Supplementary Appendix to
Search Quality and Revenue Cannibalisation
by Competing Search Engines

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A1 Expanded Discussion

A1.1 Acquiring Loyal Consumers

Building on the analysis of search engine loyalty, an interesting question concerns the effects of acquiring additional loyal users, and the incentives to do so. For the purposes of simplicity, I let $p^{\max} = 1$. To incorporate the notion that the relative degree of consumer loyalty may vary, I begin by supposing that consumers are distributed along the Hotelling line according to the linear density function $f(x) = 2ax + 1 - a$, with $a \in [-1, 1]$ a parameter that measures the distribution of loyalty. Write F for the corresponding CDF. Thus, $a = 0$ corresponds to the standard uniform case considered above, whilst $a > 0$ ($a < 0$) corresponds to a skewed distribution with most consumers being loyal to m (loyal to g).

Search engine g's profits are

$$\text{(A1)} \quad \pi_g = F\left(x^*\right)(1 - p_g)b$$

when $p_g > q$,

$$\text{(A2)} \quad \pi_g = F\left(x^*\right)b$$

when $S \le p_g < q$, and

$$\text{(A3)} \quad \pi_g = F\left(x^*\right)[(1-\lambda)(1-q) + \lambda]b$$

when $p_g = q$, where x^* is given in (4). The analogous profits for m are $\pi_m = [1 - F(x^*)](1 - p_m)b$, $\pi_m = [1 - F(x^*)]b$, and $\pi_m = [1 - F(x^*)][(1-\lambda)(1-q)b + \lambda b]$ respectively.

At this point it is useful to recall the meaning of loyalty in the present context. A consumer is loyal to a particular search engine when they would favour that search engine over its rival *in spite of* it having a lower quality. When the loyalty parameter, a, is set exogenously (and $a \neq 0$) the results are largely intuitive: search engines then set asymmetric

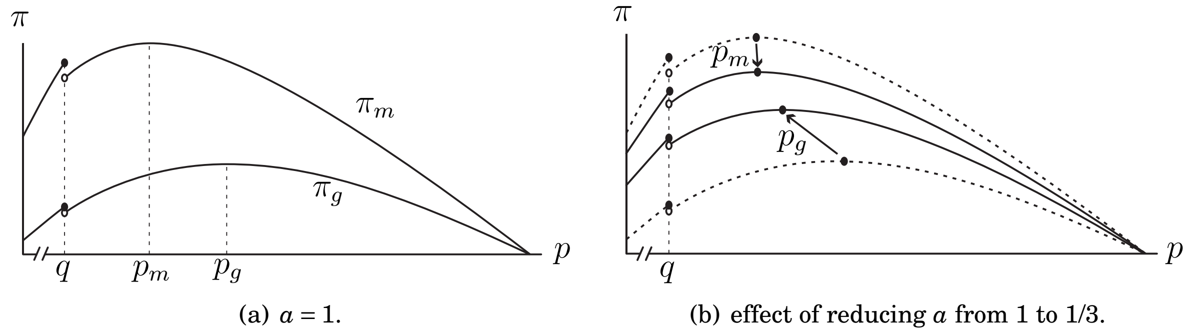

FIGURE A1 Equilibrium with $q = 0.08$.

qualities, with one search engine setting a low quality and enjoying the benefits of its installed loyal user base, whilst the other wins over consumers by virtue of offering a higher quality. An example of this kind of equilibrium is plotted in Figure A1(a), where $p_g = 0.4$ and $p_m = 0.246$ are set such that each maximises the profits of the respective search engine when m has more loyal users ($a = 1$).[1] The resulting profits for m and g are respectively $0.488b$ and $0.211b$. This kind of arrangement might reflect, for example, the early days of the search industry when, as a relatively new entrant, Google attracted users away from incumbent providers such as AltaVista or Lycos with its innovative PageRank algorithm.

Decreasing $|a|$ causes the profits of the search engine with relatively more loyal users to fall by redistributing clicks to its rival (which enjoys a corresponding increase in profits). In this fashion, equilibrium qualities converge on the symmetric case as $a \to 0$. This process is illustrated in Figure A1(b). Search engines thus have an incentive to invest in attracting loyal users. One way that this can be achieved is by investment in technologies such as toolbars for the search service, or by releasing a web browser that uses its service as the default search engine.

In the long-run, one might expect the degree of loyalty to a search engine to be determined endogenously in the sense that consumers decide which search engine to become loyal to (e.g. which toolbar to install) after considering the relative quality of the two. To model this phenomenon, suppose that $a = (p_m - p_g)/(p_m + p_g)$ so that relatively more consumers become loyal to g whenever p_g increases relative to p_m. Equilibrium A1 then describes the nature of the unique equilibrium with $p_g, p_m > q$.

Equilibrium A1 *With endogenous consumer loyalty, there exists a $\overline{q}'$ such that there is an equilibrium with organic link quality strictly greater than sponsored link quality if and only if $q \leq \overline{q}'$ (if sponsored link quality is sufficiently low). The corresponding equilibrium quality is weakly greater than that in Equilibrium 4, but is always a binding ceiling on result quality for p^{max} sufficiently high.*

[1] These equilibrium quality levels can be found by simultaneously solving the first order conditions from (A1) when $a = 1$. As in the symmetric case, for q sufficiently high, there is also an equilibrium with $p_g = p_m = q$.

Proof. Substitute $a = (p_m - p_g)/(p_m + p_g)$ into (A1) where $F(x) = x - ax + ax^2$, and x^* is given in (4). Taking a first order condition and imposing $p_g = p_m = p$ yields

$$\left.\frac{\partial \pi_g}{\partial p_g}\right|_{p_g=p_m=p} = \frac{p(3-8q) - 7p^2(1-q) + q}{8p(p(1-q)+q)} b = 0,$$

which has one root in $[0,1]$ given by

$$\text{(A4)} \quad p_g = p_m = \frac{3 - 8q + \sqrt{9 - 20q + 36q^2}}{2(7-7q)}$$

This characterises the symmetric equilibrium value of $p_g, p_m > q$. for this to be sustained in equilibrium, it must be the case that no search engine can profitable deviate to setting $p = q - \epsilon$ for ϵ small. Substituting (A4) into the profit function, (A1), reveals that profits in the putative equilibrium are

$$\text{(A5)} \quad \pi_g = \frac{11 - 6q - \sqrt{9 - 4q(5-9q)}}{28(1-q)} b.$$

If g deviate to $p_g = q - \epsilon$ for ϵ small then its profits are

$$\pi_g = \frac{196(2-q)q^2 \left(q \left(15 - 46q + 14q^2 - 3\sqrt{9-4q(5-9q)}\right) - 4\left(3 + \sqrt{9-4q(5-9q)}\right)\right)}{\left(3 + 2(3-7q)q + \sqrt{9-4q(5-9q)}\right)\left(3 + 2(17-7q)q + \sqrt{9-4q(5-9q)}\right)^2} b,$$

which is smaller than (A5) when $q < 0.174901 \equiv \overline{q}'$. ■

Relative to the case with a exogenously fixed at zero (given in Equilibrium 4), Equilibrium A1 has higher search engine qualities and can be supported for a strictly larger range of q. This is intuitive: endogenous loyalty gives search engines an additional marginal incentive to increase result quality in the short-run because doing so convinces consumers to adopt search-engine specific investments and thus become more loyal in the long-run. Although introducing endogenous loyalty causes an increases in equilibrium quality, it remains the case that cannibalisation considerations cause search engines to distort their quality downwards: (A4) is never greater than 3/7, and therefore represents a binding ceiling on equilibrium quality for $p^{\max}$ sufficiently high.

A1.2 Commercial versus informational search

It is natural to think of consumers as being heterogeneous in other dimensions besides their affinity to a particular search engine. In particular, different instances of a consumer entering a search query are likely to be associated with important differences in the entered search keywords. One might expect a sponsored result to have a higher match probability for a commercial search than for an informational one. For their part, search engines are willing and able to implement small tweaks to their algorithm, but cannot possibly hope to provide an individually tailored algorithm for each of the infinite variety of possible keywords.

To model this I suppose that, whilst search engines are constrained to offer a single p to all searchers, consumers have a two-dimensional type $\{x, q\} \in [0,1]^2$, where x is the consumer's position on the Hotelling line between the two search engines and q is the match probability of the sponsored result for the consumer's (idiosyncratic) search phrase. High-q consumers can, then, be thought of as being those who enter relatively more commercial keywords. As above, I let $s \to 0$ and $p^{\max} \to 1$ for simplicity.

Recall that the location of the consumer who is indifferent between visiting g or m first (characterised in (4)) depends upon the A-link quality; each q in the consumer population therefore gives rise to a specific indifferent consumer whose location I denote $x^*(p_g, p_m, q)$.[2] It is interesting to pause for one moment to consider the intuition for the dependence of $x^*(p_g, p_m, q)$ on q. A consumer with an overtly commercial search (very high q) will have a strong propensity to click a sponsored link first, and will almost always be satisfied when doing so. Since such consumers seldom reach the point at which they are clicking organic links, the O-link qualities are irrelevant and consumers simply visit the nearest search engine (formally, $\lim_{q\to 1} x^*(p_g, p_m, q) = 1/2$). By contrast, a consumer with a purely informational search that has no commercial value (very low q) knows that no advertiser will satisfy them and is entirely dependent upon organic links. Such consumers are therefore much more willing to travel long distances along the Hotelling line to access a better search algorithm. This intuition implies an interesting adverse selection-like effect is at work in the search industry: search engines may improve their algorithms to attract users, but the users they attract are disproportionately likely to be informational searchers of relatively low commercial value.[3] The distribution of consumer clicking behaviour is shown in the left panel of Figure A2.

Writing $\omega(q)$ for the (finite) density of q in the population of consumers, and $F(x)$ for the CDF of x, g's profits can be written as

$$\text{(A6)}\quad \pi_g = \Bigg[(1-p_g)\underbrace{\int_0^{p_g} \omega(q)\cdot F(x^*(p_g,p_m,q))\,dq}_{\substack{\text{Mass of consumers who visit}\\ g \text{ first and have } p_g > q.}} + \underbrace{\int_{p_g}^{1} \omega(q)\cdot F(x^*(p_g,p_m,q))\,dq}_{\substack{\text{Mass of consumers who visit}\\ g \text{ first and have } p_g < q.}}\Bigg]b.$$

The two integral terms respectively compute the mass of consumers in the bottom-left and top-left quadrants of the left panel of Figure A2. Note that this profit function evaluates to zero at $p_g = 1$ so that an interior solution must prevail for any distribution of q—that is to say, cannibalisation must eventually impose a competitive ceiling on equilibrium result

[2] It turns out that, for any two optimal click orders in which all three links are potentially clicked, (4) locates the indifferent consumer for any given q. Thus, for organic link qualities sufficiently large (or transport cost, S, not too large) we need only worry about a single form of indifference condition.

[3] Of course, as discussed in Section A1.1, search engines have an incentive to attract informational searchers who will later become loyal commercial searchers.

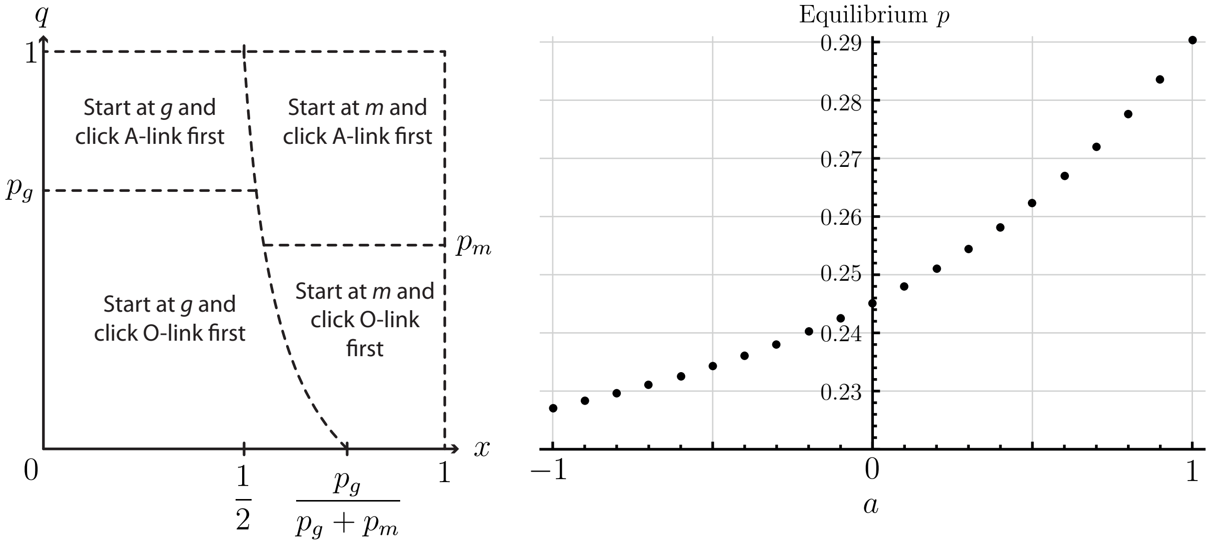

FIGURE A2 Left panel: Relationship between consumer types and clicking behaviour when $p_g > p_m$. Right panel: Equilibrium quality levels for various values of distribution parameter a when $\omega(q) = 2aq + 1 - a$.

quality regardless of technical progress or consumer characteristics.

As a concrete example, when F and ω are both uniform and S is sufficiently low to ensure that all consumers are prepared to click three links in any equilibrium the following proposition describes symmetric equilibrium quality provision and demonstrates the extent of equilibrium quality degradation.

Proposition A1 *When the commercial value of search queries, q, and consumer search engine loyalty, x, are uniformly and independently distributed in the consumer population, the unique symmetric equilibrium quality level is given by* $p_g = p_m = 0.245$.

Proof. Substituting x^* from (4) for $F(x^*(p_g, p_m, q))$ and $\omega(q) = 1$ into (A6) and evaluating the integral yields

$$\pi_g = \frac{(1-p_g)^2(2-p_g-p_m) + \ln(2)(p_g-p_m) + \ln(p_g(3-p_g-p_m)+p_m)(p_m-p_g)}{(2-p_g-p_m)^2} +$$
$$\frac{(1-p_g)\left[(1-p_g)p_g(2-p_g-p_m) - \left[\ln(p_g+p_m) - \ln(p_g(3-p_g-p_m)+p_m)\right](p_g-p_m)\right]}{(2-p_g-p_m)^2}$$

Calculating the derivative of this function and imposing $p_g = p_m = p$ yields the first order condition

$$\frac{(1-p)^2(1+5p) + (1-p)\ln(p) + p\ln((2-p)p)}{4(1-p)^2} = 0,$$

which has a unique root in $[0,1]$ at $p = 0.245$. ■

More generally, let $\omega(q) = 2aq + 1 - a$ (so that the density of q is linear but skewed) and suppose that x is distributed uniformly in the population. Equation 4 then characterises the

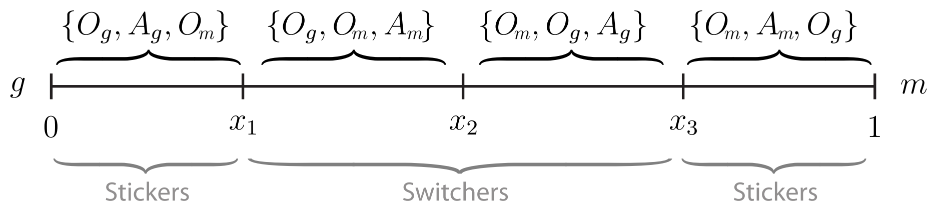

FIGURE A3 Distribution of consumer click orders along the Hotelling line when $s > 0$.

size of g's user base over the relevant range for each q, and solution of the pair of first order conditions from the search engines' profit functions yields the symmetric equilibrium O-link quality profile. These equilibrium quality levels are plotted for various values of a in the right panel of Figure A2. Increasing a corresponds to increasing the fraction of consumers with a relatively commercial search, and results in an increase in the equilibrium level of quality since cannibalisation has less bite when most searches are sufficiently commercial that consumers prefer to click A-links first.

A1.3 'Loyalty' with non-trivial clicking costs

In this subsection I relax the assumption that $s \to 0$ in the loyalty model and examine the effects of variations in the different sources of search costs on equilibrium quality provision. If at least one search engine sets $p \le q$ then all consumers will stick at the first search engine they visit. Here I focus on the characteristics of equilibria with qualities above q. I assume that the clicking cost, s, and the transport cost, S, are low enough to ensure that every consumer would, in principle, be prepared to click all three links in equilibrium. As in Section 3.2, the presence of a positive link clicking costs means that some consumers may wish to switch search engine during a given query. In such cases, consumer click orders are distributed along the Hotelling line as illustrated in Figure A3. The points of indifference identified in Figure A3 are calculated by equating the corresponding utility functions and can be expressed as

$$\text{(A7)}\quad x_1 = 1 - \frac{(p_m - q)}{q}\frac{s}{S}; \quad x_2 = \frac{p_g}{p_g + p_m} + \frac{p_g - p_m}{p_g + p_m}\frac{s}{S}; \quad x_3 = \frac{(p_g - q)}{q}\frac{s}{S}.$$

Thus, as in the simple model, it is relative rather than absolute search costs that are important for driving consumer search behaviour and I henceforth write ς for s/S (the ratio of clicking costs to transport costs).

The indifference points in (A7) apply if some consumers switch. If, on the other hand, $x_1 \ge x_3$ then all consumers prefer to use a sticking behaviour and—much as in Section 4—the relevant indifference point is then that between click orders $\{O_g, A_g, O_m\}$ and $\{O_m, A_m, O_g\}$.

In this case, the indifferent consumer is located at

$$\text{(A8)}\quad x_s^* = x^* + \underbrace{\left(p_g - p_m\right)\frac{(2-q)}{p_g + p_m + q\left(2 - p_g - p_m\right)}\varsigma}_{\text{Clicking cost adjustment.}},$$

where x^* is as in (4). This condition therefore says that, under universal sticking, g's audience is as in the $s \to 0$ case plus an adjustment that accounts for the fact that if $p_g > p_m$ then consumers will save in expected clicking costs by visiting g first, which makes g more attractive as a first port of call. This adjustment is larger when the clicking cost is large relative to the transport cost (when ς is large) as the imperative to save on clicking costs is then amplified.

When all consumers stick and both search engines set qualities above q, g's profits are $(1-p_g)x_s^* b$. Solving the search engines' maximisation problem leads to the following result:

Proposition A2 *In any symmetric equilibrium with organic link quality greater than sponsored link quality and all consumers sticking, equilibrium quality is equal to*

$$\text{(A9)}\quad \frac{1-3q+\varsigma(4-2q)}{3-3q+\varsigma(4-2q)},$$

which is greater than that in Equilibrium 4, but is nevertheless a binding ceiling on result quality for p^{max} sufficiently high.

Proof. Substituting (A8) into $(1-p_g)x_s^* b$ and calculating the FOC yields

$$\frac{b(1-p_g)(1-q+2\varsigma-q\varsigma)}{p_g+p_m+2q-p_gq-p_mq} - \frac{b(1-p_g)(1-q)(q-2p_m\varsigma+p_mq\varsigma-p_g(-1+q-2\varsigma+q\varsigma))}{(p_g+p_m+2q-p_gq-p_mq)^2}$$
$$- \frac{b(q-2p_m\varsigma+p_mq\varsigma-p_g(-1+q-2\varsigma+q\varsigma))}{p_g+p_m+2q-p_gq-p_mq} = 0.$$

Setting $p_g = p_m = p$ in this expression and solving for p yields (A9) as the unique symmetric solution. If (A9) demands a $p < q$ then the only possible symmetrically maximising $p > q$ is arbitrarily close to q by concavity; the fact that profits fall discontinuously at $p = q$ then implies that there is no equilibrium with $p > q$. ■

Equation (A9) is clearly analogous to (5), but calls for a higher level of quality. This is because offering consumers a reduction in clicking cost expenditure (by reducing the expected number of clicks necessary to attain satisfaction) is another way in which increasing quality makes a search engine more attractive and this therefore serves as an additional marginal incentive to increase quality. The larger is the clicking cost relative to the transport cost, the stronger is this effect so that equilibrium quality is increasing in $\varsigma \equiv s/S$. Note that, even with all consumers sticking, (A9) is less than 1 for every finite ς so that there is a (cannibalisation induced) competitive ceiling on equilibrium quality whenever there is a non-trivial cost of visiting at least one of the two search engines.

Next, consider the case in which all consumers use a switching strategy. This will be the case when $x_1 \leq 0$ and $x_3 \geq 1$, which is true when

$$p_g, p_m \geq \frac{1+\varsigma}{\varsigma} q.$$

Search engine g's profits can then be written as

$$\pi_g = \left[(1-p_m)(1-p_g)(1-x_2)\right] b,$$

and it can easily be verified that this is decreasing in p_g, which immediately implies that any universal switching equilibrium must be a corner case with p_g, p_m just high enough to induce all consumers to switch. The following is then immediate:

Proposition A3 *Given that i's profits are decreasing in p_i when all consumers switch, any universal switching equilibrium must have*

$$\text{(A10)} \quad p_g = p_m = \frac{1+\varsigma}{\varsigma} q = \frac{S+s}{s} q.$$

Note that (A10) entails a quality level that is entirely analogous to (3). The intuition here is also similar to that surrounding Equilibrium 2b: increasing p_g makes g more attractive as a first port of call but, when all attracted consumers will switch, g would prefer to serve as their second port of call. Thus, for ς not too low, consumer switching establishes an effective ceiling on g's quality.

When $0 < x_1 < x_3 < 1$ some, but not all, consumers prefer to use a switching behaviour—clicking both O-links before the A-link. Search engine g's profits are then of the form

$$\pi_g = \big[\underbrace{(1-p_g)x_1}_{\text{Stickers.}} + \underbrace{(1-p_m)(1-p_g)(x_3-x_2)}_{\text{Switchers from } m.}\big] b.$$

Increasing p_g makes visiting g first more attractive so that x_2 shifts to the right. It also makes switching to g more attractive for those consumers who visit m first—shifting x_3 to the right. The net effect on g's profits therefore depends upon whether the region $x_3 - x_2$ expands enough to offset the increased inframarginal degree of revenue cannibalisation that comes from satisfying more consumers with O-links. The symmetric solution to the search engines' joint profit maximisation problem in this case is too complicated to be instructive, but can be plotted as in Figure A4. Much like the universal switching case detailed in Proposition A3, equilibrium quality is decreasing in ς. This has the interesting and counter-intuitive implication that making search engines more substitutable (in the sense that the transport cost, S, is reduced) can lead to a fall in equilibrium quality when consumers are

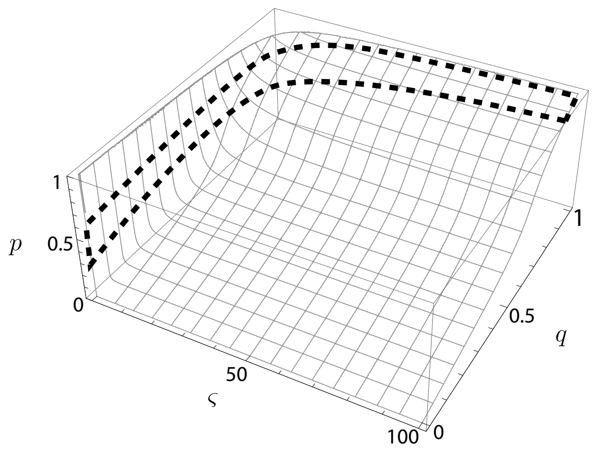

FIGURE A4 Equilibrium quality levels when some consumers switch. The dashed black line encloses the region within which $0 < x_1 < x_3 < 1$ (i.e. the region in which some, but not all consumers switch).

induced to switch search engines.

The explanation for this is as follows: Whilst a search engine has a strong incentive to compete for the attention of sticking consumers, it would prefer any switching consumers to visit its rival first. Thus, whilst the presence of sticking consumers exerts upward pressure on organic result quality, switchers have the opposite effect. From (A7) it is clear that, all else equal, increasing ς (viz. increasing clicking costs relative to the degree of differentiation) increases the size of the region $(x_3 - x_1)$ within which consumers switch search engines and thus decreases the mass of consumers over whom the search engines wish to compete.

A1.4 Market structure

In this subsection I extend the simple model to the case with an arbitrary number of search engines, assuming that whenever the consumer is indifferent between visiting two or more search engines he visits each with equal probability. In the results above, competition for visits prompts search engines to cannibalise their revenues from A-link clicks. When this competition is taken away so is the incentive to provide O-links of a high quality. Only the cannibalisation effect remains, and thus a monopolist will generally have a strong incentive to degrade its quality.[4]

With $n \geq 2$ search engines, optimal consumer behaviour is largely analogous to the duopoly case. Label the search engines so that $p_1 \geq p_2 \geq \ldots \geq p_n$. Consumers will visit search engines in order of their (organic link) quality. Since the trade-off between switching and sticking is essentially the same as in the duopoly case, consumers will switch from i to $i+1$ if $p_{i+1} > \sigma q$ (if the relative switching cost is sufficiently low), and stick at i to click the A-link if i is the first search engine such that $p_{i+1} < \sigma q$.

Given the optimal consumer strategy, it is possible to examine equilibrium search en-

[4]If $S + s > q$ for some proportion of the consumer population then the monopolist may still wish to set a $p > q$ in order to induce those consumers to search—see White (2009) for an analysis of this issue.

gine behaviour when each search engine faces $n-1 \geq 1$ competitors. It is fairly straightforward to establish that analogues of Equilibria 2a and 2b can be sustained in the n-search engine case, and that σq plays a similar role in determining equilibrium quality.

Equilibrium A2 *When $\sigma q \geq p^{max}$ (relative switching costs are high enough), there is an equilibrium in which all search engines set the maximum technologically feasible quality.*

Proof. Expected profits from compliance are $\pi_i = (1/n)(1-p^{max})b \geq 0$. Consider a deviation in which i sets $p_i < p^{max}$. A $p_i < \min\{p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, \sigma q\}$ implies that the consumer never clicks A_i, and i's profits are zero. Thus, i has no profitable deviation. ■

Equilibrium A3 *If $\sigma q < p^{max}$ (if relative switching costs are sufficiently low) an equilibrium in which $p_i = \sigma q \forall i$ can be sustained; σq is then the highest admissible equilibrium quality..*

Proof. Following the logic of Section 3.2, when $p_i = \sigma q$ the consumer is indifferent between clicking the A-link at the i^{th} search engine he visits and doing so at the $i-1^{th}$ site. If it is also the case that $p_{i+1} = \sigma q$ then the consumer is indifferent between the click orders $\{\ldots, O_{i-1}, O_i, A_i, O_{i+1}, \ldots\}$ and $\{\ldots, O_i, O_{i+1}, A_{i+1}, O_{i+2}, \ldots\}$. Thus, if $p_i = \sigma q \forall i$, it can be established by transitivity that the consumer is indifferent between any two click orders whose $n+1$ elements are the n O-links along with some search engine's A-link, and which do not have him click on that A-link first.

When i deviates by setting some $p'_i > \sigma q$, he is visited first with probability 1. Thus, profits from deviation to some $p'_i > \sigma q$ are given by $\pi'_i = (1-p'_i)\lambda b$ (where λ is the proportion of consumers that stick when indifferent), which is maximised when $p'_i \to \sigma q$. Suppose, instead, that i complies with the putative equilibrium. The probability that site i is the k^{th} site to be visited is $1/n$. Conditional on being the k^{th} site (for $k < n$), i receives b iff the first $k-1$ O-links, as well as i's own O-link fail to match the consumer's need (each link failing with probability $1-\sigma q$), and if the consumer chooses to switch at the first $k-1$ sites and stick at i's site. This gives rise to the first term in (A11). The second term comes from the fact that if i is the n^{th} site to be visited, and the consumer has switched at the first $n-1$ sites, then the consumer sticks at i with probability 1 since there are no more search engines to switch to. Profits, then, are

$$\text{(A11)} \quad \pi_i = \left[\frac{\lambda}{n}\sum_{k=1}^{n-1}(1-\sigma q)^k(1-\lambda)^{k-1}\right]b + \left[\frac{1}{n}(1-\sigma q)^n(1-\lambda)^{n-1}\right]b.$$

Note that deviation profits approach zero with λ, whilst the second term in (A11) remains positive (since $\sigma q < p^{max}$ implies $\sigma q < 1$). Thus a $p = \sigma q$ equilibrium can always be sustained by setting a low enough λ. ■

Recall that the existence of Equilibrium 1 rested upon the requirement that no search engine can obtain a big enough increase in market share from an increase in its p to offset the corresponding losses due to cannibalisation. Recalculating the condition for existence of Equilibrium 1 using the fact that each of our n search engines is visited first with probability $1/n$ when all set the same quality reveals the result described in Remark 8, whose proof follows straightforwardly from that of Equilibrium 1.

One of the issues in the regulation of Internet search has been to fully understand the effects of reduced competition in the industry. This is especially true since advertisers can often substitute to other mediums, and the ability of search engines to exercise any market power over them is thus seemingly limited. However, it is immediately apparent from Remark 8 that the condition for existence of the 'low quality' equilibrium becomes less demanding as the number of competing search engines in the industry is reduced so that less competitive industries are more susceptible to this kind of systematic quality reduction. In the oligopoly case, search engines may have an incentive to consolidate or collude since this can create new equilibria with higher total industry profits, but lower organic result quality. The above results therefore demonstrate that reduced competition may spill-over into the quality of search services enjoyed by consumers. This may prove to be an important consideration in evaluating merger proposals if the consumer's search experience is part of the regulator's objective.

An industry characterised by search engines that degrade their result quality may look vulnerable to entry by interlopers who seek to steal a large portion of the market by offering a higher quality service. There are, however, several factors that protect an incumbent against would be entrants. The most basic of these is that a search engine that is deliberately degrading its quality can reverse that degradation in the event of entry. If the technology to provide high quality results is already in place then this kind of threat to fight for the market may credibly deter entry. Section A1 speaks to a closely related point. Any incumbent is likely to have an installed base of loyal users, which puts it in a strong position vis-á-vis a would-be entrant—even if its quality continues to be considerably lower. Figure A1(a) serves to illustrate this point by highlighting the discrepancy in profitability that can come about even when an incumbent faces a rival with higher quality than itself. Although a low quality incumbent's loyal users may eventually switch if such a quality difference persists, the initial advantage enjoyed by the incumbent may suffice to allow it to force any entrant to leave the market. When these factors are put together with the high fixed costs of entry (for example, *Advertising Age* reports that Microsoft spent between 80 and 100 million dollars on advertising alone when launching search engine Bing,[5] but secured just 12% of the search market in its first year), it would seem that incumbent market leaders can enjoy some measure of security, even when degrading their result quality.

There's also another, more subtle remark to be made about firm consolidation. I have argued that, since the condition for existence of a $p_i = q \forall i$ equilibrium is more easily sat-

[5]"Microsoft Aims Big Guns at Google, Asks Consumers to Rethink Search", *Advertising Age*, 25th May 2009.

isfied as n becomes smaller, a reduction in the number of competing firms may result in the creation of low quality equilibria that could not previously be supported. The condition given in Remark 8, though, is valid only when firm's market shares are completely symmetric. Suppose, instead, that each search engine, i, is visited first by by a fraction α_i of consumers such that $\sum_i \alpha_i = 1$. The existence condition now becomes

$$\text{(A12)} \quad q \geq \max_i \left\{ \frac{1-\alpha_i}{1-\alpha_i+\alpha_i\lambda} \right\}.$$

What happens to this existence condition in the aftermath of a merger between search engines depends upon how consumers reallocate the α weights in response to such consolidation. Since it may be difficult for many small consumers to coordinate on such matters, it seems plausible that market shares may not be reallocated evenly. Consider the following example:

Example A1 *There are three competing search engines, g, y and m, with $\alpha_g = 0.65, \alpha_y = 0.25$ and $\alpha_m = 0.1$. Condition (A12) implies that a $p_i = q \forall i$ equilibrium can be sustained only if $q \geq 0.9$ (assuming $\lambda = 1$). Suppose that when two search engines merge, the new α's are simply the sums of those for the consolidated firms, and consider two possible mergers: (i) m merges with y to form my with $\alpha_{my} = 0.35$, and (ii) y merges with g to form gy with $\alpha_{gy} = 0.9$. From (A12) the corresponding existence conditions (again, taking $\lambda = 1$) are $q \geq 0.65$ and $q \geq 0.9$ respectively.*

In this example, the merger that seemingly creates a more plausible competitor for g supports a low quality equilibrium for a greater range of q than does that which consolidates g's market leadership. The intuition is straightforward: the lower is the market share of the least-favoured firm, the more it stands to gain by poaching its rivals' consumers, and hence the stronger is the incentive to deviate.

A2 Search Costs and Multiple Organic Links

Suppose that search engines offer $n \geq 1$ organic links per site and face the problem of selecting an algorithm $A = \{p_1^A, \ldots, p_n^A\}$, thereby inducing (expected) qualities $p_1^A \geq p_2^A \geq \ldots \geq p_n^A$ for the n organic links. It is useful to generically write p_m^A for the smallest p in A such that $p \geq q$, and to denote the set of feasible algorithms by $\mathscr{A} \subseteq [0,1]^n$. Write $V(A)$ for the probability of being satisfied by some link at a site offering algorithm A when links at that site are clicked in decreasing order of quality. Thus

$$V(A) = p_1^A + \left[p_2^A(1-p_1^A)\right] + \ldots + \left[p_m^A(1-p_{m-1}^A) \times \ldots \times (1-p_1^A)\right] + \left[q(1-p_m^A) \times \ldots \times (1-p_1^A)\right] + \left[p_{m+1}^A(1-q) \times \ldots\right.$$

Let $\mathscr{A}^* = \{A : A = \arg\max_{A\in\mathscr{A}} V(A)\}$ be the 'best' algorithms—i.e. those algorithms that maximise the probability of satisfaction. Similarly, write $v(A) = \Pi_{i=1}^{m}(1-p_i)$ for the probability that a consumer who resolves to click all links (until satisfied) at a search engine offering algorithm A will click on the A-link there. Let $\mathscr{A}^{**} = \{A : A = \arg\max_{A\in\mathscr{A}^*} v(A)\}$.

Proposition A4 *It is an equilibrium for both search engines to implement some $A \in \mathscr{A}^{**}$ (to implement the most profitable amongst the best algorithms that are technically feasible) if $s \to 0$ (if within-site search costs are sufficiently small).*

Proof. To see this note that as $s \to 0$ consumers exhaust all links on the first site that they visit (since doing so is costless whereas switching search engines is costly). Since consumers are risk neutral, they prefer to visit the search engine offering the higher $V(P)$ first. Thus, a search engine that deviates from the putative equilibrium to some $A \notin \mathscr{A}^*$ receives no A-link clicks and makes zero profit. A search engine that deviates to some $A \in \mathscr{A}^* - \mathscr{A}^{**}$ makes lower than equilibrium profit by definition. ■

This result has been derived under the assumption that consumers can observe the ordering of O-link qualities—because, say, a search engine puts its best links at the top of the page. When each O-link's quality is an independent draw from some distribution, the same result can easily be obtained by setting $p_1^A = p_2^A = \ldots = E(p^A)$ in the above.

If $p^{\max}$ is the expected quality of the best links that the search engine has the technology to identify then the search engine can implement any $p_k \in [0, p^{\max}]$ by using an algorithm that has $p_k = p^{\max}$ with some probability in $[0,1]$, and shows an inferior link in slot k with the complementary probability. Thus, $\mathscr{A} = [0, p^{\max}]^n$ seems like a natural way to parametrise the space of feasible algorithms. Note that given this set-up, $\mathscr{A}^* = \mathscr{A}^{**} = \{p^{\max}, p^{\max}, \ldots, p^{\max}\}$, and both search engines implementing this algorithm is therefore an equilibrium by the above proposition. Moreover, if one lets $\mathscr{A} = [0, p^{\max}]^n$ then we have the following result:

Proposition A5 *Fix a site visiting cost, $S > 0$. There exists an $\overline{s}$ such that (i) for any within-site search cost $s \le \overline{s}$ there is an equilibrium in which both search engines implement the best feasible algorithm, $\mathscr{A}^*$, and (ii) for any search cost $s > \overline{s}$ there is no such equilibrium.*

Proof. Consider a putative equilibrium with both search engines setting $\mathscr{A}^*$. All consumers begin by clicking on the organic links at the first search engine they visit. Starting from this point, their continuation utility from sticking to also click the sponsored link, before switching to continue at the second site is

$$U_{\text{stick}} = q(1-s) + \sum_{j=1}^{n}\left[(1-q)p^{\max}\left(1-p^{\max}\right)^{j-1}[1-S-(j+1)s]\right] + (1-q)(1-p^{\max})^n[-S-(n+1)s].$$

The continuation utility from switching to click the second site's organic links (before finally clicking its sponsored link) is

$$U_{\text{switch}} = \sum_{j=1}^{n} \left[p^{\max} \left(1 - p^{\max}\right)^{j-1} (1 - S - js) \right] + q(1 - p^{\max})^n [1 - S - (n+1)s] + (1 - q)(1 - p^{\max})^n [-S - (n+1)s].$$

Setting these two utilities equal yields the value

$$\overline{s} = \frac{p^{\max} q S}{(1 - (1 - p^{\max})^n)(p^{\max} - q)}$$

of s that makes the consumer indifferent. If $s < \overline{s}$ then consumers strictly prefer to stick and search engines will compete to be visited first so that the putative equilibrium can be sustained. If $s > \overline{s}$ then consumers strictly prefer to switch and some search engine would then prefer to lower the quality of some of its links in order to induce the consumers to switch to it. ■

References

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