Desargues' theorem states that in a space of more than two dimensions if two triangles are centrally perspective, they are axially perspective, that is, if \( AA' \), \( BB' \), \( CC' \) are concurrent (at \( O \)), then if \( D \) is on \( BC \) and \( B'C' \), and \( E \) is on \( CA \) and \( C'A' \), and \( F \) is on \( BA \) and \( B'A' \), \( D, E \) and \( F \) are collinear.

That is, if we have a red triangle, \( ABC \), and a green triangle, \( A'B'C' \), which are not both in the same plane,

\[ AA', BB' \text{ AND } CC' \text{ are concurrent} \]

such that \( AA', BB', CC' \) all go through the same point, say \( O \).
[Alternatively, we might start with $O$, and consider three lines (not all in the same plane) leading from it, with two points on each line, viz. $A$ and $A'$, $B$ and $B'$, and $C$ and $C'$ respectively, and then take $ABC$ as the red triangle and $A'B'C'$ as the green triangle:

Three rays from $O$, each with two points on it]

THEN if we call the point where $BB'$ and $CC'$ meet $D$,

and the point where $CC'$ and $AA'$ meet $E$,

and the point where $AA'$ and $BB'$ meet $F$,

then $D$, $E$ and $F$ are on a straight line.
Desargues' Theorem turns entirely on the intersections of the various planes: The nub of the proof is that $D, E$ and $F$ are all both in the plane of the red triangle, $ABC$ and in the plane of the green triangle, $A'B'C'$, and thus on the line common to these two planes.

In order to achieve this result, we have to establish first that the points $D, E$ and $F$ actually exist, that is to say that the lines $BC$ and $B'C'$ actually do meet (at $D$), and similarly that the lines $CA$ and $C'A'$ actually do meet (at $E$), and similarly again that the lines $AB$ and $A'B'$ actually do meet (at $F$). In each case we do this by considering the relevant planes.

Since the purple lines $BB$ and $CC'$ meet (at $O$), $B, C, B'$ and $C'$ are all in the same plane.

Because they are all on the two purple lines intersecting at $O$, the purple points, $B, B', C, C'$ are all in the same plane. Hence the purple lines $BC$ and $B'C'$ do meet. We call the point of intersection $D$. 
Likewise since the blue lines $CC$ and $AA'$ meet (at $O$), $C, A, C'$ and $A'$ are all in the same plane.

This figure shows all the points, but picks out in blue the five co-planar points, $C, C', A, A', O$ which are on the intersecting lines $C, C', O$ and $A, A', O$. Hence the blue lines $CA$ and $C'A'$ do meet. We call the point of intersection $E$.

The point where $CC'$ and $AA'$ meet $E$. An exactly similar argument shows that since $BB'$ and $CC'$ meet at $O$, $B, C, B', C'$ and $O$ are all in the same plane, and that therefore $BC$ and $B'C'$ do meet, as shown in the greenish brown diagram below.

The point where they meet is called $F$.

We have thus shown that because the three lines $AA'$, $BB'$, $CC'$ all meet in one point $O$, the three pairs of lines, $BC$ and $B'C'$, $CA$ and $C'A'$ and $AB$ and $A'B'$, intersect each other. Each point of intersection is both in the plane of the red triangle, $ABC$, and in the plane of the green triangle, $A'B'C'$. 

and since planes intersect in straight lines, the three points are on the same straight line. Q.E.D.

(See next page for larger figure)
(I could put jpeg versions of these figures onto the web, if it would be useful for making transparencies: let me know by E-mail.)
Desargues' theorem is important philosophically. It is NOT a theorem in a purely 2-dimensional projective geometry.

Usually it is postulated as an additional axiom, but that is not mandatory.

But if a 2-dimensional projective geometry is not purely 2-dimensional, but is a 2-dimensional subspace of a higher-dimensional geometry, then Desargues' theorem is a theorem (for a proof, see Desarg2d.pdf in the same sub-directory as this file): this is important philosophically because it shows that we have generality as a criterion of mathematical truth. Although we cannot prove it in purely 2-dimensional projective geometry, and no inconsistency results from our denying it, we justify our taking it as true, and postulating it as an additional axiom, because it is true if we embed our 2-dimensional geometry in a more general higher-dimensional geometry. We can call this the Bed Theory of Truth.