this is the case can easily be verified given the proof of Proposition 1 Part (ii). By the definition of $\overline{r}$ we have $1 - \mu = K(\overline{r})/\overline{r}$ so that $\overline{r}$ is decreasing in μ if and only if $K(\overline{r})/\overline{r}$ is increasing in $\overline{r}$. Differentiating $K(\overline{r})/\overline{r}$ yields $[\overline{r}K'(\overline{r}) - K(\overline{r})]/r^2$, which is positive since K is convex.

Proof of (c): By definition, we have $F_1(r,\mu) \geq F_1(r,\mu') = 0$ for all $r < \underline{r}'$, (with strict inequality for $r \in (\underline{r}, \underline{r}')$), and $F_1(r,\mu) = \min\{K(r)/(1-\mu)r, 1\} \leq \min\{K(r)/(1-\mu')r, 1\} = F_1(r,\mu')$ for all $r \geq \underline{r}'$, (with strict inequality for $r \in (\underline{r}', \overline{r})$.

We show that the effect of integration on $E(r_1)$ is ambiguous by construction. Suppose that costs have the constant-elasticity form $K(r) = r^\alpha$ (with $\alpha > 1$). Under the objective benchmark, each firm's quality is distributed according to $F(r) = r^{\alpha-1}$ and $E(r_1) =$

$\int_0^1 (\alpha-1)r^{\alpha-1}\,dr = (\alpha-1)/\alpha$. Suppose $\mu > 1 - [K(\widehat{r})/\widehat{r}]$ so that, under integration, only firm 1 is active. Its quality is given deterministically by $\widehat{r} = (1/\alpha)^{1/(\alpha-1)}$. Solving $(\alpha-1)/\alpha < (1/\alpha)^{1/(\alpha-1)}$ for α reveals that the objective benchmark results in higher quality if and only if $\alpha > 2$. ■

Proof of Proposition 1 Part (iv). The proof of parts (ii) and (iii) established that it will sometimes be the case that *both* firms' expected quality falls under integration, whence consumers must be worse off.

To see that consumers sometimes gain from integration, let $K(r) = r^\alpha$ and suppose $\alpha = 1+\epsilon$, with ϵ positive but close to zero. Because $1 - [K(\hat{r})/\hat{r}] = \lim_{\epsilon\to_+ 0}(\alpha-1)/\alpha = 0 \leq \mu$, only firm 1 is active under vertical integration and it sets quality $r_1 = \hat{r} = \lim_{\epsilon\to_+ 0}(1/\alpha)^{1/(\alpha-1)} = 1/e = 0.37$. Under objectivity, the distribution of quality is $F(r) = r^{\alpha-1}$ and

$$E(\max\{r_1, r_2\}|\text{objective}) = \lim_{\alpha\to 1}\int_0^1 2(\alpha-1)r^{2(\alpha-1)}dr = \lim_{\alpha\to 1}\frac{2\alpha-2}{2\alpha-1} = 0. \quad ■$$

Proof of Lemma 3. The proof of uniqueness uses standard arguments (see Varian, 1980, Narasimhan, 1988), and is provided in Online Appendix D.4.

Here we state the equilibrium strategies and check that they indeed constitute an equilibrium.

Define $r_u \equiv \min\{r : u(r) \leq 0\}$ where it exists, and $r_u = \infty$ otherwise (in words, u_r is the choke price above which consumers get negative utility). Let $\overline{r} \equiv \min\{u_r, \arg\max_r \mu r - K(r)\}$ and let $\underline{r}$ satisfy $\underline{r} - K(\underline{r}) = \mu\overline{r} - K(\overline{r})$. Intuitively, $\overline{r}$ is the price that firm 1 would charge if it wanted to only focus on the uninformed consumers, and $\underline{r}$ is such that firm 1 is indifferent between charging $\underline{r}$ and serving everybody and charging $\overline{r}$ and only serving the uninformed consumers. We have $\underline{r} < \overline{r}$. We also implicitly define $\widetilde{r}$ by $\underline{r}(1-\mu) - K(\underline{r}) = \widetilde{r}K'(\widetilde{r}) - K(\widetilde{r})$ (and discuss later its importance).

Our proof of equilibrium requires us to consider 3 cases.

Case 1 Suppose that $\widetilde{r} > \overline{r}$. Here we claim that (i) firm 1 randomizes over $[\underline{r}, \overline{r}]$ according to

$$F_1(r) = \frac{(r-\underline{r})(1-\mu) - K(r) + K(\underline{r})}{r(1-\mu)} \tag{16}$$

for $r \in [\underline{r}, \overline{r})$, and plays $\overline{r}$ with probability $1 - F_1(\overline{r})$; (ii) firm 2 randomizes over $[\underline{r}, \overline{r}]$ according to

$$F_2(r) = \frac{(r-\mu\overline{r}) - K(r) + K(\overline{r})}{r(1-\mu)} \tag{17}$$

for $r \in [\underline{r}, \overline{r}]$.

If firm 1 plays according to this strategy, we have, for every $r_2 \in [\underline{r}, \overline{r})$, $\pi_2 = (1-\mu)\underline{r} - K(\underline{r})$, so firm 2 is indeed indifferent over the support. Choosing $r_2 > \overline{r}$ would lead to a non-positive profit because no consumer would choose firm 2.

Choosing $r_2 < \underline{r}$, for a profit of $r_2(1-\mu) - K(r_2)$, would not be profitable either. Indeed, let $\phi(r) \equiv rK'(r) - K(r)$. ϕ is an increasing function, and $\phi(\widetilde{r}) = (1-\mu)\underline{r} - K(\underline{r})$ by definition of $\widetilde{r}$. Therefore, $\underline{r} < \widetilde{r}$ implies that $\phi(\underline{r}) < (1-\mu)\underline{r} - K(\underline{r})$, i.e. that $K'(\underline{r}) < 1-\mu$. The function $r \mapsto r(1-\mu) - K(r)$ is thus increasing for $r \leq \underline{r}$, which implies that the deviation profit $(r_2(1-\mu) - K(r_2))$ is less than $(1-\mu)\underline{r} - K(\underline{r})$.

If firm 2 plays according to the strategy, firm 1's profit, for any $r \in [\underline{r}, \overline{r}]$, is equal to $\mu\overline{r} - K(\overline{r})$, which implies that firm 1 is indeed indifferent over the support. By definition of $\overline{r}$ a deviation to $r_1 > \overline{r}$ would not be profitable, while a deviation to $r_1 < \underline{r}$ would not be profitable either (as the function $r \mapsto r - K(r)$ is increasing for $r \leq \underline{r}$).

Case 2 Suppose that $\overline{r} > \widetilde{r} > \underline{r}$. The c.d.f. given by (16) is not increasing over $[\widetilde{r}, \overline{r}]$. We claim that the following strategy profile is an equilibrium: firms play according to (16) and (17) on $[\underline{r}, \widetilde{r}]$. Firm 1 plays $\overline{r}$ with probability $1 - \frac{(\widetilde{r}-\underline{r})(1-\mu)-K(\widetilde{r})+K(\underline{r})}{\widetilde{r}(1-\mu)}$, and firm 2 plays $\widetilde{r}$ with probability $1 - \frac{(\widetilde{r}-\mu\overline{r})-K(\widetilde{r})+K(\overline{r})}{\widetilde{r}(1-\mu)}$.

Again, straightforward calculations show that firms are indeed indifferent over their support. The new deviation to check is whether firm 2 would find it profitable to choose $r_2 \in (\widetilde{r}, \overline{r})$. Over this interval, firm 2's profit is given by $\pi_2(r) = r(1-\mu)(1-F_1(\widetilde{r})) - K(r)$. Substituting $F_1(\widetilde{r})$ and using the definition of $\widetilde{r}$, one can show that this profit is decreasing over $(\widetilde{r}, \overline{r})$. Therefore, firm 2 cannot make more profit than if it were to choose $r_2 = \widetilde{r}$.

Case 3 Suppose that $\widetilde{r} \leq \underline{r}$. First, note that this implies that $\mu > 1/2$. (Suppose, instead, that $\mu \leq 1/2$; we would then have $\text{argmax}_r\{(1-\mu)r - K(r)\} \geq \overline{r} = \text{argmax}_r\{\mu r - K(r)\} > \underline{r}$. This, in turn, implies $(1-\mu) - K'(\underline{r}) > 0$, which is only true if $\widetilde{r} > \underline{r}$.[46])

We claim that $r_1 = \overline{r}$ and $r_2 = \text{argmax}_r\{(1-\mu)r - K(r)\}(< \underline{r})$ is a pure-strategy equilibrium. A deviation to $r_2 > \overline{r}$ yields zero demand for 2. We need to check that firm 1 cannot profit by deviating to $r_1 < \text{argmax}_r\{(1-\mu)r - K(r)\}$, yielding profit $r_1 - K(r_1)$. By the construction of $\underline{r}$, we know that $\mu\overline{r} - K(\overline{r}) = \underline{r} - K(\underline{r})$. We also know that $r - K(r)$ is a concave function that is increasing for every $r < \overline{r}$. It therefore follows that $\mu\overline{r} - K(\overline{r}) = \underline{r} - K(\underline{r}) > r - K(r)$ for any $r < \underline{r}$.

Stochastic dominance To complete the proof, observe that $F_2(r) \geq F_1(r)$ and that all three of the identified equilibria therefore exhibit the claimed first-order stochastic dominance ordering. ■

[46] We have $\underline{r}(1-\mu) - K(\underline{r}) = \widetilde{r}K'(\widetilde{r}) - K(\widetilde{r})$, which implies $(1-\mu) - K'(\underline{r}) = \frac{1}{\underline{r}}[\widetilde{r}K'(\widetilde{r}) - K(\widetilde{r}) + K(\underline{r}) - \underline{r}K(\underline{r})]$. The right hand side is equal to zero when $\widetilde{r} = \underline{r}$ and is increasing in $\widetilde{r}$.

B Policy Appendix

B.1 Mandatory disclosure of bias and charging consumers

Proof of Proposition 4. When the cost function is quadratic ($K(r) = r^2$), an objective intermediary ($\beta = 0$) can charge $T(0, \mu) = \frac{1}{6}$ to consumers.

If the intermediary commits to being biased ($\beta = 1$), it can extract at most

$$T(1, \mu) = \begin{cases} \frac{\mu}{4}(2 + log(\frac{1-\mu}{\mu})) & \text{if } \mu < \frac{1}{2} \\ \frac{1}{4} & \text{if } \mu \geq \frac{1}{2} \end{cases}$$

It is straightforward to check that $T(1, \beta) > T(0, \beta)$ for $\mu > \mu^* = 0.195$.

∎

B.2 Neutrality

Under neutrality, each firm receives a mass of uninformed consumers μ_i such that $\mu_1 = \mu_2 = \frac{\mu}{2}$. Here we solve for the more general case where $\mu_1 \geq \mu_2 > 0$, and $\mu = \mu_1 + \mu_2$. This analysis is also useful for the study of the case with two integrated intermediaries. Lemma 4 and Proposition 5 in the text obtain as corollaries of Lemmas 5 and 6.

Congruent payoffs Suppose $\mu_1 \geq \mu_2$. We extend the reasoning of Lemma 2. Define $\widetilde{r}_i$ as the solution to $\mu_i = K'(\widetilde{r}_i)$ (the optimal r when catering only to one's own uninformed consumers), and $\widehat{r}_i$ as the solution to $(1 - \mu_j) = K'(\widehat{r}_i)$ (the optimal r when selling to everyone other than rival uninformed consumers). If firm j plays according to a c.d.f. F_j, firm i's profit is

$$\pi_i(r_i) = (\mu_i + (1 - \mu)F_j(r_i))\, r_i - K(r_i)$$

Lemma 5. *1. If $\widehat{r}_1(1 - \mu_1) - K(\widehat{r}_1) \leq \tilde{r}_2\mu_2 - K(\tilde{r}_2)$, the unique equilibrium is in pure strategies: $r_1 = \widehat{r}_1 > r_2 = \widetilde{r}_2$.*

2. Otherwise, the unique equilibrium is in mixed strategies, with F_1 first-order stochastically dominating F_2.[47]

Proof. (1) If the condition is satisfied, $(\widehat{r}_1, \tilde{r}_2)$ is an equilibrium: firm 2 prefers to choose $\tilde{r}_2$ and focus on its μ_2 uninformed consumers rather than to "overcut" firm 1 in an attempt to serve $1 - \mu_1$ consumers. The strategy profile $(\tilde{r}_1, \widehat{r}_2)$, the only other possible pure-strategy equilibrium, is not an equilibrium as firm 1 always finds it profitable to increase r_1 to $\widehat{r}_1 > \widehat{r}_2$. Finally, one can show that no mixed strategy equilibrium exists.[48]

[47] If $\mu_1 = \mu_2$, the equilibrium is symmetric and in mixed strategies.

[48] The proof, which we omit for brevity, consists in showing that the highest element of the support of any putative mixed strategy equilibrium would be below $\widehat{r}_1$, implying that firm 1 would have a profitable deviation.

(2) If $\widehat{r}_1(1-\mu_1) - K(\widehat{r}_1) > \tilde{r}_2\mu_2 - K(\tilde{r}_2)$ then 2 can profitably over-cut $r_1 = \widehat{r}_1$ and any equilibrium must be in mixed strategies. Using arguments similar to Lemma 2, one can show that in equilibrium the supports of the mixed strategies are of the form $S(F_1) = [\underline{r}, \overline{r}]$ (with a mass point at $\underline{r}$) and $S(F_2) = \{r_0\} \cup [\underline{r}, \overline{r}]$ (with a mass point at r_0).

If firm 2 plays r_0, it cannot attract the informed consumers. Therefore r_0 must maximize $\mu_2 r - K(r)$, i.e. $r_0 = \widetilde{r}_2$. Thus, firm 2's profits are $\mu_2\widetilde{r}_2 - K(\widetilde{r}_2)$. Since profits must be constant in the support of F_2, we have $\mu_2\widetilde{r}_2 - K(\widetilde{r}_2) = r[\mu_2 + (1-\mu_1-\mu_2)F_1(r)] - K(r)$, implying

$$F_1(r) = \frac{K(r) - K(\widetilde{r}_2) + \mu_2(\widetilde{r}_2 - r)}{(1-\mu_1-\mu_2)r} \quad \text{for all } r \in [\underline{r}, \overline{r}] \tag{18}$$

Solving $F_1(\overline{r}) = 1$ implies that the maximum of the support of F_1 and F_2 solves

$$K(\overline{r}) = (1-\mu_1)\overline{r} - \mu_2\widetilde{r}_2 + K(\widetilde{r}_2). \tag{19}$$

We also know that 1's profit when $r_1 = \overline{r}$ must be equal to that for any other r in the support of F_1: $(1-\mu_2)\overline{r} - K(\overline{r}) = r[\mu_1 + (1-\mu_1-\mu_2)F_2(r)] - K(r)$. Substituting for $K(\overline{r})$ from (19) and solving the previous equation gives:

$$F_2(r) = \frac{(\mu_1-\mu_2)\overline{r} + \mu_2\widetilde{r}_2 - K(\widetilde{r}_2) + K(r) - \mu_1 r}{(1-\mu_1-\mu_2)r} \quad \text{for all } r \in [\underline{r}, \overline{r}]. \tag{20}$$

Given that $S(F_2)$ has a gap between $\widetilde{r}_2$ and $\underline{r}$, one can write firm 1's profit as:

$$\pi_1(r) = \begin{cases} r(\mu_1 + (1-\mu_1-\mu_2)F_2(r)) - K(r) & \text{if } r \in [\underline{r}, \overline{r}] \\ r(\mu_1 + (1-\mu_1-\mu_2)F_2(\widetilde{r}_2)) - K(r) & \text{if } r \in (\widetilde{r}_2, \underline{r}) \end{cases} \tag{21}$$

Firm 1 must prefer to charge $\underline{r}$ than to charge any price below, and must be indifferent between $\underline{r}$ and any $r \in (\underline{r}, \overline{r}]$. The right-hand derivative of π_1 at $\underline{r}$ must be zero, while the left-hand derivative must be non-negative. Together, these conditions imply $\mu_1 + (1-\mu_1-\mu_2)F_2(\widetilde{r}_2) - K'(\underline{r}) = 0$, which pins down $\underline{r}$.

Because F_2 has no mass at $\underline{r}$, the mass at $\widetilde{r}_2$ is equal to $F_2(\underline{r})$ in expression (20).

We can now prove the claim of the lemma. It is clear that $F_2(r) \geq F_1(r)$ for any $r < \underline{r}$. For $r \geq \underline{r}$, using (18) and (20), we have

$$F_2(r) \geq F_1(r) \Longleftrightarrow (\mu_1-\mu_2)\overline{r} - \mu_1 r \geq -\mu_2 r \Longleftrightarrow \mu_1(\overline{r}-r) - \mu_2(\overline{r}-r) \geq 0, \tag{22}$$

which is true when $\mu_1 \geq \mu_2$. If $\mu_1 > \mu_2$ then the inequalities in (22) can be made strict. ■

Conflicting payoffs

Lemma 6. *When $\mu_1 \geq \mu_2$ with conflicting payoffs, there is a unique equilibrium.*

If $\widetilde{r} > \underline{r}$,[49] the equilibrium is in mixed strategies, with $F_1(r) \leq F_2(r)$ for all r.

If $\widetilde{r} \leq \underline{r}$ the equilibrium is in pure strategies, with $r_1 > r_2$.

Proof. We follow the same steps as for Lemma 3. We now have $\underline{r}$ implicitly given by $(1-\mu_2)\underline{r} = \mu_1\overline{r} - K(\overline{r}) + K(\underline{r})$ (where $\overline{r} = \min\{\arg\max_r\{\mu_1 r - K(r)\}, r_u\}$ as in Lemma 3), and $\widetilde{r}$ solves $\underline{r}(1-\mu_1) - K(\underline{r}) + K(\widetilde{r}) - \widetilde{r}K'(\widetilde{r}) = 0$. Similar to Lemma 3, firms use mixed strategies when $\widetilde{r} > \underline{r}$. The support of F_1 is $[\underline{r}, \min\{\overline{r}, \widetilde{r}\}] \cup \overline{r}$ (with a mass point at the supremum) and the support of F_2 is $[\underline{r}, \min\{\overline{r}, \widetilde{r}\}]$. The strategies are

$$F_1(r) = \frac{(1-\mu_1)(r-\underline{r}) - K(r) + K(\underline{r})}{(1-\mu_1-\mu_2)r} \tag{23}$$

and

$$F_2(r) = \frac{(1-\mu_2)(r-\underline{r}) - K(r) + K(\underline{r})}{(1-\mu_1-\mu_2)r} \tag{24}$$

in the interior of the support. We then clearly have $F_2(r) \geq F_1(r)$ when $\mu_1 > \mu_2$.

In analogy to Lemma 3, when $\widetilde{r} \leq \underline{r}$ and $K'(r) > 0$, the equilibrium is in pure strategies with $r_1 = \overline{r}$ and $r_2 = \underline{r}$, so that again $F_2(r) \geq F_1(r)$. ■

Lemma 7. *In the model with conflicting payoffs, keeping μ constant, an increase in μ_1 (where $\mu_1 \geq \mu_2$) reduces consumer surplus.*

Proof. We start by proving the lemma in the case where firms play in mixed strategies (i.e. $\tilde{r} > \underline{r}$, see Lemma 6). Suppose $\mu_1 \geq \mu_2$. The first step is to show that $\underline{r}$ is increasing in μ_1.

We know that $\overline{r} = \min\{\text{argmax}\{r - K(r)\}, v\}$, where v is the largest revenue that can be extracted from consumers. Therefore $\overline{r}$ is independent of the allocation of uninformed consumers.

On the other hand, $\underline{r}$ solves $(1-\mu_2)\underline{r} - K(\underline{r}) = \mu_1\overline{r} - K(\overline{r})$. Letting $\mu_2 = \mu - \mu_1$ and totally differentiating, we obtain:

$$\frac{d\underline{r}}{d\mu_1} = \frac{\overline{r} - \underline{r}}{1 - (\mu - \mu_1) - K'(\underline{r})} \tag{25}$$

The denominator is positive, otherwise firm 1 would never play $r > \underline{r}$. Therefore $\frac{d\underline{r}}{d\mu_1} > 0$.

Now we check what happens to firms' equilibrium strategies as we increase μ_1. From (23):

$$\left.\frac{dF_1(r)}{d\mu_1}\right|_{\mu_2=\mu-\mu_1} = -\frac{r - \underline{r} + [1-\mu_1-K'(\underline{r})]\frac{d\underline{r}}{d\mu_1}}{r(1-\mu)}.$$

We know that $1-\mu_1-K'(\underline{r}) > 0$ by (**??**), so $\frac{dF_1(r)}{d\mu_1} < 0$ (firm 1 chooses stochastically

[49] Both are defined in the proof. Intuitively, this happens when $\mu_1 - \mu_2$ is not too large. If $\mu_1 = \mu_2$, then $F_1(r) = F_2(r)$.

higher r the larger is the asymmetry). For firm 2 from (24) (after letting $\mu_2 = \mu - \mu_1$):

$$\left.\frac{dF_2(r)}{d\mu_1}\right|_{\mu_2=\mu-\mu_1} = -\frac{\underline{r} - r + [1 - \mu + \mu_1 - K'(\underline{r})]\frac{dr}{d\mu_1}}{r(1-\mu)} = \frac{r - \overline{r}}{r(1-\mu)} < 0$$

where the second equality follows after substituting for $\frac{dr}{d\mu_1}$ from (25). Thus, increasing the asymmetry causes firm 2 to choose a stochastically higher r.

Therefore, as μ_1 increases, firms' equilibrium strategies shift to higher values of r in a first-order stochastic sense.

If we're in the region where the equilibrium is in pure strategies, $r_1 = \overline{r}$ and $r_2 = \underline{r}$. Thus r_2 increases with μ_1 while r_1 does not change.

In both cases, consumers surplus goes down following an increase in μ_1. ■

B.3 Mandated access

With both congruent and conflicting payoffs, the winner of the auction is always the firm with the highest r_i.

Congruent payoffs If $\mu \geq 1 - \frac{K(\hat{r}}{\hat{r}}$ and firm 1 plays $\hat{r}$, firm 2's best response is either $r_2 = 0$ or $r_2 = \hat{r} + \epsilon$. In the latter case, its profit is $((1-\mu)\hat{r} - K(\hat{r}) \leq 0$. Therefore $(r_1, r_2) = (\hat{r}, 0)$ is an equilibrium. It is easy to see that there cannot be another equilibrium in pure strategies (we must have $r_2 = 0$, and 1's best response is $\hat{r}$). We cannot have an equilibrium in mixed strategies either. Indeed, if that were the case let $\overline{r}$ be the (necessarily common) largest element of firms' supports. If $\overline{r} \leq \hat{r}$, firm 1 could obtain the monopoly profit by deviating to $r_1 = \hat{r}$ (or $\hat{r} + \epsilon$ if $\overline{r} = \hat{r}$). If $\overline{r} > \hat{r}$ firm 2 makes a negative profit, which cannot be true in equilibrium.

If $\mu < 1 - \frac{K(\hat{r}}{\hat{r}}$. Using arguments similar to Lemma 2, one can show that there is no equilibrium in pure strategies, and that the supports of the strategies are of the form $S_1 = [\underline{r}, \overline{r}]$ (with a mass point at $\underline{r}$) and $S_2 = \{0\} \cup [\underline{r}, \overline{r}]$ (with a mass point at 0).

Although we cannot derive the equilibrium analytically, we show how to construct it.

Denote F_1 and F_2 firms' strategies. We have, for any $r_1 \in S_1$,

$$\pi_1(r_1) = r_1 \left(\mu + (1-\mu)F_2(r_1)\right) - K(r_1) \tag{26}$$

and, for any $r_2 \in S_2 - \{0\}$,

$$\pi_2(r_2) = \int_{\underline{r}}^{r_2} (r_2 - \mu r_1) dF_1(r_1) - K(r_2) \tag{27}$$

Firm 2's profit must be constant (and equal to zero) over S_2. Differentiating (27) we then get, for any $r \in S_2 - \{0\} = S_1$

$$F_1(r) + r(1-\mu)f_1(r) - K'(r) = 0 \tag{28}$$

$\pi_1(r)$ must be constant over the support. Let π_1 be its value. Using (26), we have, for any $r \in S_1 =_2 -\{0\}$,

$$F_2(r) = \frac{\pi_1 + K(r) - \mu r}{(1-\mu)r} \tag{29}$$

If firm 2 plays $\underline{r} + \epsilon$, for ϵ arbitrarily small, its profit is $F_1(\underline{r})(1-\mu)\underline{r} - K(\underline{r})$. This should be equal to zero, which gives the weight of the mass point for F_1 as a function of $\underline{r}$:

$$F_1(\underline{r}) = \frac{K(\underline{r})}{(1-\mu)\underline{r}} \tag{30}$$

When firm 1 plays $\underline{r}$, it attracts informed consumers only when firm 2 plays $r_2 = 0$. Therefore $\underline{r}$ solves $\max_{r_1} r_1\left(\mu + F_2(0)(1-\mu)\right) - K(r_1)$. The first-order condition gives

$$K'(\underline{r}) = \mu + F_2(0)(1-\mu) \tag{31}$$

If firm 2 plays $\overline{r}$, its profit is $\overline{r} - E[r_1]\mu - K(\overline{r})$. This should be zero:

$$\overline{r} - K(\overline{r}) = E[r_1]\mu \tag{32}$$

If firm 1 plays $\overline{r}$ its profit is $\overline{r} - K(\overline{r})$.

$$\overline{r} - K(\overline{r}) = \pi_1 \tag{33}$$

Solving (28), one gets, for all $r \in S_1$,

$$F_1(r) = \frac{\int_{\underline{r}}^{r} r^{\frac{\mu}{1-\mu}} \frac{K'(r)}{1-\mu} dr + A}{r^{\frac{1}{1-\mu}}}$$

where A is a constant. In particular, $F_1(\underline{r}) = \frac{A}{\underline{r}^{\frac{1}{1-\mu}}}$. Using (30), we obtain $A = \frac{K(\underline{r})\underline{r}^{\frac{\mu}{1-\mu}}}{1-\mu}$. Therefore,

$$F_1(r) = \frac{\int_{\underline{r}}^{r} r^{\frac{\mu}{1-\mu}} K'(r) dr + K(\underline{r})\underline{r}^{\frac{\mu}{1-\mu}}}{(1-\mu)r^{\frac{1}{1-\mu}}} \tag{34}$$

Because F_2 does not put any mass on $\underline{r}$, we have $F_2(\underline{r}) = F_2(0) = \frac{K'(\underline{r})-\mu}{1-\mu}$ (the last equality comes from (31)). Taking $r = \underline{r}$ in (29), and using the previous expression for $F_2(\underline{r})$ one then gets

$$\pi_1 = \underline{r}K'(\underline{r}) - K(\underline{r}) \tag{35}$$

By (32) and (33), we see that $\pi_1 = E[r_1]\mu$. By (35) this rewrites

$$\underline{r}K'(\underline{r}) - K(\underline{r}) = E[r_1]\mu \tag{36}$$

where $E[r_1]$ depends on $\underline{r}$ and $\overline{r}$. We also have

$$F_1(\overline{r}) = 1 \tag{37}$$

Solving the system (36)-(37) gives us the values of $\underline{r}$ and $\overline{r}$, which can then be used to recover the equilibrium distributions.

Conflicting payoffs We focus on the case with pure pricing, i.e. $K(r) = 0$. $\overline{r}$ is then consumers' willingness to pay for the product. Profits are given by

$$\pi_1(r_1, r_2) = \begin{cases} \mu r_1 & \text{if } r_1 > r_2 \\ r_1 & \text{if } r_1 < r_2 \end{cases} \tag{38}$$

and

$$\pi_2(r_1, r_2) = \begin{cases} (1-\mu) r_2 & \text{if } r_1 > r_2 \\ \mu(r_2 - r_1) & \text{if } r_1 < r_2 \end{cases} \tag{39}$$

Lemma 8. *In the mandated access game with pure pricing, firms mix over the interval* $[\underline{r}, \overline{r}]$ *with*

$$\underline{r} = \mu \overline{r}, \quad F_1(r) = \frac{1 - r^{\frac{2\mu-1}{1-\mu}} \underline{r}^{\frac{1-2\mu}{1-\mu}} (1-\mu) - \mu}{1-2\mu}, \quad F_2(r) = \frac{r - \underline{r}}{r(1-\mu)}.$$

Firm 1 places mass $1 - F(\overline{r})$ *at* $\overline{r}$.

Proof. $\min S(F_1) = \min S(F_2) = \underline{r}$ because firm 1's profits when $r_1 < r_2$ are r_1, while firm 2's profits when $r_2 < r_1$ are $(1-\mu)r_2$—both are increasing in r. It must be true that $\underline{r} > 0$, otherwise profits are zero and firm 1 could profit by setting $r_1 \in (0, \overline{r}]$. Also, neither firm can have a mass point at $\underline{r}$. If there were a mass point inf F_i at $\underline{r}$ then firm j would prefer to deviate from $\underline{r}$ to some $\underline{r} - dr$. In particular, such a deviation discontinuously increases the chance of capturing the informed consumers. This more than compensates for any loss on uninformed consumers (firm 2 has to pay at least $\underline{r}$ for access to the uninformed consumers and earns zero profit on them when $r_2 = \underline{r}$; firm 1 earns μr_1 on the uninformed consumers so a small change in r_1 has negligible effect on profits).

Firm 1 must be indifferent between $\underline{r}$ and a generic $r \in S(F)_1$:

$$(\mu + (1-\mu)(1 - F_2(r)))r = \underline{r}$$

which implies

$$F_2(r) = \frac{r - \underline{r}}{r(1-\mu)}. \quad (40)$$

Firm 2's profits, $\mu \int_{\underline{r}}^{r_2} (r_2 - r) f_1(r)\, dr + (1-\mu)(1 - F_1(r_2))r_2$, must be constant in $S(F_2)$, yielding the differential equation $(1-\mu)(1-F_1(r_2)) + \mu F_1(r_2) - r_2(1-\mu)F_1'(r_2) = 0$. Along with the boundary condition $F_1(\underline{r}) = 0$, this yields

$$F_1(r) = \frac{1 - r^{\frac{2\mu-1}{1-\mu}} \underline{r}^{\frac{1-2\mu}{1-\mu}} (1-\mu) - \mu}{1 - 2\mu} \quad (41)$$

Any equilibrium must be such that $\max S(F_1) = \max S(F_2) = \overline{r}$. Since $\overline{r}$ is the monopoly price, setting $r_1 > \overline{r}$ reduces both profit per consumer and the likelihood of attracting the uninformed consumers. For similar reasons, we can't have $r_2 > \overline{r}$ given that $r_1 \leq \overline{r}$. If $\max S(F_i) < \overline{r}$ then either (i) $\max S(F_i), \max S(F_j) < \overline{r}$ and at least one of i or j has the higher r with probability 1 (and can costless increase r from $\max S(F)$ to $\overline{r}$); or (ii) $\max S(F_i) < \max S(F_j) = \overline{r}$ in which case i can costlessly increase r in $(\max S(F_i), \overline{r})$. Characterisation of the equilibrium is completed by solving $F_2(\overline{r}) = 1$ for $\underline{r}$. ■

Using the equilibrium distributions (40) and (41), we find that the expected price paid by consumers (i.e. $max\{r_1, r_2\}$ with probability μ, $min\{r_1, r_2\}$ with probability $1-\mu$) is equal to $\overline{r}(2-\mu)\mu$, which is the expected price under integration. Thus consumer surplus, $v - r$, is unchanged. However, when we compare $E[max\{r_1, r_2\}]$ under market-based access to $E[r_1]$ under integration, we find that the latter is always below the former.

B.4 Divestiture

Congruent payoffs Under divestiture, the profit of a firm is

$$\pi_i(r_i) = \int_0^{r_i} [(1-\mu)r_i + \mu(r_i - r_j)]\, dF(r_j) - K(r_i). \quad (42)$$

Focusing on symmetric equilibria, we have:

Lemma 9. *In the case with congruent payoffs, under divestiture, there is a unique symmetric equilibrium, in which firms play according to the following distribution:*

$$F(r) = \frac{\int_0^r x^{\frac{\mu}{1-\mu}} \frac{K'(x)}{(1-\mu)} dx}{r^{\frac{1}{1-\mu}}} \quad (43)$$

over a support $[0, \overline{r}]$*, where* $\overline{r}$ *is given by* $F(\overline{r}) = 1$.

Proof. After an integration by parts and some simplifications, the profit function becomes

$$\pi_i(r_i) = (1-\mu)r_i F(r_i) + \mu \int_0^{r_i} F(r_j)dr_j - K(r_i). \tag{44}$$

Over the support $[0, \overline{r}]$, profit is constant. Differentiating (44) with respect to r_i, we obtain the following first-order differential equation:

$$\frac{dF}{dr_i} + \frac{F(r_i)}{(1-\mu)r_i} = \frac{K'(r_i)}{(1-\mu)r_i}. \tag{45}$$

The solution to (45) is $F(r_i) = \frac{\int_0^{r_i} r^{\frac{\mu}{1-\mu}} \frac{K'(r)}{(1-\mu)} dr + A}{r_i^{\frac{1}{1-\mu}}}$, where A is a constant. The initial condition $F(0) = 0$ then gives $A = 0$.

With a convex constant elasticity cost function $K(r) = r^\alpha$, we obtain $F(r) = \frac{\alpha r^{\alpha-1}}{\mu+\alpha(1-\mu)}$. ■

Proof of Proposition 7. To avoid confusion, we denote $F_{\text{div}}(r)$ the equilibrium distribution under divestiture, i.e. $F_{\text{div}}(r) = \frac{\alpha r^{\alpha-1}}{\mu+\alpha(1-\mu)}$.

The equilibrium distribution of r_1 under integration and constant elasticity cost function is $F_1(r) = \frac{r^{\alpha-1}}{1-\mu}$ (see Lemma 2 for the general expression).

We observe that $\lim_{\mu\to 0} F_{\text{div}}(r) = \lim_{\mu\to 0} F_1(r)$ and

$$\lim_{\mu\to 0} \frac{d\Big[F_{\text{div}}(r) - F_1(r)\Big]}{d\mu} = \frac{(\alpha-1)\alpha r^{\alpha-1}}{\alpha^2} - r^{\alpha-1} = -\frac{r^{\alpha-1}}{\alpha} < 0.$$

Combined with Proposition 1, we therefore have $F_2(r) \geq F_1(r) > F_{\text{div}}(r)$ for $\mu \in (0, \mu')$ for some $\mu' > 0$: quality is higher under divestiture for μ small.

Under divestiture, the expected transaction r is

$$\int_0^{\overline{r}} 2F_{\text{div}}(r)F'_{\text{div}}(r) r\, dr = \frac{2(\alpha-1)\left[1-\left(1-\frac{1}{\alpha}\right)\mu\right]^{\frac{1}{\alpha-1}}}{2\alpha-1}, \tag{46}$$

(where $\overline{r}$ solves $F_{\text{div}}(\overline{r}) = 1$). For μ sufficiently large, we know (by Lemma 2) that integration yields a transaction r equal to $\hat{r} = (1/\alpha)^{1/(\alpha-1)}$. We have

$$\lim_{\mu\to 1} \frac{2(\alpha-1)\left[1-\left(1-\frac{1}{\alpha}\right)\mu\right]^{\frac{1}{\alpha-1}}}{2\alpha-1} - \hat{r} = -\frac{1}{2\alpha-1}\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} < 0$$

so, by continuity, expected r is higher under integration than divestiture over some non-degenerate interval $(\mu'', 1]$.

If we fix $\alpha = 2$ then the expected transaction r under integration is

$$(1-\mu)\left\{\int_{\underline{r}}^{\overline{r}}\Big[F_1(r)F_2'(r)+F_2(r)F_1'(r)\Big]r\,dr+F_1(\underline{r})F_2(\underline{r})\underline{r}\right\}$$
$$+\,\mu\left\{\int_{\underline{r}}^{\overline{r}}F_1(r)r\,dr+F_1(\underline{r})\underline{r}\right\}=\frac{2\left(\mu^2-\sqrt{(1-\mu)\mu^3}+\left(3\sqrt{-(\mu-1)\mu}-2\right)\mu+1\right)}{3}.$$

if $\mu < 1/2$ and equal to $\hat{r} = 1/2$ if $\mu \geq 1/2$. This is smaller (larger) than (46) if μ is greater than (less than) 1/2. ■

Conflicting payoffs Since the firm with the largest r wins the recommendation auction, the recommended firm is always that which offers the lowest utility to consumers. Both firms choose $r = 0$ under objectivity, whereas it is easy to see that $r_1 = r_2 = 0$ cannot be part of an equilibrium with mandated access.

Proof of Proposition 8. The expected profit of firm i, if it chooses a per-user revenue r_i and if firm j plays according to F, is

$$\int_0^{r_i}\mu(r_i-r_j)dF(r_j)+(1-F(r_i))\,(1-\mu)r_i$$

The equilibrium is again symmetric in mixed strategies. Differentiating with respect to r_i over the support, we find that the equilibrium distribution is given by the following differential equation:

$$F'(r)+\frac{1-2\mu}{(1-\mu)r}F(r)=\frac{1}{r} \tag{47}$$

along with a boundary condition ($F(v) = 1$), where v is consumers' willingness to pay.The solution to (47) is

$$F(r)=\frac{1-\mu-\mu\left(\frac{r}{v}\right)^{\frac{2\mu-1}{1-\mu}}}{1-2\mu}.$$

Given the distribution, F under divestiture, the expected transaction price is

$$\mu\int_{\underline{r}}^{v}2F(r)F'(r)r\,dr+(1-\mu)\int_{\underline{r}}^{v}2[1-F(r)]F'(r)r\,dr,$$

which evaluates to

$$2(1-\mu)\mu\left(\frac{\mu\left(\left(\frac{1}{\mu}-1\right)^{\frac{1-\mu}{2\mu-1}}\right)^{\frac{3\mu-1}{1-\mu}}}{(6\mu-5)\mu+1}+\frac{\left(\left(\frac{1}{\mu}-1\right)^{\frac{1-\mu}{2\mu-1}}\right)^{\frac{\mu}{1-\mu}}}{1-2\mu}-\frac{1}{1-3\mu}\right)v.$$

Under integration, the analogous expression is

$$\mu \int_{v\mu}^{v} F_1'(r) r\, dr + (1-\mu) \left[\int_{\underline{r}}^{v} [1 - F_2(r)] F_1'(r) r\, dr + \int_{\underline{r}}^{v} [1 - F_1(r)] F_2'(r) r\, dr \right] = 2v(1-\mu)\mu.$$

The v's cancel and one can readily check that divestiture yields lower average prices for any $\mu \in [0,1]$ than does integration. ■

C Omitted Proofs From Section 7

Proof of Proposition 11. (i) Integration vs partial integration, congruent payoffs If $\mu > 1 - \frac{K(\hat{r})}{\hat{r}}$, we know that full integration achieves the monopoly profit, so it is optimal.

We thus focus on smaller values of μ for which the equilibrium is in mixed strategies. Under partial integration, at $t = 2$ the intermediary chooses to make the offer to the firm with the largest r_i, and $T_b = r_i b$. Therefore, in $t = 1$, firms play as in the full integration case with a uninformed consumers and $1 - b$ total consumers. We know from the analysis of the full integration game that the profit of the integrated firm is equal to $a\overline{r}$, where $\overline{r}$ satisfies $K(\overline{r}) = (1-\mu)\overline{r}$ and is thus independent of the choice of a. The optimal T_a thus equals $a\overline{r}$, and the intermediary's expected profit is $a\overline{r} + (\mu - a)E[\max\{r_1, r_2\}]$. Because $E[\max\{r_1, r_2\}] \leq \overline{r}$, this profit is lower than $\mu\overline{r}$, which the intermediary can achieve through full integration.

(ii) Integration vs partial integration, conflicting payoffs. With conflicting payoffs, a very similar reasoning applies: for any μ the intermediary's profit is $a\overline{r} + (\mu - a)E[\max\{r_1, r_2\}] \leq \mu\overline{r}$, so again full integration ($a = \mu$) dominates partial integration.

(iii) Integration vs mandated access, congruent payoffs. Under mandated access, the profit of the intermediary is given by (33): $\pi_{1,A} = \overline{r}_A - K(\overline{r}_A)$, where the subscript A stands for *access*. Under integration, $\pi_{1,I} = \overline{r}_I - K(\overline{r}_I)$. By convexity of K, given that both $\overline{r}_I$ and $\overline{r}_A$ are larger than $\hat{r} = \text{argmax}_r\{r - K(r)\}$, $\pi_{1,I} \geq \pi_{1,A} \Leftrightarrow \overline{r}_I \leq \overline{r}_A$.

$\overline{r}_I$ satisfies $(1-\mu)\overline{r}_I - K(\overline{r}_I) = 0$ (see condition (a) in the proof of Lemma 2 part (ii)). $\overline{r}_A$ satisfies $\overline{r}_A - \mu E[r_1] - K(\overline{r}_A) = 0$ (see (32)). This latter equality implies that $(1-\mu)\overline{r}_A - K(\overline{r}_A) < 0$, i.e. $\overline{r}_A > \overline{r}_I$.

(iv) Integration vs mandated access, conflicting payoffs. The intermediary makes the same profit in both cases: $\mu\overline{r}$, where $\overline{r}$ is the same under both regimes.

(v) Integration vs divestiture, congruent payoffs. Under divestiture, the profit of the intermediary is $\pi_D = \mu E_D[\min\{r_1, r_2\}]$, where the subscript D indicates that the expectation is taken with respect to F_D, the equilibrium strategy under divestiture. Given the symmetry of strategies, $\pi_D < \mu E_D[r_2]$. The profit of firm 1 if it plays $\overline{r}_D$ is $\overline{r}_D - \mu E_D[r_2] - K(\overline{r}_D)$. The zero-profit condition then implies $\mu E_D[r_2] = \overline{r}_D - K(\overline{r}_D)$. We thus have $\pi_D < \overline{r}_D - K(\overline{r}_D)$. On the other hand, the intermediary's profit under integration is $\pi_I = \mu\overline{r}_I = \overline{r}_I - K(\overline{r}_I)$. Because $\overline{r}_I$ and $\overline{r}_D$ are both above $\hat{r}$, we just need to prove that $\overline{r}_I < \overline{r}_D$ in order to prove that $\pi_D < \pi_I$.

$\overline{r}_D - \mu E_D[r_2] - K(\overline{r}_D) = 0$ implies that $(1-\mu)\overline{r}_D - K(\overline{r}_D) < 0$. Moreover, we know that $(1-\mu)\overline{r}_I - K(\overline{r}_I) = 0$. It is then immediate to conclude that $\overline{r}_D > \overline{r}_I$.

(vi) Integration vs divestiture, conflicting payoffs. Under divestiture, the intermediary's profit is $\mu E_D[\min\{r_1, r_2\}]$, which is less than $\mu\overline{r}$, its profit under integration.

■

D For Online Publication Only: Supplementary Proofs

This appendix establishes that the equilibria constructed for the congruence and conflict games are unique.

D.1 Formal definitions

A strategy for firm i in this game is a cumulative distribution function, $F_i : \mathbb{R}_+ \to [0,1]$, such that $F_i(r) = \Pr(r_i \leq r)$. As usual, the support of F_i, $S(F_i)$, is the smallest closed set such that $\Pr(r_i \in S(F_i)) = 1$. It will be convenient to define $F_i^-(r) = \Pr(r_i < r)$.

D.2 Supplementary proofs for Lemma 1

Claims 1–8 establish that any equilibrium of the objective benchmark in the quality model has firms mixing over symmetric support $[0, \overline{r}]$.

Claim 1. *In any equilibrium, there is no r such that both firms play r with positive probability.*

Proof. Suppose there exists an r played with positive probability by both firms. A tie then occurs with positive probability. We have $\pi_i(r) = [\Pr(r_j < r) + \Pr(r_j = r)/2]r - K(r)$. On the other hand, $\lim_{\epsilon \to 0^+} \pi(r+\epsilon) = [\Pr(r_j < r) + \Pr(r_j = r)]r - K(r) > \pi_i(r)$, i.e. firm i has a profitable deviation. ■

Claim 2. *Both firms earn zero profit in any equilibrium.*

Proof. Let $r_{\min}$ be the lowest quality in the support of either firm. By Claim 1, at most one firm has a mass point at $r_{\min}$ Suppose without loss of generality that $r_{\min}$ is in firm 2's support and that firm 1 does not have a mass point at $r_{\min}$. If it plays $r_{\min}$, firm 2 does not serve any consumer and its profit is thus $-K(r_{\min})$. So we must have $r_{\min} = 0$, and firm 2 makes zero profit. Suppose now that firm 1 makes a positive profit, and let $\overline{r}_1$ be the largest quality in its support. For π_1 to be positive, we must have $\overline{r}_1[\Pr(r_2 < \overline{r}_1) + \Pr(r_2 = \overline{r}_1)/2] - K(\overline{r}_1) > 0$. Firm 2 can profitably deviate by playing a pure strategy $r_2 = \overline{r}_1 + \epsilon$ for ϵ small and positive, because the previous inequality implies that $\overline{r}_1 - K(\overline{r}_1) > 0$ and thus $\overline{r}_1 + \epsilon - K(\overline{r}_1 + \epsilon) > 0$ by continuity of K. Any equilibrium must rule-out such a deviation and thus leave 1 with zero profit. ■

Claim 3. *Both firms use mixed strategies in any equilibrium.*

Proof. Claim 1 establishes that there is no pure strategy equilibrium with $r_1 = r_2$. It suffices (by symmetry) to establish that (a) there is no pure strategy equilibrium with $r_2 < r_1$, and (b) that there is no equilibrium in which only firm 1 plays a pure strategy. To prove (a): $r_2 < r_1$ implies 1's profits are $r_1 - K(r_1)$, requiring $r_1 = \text{argmax}_x\{x - K(x)\}$.[50] A deviation by 2 to $r_1 + \epsilon$, $\epsilon \to_+ 0$, thus yields profit $\lim_{\epsilon\to 0} r_1 + \epsilon - K(r_1 + \epsilon) = \max_x\{x - K(x)\} > 0$. This is profitable since two makes zero profit in any equilibrium by Claim 2.

To prove (b), notice that firm 2's best-response to r_1 is unique (either $r_1 + \epsilon$ or 0), so that it cannot play a strictly mixed strategy in equilibrium if firm 1 plays a pure strategy. ■

The next important feature of the equilibrium that will be established is that $S(F_i)$ is an interval. To do this, let us define r_i' as the smallest strictly positive $r_i \in S(F_i)$ such that $r_i' + \epsilon \notin S(F_i)$ for $\epsilon \to_+ 0$, and r_i'' as the smallest element of $S(F_i)$ such that $r_i'' > r_i'$ (when it exists). Thus (r_i', r_i'') is the (hypothetical) first gap in $S(F_i)$ whose infimum is strictly positive.

Claim 4. *(i) In equilibrium, if $S(F_i) \cap (r_i', r_i'') = \emptyset$, then $S(F_j) \cap (r_i', r_i'')$ has at most one element. (ii) Moreover, if $\Pr(r_i = r_i'') = 0$, then $S(F_j) \cap (r_i', r_i'']$ has at most one element.*

Proof. Let $(r_j^1, r_j^2) \in (r_i', r_i'')^2$. We then have $F_i(r_j^1) = F_i(r_j^2)$. It follows that j's profit is $F_i(r_i')r_j - K(r_j)$ (which is concave in r_j) everywhere in (r_i', r_i'') and therefore that either (a) r_j^1 and r_j^2 do not yield the same profit or (b) there is an $r \in (r_j^1, r_j^2)$ that yields higher profit than either r_j^1 or r_j^2. In either (a) or (b) it is clear that we can't have $(r_j^1, r_j^2) \in S(F_j)^2$. Applying this result recursively to eliminate elements of $S(F_j)$ implies (i). The proof of part (ii) is identical with $(r_j^1, r_j^2) \in (r_i', r_i'']^2 \implies F_i(r_j^1) = F_i(r_j^2)$. ■

Claim 5. *There is no equilibrium with $r_i'' > r_i' > 0$, $\{r_i', r_i''\} \in S(F_i)^2$, $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $\Pr(r_j = r_i'') = 0$.*

Proof. Suppose that there is an equilibrium with $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $\Pr(r_j = r_i'') = 0$. By Claim 4, there exists an $\epsilon > 0$ such that $\Pr(r_i'' - \delta > r_j) = F_j(r_i'')$ for all $\delta \le \epsilon$. It follows that the left-hand derivative of π_i at r_i'' is of the form constant $\times\, F_j(r_i'') - K'(r_i'') \ge 0$;[51] to prevent i from deviating to some $r_i'' - \epsilon$ it must be the case that this is positive. Firm i's profit when setting r_i'' is constant $\times\, F_j(r_i'')r_i'' - K(r_i'') = 0$ (where the zero profit condition follows from Claim 2). Combining this zero profit condition with constant $\times\, F_j(r_i'') - K'(r_i'') \ge 0$ yields $K(r_i'') \ge K'(r_i'')r_i''$, which can never hold since K is convex. We thus have a contradiction. ■

Claim 6. *There is no equilibrium with $r_i'' > r_i' > 0$, $\{r_i', r_i''\} \in S(F_i)^2$, $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $\Pr(r_j = r_i'') > 0$.*

[50] If not, firm 1 could slightly increase or decrease r_1 and increase its profit.

[51] In this and the following proof, the value of constant is 1. It is included here to allow reuse of the proof in a later claim.

Proof. Suppose that there is an equilibrium with $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $\Pr(r_j = r_i'') > 0$. We know from Claim 1 that $\Pr(r_i = r_i'') = 0$. Thus, by Claim 4 $S(F_j) \cap (r_i', r_i'') = \emptyset$. Suppose $r_i' \notin S(F_j)$. There would then be a gap in $S(F_j)$ at $(r_i' - \epsilon, r_i')$ and Claim 4 would imply that there is also a gap in $S(F_i)$ below r_i'. For i not to want to deviate from r_i' it must be true that $\pi_i'(r_i') = \text{constant} \times F_j(r_i') - K'(r_i') = 0$. We know that i earns zero profit: constant $\times$ $F_j(r_i')r_i' - K(r_i') = 0$. Combining these two equations yields $K(r_i') = K'(r_i')r_i'$, which cannot hold since K is convex. It is thus established that $r_i' \in S(F_j)$ must hold in the putative equilibrium. That $r_i' \in S(F_j)$ implies $\Pr(r_i = r_i') = 0$—otherwise j would prefer to deviate to $r_i' + \epsilon$ for $\epsilon \to_+ 0$.

So far it has been established that an equilibrium with $S(F_i) \cap (r_i', r_i'') = \emptyset$ *and* $\Pr(r_j = r_i'') > 0$ must be such that there is an equal sized gap in each of F_i and F_j at (r_i', r_i''), and that firm i puts no mass on either endpoint of this interval. For any $r_j \in [r_i', r_i'']$, firm j's profits are therefore constant $\times$ $F_i(r_i')(r_j) - K(r_j)$, which (by concavity) can be maximised at only one point. Since j puts positive mass on r_i'', this maximum must occur at r_i'': $\pi_j(r_i') < \pi_j(r_i'')$. Thus, r_i' cannot be in the support of j—a contradiction. ■

Given that $r_i' > 0$ by assumption, Claims 5 and 6 do not rule out the case in which $S(F_i) = \{0\} \cup [\underline{r}, \overline{r}]$. This is achieved in the following Claim:

Claim 7. $S(F_i) = [\underline{r}_i, \overline{r}_i]$.

Proof. Recall that the definition of r_i' held that it be strictly positive. Claims 5 and 6 therefore rule out all gaps except those of the form $(0, r_i'')$, which we rule-out here.

Suppose that we have $S(F_i) = \{0\} \cup [\underline{r}, \overline{r}]$, $\underline{r} > 0$. This implies $\Pr(r_i = \underline{r}) > 0$. Claims 1, 5, and 6 thus imply $0 \notin S(F_j)$. We also know that $\min S(F_j) = r_i''$. Indeed, $\min S(F_j) > r_i''$ would result in negative profits for i when setting r_i''; $\min S(F_j) < r_i''$ would, by Claim 4, imply a gap in $S(F_j)$ at $(\underline{r}_j, r_i'')$ (which is impossible by Claims 5 and 6).

It must be that $Pr(r_i = r_i'') = 0$. Indeed, suppose $Pr(r_i = r_i'') > 0$. By Claim 1 and since $\min S(F_j) = r_i''$, we would then have $F_j(r_i'') = 0$, which would then imply $\pi_i(r_i'') = -K(r_i'') < 0$, which can't be optimal.

To complete the proof, note that $r_i'' \in S(F_j) \implies \pi_j(r_i'') = F_i(r_i'')r_i'' - K(r_i'') = 0$ (by Claim 2). Moreover, $\Pr(r_i = r_i'') = 0$ implies that the left hand derivative of π_j evaluated at r_i'' must be positive—otherwise j would wish to deviate to some $r < r_i''$. Thus, $F_i(r_i'') - K'(r_i'') \geq 0$. Combining this derivative with the expression for profits at r_i'' yields $K(r_i'')/r_i'' \geq K'(r_i'')$, which is never true since K is convex. ■

Claim 8. $S(F_1) = S(F_2) = [0, \overline{r}]$.

Proof. First, establish that $\underline{r}_1 = \underline{r}_2 = 0$. If $\underline{r}_1 \geq \underline{r}_2$ then 2, when setting $r_2 = \underline{r}_2$, earns profit $-K(\underline{r}_2)$. This, in turn, implies $\underline{r}_2 = 0$ and $(0, \underline{r}_1) \cap S(F_1) = \emptyset$. Such a possibility is ruled out by Claim 7. If $\underline{r}_1 = \underline{r}_2 > 0$ then, by Claim 1, at least one i has $F_j(\underline{r}_i) = 0 \implies \pi_i = -K(\underline{r}_i)$ and would prefer to deviate to a lower r.

Secondly, show that $\overline{r}_1 = \overline{r}_2 = \overline{r}$. Suppose $\overline{r}_1 > \overline{r}_2$. We would then have $\pi_1(r) = r - K(r)\ \forall r \in (\overline{r}_2, \overline{r}_1]$. Since this is concave, there can be at most one r_1 in this interval that maximises 1's profits. There are two possibilities: (a) $\widehat{r} \leq \overline{r}_2$ would imply 1's profits are decreasing for $r_1 \in (\overline{r}_2, \overline{r}_1]$ so 1 would prefer to reduce $\overline{r}_1$; (b) $\widehat{r} > \overline{r}_2$ would require $\overline{r}_1 = \widehat{r}$, but then 2 could deviate to $\widehat{r} + \epsilon$ and get the monopolist profit (for ϵ small). ■

D.3 Supplementary proofs for Lemma 2

We let $\mu < 1 - [K(\hat{r})/\hat{r}]$ and establish that the equilibrium as specified in part (ii) of Lemma 2 is unique.

Remark 1. *The proof of Claim 1 applies verbatim to the case with vertical integration.*

Claim 9. *There is no equilibrium in pure strategies.*

Proof. Pure strategy equilibria with $r_1 = r_2$ are ruled out by Claim 1. Suppose $r_1 > r_2 \implies \pi_2 = -K(r_2) \implies r_2 = 0$. It follows that $\pi_1 = r_1 - K(r_1)\ \forall r_1 > 0$ so for r_1 to be optimal $r_1 = \widehat{r}$ must hold. However, 2 could then profitably deviate to $\widehat{r} + \epsilon$ (ϵ small) and earn $\pi_2 = (1-\mu)(\widehat{r} + \epsilon) - K(\widehat{r} + \epsilon)$. This is positive when $\mu < 1 - K(\widehat{r})/\widehat{r}$. The last possibility is $r_2 > r_1 \implies \pi_2 = (1-\mu)r_2 - K(r_2)$. For 2 not to want to deviate it must be true that $(1-\mu) = K'(r_2) \implies \mu < \widehat{r}$. But then 1 could deviate to $\widehat{r}$ and capture all consumers, which must increase its profits since $\widehat{r}$ is the monopolist's profit-maximising r. ■

Claim 10. $\min S(F_1) \equiv \underline{r}_1 \neq 0$

Proof. $0 \in S(F_1) \implies \pi_1 = 0$. This cannot be part of an equilibrium since 1 could always make positive profit by catering only to its μ captive consumers and maximizing $\mu r - K(r)$. ■

Claim 11. *(1)* $S(F_2) \cap (0, \underline{r}_1) = \emptyset$ *and (2)* $\Pr(r_2 = \underline{r}_1) = 0$.

Proof. (1) $r_2 \in (0, \underline{r}_1) \implies F_1(r_2) = 0 \implies \pi_2 = -K(r_2)$ (by Claim 10). Such a strategy is therefore dominated by $r_2 = 0$. (2) A mass point in both F_1 and F_2 is ruled out by Claim 1. Thus, if $Pr[r_2 = \underline{r}_1] > 0$, $F_1(\underline{r}_1) = 0 \implies \pi_2 = -K(\underline{r}_1) < 0$. ■

Claim 12. $\underline{r}_1$ *solves* $K'(\underline{r}_1) = \mu + (1-\mu)F_2(0)$.

Proof. Claim 11 implies $F_2(\underline{r}_1) = F_2(0)$ and that firm 1's profits are $[\mu + (1-\mu)F_2(0)]\, r - K(r)$ for every $r \in (0, \underline{r}_1)$. If $K'(\underline{r}_1) > \mu + (1-\mu)F_2(0)$ then it follows that 1 could profitably deviate by setting some $r \in (0, \underline{r}_1)$. The right-hand derivative of 1's profits at $\underline{r}_1$ is

$$\mu + (1-\mu)F_2(0) - K'(\underline{r}_1) + (1-\mu)\underline{r}_1 F_2'(\underline{r}_1) \geq \mu + (1-\mu)F_2(0) - K'(\underline{r}_1).$$

Thus, if we had $K'(\underline{r}_1) < \mu + (1-\mu)F_2(0)$, firm 1's profit would be increasing in r around $\underline{r}_1$, and thus $\underline{r}_1$ would not be in $S(F_1)$. ■

Claim 13. $\max S(F_1) \equiv \overline{r}_1 = \max S(F_2) \equiv \overline{r}_2 \equiv \overline{r} > \widehat{r}$*.* $\Pr(r_i = \overline{r}) = 0 \forall i$*.*

Proof. Suppose first that $\overline{r}_1 > \overline{r}_2$. For any $r_1 > \overline{r}_2$, firm 1's profit is $r_1 - K(r_1)$. Therefore, by concavity, $S(F_1) \cap (\overline{r}_2, \overline{r}_1] = \{\overline{r}_1\}$. Indeed, no two points can generate the same level of profit unless they are dominated by a third one. There are two possibilities to consider:

1. $\overline{r}_2 \geq \widehat{r}$. In that case, because $r \mapsto r - K(r)$ is decreasing over $(\widehat{r}, +\infty)$, firm 1 would be better-off choosing a quality $r \in (\overline{r}_2, \overline{r}_1)$.
2. $\overline{r}_2 < \widehat{r}$. Firm 1's best response is to choose $\overline{r}_1 = \widehat{r}$. But then $\pi_1(\overline{r}_1) > \pi_1(r)$ for any $r \neq \overline{r}_1$, i.e. firm 1 must play a pure strategy, which is ruled out by Claim 9.

We have thus proven that $\overline{r}_1 \leq \overline{r}_2$. The proof that $\overline{r}_1 \geq \overline{r}_2$ is the mirror image of the previous one, replacing $\widehat{r}$ by the r that solves $K'(r) = 1 - \mu$. We thus have $\overline{r}_1 = \overline{r}_2 = \overline{r}$.

Suppose now that firm i has a mass point at $\overline{r}$. Then, firm j could cause an upward jump in its profit by putting some mass at $\overline{r} + \epsilon$, $\epsilon \to_+ 0$. Finally, $\overline{r} > \widehat{r}$ must hold, otherwise firm 1 could deviate and choose $r_1 = \widehat{r}$ with probability 1. ■

Claim 14. *Firm 2 plays* 0 *with positive probability, and thus makes zero profit in equilibrium.*

Proof. Suppose $F_2(0) = 0$. Claim 11 implies that $F_2(\underline{r}_1) = 0$. Since firm 1 is indifferent over its support, we must have $\pi_1(\underline{r}_1) = \pi_1(\overline{r})$, i.e. $\mu\underline{r}_1 - K(\underline{r}_1) = \overline{r} - K(\overline{r})$. Put differently,

$$K(\overline{r}) - K(\underline{r}_1) = \overline{r} - \mu\underline{r}_1. \tag{48}$$

Given that $\overline{r} \in S(F_2)$ and that F_1 has no mass at $\overline{r}$ (see claim 13), we must have $\pi_2(\overline{r}) = (1-\mu)\overline{r} - K(\overline{r}) \geq \lim_{\epsilon \to 0^+} \pi_2(\underline{r}_1 + \epsilon) = (1-\mu)\Pr(r_1 = \underline{r}_1)\underline{r}_1 - K(\underline{r}_1)$, i.e.

$$K(\overline{r}) - K(\underline{r}_1) \leq (1-\mu)(\overline{r} - Pr[r_1 = \underline{r}_1]\underline{r}_1). \tag{49}$$

Combining (48) and (49) leads to $\mu\overline{r} \leq \mu\underline{r}_1 - (1-\mu)\Pr(r_1 = \underline{r}_1)\underline{r}_1$, a contradiction. ■

Claim 15. *The common upper-bound* $\overline{r}$ *solves* $(1-\mu)\overline{r} = K(\overline{r})$*.*

Proof. If firm 2 sets $r = \overline{r}$, given that $\Pr(r_1 = \overline{r}) = 0$, firm 2's expected profit is $(1-\mu)\overline{r} - K(\overline{r})$. By Claim 14, this must be zero. ■

Claim 16. *There exists a unique value of* $F_2(0)$ *consistent with equilibrium behavior.*

Proof. By firm 1's indifference condition, we have

$$[\mu + (1-\mu)F_2(0)]\,\underline{r}_1 - K(r_1) = \overline{r} - K(\overline{r}) \tag{50}$$

Using Claim 12 (which implies a bijective relationship between $\underline{r}_1$ and $F_2(0)$), the derrivative of the left-hand side of (50) with respect to $F_2(0)$ is

$$(1-\mu)\underline{r}_1 + \underbrace{[\mu + (1-\mu)F_2(0) - K'(\underline{r}_1)]}_{=0,\text{ by claim 12}} \frac{d\underline{r}_1}{dF_2(0)} > 0.$$

The left-hand side of (50) is thus increasing in $F_2(0)$. Given that $\overline{r}$ is uniquely defined (claim 15), the right-hand side of (50) does not vary with $F_2(0)$. There is thus a unique value of $F_2(0)$ that solves (50). ■

Remark 2. *The proof of Claim 4 applies verbatim to the ex ante contracting game.*

Claim 17. *The lowest positive quality in the support of* F_2 *satisfies* $\min\{S(F_2)\backslash\{0\}\} \equiv \underline{r}_2 = \underline{r}_1 \equiv \underline{r}$.

Proof. $\underline{r}_2 < \underline{r}_1$ is ruled out by Claim 11. Suppose that $\underline{r}_2 > \underline{r}_1$. Claim 4, implies $S(F_1) \cap (0, \underline{r}_2)$ has at most one element. By definition, $\underline{r}_1 \in S(F_1)$, so $S(F_1) \cap (\underline{r}_1, \underline{r}_2) = \emptyset$. Thus, $S(F_1)$ has a mass point at $\underline{r}_1$. It cannot be the case that firm 1 also has a masspoint at $\underline{r}_2$: that would imply that there is no mass point in F_2 at $\underline{r}_2$ (Claim 1) so that firm 1's indifference would require $\mu\underline{r}_1 - K(\underline{r}_1) = \mu\underline{r}_2 - K(\underline{r}_2)$. This can only be true if $\underline{r}_1 = \underline{r}_2$—a contradiction.

The fact that no point in $(\underline{r}_1, \underline{r}_2)$ is played by either firm implies that $r_2(1-\mu)\Pr(r_1 = \underline{r}_1) - K(r_2)$ is non-decreasing to the left of $\underline{r}_2$ (otherwise firm 2 would deviate to a lower $\underline{r}_2$). Taking the derivative, we thus have $(1-\mu)\Pr(r_1 = \underline{r}_1) \geq K'(\underline{r}_2)$. Now, if $\underline{r}_2 \in S(F_2)$, we must have, by Claim 14, $(1-\mu)\Pr(r_1 = \underline{r}_1)\underline{r}_2 - K(\underline{r}_2) = 0$. Combining the two previous equations, we obtain $K(\underline{r}_2)/\underline{r}_2 \geq K'(\underline{r}_2)$, which is impossible by convexity of K. ■

Claim 18. $[\underline{r}, \overline{r}] \subseteq S(F_i)$, $i = 1, 2$.

Proof. Let $i = 2$, $j = 1$. The proofs of Claims 5 and 6 then apply verbatim to the ex ante contracting game. This implies $[\underline{r}, \overline{r}] \subseteq S(F_2)$. To show $[\underline{r}, \overline{r}] \subseteq S(F_1)$ suppose that $(r_1', r_1'') \subseteq [\underline{r}, \overline{r}]$, $(r_1', r_1'') \nsubseteq S(F_1)$. Claim 4 implies that $S(F_2)$ has at most one point in (r_1', r_1''), which implies $(r_1', r_1'') \nsubseteq S(F_2)$. But we have just seen that this contradicts Claims 5 and 6. ■

Claim 19. *Neither* F_1 *nor* F_2 *have a mass point on* $(\underline{r}, \overline{r}]$.

Proof. By claim 13, we already know that $\overline{r}$ cannot be a mass point. If $r \in (\underline{r}, \overline{r})$ was played with positive probability by firm i, firm j could increase its profit by having a mass point at $r + \epsilon$, $\epsilon \to_+ 0$. ■

Claim 20. $\underline{r}$ *solves* $\mu\overline{r} = \underline{r}K'(\underline{r}) - K(\underline{r})$

Proof. Firm 1 must be indifferent between $\overline{r}$ and $\underline{r}$: $\mu\overline{r} = [\mu + (1-\mu)F_2(0)]\underline{r} - K(\underline{r})$. Substituting $K'(\underline{r})$ for $\mu+(1-\mu)F_2(0)$ (thanks to Claim 12) yields $\mu\overline{r} = \underline{r}K'(\underline{r}) - K(\underline{r})$. ■

D.4 Supplementary proofs for Lemma 3

In this subsection we establish that the equilibrium described in the proof of Lemma 3 is unique. We make use of the variables $\widetilde{r}$, $\bar{r}$, and $\underline{r}$ as defined in that proof.

Claim 21. *If $\widetilde{r} \leq \underline{r}$ then there is a unique equilibrium as described in Case 3 of the proof of Lemma 3.*

Proof. If $\widetilde{r} \leq \underline{r}$ then $(1-\mu) - K'(\underline{r}) \leq 0$. Thus, firm 2 must always set $r_2 \leq \underline{r}$ in any equilibrium. But the proof of Lemma 3 established that having $r_1 = \bar{r}$ and attracting only uninformed consumers yields weakly higher profit for 1 than could be obtained by any $r_1 < \underline{r}$. Since firm 1 puts no mass below $\underline{r}$, the unique best response for firm 2 in any equilibrium is $\text{argmax}_r\{(1-\mu)r - K(r)\}(< \underline{r})$. Firm 1 therefore optimally maximises with respect to the uninformed: $r_1 = \bar{r}$. This is exactly the equilibrium claimed. ■

Having established that the pure strategy equilibrium of Case 3 is unique whenever it exists, we spend the rest of this subsection on Cases 1 and 2 (with $\widetilde{r} > \underline{r}$).

Claim 22. *In any equilibrium, there is no r such that both firms play r with positive probability.*

Proof. Suppose there exists an r played with positive probability by both firms. A tie then occurs with positive probability. We have $\pi_i(r) = [\Pr(r_j > r) + Pr(r_j = r)/2]r - K(r)$. On the other hand, $\lim_{\epsilon \to 0^+} \pi(r-\epsilon) = [\Pr(r_j > r) + \Pr(r_j = r)]r - K(r) > \pi_i(r)$, i.e. firm i has a profitable deviation. ■

Claim 23. *There is no equilibrium in pure strategies.*

Proof. Pure strategy equilibria with $r_1 = r_2$ are ruled-out by claim 22. Suppose $r_1 < r_2$. If $r_1 = 0$ then firm 1 makes zero profit and can profitably deviate to $r_1 \in (0, r_2)$; if $r_1 > 0$ then firm 2 makes zero profit and can profitably deviate to $r_2 \in (0, r_1)$.

Let $r_1 > r_2$. Firm 1 attracts only uninformed consumers and must solve $r_1 = \text{argmax}_r\, \mu r - K(r) = \bar{r}$ (where $\bar{r}$ is defined in Lemma 3). Firm 2 attracts the informed consumers and must solve $r_2 = \text{argmax}_r (1-\mu)r - K(r)$. The fact that $\widetilde{r} > \underline{r}$ implies $r_2 > \underline{r}$. A deviation by firm 1 to $r_2 - \epsilon$ ($\epsilon > 0$ small) yields profit $r_2 - K(r_2) > \underline{r} - K(\underline{r}) = \mu\bar{r} - K(\bar{r})$ (the inequality follows from the fact that $r - K(r)$ must be increasing below r_2 if r_2 maximises $(1-\mu)r - K(r)$; the equality follows from the construction of $\bar{r}$ and $\underline{r}$). Thus, firm 1 has a profitable deviation. ■

Claim 24. $\min S(F_1) > 0$*,* $\min S(F_2) > 0$*, and* $\Pr(r_2 < r_1 | r_2 = \max S(F_2)) > 0$*.*

Proof. $\min S(F_1) > 0$ must be true, otherwise firm 1 earns zero profits. But firm 1 could guarantee positive profit with $r_1 = \text{argmax}\, \mu r - K(r)$. Given $\min S(F_1) > 0$, $\min S(F_2) = 0$ would imply zero profit for 2. But 2 could guarantee positive profit by setting $r_2 \in (0, \min S(F_1))$.

Suppose $\Pr(r_2 < r_1 | r_2 = \max S(F_2)) = 0$. Then firm 2's profis are zero, but 2 could guarantee positive profits by deviating to $0 < r_2 < \min S(F_1)$. ■

Claim 25. $\max S(F_1) = \bar{r}$.

Proof. Since there can be no ties with positive probability, Claim 24 implies that $\Pr(r_1 < r_2 | r_1 = \max S(F_1)) = 0$. If $\max S(F_1) < \bar{r}$ then $\pi_1 = \mu r - K(r)$ is increasing in r above $\max S(F_1)$ and firm 1 profitably deviates to a higher r. If $\max S(F_1) > \bar{r}$ then π_1 is decreasing (or zero) at $\max S(F_1)$ and a deviation to a lower r would yield higher profit for 1. ■

Claim 26. *If there is a mass point in F_i at $r' > 0$ then $S(F_j) \cap [r', r' + \epsilon) = \emptyset$ for ϵ small and positive.*

Proof. If the claim were false then, for ϵ small, a deviation by j from $r_j \in [r', r' + \epsilon)$ to $r' - \epsilon'$ (ϵ' small and positive) would yield a discontinuous increase in demand but a negligible fall in revenue per consumer. Such a deviation is profitable. ■

Claim 27. $\min S(F_1) = \min S(F_2) = \underline{r}$.

Proof. Suppose $\min S(F_1) < \min S(F_2)$. Firm 1's profit is $\min S(F_1) - K(\min S(F_1))$. We know that $\min S(F_1) < \min S(F_2) < \bar{r}$ (by Claim 24). This implies $\mu - K'(\min S(F_1)) > 0 \implies 1 - K'(\min S(F_1)) > 0$. Thus, firm 1 has a profitable deviation to $r_1 \in (\min S(F_1), \min S(F_2))$.

Suppose $\min S(F_1) > \min S(F_2)$. Firm 2's profit is $(1 - \mu) \min S(F_2) - K(\min S(F_2))$ and $\min S(F_2)$ must solve $(1 - \mu) - K'(r) = 0$. Now, the fact that $\widetilde{r} > \underline{r}$ implies $(1 - \mu) - K(\underline{r}) > 0$ so $\min S(F_2) > \underline{r}$. But firm 1 must then have a profitable deviation to $r_1 = \underline{r} + \epsilon \in [\underline{r}, \min S(F_2)]$since $\underline{r} + \epsilon - K(\underline{r} + \epsilon) > \underline{r} - K(\underline{r}) = \mu\bar{r} - K(\bar{r}) = \pi_1$.

It is thus established that $\min S(F_1) = \min S(F_2)$. This result in combination with Claims 22 and 26 implies that neither firm has a mass point at $\min S(F_i)$. Firm 1 must be indifferent between $\bar{r}$ and $\min S(F_1)$, implying $\mu\bar{r} - K(\bar{r}) = \min S(F_1) - K(\min S(F_1))$. This pins down $\min S(F_1) = \underline{r}$. ■

Claim 28. *(i) In equilibrium, if for some $r_i' < r_i''$, $S(F_i) \cap (r_i', r_i'') = \emptyset$, then $S(F_j) \cap (r_i', r_i'')$ has at most one element. (ii) Moreover, if $\Pr(r_i = r_i') = 0$, then $S(F_j) \cap [r_i', r_i'')$ has at most one element.*

Proof. Let $(r_j^1, r_j^2) \in (r_i', r_i'')^2$. We then have $F_i(r_j^1) = F_i(r_j^2)$. It follows that j's profit is $[1 - F_i(r_i')] r_j - K(r_j)$ (which is concave in r_j) everywhere in (r_i', r_i'') and therefore that either (a) r_j^1 and r_j^2 do not yield the same profit or (b) there is an $r \in (r_j^1, r_j^2)$ that yields higher profit than either r_j^1 or r_j^2. In either (a) or (b) it is clear that we can't have $(r_j^1, r_j^2) \in S(F_j)^2$. Applying this result recursively to eliminate elements of $S(F_j)$ implies (i). The proof of part (ii) is identical with $(r_j^1, r_j^2) \in [r_i', r_i'')^2 \implies F_i(r_j^1) = F_i(r_j^2)$. ■

Claim 29. *$S(F_2)$ is an interval with no gaps.*

Proof. Suppose $S(F_2) \cap (r_2', r_2'') = \emptyset$ and $\{r_2', r_2''\} \subseteq S(F_2)$. By Claim 28, either F_1 has a mass point at r_2' or there exists an $\epsilon > 0$ such that $\Pr\{r_1 \in [r_2', r_2' + \epsilon)\} = 0$.

If $\Pr\{r_1 \in [r_2', r_2' + \epsilon)\} = 0$ then it must be that $(1-\mu)[1 - F_1(r_2')] - K'(r_2') \leq 0$—otherwise, firm 2 would wish to deviate to some $r_2 \in (r_2', r_2' + \epsilon)$. But $(1-\mu)[1 - F_1(r_2')] - K'(r_2') \leq 0$ implies that the marginal payoff to increasing r_2, $(1-\mu)[1 - F_1(r)] - (1-\mu)F_1'(r)r - K'(r)$, is negative everywhere above r_2'. It must then be true that $r_2'' \notin S(F_2)$, contradicting the existence of a gap.

Alternatively, suppose that there is a mass point in F_1 at r_2'. Then we must have $\mu + (1-\mu)[1 - F_2(r_2')] - K'(r_2') \leq 0$ (or else firm 1 would prefer to deviate to some $r_2' + \epsilon$). We also know, by the definition of $\overline{r}$, that $\mu - K'(\overline{r}) \geq 0$. These two inequalities are impossible to satisfy if $r_2' \leq \overline{r}$ and there is a gap at (r_2', r_2'') (where readers are reminded that a gap would require $F_2(r_2') < 1$). ■

Claim 30. *Suppose $S(F_1) \cap (r_1', r_1'') = \emptyset$ and $\{r_1', r_1''\} \subseteq S(F_1)$ (so that there is a gap in $S(F_1)$ at (r_1', r_1'')). Then (i) $r_1' = \max S(F_2)$, (ii) there is a mass point in F_2 at r_1', and (iii) $r_1'' = \overline{r}$.*

Proof. Suppose such a gap exists. By Claim 28, either F_2 has a mass point at r_1' or there exists an $\epsilon > 0$ such that $\Pr\{r_2 \in [r_1', r_1' + \epsilon)\} = 0$.

If $\Pr\{r_2 \in [r_1', r_1' + \epsilon)\} = 0$ then it must be that $\mu + (1-\mu)[1 - F_2(r_1')] - K'(r_1') \leq 0$—otherwise, firm 1 would wish to deviate to some $r_1 \in (r_1', r_1' + \epsilon)$. But $\mu + (1-\mu)[1 - F_2(r_1')] - K'(r_1') \leq 0$ implies that the marginal payoff to increasing r_1, $\mu + (1-\mu)[1 - F_2(r)] - (1-\mu)F_2'(r)r - K'(r)$, is negative everywhere above r_1'. It must then be true that $r_1'' \notin S(F_1)$, contradicting the existence of a gap.

Suppose, instead that there is a mass point in F_2 at r_1'. This implies that there is no mass point in F_1 at r_1' (claim 22) and that $S(F_2) \cap (r_1', r_1'') = \emptyset$ (claim 28). Since gaps in $S(F_2)$ have been ruled-out (Claim 29), it must be that $r_1' = \max S(F_2)$. Lastly, since $r_1' = \max S(F_2)$, any $r > r_1'$ results in 1 attracting only uninformed consumers—implying (i) that $S(F_1) \cap (r_1', \infty) = r_1''$, and (ii) that $r_1'' = \overline{r}$. ■

To recap, it has so far been shown that $\widetilde{r} > \underline{r}$ implies that firm 2 mixes over an interval $[\underline{r}, \max S(F_2)]$ and firm 1 mixes over $[\underline{r}, \max S(F_2)] \cup \bar{r}$, where $\bar{r}$ may or may not equal $\max S(F_2)$. Uniqueness is finally established by showing $\max S(F_2) = \min\{\widetilde{r}, \overline{r}\}$, pinning down cases 1 and 2 in the proof of Lemma 3.

Claim 31. $\max S(F_2) = \min\{\widetilde{r}, \overline{r}\}$.

Proof. Firstly, show that $\max S(F_2) \leq \min\{\widetilde{r}, \overline{r}\}$. If $\max S(F_2) > \overline{r}$ then $\Pr(r_2 < r_1 | r_2 = \max S(F_2)) = 0$, which contradicts Claim 24. Suppose $\max S(F_2) > \widetilde{r}$. Then, by Claim 29, $(\widetilde{r}, \max S(F_2)) \subseteq S(F_2)$. But this requires 2 to be indifferent between any $r_2 \in (\widetilde{r}, \max S(F_2))$ and $\underline{r}$ (by Claim 27), i.e. we must have

$$(1-\mu)[1-F_1(r_2)]r_2 - K(r_2) = (1-\mu)\underline{r} - K(\underline{r}). \tag{51}$$

This condition implies (16), which is decreasing for $r_2 > \widetilde{r}$. We therefore cannot find a valid F_1 that makes firm 2 indifferent.

Secondly, we prove that $\max S(F_2) \geq \min\{\widetilde{r}, \bar{r}\}$. Suppose that $\max S(F_2) < \widetilde{r}, \bar{r}$. Then there must be a gap $(\max S(F_2), \bar{r})$ in $S(F_1)$. For firm 2 not to deviate to some $\max S(F_2) + \epsilon$ it must be that, at $r_2 = \max S(F_2)$, we have $(1-\mu)[1-F_1(r_2)] - K'(r_2) \leq 0$. Combining this with (51) yields

$$(1-\mu)[1-F_1(r_2)] - K'(r_2) = \frac{(1-\mu)\underline{r} - K(\underline{r}) + K(r_2) - r_2 K'(r_2)}{r_2} \leq 0.$$

Using the definition of $\widetilde{r}$, this becomes $[\widetilde{r}K'(\widetilde{r}) - K(\widetilde{r}) + K(r_2) - r_2 K'(r_2)]/r_2 \leq 0$. Evaluated at $r_2 = \max S(F_2)$, this can be true only if $\widetilde{r} \leq \max S(F_2)$—a contradiction. ■