Application bundling in system markets*

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Abstract

We study markets where consumers buy a device that allows them to use applications. Application developers earn revenues by interacting with consumers, and are willing to pay device manufacturers to be installed exclusively on the devices. A firm that controls multiple applications can license them to device manufacturers either individually or as a bundle. We show that this yields a new explanation for why a firm might choose to bundle: Bundling reduces rival application developers' willingness to bid for inclusion on the device and allows a multi-application developer to capture a larger share of industry profit. Application bundling can also strengthen competition between manufacturers and thereby increase consumer surplus, even if it leads to foreclosure of application developers and a loss in product variety.

1 Introduction

This paper studies bundling in markets where (i) a *manufacturer* must carry *applications* in order to be attractive to consumers, and (ii) application makers profit from consumers they access via a manufacturer (for example, by showing ads to those consumers). In such markets it is fairly common for application firms to sell or license several applications to a manufacturer as a bundle—a practice that has attracted scrutiny from regulators. For example, in 2016 the European Commission issued a "Statement of Objections" concerning the Android mobile telephone operating system.[1] A key source of concern is the use of contracts that require handset manufacturers wishing to install one Google application (such as the Google Play marketplace or Google search) to install a whole suite of other apps supplied by Google.[2] A second example is channel bundling in cable television.

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[1] See `http://europa.eu/rapid/press-release_IP-16-1492_en.htm`, accessed 18 October 2016.

[2] See Edelman and Geradin (2016) for a critical antitrust analysis.

Conglomerate media firms often own both highly attractive and less popular channels. A common practice is to license these channels as a bundle, so that any distributor wishing to broadcast the most popular channels must also agree to offer the less popular ones. This has drawn criticism from some quarters (see, e.g., Crawford, 2015, for an overview).

An important theme in the bundling literature is the leverage theory of foreclosure, which holds that a firm with a dominant position in one market can extend that market power into an adjacent market by bundling its products in the two markets together. This theory was dealt a heavy blow when scholars of the Chicago School (e.g., Director and Levi, 1956) pointed out that firms could achieve the same profit without recourse to bundling through appropriate choice of prices.[3] More recently, scholars have come to reexamine the Chicago critique and have shown that bundling can be profitable if it succeeds in foreclosing competition (e.g., Carlton and Waldman, 2002; Choi and Stefanadis, 2001; Nalebuff, 2004; Whinston, 1990) or softening competition between firms (e.g., Chen, 1997; Carbajo, De Meza, and Seidmann, 1990).[4] Since bundling in these models is used as a tool to weaken competition, it is typically detrimental to consumers.

We extend the analysis of bundling to deal with situations in which firms can license bundles of applications to manufacturers, and show that this gives rise to a novel explanation for why firms might bundle. In our model, two application developers (firms 1 and 2) produce applications that can be installed on a manufacturer. There are two types of application: A and B, and the manufacturer can install one application of each type. Firm 1 is the only one to offer an A-application, and both firms offer a B-application. Applications generate direct per-consumer revenues for their developers, which induces developers to offer payments to the manufacturer in exchange for being installed. Bundling by firm 1 rules-out the possibility of firm 2's B application being installed alongside firm 1's A application. We show that this can reduce firm 2's willingness to bid for inclusion on the manufacturer through two distinct but related channels. Firstly, a manufacturer that installs firm 2's application will attract fewer consumers (because it must forgo application A), which reduces the value to firm 2 of being on the manufacturer. Secondly, there may be complementarity between applications;[5] but bundling ensures that firm 2 has no way

[3]Suppose firm 1 supplies good A as a monopolist, while firms 1 and 2 compete to supply good B. Consumers value good i at v_i. When A_1 and B_1 are bundled, firm 2 will bid the price of its B-market good down to zero, leaving consumers who buy it with utility v_B. A consumer who buys the bundle at price p_{AB} receives utility $v_A + v_B - p_{AB}$, implying that the bundling firm can charge up to $p_{AB} = v_A$. If, instead, firm 1 does not bundle then Bertrand competition forces prices and profits in market B to zero. Thus, firm 1 is reduced to acting as a monopolist in market A and charging $p_A = v_A$. We therefore observe that firm 1's profit ($\pi_1 = v_A$) is the same regardless of whether it bundles or not.

[4]A separate literature studies the role of bundling in facilitating price discrimination (e.g., Adams and Yellen, 1976; Schmalensee, 1984; Zhou, forthcoming) by reducing dispersion in consumers' willingness to pay and thereby facilitating extraction of their surplus.

[5]For example, application A may provide infrastructure that helps application B to function effectively (such as when application A is a smartphone app store that distributes application updates). Alternatively, application A might, by making the manufacturer more valuable, increase overall usage, with the upshot that application B also receives more users.

to benefit from any complementary spillovers generated by application A. We show that when payments from application firms to the manufacturer are lump-sum, firm 1 can profit from bundling via either channel thanks to the reduction in the payment offered by firm 2 to the manufacturer. We also examine the case where payments are royalties or two-part tariffs, in which case the first channel is inactive but complementarity can still generate profitable bundling opportunities.

We then extend the analysis to a set-up with two competing manufacturers, and discuss the conditions for bundling to be profitable. We find that bundling is more likely to be profitable when applications' revenues are large and when differentiation with respect to which B-application is installed is not too profitable. Finally, in a framework with Hotelling-like horizontal differentiation both at the application level and at the manufacturer level, we look at the effect of bundling on equilibrium manufacturer pricing, consumer surplus and total welfare, and we show that bundling, by reducing manufacturer differentiation, can increase consumer surplus, even if the loss in equilibrium variety reduces total welfare.

Unlike in Carlton and Waldman (2002), Whinston (1990), or Chen (1997), neither foreclosure of competition nor the ability to commit to bundling are necessary for bundling to be profitable. Moreover, bundling can increase consumer surplus even when it does result in exclusion (by reducing differentiation and forcing manufacturers to compete more fiercely).

Several other recent papers consider bundling in platform markets. Amelio and Jullien (2012) and Choi and Jeon (2016) consider models with platforms that are unable to charge negative prices. As in many models of two-sided markets, platforms would like to subsidize one side in order to capture profit on the other, but the non-negative price constraint limits their ability to do this. A platform owner, though, can implicitly subsidize participation by tying the platform to another product and then reducing the price charged for that product. By relaxing the zero price constraint, bundling can therefore be profitable for the firm. Indeed, Choi and Jeon (2016) show that, consistent with the leverage theory, a monopolist in one market can exploit this mechanism to profitably extend its market power. Another paper that studies bundling in a two-sided context is Choi (2010). Suppose that there is some enabling good that is necessary to use a platform, and that this good is bundled with one of two competing platforms. This puts the rival platform at a disadvantage and causes it to reduce its price. Choi shows that the result can be an increase in welfare because more consumers choose to multi-home and consume exclusive content only available at the (now cheaper) rival platform. The mechanism we study is distinct from that at work in these papers: we do not impose a non-negative price constraint,[6] and focus on environments

[6]Indeed, when applications are licensed to platforms rather than end users, negative prices are fairly common. For example, court proceedings revealed that, in 2014, Google paid Apple $1bn for the right to be installed as the default search engine on iPhone devices. See https://www.bloomberg.com/news/articles/2016-01-22/google-paid-apple-1-billion-to-keep-search-bar-on-iphone, accessed 31 October 2016.

in which buyers of the potentially bundled products are themselves manufacturers who subsequently interact with consumers (and compete).

Like us, Ide and Montero (2016) consider firms that sell to consumers through an intermediary. Bundling can profitably exploit consumer heterogeneity (as in Nalebuff, 2004), but only if intermediaries don't have too much market power (indeed, a monopolist intermediary acts like a single, large buyer who therefore exhibits no heterogeneity). We put aside the issues of consumer heterogeneity and intermediary market power, and instead focus on a different mechanism. In particular, the two-sidedness of the market in our model means payments flow in the opposite direction: from producers to the intermediary. This creates a new opportunity for bundling to be profitable through its effects on these payments.

Another paper which deals with bundling in vertical relations is O'Brien and Shaffer (2005), who ask how the welfare consequences of a merger between two wholesalers depend upon whether the merged unit can bundle its products for sale to a retailer. Post merger, a wholesaler uses bundling to prevent the retailer from carrying only one of its products. Denied the possibility to do this, it instead distorts marginal prices to achieve a similar objective. These price distortions harm consumers and reduces welfare so society is better-off is bundling is not prohibited.

Although reminiscent of the literature on compatibility in systems markets (Matutes and Regibeau (1988), Kim and Choi (2015)), our paper differs from it in the sense that the choice of which applications to install is made by the manufacturers rather than the consumers. Therefore, with or without bundling, consumers must choose between "integrated" systems, and can never "mix and match".[7]

2 Model setup

The market is composed of one manufacturer, which allows consumers to use applications. There are two categories of applications, A and B (for instance email and maps), and two application developers, 1 and 2. Firm 1 offers one application in each category (A_1 and B_1), whereas firm 2 only offers an application of category B (B_2). Because of capacity constraints, the manufacturer can offer at most one application of each category.[8]

Applications differ from standard components of a final product in the sense that they generate direct revenues from their interactions with consumers. These revenues may come from advertising, sale of consumer data to third parties, or "in-app purchases". We

[7] We do allow for costly mix and match in section 5.

[8] The debate around bundling of smartphone applications has mostly focused on the manufacturer's choice of a default application (or on which application makes it onto the phone's home screen). Capacity is constrained because there can be only one default for each task and space on the home screen is limited. Likewise, digital TV broadcasts can carry many channels, but low-numbered channel slots (which receive more viewers) are scarce. In Section 5 we allow consumers to change the default application configuration.

normalize application A_1's revenue to zero[9] but allow B-applications to be asymmetric. If the manufacturer installs (only) application B_i and serves q consumers, i's revenue is $r_i q$. If both applications A_1 and B_i are installed then i's revenue is $(r_i + \lambda)q$.

The parameter λ allows for the possibility that A may help enable B to generate revenue. For example, in the smartphone market application A might be Google's Play store, which supports other applications' revenue generation in various ways. Firstly, it provides a secure and trusted payment infrastructure for in-app purchases—which are a key source of revenue for many apps. Secondly, it provides a central clearinghouse for various kinds of user data that can be exploited to enhance the profitability or usefulness of applications. Thirdly, the application store provides infrastructure for the automatic distribution of software updates, which extend an application's usefulness and usable lifetime. Lastly, by increasing the overall usefulness of a phone, application A might induce consumers to use their phone more, with spillover benefits for other applications on the phone. We assume $\lambda \geq 0$, with $\lambda = 0$ corresponding to the case with no complementarity.

These revenues may induce application developers to offer payments to the manufacturer in exchange for being installed. Such payments may have both lump-sum and royalty components and we study in turn lump-sum contracts, royalty contracts, and two-part tariffs.

Our focus in this paper is on firm 1's decision to offer its two applications A_1 and B_1 as a bundle or separately. We assume that this decision is made before any payments are offered to the manufacturer. The timing is thus the following: At $t = 0$, firm 1 decides whether to bundle its applications. At $t = 1$, application developers offer payments (we will discuss various types of contracts). At $t = 2$ the manufacturer chooses which application(s) to install, and profits and payments are realized. We focus on sub-game perfect equilibria that do not involve weakly dominated strategies, and simply refer to them as *equilibria* throughout the paper.

3 Model with a single manufacturer

In this section we examine conditions under which bundling can be a profitable strategy for firm 1 when there is a single manufacturer. We first analyze an environment in which firms offer lump-sum payments. As these payments are non-distortive, this allows us to expose our logic in a rather general way. We then turn to linear per-unit contracts in a set-up with symmetric applications and linear demand. We discuss two-part tariffs at the end of the section.

[9]But our analysis easily extends to positive revenues for A_1.

3.1 Lump-sum payments

We begin with the case in which that manufacturer and application firms sign contracts with lump-sum payments. Denote by T_{Xi} the payment offered by i to the manufacturer in return for installation of application X_i (with the convention that $T_{Xi} < 0$ means that the manufacturer pays i).

If the manufacturer installs A_1 and either B_1 or B_2, its sales to consumers allow it to generate a profit π_{AB}.[10] The number of consumers who use the manufacturer in this case is Q. If the manufacturer only installs a B application, it serves $q < Q$ consumers for a sales-profit of $\pi_B < \pi_{AB}$. For the sake of brevity we assume that not installing a B application leads to zero profit. Let $\alpha \equiv \pi_{AB} - \pi_B > 0$ be the value of application A_1 to the manufacturer, and $\Delta q \equiv Q - q$.

No bundling Suppose that in equilibrium the manufacturer installs A_1 and B_i. Then the following inequalities must hold (where $j \neq i$):

$$\pi_{AB} + T_{A1} + T_{Bi} \geq \pi_{AB} + T_{A1} + T_{Bj} \tag{1}$$

$$\pi_{AB} + T_{A1} + T_{Bi} \geq \pi_B + T_{Bi} \tag{2}$$

$$\pi_{AB} + T_{A1} + T_{Bi} \geq \pi_B + T_{Bj} \tag{3}$$

Inequalities (1), (2) and (3) respectively ensure that the manufacturer prefers to install A_1 and B_i rather than A_1 and B_j, B_i alone, and B_j alone.

Our first result will be useful throughout the paper, as its logic extends to more than one manufacturer.

Lemma 1. *In any equilibrium where the manufacturer installs A_1 and B_i, application B_j (with $j \neq i$) offers a payment $T_{Bj} = (r_j + \lambda)Q$.*

Proof. First, any $T' > (r_j + \lambda)Q$ is at least weakly dominated by $T_{Bj} = (r_j + \lambda)Q$, because paying T' would lead to a loss.

Notice that, in equilibrium, (1) must hold with equality. Indeed, (1) or (3) must bind (otherwise B_i could offer slightly less without inducing the manufacturer to switch to B_j), but (3) cannot be the only one binding (as this would violate (2)). Suppose now that $T_{Bj} < (r_j + \lambda)Q$. Because (1) is binding, application B_j could induce the manufacturer to install B_j along with A_1 by offering $T_{Bj} + \epsilon > T_{Bj}$. Therefore we wouldn't be in an equilibrium. ■

[10] Applications B_1 and B_2 are symmetric in their effect on the manufacturer's profit. An analysis with asymmetric applications does not bring much extra insight.

The following lemma characterizes the equilibrium under no bundling:

Lemma 2. *Suppose that $r_i > r_j$. In equilibrium, the manufacturer installs A_1 and B_i. Equilibrium offers are given by*

$$T_{A1} = -\alpha, \quad T_{Bi} = (r_j + \lambda)Q, \quad T_{Bj} = (r_j + \lambda)Q$$

Proof. $T_{A1} = -\alpha$ is the largest payment that A_1 can ask in exchange for being installed. Clearly, not being installed would be less profitable for A_1.

If B_i is not installed in equilibrium, we must have $T_{Bi} = (r_i + \lambda)Q$ by Lemma 1. For B_j to be installed, inequality (1) (where i and j are reversed) shows that T_{Bj} must be larger than T_{Bi}. But in that case firm j would make a loss, which is incompatible with equilibrium behavior.

Given that A_1 and B_i must be installed, and that $T_{Bj} = (r_j + \lambda)Q$ by Lemma 1, the constraints (1), (2) and (3) become

$$T_{Bi} \geq (r_j + \lambda)Q \tag{4}$$

$$T_{A1} \geq -\alpha \tag{5}$$

$$T_{A1} + T_{Bi} \geq -\alpha + (r_j + \lambda)Q \tag{6}$$

Clearly (6) is implied by (4) and (5), so that we can ignore it. The optimal offers are then such that the two remaining constraints bind. ■

Firm 1's monopoly allows it to extract all of the joint profit attributable to application A. Competition for access to B-market consumers, on the other hand, means that the joint profit in this market is largely captured by the bottleneck manufacturer. Indeed, if $r_1 = r_2$ then the manufacturer captures all of the B-market profit. We will see that this situation can change markedly when firm 1 bundles its two applications.

Firm 1's profits are given by revenues from its applications less the total payment made to the manufacturer. Thus, Lemma 2 immediately implies the following result.

Corollary 1. *If $r_1 > r_2$, firm 1's profit is $\pi_1 = (r_1 + \lambda)Q - T_{A1} - T_{B1} = \alpha + (r_1 - r_2)Q$. If $r_1 < r_2$, $\pi_1 = -T_{A1} = \alpha$.*

Bundling Suppose now that firm 1 offers A_1 and B_1 as a bundle, along with a transfer T_1. The manufacturer installs the bundle if and only if $\pi_{AB} + T_1 \geq \pi_B + T_{B2}$. The maximal T that firm 1 would be prepared to offer is $(r_1 + \lambda)Q$, which corresponds to all the revenues generated by application B_1. As for firm 2, its willingness to pay to have B_2 installed is $r_2 q$. Indeed, firm 2 knows that if it were to successfully outbid firm 1 and be chosen by the manufacturer, the manufacturer would not be able to install A_1. This would mean that the number of consumers served is q instead of Q, and the complementarity effects of A are not realized.

We therefore have

Lemma 3. *When firm 1 offers A_1 and B_1 as a bundle:*

1. *If $\pi_{AB} + (r_1 + \lambda)Q \geq \pi_B + r_2 q$, equilibrium offers are $T_1 = -\alpha + r_2 q$ and $T_{B2} = r_2 q$. The manufacturer installs $A_1 - B_1$, and firm 1's profit is $\pi_1 = (r_1 + \lambda)Q - T_1 = \alpha + (r_1 + \lambda)Q - r_2 q$.*

2. *If $\pi_{AB} + (r_1 + \lambda)Q < \pi_B + r_2 q$, equilibrium offers are $T_1 = (r_1 + \lambda)Q$ and $T_{B2} = \alpha + (r_1 + \lambda)Q < r_2 q$. The manufacturer installs B_2, and firm 1's profit is zero.*

We can now state the condition for bundling to be profitable by comparing Lemma 2 and Lemma 3.

Proposition 1. *Bundling is strictly profitable if and only if*

$$\frac{qr_2}{Q} < \min\{r_1, r_2\} + \lambda.$$

Bundling is more likely to be profitable if application A significantly boosts demand for the manufacturer (i.e., if $Q \gg q$). Intuitively, bundling implies that B_2 and A_1 cannot be installed on the manufacturer at the same time, so firm 2 cannot benefit from additional consumers attracted to the manufacturer by application A. This reduces firm 2's willingness to pay to be on the manufacturer's device, and hence the amount that firm 1 must pay to have B_1 installed. Similarly, bundling tends to be profitable in the presence of strong complementarity ($\lambda \gg 0$) because it denies firm 2 the complementary benefits of being installed alongside application A, and thereby reduces 2's bid for the right to be on the device.

If $r_1 > r_2$ bundling results in the efficient application configuration and is therefore non-exclusionary (but is, nevertheless, profitable). However, bundling can also lead to inefficient exclusion.[11] Indeed, if $r_2 > r_1 > \frac{qr_2}{Q} - \lambda$ then bundling is profitable and results in application B_1 being installed, even tough application B_2 yields a higher industry revenue.

The fact that bundling allows firm 1 to capture more of the surplus generated by its applications has implications for its incentives to invest in them. Suppose, for example, that firm 1 can improve the quality of A_1 by investing according to some convex-increasing cost

[11]Here we measure efficiency in terms of industry profit (i.e. excluding consumers' surplus). But there are natural contexts in which a high revenue firm also generates more consumer surplus. For example, suppose a B application offers an in-app purchase of quality q and price p. Demand is $q - p$. Then the optimal price is $p^* = q/2$, revenue per-consumer is $r(q) = q^2/4$, and consumer surplus is $S(q) = q^2/8$. We see that $r'(q)$ and $S'(q)$ have the same sign so the globally efficient allocation has the high-r application installed.

function $k(\cdot)$. An investment of $k(I)$ results is a demand of $Q = q + I$ and a platform profit of $\pi_{AB}(I)$. Without bundling, the optimal investment solves $\pi'_{AB}(I) + \max\{r_1 - r_2, 0\} = k'(I)$. If it anticipates that it will bundle then firm 1 solves $\pi'_{AB}(I) + r_1 + \lambda = k'(I)$. Thus, the firm invests more under bundling. A similar point holds for investments that increase λ.

3.2 Royalties

While lump-sum payments are a convenient type of contract to illustrate our point, in practice firms often use on linear contracts, where payments are proportional to the actual number of units sold. Such contracts introduce an additional consideration, namely that softer bidding by firm 2 may lead the manufacturer to charge a higher price for its device.

To see this in a simple way, suppose that $r_1 = r_2 = r$, and that the final demand is linear. More specifically, if the manufacturer sets a price P, demand is $q(P) = \max\{1 - P, 0\}$ if only application B is installed, and $Q(P) = \max\{1 + A - P, 0\}$ if application A is also installed. Contracts are linear: firm i offers w_{X_i} to the manufacturer for each device sold in exchange for application X_i to be installed (where $w_{X_i} < 0$ means that i require the manufacturer to pay to have the right to install X_i).

Suppose that the manufacturer chooses to install A_1 and B_i, with $w_{A_1} + w_{B_i} = W$. Then the optimal price of the device is $P(W) = \frac{1+A-W}{2}$, and the manufacturer's profit is $\Pi_0(W) = \left(\frac{1+A+W}{2}\right)^2$. If the manufacturer only installs B_i with a unit payment w_{B_i}, the optimal price is $p(w_{B_i}) = \frac{1-w_{B_i}}{2}$, for a profit $\pi_0(w_{B_i}) = \left(\frac{1+w_{B_i}}{2}\right)^2$. Without application B, demand for the device is assumed to be zero.

No bundling We start by assuming that A_1 and B_1 are offered separately. By inspection of the manufacturer's profit function, we can see that, if it decides to install a B application, the manufacturer selects B_i such that $w_{B_i} = \max\{w_{B_1}, w_{B_2}\}$. As a tie breaking rule, we assume that if $w_{B_1} = w_{B_2}$, the manufacturer installs B_1. We can also see that the manufacturer installs A_1 if and only if $w_{A_1} \geq -A$.

Lemma 4. *In any equilibrium under no bundling, the manufacturer installs A_1.*

Indeed it is profitable for firm 1 to make an offer $w_{A_1} \in (-A, 0)$, which the manufacturer always accepts.

If $w_{A_1} \geq -A$, firm 1's profit is

$$\pi_1 = \begin{cases} \frac{1+A+w_{A_1}+w_{B_1}}{2}(r + \lambda - w_{A_1} - w_{B_1}) & \text{if } w_{B_1} \geq w_{B_2} \\ \frac{1+A+w_{A_1}+w_{B_2}}{2}(-w_{A_1}) & \text{otherwise} \end{cases} \tag{7}$$

Lemma 5. *Under no bundling, there is a unique equilibrium profit for all the firms. This equilibrium profit can be implemented by $w^*_{B_2} = r + \lambda$, $w^*_{B_1} = r + \lambda$ and $w^*_{A_1} = \max\{r + \lambda - A, \frac{r+\lambda-(1+A)}{2}\} - w^*_{B_1}$.*

Proof. There are two possible equilibrium configurations: either the manufacturer installs A_1 and B_1, or it installs A_1 and B_2.

A1B1 equilibria. Let $w^*_{X_i}$ be the corresponding equilibrium offers. Several conditions must be met. (i) We must have $w^*_{B_1} \geq w^*_{B_2}$, so that the manufacturer prefers B_1 to B_2. (ii) Firm 1 should not be willing to increase $w^*_{A_1} + w^*_{B_1}$. Using the first expression in (7), this is equivalent to $w^*_{A_1} + w^*_{B_1} \geq \frac{r+\lambda-1-A}{2}$. (iii) We must have $w^*_{B_1} \geq r + \lambda$, otherwise firm 2 could profitably deviate by offering $w_{B_2} = w^*_{B_1} + \epsilon < r + \lambda$. (iv) We must have $w^*_{A_1} \geq -A$, for the manufacturer to install A_1. Taken together, these conditions imply $W^*_1 = w^*_{A_1} + w^*_{B_1} \geq \max\{r + \lambda - A, \frac{r+\lambda-(1+A)}{2}\}$. Note also that any $W_1 > \max\{r + \lambda - A, \frac{r+\lambda-(1+A)}{2}\}$ cannot be an equilibrium, because firm 1 could slightly reduce its payment to the manufacturer without inducing the manufacturer to change its behavior, and $W \geq \frac{r+\lambda-(1+A)}{2}$ implies that payments are excessive conditional on the manufacturer installing A_1 and B_1.

Any $w^*_{A_1}, w^*_{B_1}$ and , $w^*_{B_2}$ satisfying $w^*_{A_1} + w^*_{B_1} = \max\{r + \lambda - A, \frac{r+\lambda-(1+A)}{2}\}$, $w^*_{A_1} \geq -A$, $w^*_{B_1} \geq r + \lambda$ and $w^*_{B_2} \leq r + \lambda$ therefore constitute an $A1B1$ equilibrium.

A1B2 equilibria. Again, several conditions must hold for $w^*_{A_1}, w^*_{B_1}$ and $w^*_{B_2}$ to constitute an $A1B2$ equilibrium. (i) $w^*_{B_2} = r + \lambda$. Indeed if $w^*_{B_2} < r + \lambda$ then firm 1 could profitably deviate by keeping $w^*_{A_1}$ the same and setting $w^*_{B_1} = w^*_{B_2} + \epsilon$.(ii) $w^*_{B_1} < w^*_{B_2}$, for the manufacturer to choose B_2. (iii) $w^*_{A_1} = \max\{-A, -\frac{1+A+r+\lambda}{2}\}$. Indeed, $w_A < -A$ leads the manufacturer to not install A_1, and $-\frac{1+A+r+\lambda}{2}$ is the value of w_A that maximizes the profit conditional on $w^*_{B_1} < w^*_{B_2} = r + \lambda$ (see (7)).

One can then readily check that firm 2's profit is zero in both $A1B1$ and $A1B2$ equilibria. The sum of payments offered to the manufacturer $w^*_{A_1} + w^*_{B_i}$ is equal to $\max\{r + \lambda - A, \frac{r+\lambda-(1+A)}{2}\}$ in both types of equilibria, which implies that the number of units sold and the manufacturer's profits are the same. Finally the net profit per unit for firm 1 is identical as well ($r + \lambda - \max\{r + \lambda - A, \frac{r+\lambda-(1+A)}{2}\}$ in any $A1B1$ equilibrium, $\max\{-A, -\frac{1+A+r+\lambda}{2}\}$ in any $A1B2$ equilibrium). ■

Bundling Suppose now that A_1 and B_1 are offered together, and let W_1 be the associated payment to the manufacturer. The manufacturer installs the bundle if and only if $A + W_1 \geq w_{B_2}$. Conditional on this, firm 1's profit is

$$\pi_1(W_1) = \frac{1 + A + W_1}{2}(r + \lambda - W_1) \tag{8}$$

Now, firm 2 is only willing to bid up to r because it anticipates that the manufacturer will not install A_1 if it chooses B_2. A similar analysis to the above leads to the following lemma.

Lemma 6. *When firm 1 bundles A_1 and B_1, there is a unique equilibrium profit for all the firms. This profit can be implemented with $W^{**}_1 = \max\{r - A, \frac{r+\lambda-(1+A)}{2}\}$ and $w^{**}_{B_2} = r$.*

Profitability of bundling Using Lemmas 5 and 6, we can state the following result:

Proposition 2. *Bundling is strictly profitable for firm 1 when $\lambda > 0$ and $r + \lambda > A - 1$. When this is the case, bundling reduces total welfare and consumer surplus.*

Proof. If $\lambda = 0$, or if $r + \lambda - A \leq \frac{r+\lambda-(1+A)}{2}$, then profits are equivalent because $W_1^* = W_1^{**}$. However, if $\lambda > 0$ and $r + \lambda - A > \frac{r+\lambda-(1+A)}{2}$ (i.e.$r + \lambda > A - 1$), then $W_1^* > W_1^{**} \geq \frac{r+\lambda-(1+A)}{2}$. Because firm 1's profit in both cases is $\frac{1+A+W_1}{2}(r + \lambda - W_1)$, which is decreasing for $W \geq \frac{r+\lambda-(1+A)}{2}$, profit is higher under bundling.

Because bundling is strictly profitable only when it reduces the total payments to the manufacturer, it leads the manufacturer to charge a higher price ($P(W)$ is increasing), which reduces welfare and consumer surplus. ■

Discussion In a similar way to the lump-sum case, bundling makes firm 2 less aggressive in its bidding, which benefits firm 1 when competition from 2 is "binding" (i.e. competition forces firm 1 to offer larger payments than what it would like to do if it was a monopoly on both markets). However, unlike in the lump-sum case, the reduction in payments to the manufacturer generates an inefficiency, as the manufacturer is less eager to price low.

3.3 Two-part tariffs

With two-part tariffs $T_{X_i} + w_{X_i} q$, we again find that bundling can be profitable if λ is large enough. Unlike with linear contracts, the unit fee is always efficient ($w_{X_i} = r_i + \lambda$), but a distortion can still emerge when r_2 is larger than r_1, but not too much so that bundling is still profitable. In this case it would be efficient for B_2 to be installed. The full analysis of two-part tariffs is available upon request.

4 Competition between manufacturers

The mechanisms that can make bundling profitable with a monopoly manufacturer continue to operate when we introduce competition. Competition also introduces new considerations, namely that applications become a potential source of differentiation. To see this clearly and economize on notations we focus on a symmetric setup ($r_1 = r_2 = r$) and let $\lambda = 0$. We also restrict attention to lump-sum payments.

There are two symmetric manufacturers, L and R, and two application developers, 1 and 2. Firm 1 provides applications A_1 and B_1, whereas firm 2 only provides B_2. If both manufacturers install A_1 and choose the same B application, their profit is π_S (S stands for Same), and they each serve Q_S consumers. If they both install application A_1 but choose different B-applications, their profit is π_D, with $\pi_D - \pi_S = \delta$, and they each serve Q_D consumers. If a manufacturer only installs an application of category B

whereas the other manufacturer installs two applications, its profit is $\underline{\pi}_S$ or $\underline{\pi}_D$, depending on whether the B-applications are the same or different, and its consumer base is $q_S < Q_S$ or $q_D < Q_D$. Let $\alpha_D = \pi_D - \underline{\pi}_D$ and $\alpha_S = \pi_S - \underline{\pi}_S$ be the value to a manufacturer of installing application A_1 when the other manufacturer has it, when their B-applications are respectively differentiated or not. We further assume that $\min\{\alpha_S, \alpha_D\} > \delta$, and that $\max\{q_D, q_S\} < \min\{Q_D, Q_S\}$. For any rival strategy, a manufacturer's gross profit is assumed to be higher when it installs A_1 than when it does not.

Offers by the developer of application i to manufacturer K are denoted T_i^K, where a positive value means that the developer pays the manufacturer. We assume that offers are secret, i.e. manufacturer L cannot observe the offers made to manufacturer R before choosing which applications to install.

When firm 1 offers A_1 and B_1 separately, we have the following:

Lemma 7. *The equilibrium has the following features:*

- *Both manufacturers install application A_1.*
- *If $rQ_D + \pi_D > rQ_S + \pi_S$, manufacturers install a different B-application from each other.*
- *If $rQ_D + \pi_D < rQ_S + \pi_S$, both manufacturers install the same B-application.*

Proof. Suppose that one manufacturer, say L, does not install A_1. Then, because offers are secret, firm 1 could increase its profit by requiring a small payment from L in exchange for installing A_1. This offer would be accepted by L.

Suppose now that L expects R to choose A_1 and B_i. If firms expect L to install A_1, firm i is willing to pay up to rQ_S, and firm j is willing to pay rQ_D. If $rQ_S + \pi_S > rQ_D + \pi_D$, then i can always offer enough to be chosen by L, so that both manufacturer must install the same B application in equilibrium. If the inequality is reversed, the application j can induce L to install it over B_i. ■

We now study two cases, depending on whether $rQ_D + \pi_D > rQ_S + \pi_S$ or not.

4.1 Case 1: B-differentiation is efficient $(rQ_D + \pi_D > rQ_S + \pi_S)$

No bundling By Lemma 7, manufacturers must install different B-applications in equilibrium. Since payoffs do not depend on which firm installs B_1, we focus on the case where L installs B_1.

Facing offers T_{A1}^L, T_{B1}^L and T_{B2}^L, and expecting R to install A_1 and B_2, L chooses to install A_1 and B_1 if

$$\pi_D + T_{A1}^L + T_{B1}^L \geq \pi_S + T_{A1}^L + T_{B2}^L \tag{9}$$

$$\pi_D + T_{A1}^L + T_{B1}^L \geq \underline{\pi}_D + T_{B1}^L \tag{10}$$

$$\pi_D + T_{A1}^L + T_{B1}^L \geq \underline{\pi}_S + T_{B2}^L \tag{11}$$

By the same logic as Lemma 1, we must have $T_{B2}^L = rQ_S$: given that L installs A_1, B_2 is willing to pay up to rQ_S to convince L to switch from B_1 to B_2 (L and R would then both install B_2 and thus serve Q_S consumers).

Using the fact that $\pi_D - \pi_S = \delta$, $\pi_S - \underline{\pi}_S = \alpha_S$ and $\pi_D - \underline{\pi}_D = \alpha_D$, the constraints (9), (10) and (11) therefore rewrite

$$T_{B1}^L \geq rQ_S - \delta \tag{12}$$

$$T_{A1}^L \geq -\alpha_D \tag{13}$$

$$T_{A1}^L + T_{B1}^L \geq rQ_S - \delta - \alpha_S \tag{14}$$

If $\alpha_S \geq \alpha_D$, (12) and (13) imply (14). Therefore the total payment from firm 1 to manufacturer L is $T_{A1}^L + T_{B1}^L = rQ_S - \alpha_D - \delta$. The same set of equations hold for manufacturer R, but this time firm 1's payment is $T_{A1}^R = -\alpha_D$. Firm 1's profit is therefore $r(Q_D - Q_S) + 2\alpha_D + \delta$.

If $\alpha_S < \alpha_D$, (14) is binding and payment to L is thus $rQ_S - \delta - \alpha_S$. Payment from 1 to R is at least $-\alpha_D$, so that firm 1's profit is at most $r(Q_D - Q_S) + \alpha_D + \alpha_S - \delta$.

Proposition 3. *Suppose that $rQ_D + \pi_D > rQ_S + \pi_S$.*

In equilibrium without bundling, one manufacturer (say L) installs A_1 and B_1 and the other installs A_1 and B_2.

Firm 1 receives a payment equal to α_D from manufacturer R ($T_{A1}^R = -\alpha_D$), and makes a total payment $T_{A1}^L + T_{B1}^L = rQ_S - \delta - \min\{\alpha_S, \alpha_D\}$.

Profits are given by

$$\pi_1 = r(Q_D - Q_S) + \delta + \alpha_D + \min\{\alpha_D, \alpha_S\}$$

$$\pi_2 = r(Q_D - Q_S) + \delta + \max\{\alpha_D - \alpha_S, 0\}$$

$$\pi_L = \pi_R = \underline{\pi}_S + rQ_S$$

Notice that the equilibrium features a coordination failure by the manufacturers. Indeed, they would be better-off if they were to swap their choice of B-applications (but this would not be an equilibrium).

Bundling Suppose now that firm 1 offers A_1 and B_1 as a bundle, along with transfers T_1^L and T_1^R.

There are three potential outcomes of the ensuing subgame: one in which both manufacturers choose the bundle, one in which one manufacturer (say L) installs the bundle while the other chooses to install only B_2, and one in which both manufacturers choose B_2 over $A_1 - B_1$.

Suppose that manufacturer L chooses to install the bundle $A_1 - B_1$. manufacturer R will also opt for the bundle if

$$\pi_S + T_1^R \geq \underline{\pi}_D + T_{B2}^R$$

The highest payment that B_2 can offer to R, given that L installs firm 1's bundle, is rq_D. Therefore, any $T_1^R \geq rq_D + \delta - \alpha_D$ induces R to install the bundle. Firm 1's willingness to pay to be installed on manufacturer R given that it's installed on L is $rQ_S > rq_D + \delta - \alpha_D$ (by our assumptions that $\delta < \min\{\alpha_D, \alpha_S\}$ and that $\max\{q_D, q_S\} < \min\{Q_D, Q_S\}$). Therefore, if L installs the bundle, firm 1 always finds it profitable to offer R a payment such that R installs the bundle.

By a similar reasoning, one can show that if L does not install the bundle then firm 1 would find it profitable for R to install the bundle.[12]

We thus have

Proposition 4. *Suppose that $rQ_D + \pi_D > rQ_S + \pi_S$. If firm 1 offers A_1 and B_1 as a bundle, both manufacturers install the bundle. Firm 1's profit is*

$$\pi_1 = 2\left(r(Q_S - q_D) + \alpha_D - \delta\right)$$

manufacturers' profit is

$$\pi_L = \pi_R = \underline{\pi}_D + rq_D$$

Comparing profits with and without bundling, we obtain the following:

Proposition 5. *When $rQ_D + \pi_D > rQ_S + \pi_S$, bundling is profitable for firm 1 if*

$$r(3Q_S - Q_D - 2q_D) \geq 3\delta. \tag{15}$$

Both manufacturers' and total industry profits are lower under bundling.

One downside to bundling is that it eliminates manufacturers' ability to differentiate through their choice of installed applications. The bundling firm must compensate manufacturers for this loss of differentiation in order to induce them to install the bundle. Bundling therefore tends to be profitable when differentiation is not too important (i.e.,

[12] Proving this claim requires to introduce an extra set of notations, corresponding to a manufacturer's profit and output when the other manufacturer does not install A_1. We omit this step for the sake of brevity.

when δ is not too large and when Q_S is fairly large compared to Q_D, q_D). Bundling provides a mechanism for application B_1 to be installed by more manufacturers. Firm 1 therefore tends to find bundling most profitable when its revenue from B_1 is large.

Example Suppose that manufacturers are horizontally differentiated à la Hotelling, with a transportation parameter τ_p, and that applications B_1 and B_2 are also horizontally differentiated, with a transportation cost τ_b, that $\tau_b > \tau_p$, and that τ_b is small enough so that the market is covered. As we show in section 5, this implies that $\delta = \frac{\tau_b - \tau_p}{2} > 0$. Suppose also that application A is so important that $q_S = q_D = 0$. Then condition (15) rewrites $\frac{2}{3}r \geq \tau_b - \tau_p$.

4.2 Case with $rQ_D + \pi_D < rQ_S + \pi_S$

We now turn to the case where differentiation at the level of B-applications does not increase industry profits.

Example Suppose now that while manufacturers are still horizontally differentiated with a transportation cost τ_p, B applications are no longer horizontally differentiated but instead exhibit network externalities: if a mass n_i of consumers use application i (irrespective of which manufacturer they use), the utility from using i is γn_i, with $\gamma \in (0, \tau_p)$. If both manufacturers install the same B application, and if τ_p is small enough that the market is covered, the network externalities cancel out (consumers get the same benefit on either manufacturer), and equilibrium (gross) profit is $\pi_S = \tau_p/2$. If, on the other hand, they install a different B application, network effects intensify competition and lead to a gross profit of $\pi_D = \frac{\tau_p - \gamma}{2} < \pi_S$. In this market, the condition $rQ_D + \pi_D < rQ_S + \pi_S$ holds.

Profitability of bundling We now return to the general case. We have the following:

Proposition 6. *When $rQ_D + \pi_D < rQ_S + \pi_S$, bundling is always profitable.*

Proof. We use a similar reasoning as the case where $rQ_D + \pi_D > rQ_S + \pi_S$

Without bundling, there are two equilibria of the subgame: one in which both manufacturers install B_1, in which case firm 1's profit is $\pi_1 = 2\left(\min\{\alpha_D, \alpha_S\} + r(Q_S - Q_D) - \delta\right)$; and one in which manufacturers install B_2 leading to a profit $\pi_1 = 2\alpha_S$.

With bundling, firm 1's profit is $\pi_1 = 2\left(\alpha_D + r(Q_S - q_D) - \delta\right)$.

It is straightforward to see that firm 1's profit is always larger with bundling. ■

5 Hotelling model

The results so far have been mostly couched in terms of general profit levels. This approach has allowed us to produce general results on the optimality of bundling, but is not wellsuited to study the broader welfare effects of bundling or the relationship between bundling and consumer prices. To examine such questions, we add additional structure of the model by adopting a variant of the familiar Hotelling framework. Making consumers' payoffs explicit also allows us to address a question of practical import: what happens when consumers are able to circumvent bundling by installing applications themselves.

To be more precise, suppose that each consumer has a type $\mathbf{x} = (x_p, x_b)$, uniformly distributed in $[0, 1]^2$. manufacturers L and R are respectively located at distance $d_L = x_p$ and $d_R = 1 - x_p$ from the consumer's ideal. Similarly, the distance to applications B_1 and B_2 is x_b and $1 - x_b$ respectively. Let d_{ia} denote the distance to the B-market application installed by manufacturer i. A consumer of manufacturer i obtains, by default, utility

$$V - \tau_p d_i - \tau_b d_{ia} - P_i,$$

where V is the standalone value of the consumer's ideal manufacturer, τ_p and τ_b are transport costs, and P_i is the price charged by manufacturer i.

Lastly, we allow consumers to incur a cost, Δ to install a different B-market application on their manufacturer of choice. If the consumer chooses to do this then his utility becomes

$$V - \tau_p d_i - \tau_b(1 - d_{ia}) - P_i - \Delta.$$

We assume that V is large enough to guarantee the market is covered and study a game with the following timing:

1. Firms make (lump-sum) offers to the manufacturers.
2. manufacturers choose which applications to install.
3. manufacturers set their prices, P_L and P_R.
4. Consumers choose a manufacturer and decide whether to change the default application configuration.

We focus on the case in which application A_1 is essential and is installed by both manufacturers.

5.1 Equilibrium when manufacturers install the same applications

Begin by supposing that both manufacturers have identical application configurations (both $A_1 - B_1$ or both $A_1 - B_2$)—noting, in particular, that $A_1 - B_1$ would be the configuration if bundling forced both manufacturers to install B_1.

Given that manufacturers have the same applications, any consumer who would install a custom application on manufacturer L would also do so on R. Thus, for every consumer, the relevant comparison is between two manufacturers with identical applications. The indifferent consumer has

$$\tau_p x_p^* + P_L = \tau_p(1 - x_p^*) + P_R \implies x_p^* = \frac{P_R - P_L + \tau_p}{2\tau_p}.$$

manufacturer L chooses P_L to maximize $P_L x_p^*$. This gives rise to the symmetric equilibrium price $P_L = P_R = \tau_p$, with corresponding equilibrium profit $\pi_L = \pi_R = \tau_p/2$. This is just the standard symmetric Hotelling outcome when firms have a single dimension of differentiation.

Suppose both manufacturers install B_1 by default (the case where both install B_2 is symmetric). A consumer finds it worthwhile to switch to application B_2 if

$$x_b\tau_b > (1 - x_b)\tau_b + \Delta \iff x_b > \frac{\tau_b + \Delta}{2\tau_b}. \tag{16}$$

If $\Delta < \tau_b$ (which is a necessary condition for some consumers to switch), consumer surplus is therefore

$$V - 2\int_0^{1/2}\left(\int_0^{\frac{\tau_b+\Delta}{2\tau_b}} (\tau_p x_p + \tau_b x_b + P_L)\, dx_b + \int_{\frac{\tau_b+\Delta}{2\tau_b}}^1 (\tau_p x_p + \tau_b(1 - x_b) + P_L + \Delta)\, dx_b\right) dx_p.$$

Substituting $P_L = P_R = \tau_p$ yields the equilibrium consumer surplus:

$$V - \frac{1}{4}\left(\tau_b + 2\Delta - \frac{\Delta^2}{\tau_b} + 5\tau_p\right).$$

If $\tau_b \leq \Delta$ then installing applications is prohibitively costly. Consumers then stick with the default and obtain surplus

$$V - 2\left(\int_0^{1/2}\int_0^1 \tau_p x_p + \tau_b x_b + P_L\, dx_b\, dx_p\right) = V - \frac{1}{4}(2\tau_b + 5\tau_p).$$

Lemma 8 summarises these results (proofs for this section are in the Appendix).

Lemma 8. *If both manufacturers install the same applications then prices in a symmetric equilibrium are $P_L = P_R = \tau_p$. manufacturer gross profit is $\pi_L = \pi_R = \tau_p/2$. Consumer surplus is*

$$CS = \begin{cases} V - \frac{1}{4}\left(\tau_b + 2\Delta - \frac{\Delta^2}{\tau_b} + 5\tau_p\right) & \text{if } \Delta < \tau_b \\ V - \frac{1}{4}\left(2\tau_b + 5\tau_p\right) & \text{if } \tau_b \leq \Delta. \end{cases} \tag{17}$$

5.2 Equilibrium when manufacturers differentiate in applications

Now suppose that the manufacturers differentiate in their choice of B market applications: manufacturer L installs $A_1 - B_1$ and R installs $A_1 - B_2$. By analogy to (16), we can

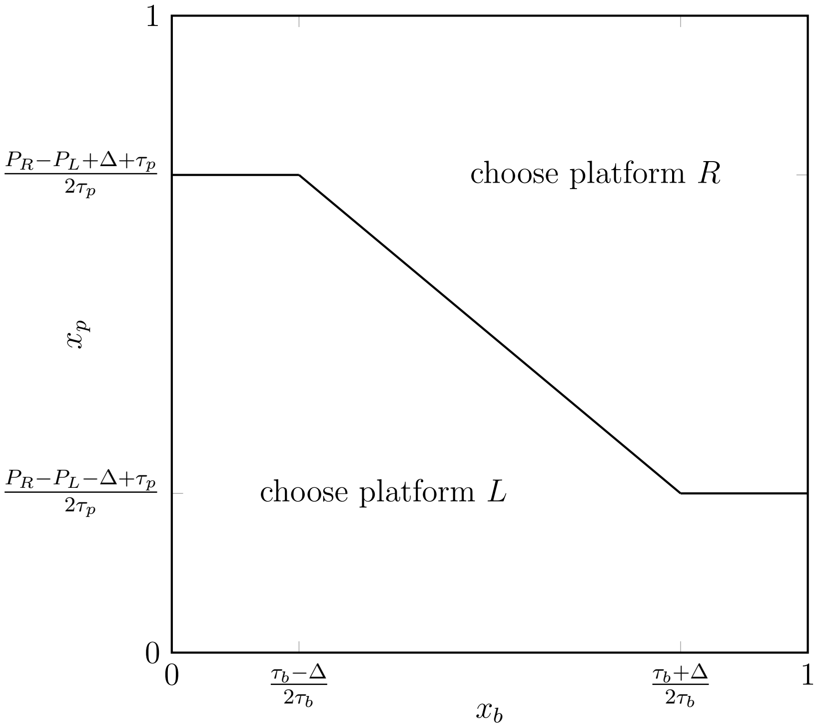

Figure 1: manufacturer demand when installing different B-market applications.

determine when consumers find it worthwhile to change the default applications on their chosen manufacturer and identify three types of consumer:

1. Consumers with $x_b < \frac{\tau_b - \Delta}{2\tau_b}$ who use manufacturer R install application B_1. Thus, they are indifferent between L and R if

$$\tau_p x_p + \tau_b x_b + P_L = \tau_p(1 - x_p) + \tau_b x_b + P_R + \Delta. \tag{18}$$

2. Consumers with $x_b \in [\frac{\tau_b - \Delta}{2\tau_b}, \frac{\tau_b + \Delta}{2\tau_b}]$ never choose to change the default application. Thus, they are indifferent between L and R if

$$\tau_p x_p + \tau_b x_b + P_L = \tau_p(1 - x_p) + \tau_b(1 - x_b) + P_R. \tag{19}$$

3. Consumers with $x_b > \frac{\tau_b + \Delta}{2\tau_b}$ who use manufacturer L install application B_2. Thus, they are indifferent between L and R if

$$\tau_p x_p + \tau_b(1 - x_b) + P_L + \Delta = \tau_p(1 - x_p) + \tau_b(1 - x_b) + P_R. \tag{20}$$

We thus obtain demand for the two manufacturers as illustrated in Figure 1. Given manufacturers' demand, we can compute the implied equilibrium prices, profits, and consumer surplus. Lemma 9 summarises these quantities; the proof is in the Appendix.

Lemma 9. *If manufacturer L installs $A_1 - B_1$ and R installs $A_1 - B_2$ then prices in a symmetric equilibrium are*

$$P_L = P_R = \begin{cases} \tau_b \text{ if } \tau_p \leq \min\{\tau_b, \Delta\} \\ \tau_p \text{ otherwise.} \end{cases}$$

manufacturer gross profit is $\pi_L = \pi_R = P/2$. Consumer surplus is

$$CS = \begin{cases} V - \frac{1}{12}\left(6\tau_b - \frac{\tau_b^2}{\tau_p} + 15\tau_p\right) \text{ if } \tau_b < \min\{\tau_p, \Delta\} \\ V - \frac{1}{12}\left(6\tau_p - \frac{\tau_p^2}{\tau_b} + 15\tau_b\right) \text{ if } \tau_p \leq \min\{\tau_b, \Delta\} \\ \frac{\Delta^2(3\tau_p - 2\Delta) + 3\tau_b\left(\Delta^2 - 2\Delta\tau_p + (4V - 5\tau_p)\tau_p\right) - 3\tau_b^2\tau_p}{12\tau_b\tau_p} \text{ if } \Delta < \min\{\tau_p, \tau_b\}. \end{cases}$$

5.3 Overall equilibrium and welfare effects of bundling

If $\tau_p > \min\{\tau_b, \Delta\}$ then the gross manufacturer profit is $\tau_p/2$ regardless of whether manufacturers are differentiated or not. manufacturers are therefore indifferent and will install whichever B-application offers the largest transfer. In equilibrium this will be the application with the largest r_i (note that this implies both manufacturers will install the same application). If $r_1 = r_2$ then there are two equilibria—one with and one without manufacturer differentiation. We focus on the undifferentiated equilibrium because the equilibrium with differentiation is a knife-edge case that disappears if there is an ϵ perturbation in r_i.

Proposition 7. *Suppose there is no bundling, that $r_i \geq r_j$, and $\tau_p > \min\{\tau_b, \Delta\}$. Then there is an equilibrium where both manufacturers install B_i. If $r_1 \neq r_2$ then this is the only equilibrium configuration.*

Now turn to the case with $\tau_p \leq \min\{\tau_b, \Delta\}$. Application differentiation yields higher gross manufacturer profit ($\tau_b/2$) than does installing the same applications ($\tau_p/2$). However, if the difference in profits ($\frac{\tau_b - \tau_p}{2}$) is smaller than the difference in firms' willingness to pay to be installed ($\left|\frac{r_1 - r_2}{2}\right|$) then any equilibrium must again have both manufacturers installing the application with the largest r_i. When, on the other hand, the profits from differentiation are sufficiently large, equilibrium must involve manufacturer differentiation.

Proposition 8. *Suppose there is no bundling, that $r_i \geq r_j$, and that $\tau_p \leq \min\{\tau_b, \Delta\}$. In any equilibrium,*

1. *if $r_i - r_j > \tau_b - \tau_p$ then both manufacturers install B_i,*

2. *if $r_i - r_j < \tau_b - \tau_p$ then one manufacturer installs B_i and the other installs B_j,*

3. *if* $r_i - r_j = \tau_b - \tau_p$ *then either both manufacturers install* B_i*, or one manufacturer installs* B_i *and the other installs* B_j*.*

For the remainder of the section we focus on the most interesting case where firms are not too asymmetric and differentiation can arise in equilibrium.

Assumption 1. $r_i - r_j < \tau_b - \tau_p$.

The effect of bundling is then to prevent manufacturers from differentiating through applications and its welfare implications can therefore be found by comparing Lemmas 8 and 9. The effect of bundling on consumer surplus is described in the following result:

Proposition 9. *Bundling causes consumer surplus to*

1. *increase if*
$$\tau_p \leq \min\left\{\Delta, \frac{1}{2}\left(\sqrt{3}\sqrt{43\tau_b^2 + 4\widetilde{\Delta}^2 - 8\widetilde{\Delta}\tau_b} - 9\tau_b\right)\right\} \tag{21}$$
(where $\widetilde{\Delta} = \min\{\Delta, \tau_b\}$*),*

2. *decrease if*
$$\frac{1}{2}\left(\sqrt{3}\sqrt{43\tau_b^2 + 4\widetilde{\Delta}^2 - 8\widetilde{\Delta}\tau_b} - 9\tau_b\right) < \tau_p \leq \min\{\tau_b, \Delta\}, \tag{22}$$

3. *remain unchanged if* $\min\{\tau_b, \Delta\} < \tau_p$.

Figure 2 illustrates Proposition 9. If installing applications is not feasible for consumers ($\Delta > \tau_b$) then (21) simplifies to $\tau_p < 0.91\tau_b$.

Bundling can benefit consumers. This makes for a striking contrast with most extant theories, where bundling is used as a tool to either foreclose or soften competition—typically resulting in higher prices and lower consumer surplus. Here, bundling has the opposite effect: installing different applications is a way for manufacturers to differentiate and thus make their residual demand less elastic. But bundling prevents such differentiation. While this harms consumers through a loss of product variety, the net effect can be beneficial because consumers pay lower prices for access to the manufacturer.[13] This effect is strongest when manufacturers are not intrinsically very differentiated because this is when the inability to differentiate through applications forces manufacturers into fierce price competition.

Consumers are more likely to benefit from bundling when there are sufficient barriers to end-user installation of applications (i.e., when Δ is not too small). If consumers can easily

[13]This also suggests an additional mechanism through which bundling can be profitable. Suppose that manufacturers L and R produce telephones within the Android ecosystem, while a vertically-integrated firm ("Apple") produces telephones that are completely incompatible. By bundling, firm 1 can induce L and R to lower the price of an Android phone. This will typically cause users to switch from Apple to Android, increasing firm 1's application revenues.

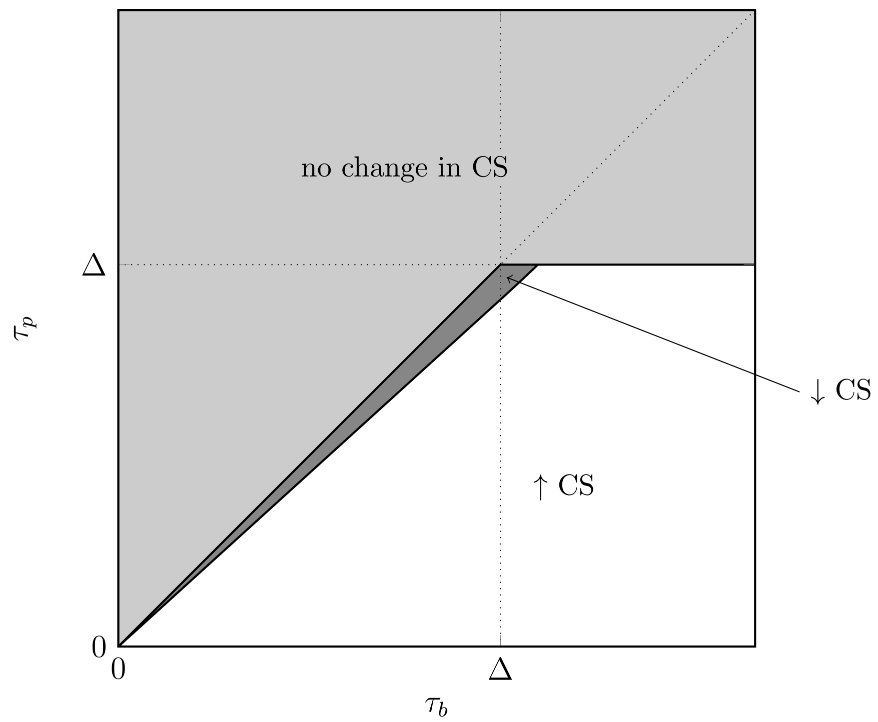

Figure 2: Effect of bundling on consumer surplus. Consumer surplus increases in the unshaded region, decreases in the dark gray region, and remains unchanged in the light gray region.

undo differentiation by installing alternative applications then manufacturers compete *as if* there were no application differentiation at all. Thus, prices do not fall further when differentiation is actually eliminated.

One caveat to Proposition 9 is that lump-sum transfers have no distortionary effect on manufacturers' prices. If payments were royalties then, by causing these payments to fall, bundling could result in higher prices. The overall impact of bundling on consumer surplus would then depend on the relative strength of this effect compared to the competition-enhancing effect of reduced differentiation.

6 Conclusion

To be written.

A Omitted Proofs

A.1 Proof of Lemma 9

Let $\widetilde{\Delta} = \min\{\Delta, \tau_b\}$. Solving the three indifference conditions in (18)–(20) yields a function for the indifferent consumer's x_p:

$$x_p^*(x_b) = \begin{cases} \dfrac{P_R - P_L + \Delta + \tau_p}{2\tau_p} \text{ if } x_b < \dfrac{\tau_b - \widetilde{\Delta}}{2\tau_b} \\ \dfrac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_p}{2\tau_p} \text{ if } x_b \in \left[\dfrac{\tau_b - \widetilde{\Delta}}{2\tau_b}, \dfrac{\tau_b + \widetilde{\Delta}}{2\tau_b}\right] \\ \dfrac{P_R - P_L - \Delta + \tau_p}{2\tau_p} \text{ if } x_b > \dfrac{\tau_b + \widetilde{\Delta}}{2\tau_b}. \end{cases} \tag{23}$$

The smallest x_b for which some consumers buy R is the $\underline{x}_b$ solving $\underline{x}_b = \min\{x_b \geq 0 \colon x_p^*(\underline{x}_b) \leq 1\}$. Similarly, define $\overline{x}_b$ as the largest x_b for which some consumers buy L: $\overline{x}_b = \max\{x_b \leq 1 \colon x_p^*(\underline{x}_b) \geq 0\}$.

Firm L's profit is

$$\pi_L(P_L, P_R) = P_L\left(\underline{x}_b + \int_{\underline{x}_b}^{\overline{x}_b} x_p^*(\underline{x}_b)\, dx_x\right). \tag{24}$$

At a symmetric equilibrium we face two possibilities: Firstly, if $\Delta < \tau_b$ then $\underline{x}_b, \overline{x}_b \in (0,1)$ if and only if $\tau_p < \Delta$ (this can be seen by looking at the first and last lines of (23)). The second possibility is $\Delta \geq \tau_b$, in which case $\underline{x}_b, \overline{x}_b \in (0,1)$ if and only if $\tau_p < \tau_b$ (this can be seen by looking at the middle line of (23) and noting that the first and last lines of (23) are irrelevant when $\Delta \geq \tau_b$).

Evaluating (24) accounting for the possibility of corner solutions, the profit around a symmetric equilibrium is therefore

$$\pi_L(P_L, P_R) = \begin{cases} \dfrac{P_R - P_L + \tau_b}{2\tau_b} P_L \text{ if } \tau_p \leq \widetilde{\Delta} \\ \dfrac{P_R - P_L + \tau_p}{2\tau_p} P_L \text{ if } \tau_p > \widetilde{\Delta}. \end{cases} \tag{25}$$

We now proceed as follows: Firstly, we identify a putative symmetric equilibrium such that a small deviation is not profitable—where "small" means that we remain within the same piecewise case in (25). Secondly, we verify that this is indeed an equilibrium be checking whether a firm could profit from a larger deviation that causes us to switch between the two cases in (25).

For the first step, taking a first-order condition from (25) and imposing symmetry yields

$$P_L = P_R = \begin{cases} \tau_b \text{ if } \tau_p \le \widetilde{\Delta} \\ \\ \tau_p \text{ if } \tau_p > \widetilde{\Delta}, \end{cases} \qquad \pi_L = \pi_R = \begin{cases} \tau_b/2 \text{ if } \tau_p \le \widetilde{\Delta} \\ \\ \tau_p/2 \text{ if } \tau_p > \widetilde{\Delta}. \end{cases}$$

We now need to check whether this putative equilibrium is robust to larger deviations. There are four cases: $\Delta > \tau_b, \tau_p > \tau_b$; $\Delta > \tau_b \ge \tau_p$; $\tau_b \ge \Delta \ge \tau_p$; and $\tau_b \ge \Delta, \tau_p > \delta$. In each case, there are two large deviations to consider: an increase and a decrease. Here we show how to rule-out these two deviations for the first of the four cases; the other three cases follow a similar logic, with details available on request.

Suppose $\Delta > \tau_b, \tau_p > \tau_b$. This implies that

$$x_p^*(x_b) = \frac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_p}{2\tau_p}$$

for every $x_b \in [0, 1]$ and the putative equilibrium price identified above is $P_L = P_R = \tau_p$.

If L cuts P_L such that $P_L < \tau_b$ then $x_p^*(x_b) = 1$ for $x_b < (\tau_b - P_L)/2\tau_b$. L's profit is therefore

$$\left[\frac{\tau_b - P_L}{2\tau_b} + \int_{\frac{\tau_b - P_L}{2\tau_b}}^{1} \frac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_p}{2\tau_p}\, dx_b\right] P_L = \frac{P_L\left[(8\tau_p - \tau_b)\tau_b - 2P_L\tau_b - P_L^2\right]}{8\tau_b\tau_p}.$$

One can check that this is increasing for every $P_L < \tau_b$ so the best such deviation is to $P_L = \tau_b$, which yields lower than the putative equilibrium profits.

If P_L is increased such that $P_L > 2\tau_p - \tau_b$ then $x_p^*(x_b) = 0$ for $x_b > (2\tau_p + \tau_b - P_L)/2\tau_b$. This implies L's profits are

$$P_L \int_0^{\frac{2\tau_p + \tau_b - P_L}{2\tau_b}} \frac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_p}{2\tau_p}\, dx_b = \frac{P_L(2\tau_p + \tau_b - P_L)}{8\tau_b\tau_p}.$$

One can check that this is decreasing for every $P_L > 2\tau_p - \tau_b$ so the best such deviation is to $P_L = 2\tau_p - \tau_b$, which yields lower than the putative equilibrium profits.

If we let $\widetilde{x}_p(x_b) = \min\{1, \max\{0, x_p^*(x_b)\}\}$, consumer surplus can be written as

$$2\left[\int_0^1 \int_0^{\widetilde{x}_p(x_b)} \Big(V - \tau_p x_p - \tau_b x_b - P_L\Big)\, dx_p\, dx_b - \int_{\frac{\tau_b + \widetilde{\Delta}}{2\tau_b}}^{1} \int_0^{\widetilde{x}_p(x_b)} \Delta\, dx_p\, dx_b\right]. \qquad (26)$$

We have four cases depending on whether $\tau_p \le \widetilde{\Delta}$ and whether $\tau_b \le \Delta$. Case 1: if $\tau_p \le \widetilde{\Delta}$

and $\tau_b \leq \Delta$ then (26) becomes

$$2\int_0^{\frac{\tau_b-\tau_p}{2\tau_b}}\int_0^1 \left(V-\tau_p x_p-\tau_b x_b-\tau_b\right)dx_p dx_b+$$
$$2\int_{\frac{\tau_b-\tau_p}{2\tau_b}}^{\frac{\tau_b+\tau_p}{2\tau_b}}\int_0^{\frac{\tau_b-2x_b\tau_b+\tau_p}{2\tau_p}} \left(V-\tau_p x_p-\tau_b x_b-\tau_b\right)dx_p dx_b = V-\frac{1}{12}\left(6\tau_p-\frac{\tau_p^2}{\tau_b}+15\tau_b\right). \quad (27)$$

Case 2: if $\tau_p \leq \widetilde{\Delta}$ and $\tau_b > \Delta$ then (26) is again given by (27).

Case 3: $\tau_p > \widetilde{\Delta}$ and $\tau_b \leq \Delta$ then (26) becomes

$$\int_0^1\int_0^{\frac{\tau_b-2x_b\tau_b+\tau_p}{2\tau_p}} \left(V-\tau_p x_p-\tau_b x_b-\tau_p\right)dx_p dx_b = V-\frac{1}{12}\left(6\tau_b-\frac{\tau_b^2}{\tau_p}+15\tau_p\right).$$

Case 4: $\tau_p > \widetilde{\Delta}$ and $\tau_b > \Delta$ then (26) becomes

$$2\int_0^{\frac{\tau_b-\Delta}{2\tau_b}}\int_0^{\frac{\tau_p+\Delta}{2\tau_p}} \left(V-\tau_p x_p-\tau_b x_b-\tau_p\right)dx_p dx_b+$$
$$2\int_{\frac{\tau_b-\Delta}{2\tau_b}}^{\frac{\tau_b+\Delta}{2\tau_b}}\int_0^{\frac{\tau_b-2x_b\tau_b+\tau_p}{2\tau_p}} \left(V-\tau_p x_p-\tau_b x_b-\tau_p\right)dx_p dx_b+$$
$$2\int_{\frac{\tau_b+\Delta}{2\tau_b}}^{1}\int_0^{\frac{\tau_p-\Delta}{2\tau_p}} \left(V-\tau_p x_p-\tau_b(1-x_b)-\tau_p\right)dx_p dx_b$$
$$=\frac{\Delta^2(3\tau_p-2\Delta)+3\tau_b\left(\Delta^2-2\Delta\tau_p+(4V-5\tau_p)\tau_p\right)-3\tau_b^2\tau_p}{12\tau_b\tau_p}.$$

Combining cases 1–4 yields the expression for consumer surplus in the statement of the lemma.

A.2 Proof of Proposition 7

First show by construction that there is such an equilibrium. Suppose $r_i > r_j$, that both platforms install B_i, and that $T^k_{B1} = T^k_{B2} = \frac{r_2}{2}$. Neither platform can profitably deviate because gross profit is $\tau_p/2$ regardless of whether they differentiate or not (and because both firms offer the same payment). Increasing T_{B2} would result in negative profit, while decreasing T_{B1} would result in application B_2 being installed, resulting in a loss of profit for firm 1 of $(r_1 - r_2)/2$ per-platform.

To show that this equilibrium configuration is unique when $r_1 \neq r_2$, suppose that some platform installed B_j. Note that j's payment to L must be less than $r_j/2$ since any higher offer is dominated. Now consider an offer of $(r_i - \epsilon)/2 > r_2/2$ from i to L. If L accepts this offer it gains at least $(r_i - \epsilon - r_j)/2$ in extra payments. If L accepts the offer then i's profits also increase by $\epsilon/2$. Thus, the putative allocation either has some firm failing to play a best response or is sustained by a non-credible threat from a platform to refuse an offer of $(r_i - \epsilon)/2$. In either case it is not an equilibrium.

A.3 Proof of Proposition 8

Case 1: Let $r_i - r_j \geq \tau_b - \tau_p$. Suppose firm i offers $T^k_{Bi} = \frac{\tau_b - \tau_p + r_j}{2}$, firm j offers $T^k_{Bi} = \frac{r_j}{2}$, and both platforms install application B_i. Since T^k_{Bi} is set to make platforms indifferent between installing B_i and B_j, neither platform can profitably deviate. Increasing T_{Bj} would result in negative profit, while decreasing T_{Bi} would result in application B_j being installed (yielding a loss of profit for firm 1 of $(r_1 - r_2)/2$ per-platform). Thus, we have an equilibrium where both platforms install B_i.

To see that *every* equilibrium has both platforms install B_i when $r_i - r_j > \tau_b - \tau_p$, suppose on the contrary that some platform (say, L) installs B_j. Note that j's payment to L must be less than $r_j/2$ since any higher offer is dominated. Now, consider an offer of $(r_i - \epsilon)/2$ from i to L (ϵ small and positive). If L accepts this offer it gains at least $(r_i - \epsilon - r_j)/2$ in extra payments, and loses at most $(\tau_b - \tau_p)/2$ in foregone differentiation. If L accepts the offer then i's profits also increase by $\epsilon/2$. Thus, the putative equilibrium either has some firm failing to play a best response or is sustained by a non-credible threat from a platform to refuse an offer of $(r_i - \epsilon)/2$.

Case 2: Suppose that $r_i - r_j \leq \tau_b - \tau_p$. We can construct an equilibrium in which platforms differentiate as follows: Platform L installs B_i and is offered $T^L_{Bi} = \frac{r_j + \tau_p - \tau_b}{2}$, $T^L_{Bj} = \frac{r_j}{2}$. Platform R installs B_j and is offered $T^L_{Bj} = \frac{r_i + \tau_p - \tau_b}{2}$, $T^L_{Bj} = \frac{r_i}{2}$. Given these offers, platforms are indifferent between the application they install and its competitor. i's accepted offer, $T^L_{Bi} = \frac{r_j + \tau_p - \tau_b}{2}$, is such that j would have to pay more than $r_j/2$ to be installed by L, which is clearly dominated (a symmetric argument holds for j's accepted offer). Lastly, $|r_1 - r_2| \leq \tau_b - \tau_p$ implies that $\frac{r_j + \tau_p - \tau_b}{2} \leq \frac{r_i}{2}$ so firms make non-negative profits from being installed and cannot, therefore, profit from a deviation to some lower offer.

To complete the proof, we show that $r_i - r_j < \tau_b - \tau_p$ implies that all equilibria have platform differentiation. Suppose, on the contrary, platforms both install B_i. An offer by i greater than $r_i/2$ is strictly dominated. Thus, each platform's profit is no greater than $(\tau_p + r_i)/2$. Now, consider an offer from j to L of $(r_j - \epsilon)/2$, with ϵ small and positive. L would find it worthwhile to accept this offer because it would imply profit $(\tau_b + r_j - \epsilon)/2 > (\tau_p + r_i)/2$. The offer is also profitable for j (it yields profit $\epsilon/2$, whereas the putative equilibrium implies zero profit). A symmetric argument holds when both platforms install j.

A.4 Proof of Proposition 9

We can rewrite (17) as

$$CS = V - \frac{1}{4}\left(\tau_b + 2\widetilde{\Delta} - \frac{\widetilde{\Delta}^2}{\tau_b} + 5\tau_p\right), \tag{28}$$

which is the consumer surplus under bundling (i.e., absent differentiation).

If $\tau_p > \min\{\tau_b, \Delta\}$ then the equilibrium involves no application differentiation, regardless of whether there is bundling or not. Thus, the price (and hence consumer surplus) are unchanged.

If $\tau_p \leq \min\{\tau_b, \Delta\}$ then, by Proposition 8, there will be application differentiation in the absence of bundling. Thus, (28) should be compared with the expression for consumer surplus found in Lemma 9. The change in consumer surplus from bundling is therefore

$$\left[V - \frac{1}{4}\left(\tau_b + 2\widetilde{\Delta} - \frac{\widetilde{\Delta}^2}{\tau_b} + 5\tau_p\right)\right] - \left[V - \frac{1}{12}\left(6\tau_p - \frac{\tau_p^2}{\tau_b} + 15\tau_b\right)\right],$$

which is positive if (in addition to $\tau_p < \Delta$) we have

$$\tau_p < \frac{1}{2}\left(\sqrt{3}\sqrt{43\tau_b^2 + 4\widetilde{\Delta}^2 - 8\widetilde{\Delta}\tau_b} - 9\tau_b\right).$$

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