Raising Search Costs To Deter Window Shopping Can Increase Profits and Welfare

—Supplementary Appendix—

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B-1 Timing, commitment, and alternative screening instruments

The timing of the model is such that the firm can commit to—and consumers can observe—s, but that p and μ are not observed prior to search. One implication is that the firm can screen consumers by changing s but not by varying p or μ. This set-up is, in part, motivated by the reality that a consumer must typically decide first whether to browse (*i.e.*, whether to enter the store), and only then learns about what he will experience.

There are also good formal reasons to favor this particular timing. Firstly, the firm will often face commitment problems when attempting to use p or μ to screen consumer interest. Fix an s and μ and suppose that the firm in the model above posts a "sticker-price" greater than p^*, which can be observed ex ante. If consumers believe that the sticker-price is non-negotiable then the increased price causes the surplus from browsing to fall and, provided $s > 0$, low-interest consumers will be screened out. However, once a consumer is in the store both he and the firm have a shared interest in negotiating a reduction in the price to p^*. The sticker price should therefore be regarded as a non-credible threat to refuse such negotiations. Likewise, to deter low-interest consumers the firm would have to promise to invest less in sales than it would otherwise like to. Again, once the decision to browse is made, the consumer and firm alike would prefer this promise to be broken (e.g., for a salesperson to appear from "the back of the store"). The same commitment problems do not arise for search costs. One interpretation of the analysis above is therefore as a reduced form of a model in which browsing consumers can negotiate with the firm prior to purchasing.

Secondly, suppose that the firm can fully-commit to p and μ at the first stage of the game. It will nevertheless wish to rely solely on search costs to screen consumers. In other words, allowing full commitment does not change the firm's optimal strategy. To see this,

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note that the firm's strategy must satisfy three first-order conditions: on p, μ, and s:

$$(1)\quad \frac{\partial \Pi}{\partial p} + \underbrace{\frac{\partial \Pi}{\partial \overline{\pi}}\frac{\partial \overline{\pi}}{\partial p}}_{=0} = 0, \qquad \frac{\partial \Pi}{\partial \mu} + \underbrace{\frac{\partial \Pi}{\partial \overline{\pi}}\frac{\partial \overline{\pi}}{\partial \mu}}_{=0} = 0, \qquad \frac{\partial \Pi}{\partial \overline{\pi}}\frac{\partial \overline{\pi}}{\partial s} = 0 \implies \frac{\partial \Pi}{\partial \overline{\pi}} = 0.$$

The second term in each of the first two first-order conditions reflects the marginal benefit of using p or μ respectively as a screening device. These terms are equal to zero by the third first-order condition—implying that price and sales intensity are set in ignorance of any screening considerations.

Intuitively, because s is an instrument that affects profits only through the selection of consumers who browse, whereas p and μ also distort the profits associated with any given group of browsers, the optimal scheme is to set p and μ to maximize profits (taking $\overline{\pi}$ as given) and then to use (only) s to fine-tune the value of $\overline{\pi}$.

Now, suppose that we exogenously constrain $s = 0$ such that $\partial \Pi / \partial \overline{\pi} > 0$. It then follows from (1) that the firm will respond to such an intervention by increasing price or reducing sales intensity or both. In other words, preventing the creation of search costs will typically induce the firm to substitute to other means of screening and is likely to distort prices and sales in a manner that is harmful for consumers. Any such distortion is in addition to the fact that p and μ are already set inefficiently high and low respectively.

How does this relate to consumer surplus? Search costs are borne by every browsing consumer, whereas the price is paid only by those who buy. Might it therefore be possible to make consumers better-off by replacing search costs with other instruments for screening? Suppose that there are two schemes, (s, μ, p) and (s', μ', p'), that both screen-out the same consumers (*i.e.*, that induce the same $\overline{\pi}$). Let $s > s'$ so that the first scheme can be said to be more reliant on search costs to achieve the desired screening outcome. Because $s/\sigma(p, \mu) = \overline{\pi} = s'/\sigma(p', \mu')$, it must be that $\sigma(p, \mu) > \sigma(p', \mu')$. Under both schemes, the set of excluded consumers (who receive zero utility) is the same, so any effect on consumer surplus must come from the non-excluded inframarginal consumers. We have

$$\begin{aligned} \overline{\pi} = \frac{s}{\sigma(p, \mu)} = \frac{s'}{\sigma(p', \mu')} &\implies \overline{\pi}\big[\sigma(p, \mu) - \sigma(p', \mu')\big] = s - s' \\ &\implies \pi\big[\sigma(p, \mu) - \sigma(p', \mu')\big] > s - s' \quad \forall \pi > \overline{\pi}. \end{aligned}$$

This gives rise to the following result:

Proposition B-1. *Suppose that $s > s'$ and that (s, μ, p) and (s', μ', p') yield the same $\overline{\pi}$. Then consumer surplus is higher in the scheme with larger search costs, (s, μ, p).*

Conditional on achieving a desired level of screening, consumer surplus is maximized when screening is achieved (to the greatest extent possible) through search costs. Intuitively, low-interest consumers' browsing decisions are hardly affected by an increase in the firm's price because they expect to buy (and hence pay that price) with low probability. It therefore takes a relatively large increase in price to achieve the same screening result as could be managed with only a modest increase in search costs.

B-2 Browsing fees can result in higher or lower s

The following result establishes that a firm setting a browsing fee, s_b, may choose to set either a higher or lower equilibrium s than one that sets a wasteful search cost, s_w. In order to give more primitive conditions for this to be the case, it is necessary to specify more of the model's structure. For example, one can show that $s_b^* > s_w^*$ always holds under the specification of Corollary 1. More interestingly, a parametrized example in which both outcomes are possible can be obtained by taking the specification of Corollary 1 and modifying the demand function such that μ is interpreted as measuring the probability that a trade can take place: $D(p,\mu) = \mu D(p)$, with $\mu \in [0,1]$.

Proposition B-2. *Suppose that demand is $D(p,\mu) = \mu D(p)$ (with $\mu \in [0,1]$), $G(\pi) = \pi$, and $C(\mu) = \mu^k$ (with $k > 1$). Then (i) if $D(p^*)p^* < 2^{2-k}k$ we have $s_b^* < s_w^*$, and (ii) if $D(p^*)p^* > 2^{2-k}k$ we have $s_b^* > s_w^*$.*

Proof of Proposition B-2. Step 1: Begin by considering the wasteful search cost case. (4) implies that the firm increases s (and hence $\overline{\pi}$) at least until $\mu^* = 1$. Profit then becomes

$$E(\pi|\pi > \overline{\pi})mD(p^*)p^* - C(m) = \left(\frac{1+\overline{\pi}}{2}\right)(1-\overline{\pi})D(p^*)p^* - (1-\overline{\pi})^k.$$

Thus, the first-order condition for profit maximisation is to set s such that $\overline{\pi}$ solves $k(1-\overline{\pi})^{k-1} - D(p^*)p^*\overline{\pi} = 0$ (one can check that the μ^* associated with this choice is indeed 1). It will also be useful to check the second-order condition. Profit is increasing in $\overline{\pi}$ if $\mu^* < 1$ and once $\mu^* = 1$ we have $\partial^2\Pi/\partial\overline{\pi}^2 = -2D(p^*)p^* - (k-1)k(1-\overline{\pi})^{k-2} < 0$ so profits are quasi-concave.

Step 2: Now consider the case of a browsing fee and begin by showing that the firm again wishes to increase s at least until $\mu^* = 1$. If $\mu^* < 1$ we obtain

$$(2) \qquad \mu^* = 2^{\frac{1}{1-k}}\left(\frac{D(p^*)p^*}{k}\right)^{\frac{1}{k-1}}\frac{(1+\overline{\pi})^{\frac{1}{k-1}}}{1-\overline{\pi}}.$$

from (3). Profits are

$$(3) \qquad E(\pi|\pi > \overline{\pi})\mu m D(p^*)p^* - C(m\mu) + ms$$
$$= \left(\frac{1+\overline{\pi}}{2}\right)(1-\overline{\pi})\mu D(p^*)p^* - [(1-\overline{\pi})\mu]^k + \underbrace{(1-\overline{\pi})}_{=m}\underbrace{\overline{\pi}\sigma}_{=s}.$$

Substituting from (2) and differentiating with respect to $\overline{\pi}$ yields

$$\left.\frac{d\Pi}{d\overline{\pi}}\right|_{\mu=\mu^*} = \frac{2^{\frac{k}{1-k}}\left(2u\left(\frac{D(p^*)p^*(1+\overline{\pi})}{k}\right)^{\frac{1}{k-1}}(k-1+k\overline{\pi})+k(k-1)\left(\frac{D(p^*)p^*(1+\overline{\pi})}{k}\right)^{\frac{k}{k-1}}\right)}{(k-1)(1+\overline{\pi})},$$

where $u=\int_{p^*}^{\infty}D(p)\,dp$. This is positive (recall $k>1$) so no equilibrium can have $\mu^*<1$.

Step 3: Define $\overline{s}=\min\{s:\mu^*(s)=0\}$ ($\overline{s}$ is the smallest s such that the firm wishes to set $\mu=1$). Suppose that (under a browsing fee) the firm finds it optimal to set $s^*>\overline{s}$. Letting $\mu=1$ in (3) and calculating the first-order condition for the optimal $\overline{\pi}$ yields

$$(4)\qquad k(1-\overline{\pi})^{k-1}-D(p^*)p^*\overline{\pi}+\sigma(1-2\overline{\pi})=0.$$

Comparison with the first-order condition in step 1 (along with quasi-concavity) reveals that the equilibrium $\overline{\pi}$ is higher (lower) under a browsing fee if $\overline{\pi}(s_w^*)<(>)1/2$.

Given quasi-concavity, we know that $\overline{\pi}(s_w^*)<1/2$ will hold if the left-hand side of the first-order condition from step 1 is negative at $\overline{\pi}=1/2$. This is the case if and only if $D(p^*)p^*>2^{2-k}k$.

Step 4: Step 3 was predicated on the assumption that $s^*>\overline{s}$. We now examine what happens if $s^*=\overline{s}$. From (2), this requires $\overline{\pi}$ to satisfy

$$(5)\qquad \frac{2^{\frac{1}{1-k}}\left(\frac{D(p^*)p^*(1+\overline{\pi})}{k}\right)^{\frac{1}{k-1}}}{1-\overline{\pi}}=1 \implies 1-\overline{\pi}=2^{\frac{1}{1-k}}\left(\frac{D(p^*)p^*(1+\overline{\pi})}{k}\right)^{\frac{1}{k-1}}.$$

Substituting this expression for $1-\overline{\pi}$ into the left-hand side of the first-order condition calculated in step 1 and simplifying yields $(1-\overline{\pi})D(p^*)p^*/2$, which is positive. The quasi-concavity established in step 1 therefore implies that the $\overline{\pi}$ which just induces $\mu^*=1$ is lower than the equilibrium $\overline{\pi}$ under a wasteful search cost.

Step 5: Lastly, establish that the case in step 4 is only ever relevant if $D(p^*)p^*<2^{2-k}k$ (this implies that the $D(p^*)p^*=2^{2-k}k$ cut-off calculated in step 3 is the relevant cut-off beyond which $s_b^*>s_w^*$). If $s^*=\overline{s}$ then we know that $\overline{\pi}$ is such that (5) is satisfied. This equation can be rewritten as $k(1-\overline{\pi})^{k-1}=D(p^*)p^*(1+\overline{\pi})/2$. For the firm not to want to increase $\overline{\pi}$ further, it must be that the right-hand side of (4) is negative. Substituting $k(1-\overline{\pi})^{k-1}=D(p^*)p^*(1+\overline{\pi})/2$, the right-hand side of (4) is negative if $\left[D(p^*)p^*(1-\overline{\pi})+(2-4\overline{\pi})u\right]/2<0$. This condition can be satisfied only if $\overline{\pi}>1/2$.

But $\overline{\pi}>1/2$ holds only if $D(p^*)p^*<2^{2-k}k$. To see this, observe that the $2^{\frac{1}{1-k}}\left(D(p^*)p^*/k\right)^{\frac{1}{k-1}}$ term in (2) (the equation that pins-down $\overline{\pi}$) is increasing in $D(p^*)p^*$, so force $\overline{\pi}$ as large as possible by taking $D(p^*)p^*=2^{2-k}k$. We then have $\overline{\pi}$ determined by $(1+\overline{\pi})^{\frac{1}{k-1}}=(1-\overline{\pi})/2$. The left-hand side is decreasing in k so, again, force $\overline{\pi}$ to be as large as possible by letting $k\to\infty$. We are left with $1/2(1-\overline{\pi})=1$, which is solved by $\overline{\pi}=1/2$. ■

B-3 Omitted proofs from the oligopoly model

Calculations for Example 1. *Proof of part 1.* When $D(p) = 1 - p$ the Diamond equilibrium price is the normal monopoly price, $p^* = 1/2$. A firm's profits are

$$\Pi_i = E(\pi_i|\text{consumer visits } i)m_i\mu_i D(p^*)p^* - C(m_i\mu_i), \tag{6}$$

where m_i is the number of consumers who visit i.

Begin by supposing that firms are constrained to use $\mu_1 = \mu_2 = 1$ (this, along with $p^* = 1/2$, implies $\sigma_1 = \sigma_2 = \sigma = 1/8$). From (15), a consumer visits i first if $\pi_i\sigma_i - s_i + [1 - \pi_i][\pi_j\sigma_j - s_j] \geq \pi_j\sigma_j - s_j + [1 - \pi_j][\pi_i\sigma_i - s_i]$. It will be useful to write

$$\widetilde{\pi}_i = \min\left\{1, \frac{s_i\pi_j}{s_j + \pi_j(\sigma_i - \sigma_j)}\right\} = \min\left\{1, \frac{s_i\pi_j}{s_j}\right\}.$$

Thus, if $\widetilde{\pi}_i < 1$, it represents the π_i that is indifferent between visiting $i \to j$, and visiting $j \to i$, for a given π_j. We can calculate m_1 by integrating over the relevant areas in Figure 3:

$$m_1 = \int_{s_1/\sigma}^{1}\int_{0}^{\widetilde{\pi}_2} g(\pi_1, \pi_2)\, d\pi_2\, d\pi_1 + \int_{s_1/\sigma}^{1}\int_{\widetilde{\pi}_2}^{1} g(\pi_1, \pi_2)(1 - \pi_2)\, d\pi_2\, d\pi_1.$$

Likewise, we have

$$E(\pi_1|\text{consumer visits } 1) = \frac{1}{m_1}\left[\int_{s_1/\sigma}^{1}\int_{0}^{\widetilde{\pi}_2} g(\pi_1, \pi_2)\pi_1\, d\pi_2\, d\pi_1 + \int_{s_1/\sigma}^{1}\int_{\widetilde{\pi}_2}^{1} g(\pi_1, \pi_2)(1 - \pi_2)\pi_1\, d\pi_2\, d\pi_1\right].$$

Assume that $s_1 \geq s_2$ so that $\widetilde{\pi}_2 < 1$ holds for all π_1. Substituting the above expressions for m_1 and $E(\pi_1|\text{consumer visits } 1)$ into (6) and differentiating with respect to s_1 yields the first-order condition

$$\left(4 + \frac{256s_2^2}{3} + \frac{s_2^2}{3s_1^3}\right)\left(\frac{1}{2} - 4s_1 + \frac{s_2^2}{6s_1^2} - \frac{256}{3}s_1s_2^2\right) - \frac{1}{4}\left(32s_1 + \frac{s_2^2}{4s_1^3} + 1024s_1s_2^2\right) = 0.$$

Imposing $s_1 = s_2 = s$, this implies $s^* = 0.097$. It is easily verified that Π_1 is concave in s_1 when $s_2 = 0.097$ and $\mu_1 = \mu_2 = 1$.[1]

By assuming that $s_1 \geq s_2$ and then differentiating with respect to s_1 (before imposing symmetry) we have identified the $s_1 = s_2 = s$ such that neither firm can profit by *increasing* its s. To check that firm 1 cannot profit by decreasing its s, the relevant

[1] Differentiating the first-order condition yields $[-0.0000245971 - 0.00294065s_1^2 + 0.0150655s_1^3 - 33.4777s_1^6]/s_1^6$, which is negative for $s_i \geq 0$.

profit expression is that which arises when $s_1 \leq s_2$ so that $\widetilde{\pi}_2 = 1$ when π_1 is close to 1.[2] Differentiating profit with respect to s_1 then yields the first-order condition

$$\left(4+\frac{1}{3s_2}+\frac{256s_2^2}{3}\right)\left(1-4s_1-\frac{s_1}{3s_2}-8s_2+8\left(1-8s_1\right)s_2+64s_1s_2-\frac{256}{3}s_1s_2^2\right) \\ -\frac{1}{4}\left(32s_1+\frac{s_1}{4s_2^2}+1024s_1s_2^2\right)=0.$$

This equation is satisfied at $s_1 = s_2 = s^* = 0.097$, and the left-hand side is again decreasing so that $s_1 = s_2 = s^* = 0.097$ is indeed an equilibrium conditional on constraining $\mu_1 = \mu_2 = 1$.

To verify that this really is an equilibrium we must check that the $\mu = 1$ constraint introduced above is non-binding. Firstly, in analogy to (3), firm 1 wishes to increase μ if and only if

$$E(\pi_1|\text{consumer visits 1})D(p^*)p^* - C'(m_1\mu_1) > 0. \tag{7}$$

Substituting $s_1 = s_2 = 0.097$, $\sigma = 1/8$ into the expressions for $E(\pi_1|\text{consumer visits 1})$ and m_1 and evaluating the left-hand side of (7) yields $0.22 - 0.2\mu$, which is positive so $\mu = 1$ is indeed optimal given $s_1 = s_2 = 0.097$. Secondly, we need to check that a firm can't profit by a joint deviation in s and μ. Suppose that the firm could directly choose *any* value for $E(\pi_1|\text{consumer visits 1})$ (by directly excluding the least interested consumers). Differentiating (6) with respect to $E(\pi_1|\text{consumer visits 1})$ yields

$$\overbrace{m_1\mu_1 D(p^*)p^*}^{>0} \\ + \frac{\partial m_1}{\partial E(\pi_1|\text{consumer visits 1})}\underbrace{\left[E(\pi_1|\text{consumer visits 1})\mu_1 D(p^*)p^* - \mu_1 C'(m_1\mu_1)\right]}_{A}. \tag{8}$$

If $\mu^* < 1$ then term A is equal to zero (we can substitute for $C'(m_1\mu_1)$ from the first-order condition on μ implied by (7)). But if A is equal to zero then (8) is strictly positive. Thus, any deviation with $\mu < 1$ is dominated by some other deviation in which $E(\pi_1|\text{consumer visits 1})$ is just high enough to induce $\mu^* = 1$. Such a deviation is, in turn, dominated by the putative equilibrium because there is a bijective relationship between s and $E(\pi_1|\text{consumer visits 1})$ once $\mu = 1$ and we have seen that profits are concave.

To complete the proof of part 1, note that a consumer is excluded if $\pi_i\sigma_i - s_i < 0$ for $i = 1, 2$. Letting $s_i = 0.097$ and $\sigma_i = 1/8$ and solving for π yields $\pi = 0.78$.

Proof of part 2. In the equilibrium derived in part 1, we have $s_1 = s_2 = s^* = 0.097$, $\mu_1 = \mu_2 = \mu^* = 1$, $p_1 = p_2 = p^* = 1/2$. This implies that each trade yields revenue of $1/4$,

[2]Geometrically, this means that the diagonal curve in Figure 3 intersects the top of the frame.

consumer surplus of 1/8 and total surplus of 3/8, and that $s^*/\sigma = 8s^*$. The total welfare stemming from consumers who visit only firm 1 is

$$W_1 = \int_{8s^*}^{1} \int_{0}^{8s^*} g(\pi_1, \pi_2) \left(\frac{3}{8}\pi_1 - s^* \right) d\pi_2 \, d\pi_1.$$

Total welfare from consumers who visit 1 and then 2 is

$$W_{1\to 2} = \int_{8s^*}^{1} \int_{8s^*}^{\pi_1} g(\pi_1, \pi_2) \left[\frac{3}{8}[\pi_1 + (1-\pi_1)\pi_2] - s^* - (1-\pi_1)s^* \right] d\pi_2 \, d\pi_1.$$

Total welfare is thus given by $W(s^*) = 2[W_1 + W_{1\to 2}]$ (multiplying by 2 because for every consumer who visits i first, there is a symmetric consumer who visits j first). Substituting $s^* = 0.097$ and $g(\cdot, \cdot) = 1$ yields $W(s^*) = 0.095$. Equilibrium consumer surplus is found analogously by replacing 3/8 with 1/8 in W_1 and $W_{1\to 2}$. We thus obtain $\text{CS}(s^*) = 0.0058$.

Now consider the frictionless case with $s_1 = s_2 = 0$ and all consumers visiting both firms. Since a firm is indifferent over all strategies in the equilibrium support, profits are given by those that arise when the firm plays the monopoly price, $p = 1/2$, and sells only to captive consumers:

$$\Pi_1 = E[\pi_1(1 - \pi_2\mu_2)]\mu_1 \frac{1}{4} - C(\mu_1) = \frac{1}{4}\mu_1 \left[\frac{1}{2} - \frac{\mu_2}{4} \right] - \frac{(\mu_1)^2}{2}.$$

Maximizing with respect to μ_1 and imposing $\mu_1 = \mu_2 = \mu^*$ yields $\mu^* = 2/17$. We can substitute this into (14) and solve $F(p) = 0$ for p to find the minimum of the support of the price distribution, which is 0.38. The expected total surplus *from each trade* is

$$\int_{0.38}^{1/2} \left(\int_{\widetilde{p}}^{\infty} D(p)\, dp + D(\widetilde{p})\widetilde{p} \right) F'(\widetilde{p}) \; d\widetilde{p} = 0.412$$

if one firm is in the consideration set. If both firms are in the consideration set, the price paid is the minimum of two draws from F and surplus per-trade is

$$\int_{0.38}^{1/2} \left(\int_{\widetilde{p}}^{\infty} D(p)\, dp + D(\widetilde{p})\widetilde{p} \right) 2[1 - F(\widetilde{p})]F'(\widetilde{p}) \; d\widetilde{p} = 0.419$$

This allows us to calculate the aggregate total surplus across all trades:

$$2\int_0^1 \int_0^1 0.412 \times g(\pi_1, \pi_2) \left(1 - \frac{2}{17}\pi_1\right) \frac{2}{17}\pi_2 \, d\pi_2 \, d\pi_1 + \int_0^1 \int_0^1 0.419 \times g(\pi_1, \pi_2) \left(\frac{2}{17}\pi_1\right) \left(\frac{2}{17}\pi_2\right) d\pi_2 \, d\pi_1 = 0.047.$$

Similarly, consumer surplus *per-trade* is

$$\int_{0.38}^{1/2}\left(\int_{\widetilde{p}}^{\infty} D(p)\,dp\right) F'\left(\widetilde{p}\right)\, d\widetilde{p} = 0.170$$

when the consideration set is a singleton and

$$\int_{0.38}^{1/2}\left(\int_{\widetilde{p}}^{\infty} D(p)\,dp\right) 2[1-F\left(\widetilde{p}\right)]F'\left(\widetilde{p}\right)\, d\widetilde{p} = 0.179$$

when both firms are in the consideration set. This implies aggregate consumer surplus of

$$2\int_0^1\int_0^1 0.170\times g(\pi_1,\pi_2)\left(1-\frac{2}{17}\pi_1\right)\frac{2}{17}\pi_2\,d\pi_2\,d\pi_1 + \int_0^1\int_0^1 0.179\times g(\pi_1,\pi_2)\left(\frac{2}{17}\pi_1\right)\left(\frac{2}{17}\pi_2\right)d\pi_2\,d\pi_1 = 0.019.$$

Comparing the calculated welfare and consumer surplus between the equilibrium and the frictionless case yields the result.

Proof of part 3. If firm 2 exits the market then the problem collapses back to the baseline monopoly case. From (5), if $D(p,\mu)=\mu(1-p)$, the firm finds it optimal to increase s_1 whenever $\mu^*<1$. Consumers visit the firm if and only if $\pi_1>\overline{\pi}=s_1/\sigma_1=8s_1$. Thus, $m_1=1-8s_1$. The firm's profit is

$$E(\pi_1|\pi_1>\overline{\pi})m_1\frac{1}{4}-C(m_1)=\frac{1-64s^2}{8}-\frac{(1-8s)^2}{2}.$$

Differentiating with respect to s_1 and solving the resulting first-order condition yields $s_1=1/10$. This implies $\overline{\pi}=(1/10)/(1/8)=8/10$. ■

Proof of Proposition 7. In a symmetric frictionless equilibrium, consumer surplus is no greater than[3]

$$\mathrm{CS}(0)=(1-\omega)\underbrace{\left\{1-\int_0^1\cdots\int_0^1 g(\boldsymbol{\pi})\prod_{i=1}^{n}(1-\pi_i\mu(0))\,d\pi_n\ldots d\pi_1\right\}}_{\text{probability at least one firm is in consideration set}}\int_{\underline{p}}^{\infty} D(p)\,dp,$$

where the symmetric $\mu(0)$ solves firms' first-order condition:

$$(1-\omega)\underbrace{\left\{\int_0^1\cdots\int_0^1 g(\boldsymbol{\pi})\pi_1\prod_{i=2}^{n}(1-\pi_i\mu)\,d\pi_n\ldots d\pi_1\right\}}_{\text{probability that (only) firm 1 is in consideration set}} D(p^*)p^*-C'(\mu)=0. \tag{9}$$

[3] "No greater than" because the following expression assumes all firms set price $\underline{p}$ whereas they in fact mix on support $[\underline{p},p^*]$.

Note, in particular, that the first term in (9) is proportional to $1-\omega$ since the firm is visited by all consumers but only has a chance of selling to those with $\pi > 0$.

Now suppose that $s_1 = \ldots = s_n = \epsilon > 0$ and let

$$\xi_i(\pi_i) = \begin{cases} \pi_i \text{ if } \pi_i \geq \epsilon/\sigma \\ 0 \text{ if } \pi_i < \epsilon/\sigma. \end{cases}$$

Thus, a consumer will never visit firm i if $\xi_i = 0$. From (15), in a symmetric equilibrium the consumer prefers to visit firms in descending order of their π. Consumer surplus is

$$\begin{aligned}\mathrm{CS}(\epsilon) = (1-\omega) & \overbrace{\left\{1 - \int_0^1 \cdots \int_0^1 g(\boldsymbol{\pi}) \prod_{i=1}^n \left[1 - \xi_i(\pi_i)\mu(\epsilon)\right] \, d\pi_n \ldots d\pi_1\right\}}^{\text{probability at least one firm is in consideration set}} \int_{p^*}^{\infty} D(p)\, dp \\ & - (1-\omega)\epsilon E(\text{number of firms visited}).\end{aligned}$$

As $\epsilon \to 0$, this becomes

$$\mathrm{CS}(\epsilon) = (1-\omega) \left\{1 - \int_0^1 \cdots \int_0^1 g(\boldsymbol{\pi}) \prod_{i=1}^n \left[1 - \pi_i(\pi_i)\mu(\epsilon)\right] \, d\pi_n \ldots d\pi_1\right\} \int_{p^*}^{\infty} D(p)\, dp$$

and the symmetric $\mu(\epsilon)$ solves firms' first-order condition:

$$\begin{aligned}(10) \quad & \overbrace{(1-\omega)\left\{\frac{1}{n}\sum_{x=1}^n \left[\prod_{z=x+1}^n \left[1 - E\left(\pi_{(z)}\right)\mu\right] E\left(\pi_{(x)}\right)\right]\right\}}^{\text{mass of consumers who visit and are interested in a given firm}} D(p^*)p^* \\ & - \underbrace{(1-\omega)\left\{\frac{1}{n}\sum_{x=1}^n \left[\prod_{z=x+1}^n \left[1 - E\left(\pi_{(z)}\right)\mu\right]\right]\right\}}_{m} C'(m\mu) = 0,\end{aligned}$$

where $E\left(\pi_{(x)}\right)$ is the expected value of the x^{th} order statistic of $\boldsymbol{\pi}$ (i.e. the x^{th} smallest π_i) and m is the number of consumers expected to visit a given firm.

Thus, as $\epsilon \to 0$, the sign of $\mathrm{CS}(\epsilon) - \mathrm{CS}(0)$ is positive if $\mu(0)$ is sufficiently small and $\mu(\epsilon)$ is not too small.[4] From (9) we see that $\mu(0) \to 0$ as $\omega \to 1$. From (10), on the other hand, $\mu(\epsilon) \to 1$ as $\omega \to 1$ (because the $1-\omega$ terms cancel and $\lim_{\omega \to 1} m = 0$). ∎

[4]In particular, note that $\mathrm{CS}(0) \to 0$ as $\mu \to 0$.