Chapter 2
The Development of Normative Reason

§2.1 Noncontradiction
One can think wrong. The fact that after much thought one has reached a conclusion is no guarantee that the conclusion reached is right. Only a very opinionated man would refuse to concede the possibility of error, and once the admission of fallibility is made, the problem of justifying one's beliefs becomes acute. So we formulate our reasons as best we can. But even when formulated, they may fail to convince. Only if people are willing to be reasonable can they be reasoned with. None so obdurate as those who will not listen to reason, and with them at least it is better to save one's breath than to attempt to convince them. You just cannot argue with people who will not be argued with. With them we can only let them go their way, as did Socrates; ἐν χαὶ ςεϊν (en chaiein)

And yet. Sometimes even the most unreasonable sophist can be caught out, and made to admit the force of an argument. Socrates was able to make Thrasyrmachus blush, and Aristotle gave some general rules for arguments which are absolutely incontrovertible. If I admit that All men are mortal and that Socrates is a man, I must then concede that Socrates is mortal. Why? What happens if I do not? If I do not concede that Socrates is mortal but say instead that he is not mortal, having already said that All men are mortal and that Socrates is a man, then I am contradicting myself.

The traditional syllogism

All men are mortal
Socrates is a man

therefore Socrates is mortal
is valid\(^1\) because the conjunction of the three propositions

\[
\text{All men are mortal} \\
\text{Socrates is a man} \\
\text{Socrates is not mortal}
\]

is inconsistent. A man who utters this inconsistent triad of propositions is guilty of a self-contradiction. Having affirmed any two, he must not go on to affirm the third on pain of inconsistency, but must contrariwise concede the negation of that third proposition. If he affirms the first two he must concede the negation of the third, as in the familiar syllogism. If he had affirmed the second and the third, he would have to deny the first, as in the syllogism

\[
\text{Socrates is a man} \\
\text{Socrates is not mortal}
\]

\textit{therefore} Not all men are mortal,

while if he had affirmed the first and the third, he would have to deny the second, as in the syllogism

\[
\text{All men are mortal} \\
\text{Socrates is not mortal}
\]

\textit{therefore} Socrates is not a man.

Simple informal deductive arguments can be defined in terms of inconsistency in much the same way as analytic propositions can. A proposition is analytic if its negation is inconsistent, and similarly an argument is deductive if it would be inconsistent to affirm the conjunction of the premises and the negation of the conclusion; or, more colloquially, if a man would be contradicting himself if he affirmed the premises and denied the conclusion. We are bound to admit the force of deductive arguments because we should be contradicting ourselves if we did not. Having said that All men are mortal and that Socrates is a man, we cannot refuse to allow

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\(1\) The use of the word ‘valid’ has caused difficulties. It is sometimes used as a general term of appraisal, at other times as applying only to deductive arguments. See J.O. Urmson, “Some Questions Concerning Validity”, \textit{Revue Internationale de Philosophie}, 25, 1953, pp.217-229; reprinted in R.G. Swinburne, \textit{The Justification of Induction}, Oxford, 1974, pp.74-84. In this book it will be used only of deductive arguments, which are either valid, or invalid, with no further value in between. Inductive and other arguments will be assessed as being of greater or less cogency. It follows that an “invalid” (i.e. not deductively valid) argument may nevertheless be extremely cogent. See further below §3.4.
that Socrates is mortal, any more than we can resist the claim that All red things are coloured, that All uncles are brothers, that All bachelors are unmarried, or that Either it is raining or it is not. Deductive arguments, like analytic propositions, flow from the Law of Non-contradiction, and thence obtain their incontrovertible validity.

But why should I not contradict myself? It is a free country, and it would be uncivilised to make me bridle my tongue out of deference to Plato or Aristotle. And indeed there are no legal penalties for inconsistency. The sanction is quite another one, namely that if I contradict myself I make myself unintelligible. A speaker must be consistent, or communication breaks down. In most systems of formal logic it is easily shown that if both \( p \) and not-\( p \) are given, we can prove any other proposition we like; and one definition of the consistency—the “absolute consistency” of a system—is that not every proposition can be proved in it. The same thought is expressed in the colloquial rejoinder “If you would say that, you would say anything”. And this is a rebuke, because if a person is prepared to say anything, then anything he says is no better than anything else. Only if there are some things he is not prepared to say does the fact of his saying some other thing signify. Where everything is free, nothing is of value. Propositions acquire meaning in as much as they have scarcity value. Only if Thrasy medicines is not prepared to say absolutely anything, will people attend to what he actually does say. Else his words cease to have scarcity value or significance, and cease to be words at all, and become just babble.

Simple informal deductive arguments, therefore, are valid. Anybody who refuses to accept a deductive argument, puts himself out of court by making himself unintelligible. If he wishes to communicate, he must use a language, and abide by the rules of that language, which alone constitute it as a language and not a sequence of meaningless noises. They are therefore utterly incontrovertible,

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\[ \text{The word ‘logic’, like the word ‘valid’ is the cause of much confusion.} \]

\[ \text{Often it is taken to mean ‘deductive logic’, but philosophers used to talk of ‘inductive logic’, without its being a contradiction in terms. Scientists sometimes speak of the logic of an experiment, and historians of the logic of a situation, while feminists are furious when men say that women are emotional rather than logical. Confusion is best avoided by always asking what the words ‘logic’ and ‘logical’ are being contrasted with.} \]
since only if he accepts them can he make himself understood. Even a sophist, even Thrasymachus, could not resist deductive arguments. They are absolutely cogent, and make no demands on a person’s being reasonable, but only on his being able to use language. In that sense, man is a λογιστικόν ζώον (logistikon zoon), a talking animal, even more ineluctably than that he is rational. Not all men are reasonable, but all who argue with us are of necessity language-users.

Although informal deductive arguments are valid, there may be in some cases a problem of recognising them for what they are. The informal deductive arguments which Plato has Socrates use against his opponents obtain their force from the rules of correct English (or Greek) usage. Sometimes it is the rules for certain key words, ‘not’, ‘are’, ‘is’, ‘all’, ‘some’, ‘more’: at other times it is the rules for some specialised word; for example, instead of the analytic proposition All red things are coloured, we may have the deductive argument

\[
\text{This is red} \\
\text{therefore This is coloured}
\]

where the validity of the argument, like the truth of the analytic proposition, turns on the meaning of the words ‘red’ and ‘coloured’. Similarly the arguments

\[
\text{He is an uncle} \\
\text{therefore He is not an only child}
\]

and

\[
\text{He is a bachelor} \\
\text{therefore He is not married}
\]

turn on the meanings of the words ‘uncle’, ‘only’, ‘child’, ‘bachelor’, ‘not’ and ‘married’. But the meanings of words are not always clear. Although we often know how to use words we seldom know how they are used, in the sense of being able to give explicit definitions of them; indeed, we find it very hard to formulate adequate definitions, and normally we just rely on our unformulated “know-how”. We are none the worse for that, so long as usage is clear and we agree about it. But often usage is not clear, and we are not agreed about how to use a crucial word. In the United States at one time to say ‘He is red’ meant that he had left-wing sympathies and to say ‘He is coloured’ meant that he numbered Negroes among his ancestors. But the inference from the former proposition to the latter is not deductively valid. Again the word
'uncle' is used in England not only of a parent's brother but of any middle-aged friend of the family, and also of pawnbrokers. It is not clear whether these peripheral uses of the word 'uncle' are standard ones or not. Certainly, we could not convict a man who used the word 'uncle' in either of these senses of not knowing English, although if he refused to distinguish the different senses of the word, communication would break down. But the criterion for distinguishing senses is simply the validity or invalidity of various patterns of deductive argument, and so our appeal is no longer to a common language but simply to a willingness to accept certain types of argument as valid. If the only cogent arguments were deductive ones, it might not matter making a willingness to accept them a precondition of communication: but since there are other arguments, many of which are felt by many people to be cogent, we shall be arguing in a circle if we explain deductive arguments in terms of linguistic usage, and linguistic usage in terms of deductive argument.

Language cannot be separated from the rest of life and thought. The meaning of the word 'gentleman' depends on an understanding of the considerations a gentleman is guided by. We speak the same language only because we share to some extent a common life and a common standard of rationality. We often say of a person who completely rejects our assumptions or refuses to acknowledge the cogency of any of the arguments commonly regarded as cogent that "he does not speak the same language as we do". Our language is not a formal logistic calculus but something much more flexible, often vague, sometimes shifting, impregnated with implicit assumptions, something almost alive. Sometimes its shifts reflect merely a social change. At one time the argument

This person is a bachelor

\[ \text{therefore} \quad \text{This person is a male} \]

was a valid deductive argument, because unmarried females were called spinsters. But now we have the term 'bachelor girl' and the validity of the argument is in doubt. The meaning of the word 'bachelor' is shifting, and now has connotations of bed-sits, doing for oneself and cooking for oneself, rather than of being as yet uncommitted and fancy-free. There may even come a day when the word 'bachelor' is so completely anchored in, say, a residential context, that from someone's being a bachelor it will no longer follow as an inference of deductive logic that he or she is unmarried. More serious, from our point of view, than social
changes are intellectual ones. Our thoughts change, and our theories have to accommodate new insights or new facts. At one time

This is gold

therefore This is soluble in *aqua regia* but not in *aqua fortis*

would have been a reasonable argument, but not a deductive one, at another it might have been deductive, and at yet another it might have no longer been deductively valid. As our knowledge of chemistry has deepened, and the concepts of allomorph and isotope have been introduced, the meaning of other concepts has been changed too. The change is sometimes gradual, and we cannot say whether some inference is or is not valid in consequence of the way we use words or in consequence of the way, we believe, the laws of nature operate. Similarly in moral arguments the distinction between moral and purely deductive arguments is sometimes difficult to draw. A man who says that it is always wrong to kill people but that capital punishment is quite all right is clearly failing to use the word ‘always’ correctly. But is a person who says ‘I promise to marry you’ and later denies that he ought to marry her failing on a point of linguistic usage or only of morality? If he gave some reason for not carrying out his promise, we should acquit him on the linguistic charge, even if we regarded his reasons as inadequate for going back on his obligations. But if he failed to see that saying the words ‘I promise’ put him under any obligation whatever, we might also say that he did not know what the word ‘promise’ meant. There are many other words—*justice, mercy, duty, love*—whose meaning is not constituted by any particular patterns of inference but rather to the recognition of the force of some lines of argument. We cannot separate considerations of language from those of reasonableness. A man who never sees reason to do justice or love mercy, altogether lacks these concepts and does not really understand the meaning of the words.

These difficulties tell not against our drawing the distinction between deductive and other agreements but against the claim that the distinction is always a clear one. Often we can draw the distinction, and it is useful to pick out those arguments which are basically verbal since they turn on the meaning of words, from other more substantial arguments. But we cannot always draw the distinction, and therefore in practice cannot wield simple informal deductive arguments as effectively as we should like, to force recalcitrant reasoners to concede conclusions.
§2.2 ‘Not’ and ‘And’

One natural response is to formalise. We pick out certain basic patterns of inference, and spell out explicitly what the essential pattern is. We choose basic patterns which are, beyond doubt, purely deductive. And if anyone denies their validity, he can be convicted by appeal to the explicit rules of formal logic. We might lay down that the following forms of argument are valid:

All Bs are C
X is a B
therefore X is a C

and if ever our opponent refuses to accept an argument of this form, we no longer need appeal to his unformulated sense of the meaning of the words ‘all’, ‘are’, ‘is’ etc., but can simply point out that the argument in question is of this form, and therefore must be valid.

Forms of valid inference have not only been specified explicitly, but have been systematized, so that granted only a few, simple, patterns of inference, others can be validated by a succession of simple steps. Deductive logic is thus very like geometry. At first, in Aristotle’s day, there were a number of separate patterns of inference, each recognised as valid, but only a few having been “reduced” to simpler forms. Now, however, there are many axiomatizations of formal logic with only three axioms, some with only one, and correspondingly few rules of inference. The difficulty is that a recalcitrant listener cannot be forced to accept cogent deductive inferences in these formal logics on pain of self-contradiction: If I fail to concede that once you have established the two premises $p$ and $p \rightarrow q$, I cannot deny you the conclusion $q$. I do not thereby show myself ignorant of English. Rather, I am failing to play the game. The analogy is with cricket: it is as though I refused to leave the wicket after having been bowled. A person who refuses to leave the wicket when out is just not playing cricket; there is no law against it—the pitch may be in his own garden—but nobody will play with him. The sanction is the same: being ignored by others if we ignore the rules of the game. Formal deductive arguments thus appear as a species of rule-observance: we don’t have to observe rules, but if we do not, we are not observing them. But cricket is optional, whereas logic is not. We could change the rules of cricket, and have bowlers bowl eight times in an over, instead of only six, but we should be chary of recognising “Australian logic” as an equally good way of arguing, simply because the Australians chose to adopt it. Although there is some room for dispute at the
margins, the main body of logic is what it is for good reason. Its rationale arises from the need to avoid self-contradiction.

If I have not both \((p \land \lnot q)\), or not \(p\) without \(q\), then if I am given \(p\), I can infer \(q\), because else I should have \(p\) and not \(q\) as well as not both \((p \land \lnot q)\), which would be an evident contradiction.

We can express this in symbols. The older logicians wrote not both \((p \land \lnot q)\), as \(\lnot(p \land \lnot q)\), but it is better to have a distinct sign for negation, \(\lnot\), which will not be mistaken for a dash. For reasons that will emerge later, instead of the simple ampersand, \&, logicians now prefer to use a wedge-shaped symbol, \(\land\). Then granted a very minimal sense of the word ‘and’, namely that if we have \(p\) and \(\lnot q\), we have \((p \land \lnot q)\), it follows that if we have \((p \land \lnot q)\) and \(\lnot(p \land \lnot q)\) we have \((p \land \lnot q) \land \lnot(p \land \lnot q)\) which is evidently a self contradiction. That is, if I have \(\lnot(p \land \lnot q)\), then I can infer \(q\) from \(p\), or granted \(\lnot(p \land \lnot q)\), I have ‘if \(p\) then \(q\)’. This suggests that we write \(\lnot(p \land \lnot q)\) as \(p \rightarrow q\); \(p \rightarrow q\) is called material implication, and is often read as ‘\(p\) implies \(q\)’. But it is important to note that in most ordinary uses of ‘\(p\) implies \(q\)’ there is a suggestion of some connexion between \(p\) and \(q\) justifying the inference.

Material implication does not carry this connotation. We can see this if we note that \(\lnot(p \land \lnot q)\) is true automatically if \(p\) is false, so that if we know that \(p\) is false, we can assert \(p \rightarrow q\) whatever \(q\) is, and without there being any connexion between \(p\) and \(q\). Similarly if \(q\) is true, \(\lnot(p \land \lnot q)\) must be true too, so that \(p \rightarrow q\) holds vacuously. Material implication does not give the full sense of ‘if , then ’, but only the minimal truth conditions it must satisfy. But these are important. They are the minimal conditions, and if an implication does not satisfy them, it cannot be an implication as we know it. We can express the force of material implication by the inference it legitimises. If I have \(p\) and \(p \rightarrow q\), then I can conclude \(q\), or, representing inference by \(\vdash\), \(p, p \rightarrow q \vdash q\).

In reaching this formulation, we have implicitly appealed to inference patterns expressing the force of ‘and’, \(\land\), namely that \(p, q \vdash p \land q\). The complete specification of the logical force of \(\land\) has in addition to introduction rules, the elimination rules: \(p \land q \vdash p\) and \(p \land q \vdash q\). It follows that \(p \land q \vdash q \land p\). Here, as with material implication, the logical symbol carries only part of the sense of ordinary language. \(p \land q\) means the same as \(q \land p\), whereas “They got married and had a baby” means something different from “They had a baby and got married”.

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**Reason and Reality**

[2.2]
Negation, $\neg$, is similarly defined by inference patterns. In addition to the ban on self-contradiction, it is commonly held that $p \vdash \neg \neg p$ and $\neg \neg p \vdash p$, though the latter inference is controversial. Some logicians work with an "intuitionistic logic" in which only $\neg \neg p \vdash \neg p$ holds. Such a logical system can be developed without inconsistency, but fails to capture the two-sided nature of contradiction. If I say $p$, you can contradict me by saying 'not-$p$, $\neg p$, but I can then contradict you by saying $\neg \neg p$, thereby re-asserting my original claim. Although logical systems can be devised with only a truncated form of negation, or, indeed, without any form of negation whatsoever, they fail to capture the full force of dialogue, in which assertions on the one hand can be confronted by denials on the other, followed by a ding-dong of argument in which claim and counter-claim are made and defended.

Just as we can define $\rightarrow$ in terms of $\neg$ and wedge, so can we similarly define $\land$ in terms of $\rightarrow$ and $\neg$. We can also define 'if and only if', the inclusive 'or', often now written 'and/or', in Latin $\vee$, and the exclusive 'or', that is 'either or', but not both', in Latin $\lor$, in terms of $\neg$ together with $\land$, or $\lor$, or $\rightarrow$.

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3 Indeed, we can generate all the truth functions from a single binary 'neither...nor...', $\downarrow$, joint denial, where $p \downarrow q$ is true just in case both $p$ and $q$ are false, or from $\downarrow$, alternative denial, which is false just in case both $p$ and $q$ are true. Whitehead and Russell took $p \lor q$ as basic, defining $p \rightarrow q$ as $\neg p \lor q$, and $p \land q \equiv \neg (\neg p \lor \neg q)$.
§2.3 Gödelian Arguments

Formal logic saves us from being swamped by the fluidity of ordinary usage. We may disagree about whether ‘or’ should be construed in an inclusive or exclusive sense, but the rules for $\land$ and for $\not\lor$ (which captures the sense of the exclusive ‘or’, aut in Latin) are unambiguous. It would seem natural, therefore, in carrying out the programme of formalisation, to seek total explicitness, with all the agreements expressed in propositions, so that everyone could know that they had been agreed, with nothing depending on an implicit know-how. But that cannot be achieved. As Lewis Carroll pointed out,\footnote{Lewis Carroll, “What the Tortoise Said to Achilles”, Mind, 10, 1895, pp.278-290; reprinted in The works of Lewis Carroll, ed. R.L. Green, London, 1965, pp.1049-51. See also, Gilbert Ryle, The Concept of Mind, London, 1949, ch.2, pp.25-61; or “Knowing How and Knowing that”, Proceedings of the Aristotelian Society, 48, 1945-1946, pp.1-16; and Kant, Critique of Pure Reason, tr. Kemp Smith, A132-3/b171-2.} we cannot replace all rules of inference by propositions, but need at least one rule, telling us how to operate with the propositions we are given. It is not enough to have the explicit statement

$$(((p \rightarrow q) \land p) \rightarrow q$$

which expresses the truth enshrined in the rule of inference

$p \rightarrow q$

\hline$p$

$q$

We need also to be able to apply it, which involves being able to recognise symbols, and to see when we have premises of the form $p \rightarrow q$ and $p$, and then allow in that case that $q$ also holds. There was implicit appeal to a rule of inference in the previous section, in recognising $(p \land \neg q) \land \neg (p \land \neg q)$ as a contradiction of the same form as $p \land \neg p$. Formally, the recognition is justified by a rule of substitution, which is taken for granted in all formal systems. Formal logic is concerned with formal patterns of inference, and has to be able to classify different particular inferences as being of the same general type. Know-how cannot be entirely reduced to know-that: there remains an ineliminable element of know-how: logic is ultimately about how to reason, not a set of formulas in a calculus.
None the less, by formalising we hope to reduce the element of know-how to a minimal ability to recognise shapes, and we can go a very long way in formalising logic in terms of a finite number of axioms and rules of inference. Rules can be applied over and over again. There is a potential infinity of applications. We begin to go from simple deductive logic to mathematics.

Mathematics has long been regarded the paradigm of rigorous deductive reasoning. Many philosophers have sought to cast all reasoning into an incontrovertible chain of rule-validated inference more geometrico, supposing that only so could they establish thought on sure foundations. But much to everyone's surprise, mathematics itself has shown that such an ideal is impossible. This highly unexpected result was proved by Gödel in 1929.\(^5\) Granted that we can represent both addition and multiplication in our formal system—it would be a very defective one, if we could not—we can prove in the system that, provided it is a consistent system, it will contain some formula which can be neither proved nor disproved in the system. When we examine the proof we come to the even more surprising conclusion that the proposition expressed by that formula must in fact be true. The Gödelian formula is considered both as an abstract formula in an abstract system, and as an arithmetical proposition with its ordinary meaning. Looking at it in the former way, we prove that, granted the consistency of elementary number theory, it is unprovable; and looking at it the latter way we see that it is true. Thus we have found a new way of establishing the truth of certain propositions of elementary number theory which could not be proved within the standard axiomatisation of the theory.

Gödel's theorem is a variant of the Epimenides—or "liar"—paradox, "this statement is untrue", in which the wide-ranging

'untrue' is replaced by a tightly defined 'unprovable-in-the-given-system'. Provided the system is a formal system, and contains enough simple arithmetic to define addition and multiplication, we can code the concept of being 'unprovable-in-the-given-system' into an arithmetical formula, and assign numbers ('Gödel numbers') to each formula of the system. It is then possible to find a number (in practice enormously large, but here pretended to be 1024) which is the Gödel number of the formula 'the formula with the number 1024 is "unprovable-in-the-given-system" ', where the property of being 'unprovable-in-the-given-system' has been given its appropriate code. It follows that the formula no.1024, which we shall call G, must be true, but unprovable-in-the-given-system, since if it were false, it would be provable-in-the-given-system, which would mean that elementary number theory was false.

The actual argument is enormously long, and needs to be stated with great care. But the upshot is that if we formalise any system rich enough to contain ordinary arithmetic, there will be some formula (in fact an infinite number of them) which cannot be proved in it but is obviously true. An obvious retort is that if the proof is valid, it can be formalised, so that a heuristic, informal proof at one stage can be replaced by a proper, formally valid one in due course. It can. But Gödel's theorem applies to any formal system strong enough for elementary number theory: and if we try to complete such a system by adding the Gödelian formula as an additional axiom, we can apply Gödel's theorem to this new system to find a new Gödelian formula which is unprovable in the new system. Even if we add a new rule of inference, which would be equivalent to adding an infinite sequence of axioms, we still should not have secured a system that was complete, because Gödel's theorem would again apply to the new system, with its new rule of inference and all, and would again enable us to find a formula which could not be proved or disproved in the new system, although we would be quite clear that in fact it was true.

Gödel's theorem is difficult to understand, easy to misunderstand. The formal proof shows only that if the system is consistent, then the Gödelian formula is unprovable in the system. Some philosophers have suggested that the system itself is inconsistent. But that is a counsel of despair; and if elementary number theory were inconsistent, deduction would cease to be a paradigm of valid argument. Other philosophers stick their feet in and refuse to recognise the Gödelian proposition as true. And of course if a man
refuses to concede as true anything except what can be proved in a formal system, he can avoid any further embarrassment, beyond that of having to be remarkably obdurate to cogent mathematical reasoning. Essentially what he is doing is to deny any application of the word ‘true’. He can understand what the word ‘provable’ means, and can be forced to acknowledge that in a consistent formalisation of elementary number theory there must be formulas which can be neither proved nor disproved in the formalised system. But he refuses to understand what the word ‘true’ means, and see that the proposition expressed by the Gödelian formula must in fact be true.

But we do understand what ‘true’ means, and we learn from Gödel’s theorem that truth outruns provability. That has always been believed by some thinkers, though denied by others, who have shied away from any concept of truth that could not be established against the sceptic by some copper-bottomed proof. But once the concept of proof has been made explicit, and the criteria for being provable clearly laid down, Gödel’s theorem shows that there are truths which go beyond that concept of provable.

We are led, therefore, to reject certain minimalising views of truth and reason. It is possible for propositions to be true, even though we cannot verify them. It make sense to claim truth, to wonder about truth, to seek truth, beyond the limits of assured knowledge. Equally with reason, to be reasonable is not just to be in accordance with a rule. Aristotle sometimes talks of ὀκτὼ οὕτως λόγω καλα ἐν ὀρθω λόγω, in accordance with correct reason, sometimes of ἐπεὶ λόγῳ μετὰ λόγον, with reason; Gödel’s theorem underlines the difference, and extends the point made by the tortoise to Achilles in the previous section: however carefully or fully we specify rules of inference, not only is inference something different, but it is not always just rule-observance and can go beyond mere conformity to rules.

Many philosophers have sought to formalise inference. Rather than have incoherent appeals to reason, they seek a few rules of inference which can be precisely formulated, or better still, articulate them as axioms, which can act as premises for simple applications of the *Modus Ponens* rule. Formalisation along these lines can be useful. If the inference itself is in dispute, it helps to try and formulate it precisely, so that each party can spot hidden assumptions, or unjustified claims, made by the other. And treating a system
purely formally enables important questions of consistency, completeness, or independence to be asked and sometimes answered. But the programme—we may call it "deductivism"—of always formalising every inference cannot be carried through. Deductivists in moral philosophy want to replace every moral inference by a moral major premise, which will apply syllogistically to a proposition describing a particular case. In the philosophy of history, they construe particular inferences about particular situations as covertly universal propositions saying that whenever a situation of the one sort arises, it will be followed by a situation of the other sort. These rational reconstructions always were implausible. Now that we see that the programme cannot in any case be carried through, we need feel no compunction in not formalising in cases where there is no special reason for doing so.

If reason transcends rules, we need to alter many of our views of rational activity. Rules will still be important, but not all important. They provide a useful check on the fallibility of individual reasoners, and a means of agreeing among ourselves about what may be commonly taken for granted. But they need validation by reasonable men recognising that the rules are in fact reasonable, and they are open to criticism. If to be reasonable was simply to follow a rule, then it would be self-contradictory to hold that a rule was unreasonable; but it is always intelligible to say that, and sometimes correct.

Rules should be evaluated at the bar of reason: reason need not necessarily be called to account in terms of rule-following. Often, of course, we are concerned with a rule-governed activity, but not of necessity in all cases. Judgement may be called for, and we may have to decide whose judgment is most to be respected in cases where the rules conflict, or have run out, or do not exist. Although there are disciplines, such as mathematics or mathematical logic, where the traditional aim has been to prove everything more geometrically, we do not need, nor should attempt, to reconstruct other disciplines on that model. In history or literary criticism it may be right to recognise the authority of great historians or sensitive critics without being able to reduce their reasoning to explicit syllogistic form. We often talk of originality and creativity in artistic work, and hold that the great artist breaks out from the canons of correct taste and achieves something that, although not conforming to them, is nevertheless absolutely right. Gödel's theorem underwrites this possibility, as one obtaining not only in artistic creation, but throughout the whole realm of rationality, even in the austere field of mathematical logic.
Analyticity

Gödel proved another theorem, a completeness theorem. He proved that first-order logic is complete, that is that every well-formed formula in first-order logic is a theorem. This is most easily understood if we consider propositional calculus. Propositional calculus can be treated axiomatically, as it was by Russell and Whitehead, who postulated five axioms, and certain rules of inference, and proved well-formed formulas by deriving them from the axioms by means of the rules of inference. But we can also use truth-tables to work out the truth-value of a well-formed formula from the truth-values of its components; and if a well-formed formula comes out true whatever the truth-values of its components, we call it a tautology. It is a fairly easy exercise to check that the axioms are tautologies, and that the rules of inference are such that if the premise or premises are tautologies, then the conclusion must be one too. It is much harder to prove the converse of this, but it can be done: every tautology can be proved from the axioms. We say that propositional calculus is *complete* (or sometimes “semantically complete”). Gödel proved that the same holds good for first-order logic, the logic in which we have quantifiers, All, Some, None and Not All, with the quantifiers ranging over individuals, so that we can talk of All men being mortal, and the like. We can construe this as showing that the quantifiers, when they range over individuals, are subject to the same discipline as we have worked out for And and Not. If you deny the conclusion of some syllogism that holds whatever individual cases it is about, you are not only guilty of contradicting yourself, but can be brought step by step to a self-evident self-contradiction. This supports the claim by Kant and many modern philosophers that deductive logic is purely analytic, based on the principle of non-contradiction, and yielding only empty tautologies.

But Gödel’s completeness theorem holds only for first-order logic, in which quantification is allowed only over individual variables, and not over predicate variables. In first-order logic we can talk about all Peter’s children, and say that they have blue eyes, but not about all Peter’s features, which none of his children possess in their entirety. Dedekind needed the latter sort of location in order to define the natural numbers, and it is similarly needed to define the concept of being finite, and to formulate various different axioms of infinity. Second-order logic allows quantification over predicate variables, so that we can talk about all features,
all properties, all qualities, and the like, but second-order logic is not complete in Gödel's sense, but only Henkin-complete, which secures completeness only by misinterpreting key logical terms. In second-order logic not all well-formed formulas that are true under all (reasonable) interpretations are theorems. Logical truths are not always provable by a step-by-step ("algorithmic") derivation. A sceptic can deny them without being led into patent self-contradiction. Deductive logic is not necessarily analytic: synthetic \textit{a priori} propositions are possible.

And we need to have them.

\S 2.5 Mathematical Dialogues
The argument of the previous section is cogent, but not incontrovertible. All that has been formally proved in any system which contains simple arithmetic is that there is no formal proof in the system of the Gödelian formula or its negation. A determined sceptic could maintain the negation of the Gödelian formula without contradicting himself—just. We could argue with him—there are cogent arguments for not accepting the negation of the Gödelian formula—but his position, though weird, is tenable: we can understand what is being claimed; it is not a contradiction in terms, and if put forward, would need to be argued against, not simply dismissed as unintelligible. It follows that even this austere mathematical reasoning is two-sided. And, contrary to orthodox opinion, much else in mathematics is best understood in terms of a two-sided dialogue between seekers after truth.\footnote{6 See above, §1.5.}

Mathematics, with its unexpected incompleteness and unprovable consistency, is different from simple, first-order logic, which is provably consistent and complete. Poincaré\footnote{7 H.Poincaré, \textit{Science and Hypothesis}, pbk ed., New York, 1952, §§IV-VII, pp.8-13.} regarded the Rule of Recursion, often misleadingly called the Principle of Mathematical Induction,\footnote{8 Although it may be advantageous, for the purposes of logistic analysis, to articulate an axiom, which can be added or not added to an axiomatic system, we gain a deeper understanding of the principle involved, if we view it as a rule of inference. If we do regard it as an axiom, it is evidently a synthetic \textit{a priori} one, stating some fact about a strange universe of enormous size: we wonder not only whether it is true or false, but also how we could ever come to know its truth or falsity. A rule arising from a dialogue between two truth-seekers is much easier to understand and to justify.} as the key difference. In Peano's first-order axiomatization of arithmetic, it is expressed by the axiom schema

\footnote{6 See above, §1.5.}
The Development of Normative Reason

\( F(0) \land (\forall n)(F(n) \rightarrow F(n+1)) \rightarrow (\forall x)F(x) \),

which lays down that if the property \( F \) holds for 0, and is such that if it holds for any one natural number it holds for its successor, then it holds for all natural numbers. It seems a very reasonable rule to adopt. It does not seem to add anything that could be questioned to what we already have. If we had Peano’s other four postulates without this axiom schema, we should have a system that could be called “Sorites Arithmetic”, in which, granted the antecedents of the axiom schema, namely \( F(0) \) and \( (\forall n)(F(n) \rightarrow F(n+1)) \), we could establish for each natural number \( n \) that \( F(n) \). There could not be a counter-example within what we normally regard as the natural numbers. If anyone were to maintain that there was, and that for some number, say 257, it was not the case that \( F(257) \), we could construct a Sorites argument

\[
\begin{align*}
F(0) \\
F(0) \rightarrow F(1) \text{ (particular case of } (\forall n)(F(n) \rightarrow F(n+1)) \text{)} \\
F(1) \text{ by Modus Ponens} \\
F(1) \rightarrow F(2) \text{ (particular case of } (\forall n)(F(n) \rightarrow F(n+1)) \text{)} \\
F(2) \text{ by Modus Ponens} \\
F(2) \rightarrow F(3) \text{ (particular case of } (\forall n)(F(n) \rightarrow F(n+1)) \text{)} \\
F(3) \text{ by Modus Ponens} \\
F(3) \rightarrow F(4) \text{ (particular case of } (\forall n)(F(n) \rightarrow F(n+1)) \text{)} \\
F(4) \text{ by Modus Ponens} \\
\vdots \\
F(256) \text{ by Modus Ponens} \\
F(256) \rightarrow F(257) \text{ (particular case of } (\forall n)(F(n) \rightarrow F(n+1)) \text{)} \\
F(257) \text{ by Modus Ponens}
\end{align*}
\]

Thus in the end he would be forced to withdraw his putative counter-example, on pain of self-contradiction. In fact, he could see that this would be the outcome long before we had finished the proof, and would concede rather than waiting to be checkmated. And he would see that the same would happen if he were to suggest any other putative counter-example. It would be reasonable, therefore, for him to concede the claim that \( F(n) \) holds for all \( n \).

But we have not actually proved that. We have shown how, for each \( n \), we can prove \( F(n) \); but without the Rule of Recursion, or some equivalent, we cannot actually produce a formal proof, the last line of which is \( (\forall x)(F(x)) \), expressing the proposition that all natural numbers \( x \) have the property \( F \). There is a subtle difference
between being able to prove, given any particular natural number \( n \), that \( F(n) \) holds, and actually proving that all numbers possess the property in question. Nevertheless, we are confident of the validity of the Rule of Recursion, and generally regard arguments invoking it, along with the rest of mathematics, as deductive.

But there is a difference. The difference shows up in a difference of sanction. The sanction against those who refuse to recognize the cogency of arguments by recursion is no longer that they are contradicting themselves or failing to abide by the rules of the calculus they are operating. The Rule of Recursion is independent of the other axioms and rules of inference of first-order Peano arithmetic (because our concept of natural number is not sufficiently tightly formulated to exclude the possibility of there being some "inaccessible" numbers, which could not ever be reached by a chain of argument, no matter how long it was). Someone who refuses to concede that every number has the property \( F \), granted that \( 0 \) does, and that if any number does, so too does its successor, cannot be convicted of a straight contradiction, however unreasonable his stance seems, provided he sticks to his position without making any further move. Although it is very natural to give reasons for denying a claim made by a friend, it is not absolutely obligatory, and an antagonist who plays with his cards very close to his chest can avoid being brought up short in a self-contradiction. But once he starts considering the possibility of counter-examples, he is lost.

Why should he be reasonable? Or better, Why should I be reasonable? If my sole aim is the polemical one of not losing the contest, there is no reason why I should expose myself to being worsted in the argument. But the case is altered if I want to know the truth. In that case I value discovering what actually is true more than winning the argument, and indeed would rather be shown wrong in what I had previously believed, if by that means I come to exchange wrong opinions for better ones. If I have any idea of truth, it is something that my beliefs should conform to, rather than something that should be conformed to what I think. Truth is independent of me, or it is nothing. And however good an opinion I have of my opinions, once I am possessed of the idea of truth, I know that the opinions which are worthy to be believed are the true ones, not those that happen to be believed by me.

A reasonable man, therefore, will make the moves necessary for ascertaining the truth, even if he thereby exposes himself to being refuted. He will, "for the sake of argument", suppose that
some particular natural number was a counter-example to the thesis being put forward, and, then seeing that any such supposition would involve him in inconsistency, allow that no counter-example could exist, and hence that the thesis must be true. The love of truth makes one vulnerable to pressures to which the purely contentious man is immune. Although the sanction against rejecting the Rule of Recursion is different from the sanction for rejecting Modus Ponens, arguments invoking recursion are properly regarded as deductive.

\[ \text{§2.6 All, Any, Every and Each} \]

The terms 'all', 'any', 'every' and 'each', normally interchangeable when we are talking about the finite cases of ordinary converse, reveal logical differences when we are deploying formal proofs over infinite domains.

'All' is the strongest claim. Once admitted, it applies universally without more ado; I defy you to produce a counter-example; and—if my claim is admitted—you know you cannot. But before it has to be admitted, it needs to be justified to the hilt. In order to justify the claim 'all . . . ', I must produce a straightforward derivation, the last line of which is \((\forall x)(F(x))\). 'Any . . . ' is not so forthright. Instead of saying that I have got a proof, I invite you to choose some particular instance, \(x\), and then show \(F(x)\) by an incontrovertible derivation of a standard form. Since it is a standard-form derivation, I can myself apply it mechanically, and hence instruct you to apply it mechanically, and thus lead from an assured ability to prove any particular case you happen to choose, to a straightforward proof that my claim does, indeed, hold for all cases. Any entails all when the proof of any is always of the same form.

With recursive reasoning, the inference from every to all, the proof in each case is indeed valid, but not quite the same. The number of \(\text{Sorites}\) steps needed depends on the particular example chosen. The proof of \(F(257)\) outlined in the previous section was similar to that of \(F(4)\), but not precisely the same, inasmuch as it had 515 steps, whereas only 9 were needed to prove it for \(F(4)\). Similarly the contra-positive disproof of the denial of \(F(257)\) is much longer than that for \(F(4)\). Any entails all because

\[ (\forall x)(\exists \text{Disproof}_{\text{standard-issue}} \neg F(x)) , \]
whereas \textit{every} entails \textit{all} because

\[(\forall x)(\exists \text{Disproof}_x \rightarrow F(x)).\]

Since the disproof is an incontrovertible derivation, it is reasonable to expect any reasonable person to recognise that fact, and to concede without more ado, but if it is to be carried through to the bitter end, the respondent has to make the necessary adjustments to the length of the derivation, before he sees the inevitable outcome. The cooperation required of the respondent is minimal, but enough to mark the distinction between the inference from \textit{any} to \textit{all} and the inference from \textit{every} to \textit{all}.

Recursion is implicit in our ideal of a formal proof. A formal proof consists in a finite number of steps, each of which is a valid inference, in virtue of some stated rule of inference, of the proposition (or formula or sentence) in question from some proposition(s) which either is an axioms or is already established at some earlier stage in the proof. It must consist of only a finite number of steps, but can consist of any finite number of steps. That is, it can be as long as we please, but not infinitely long. It follows that as soon as a number, any number (but some definite one), is picked, we can prove our formula for that number; but that as long as the number has not been fixed, there is no guarantee that any particular proof we have produced will have reached that number.

The derivations adduced to make good a claim that ‘\textit{every} . . . \textit{.}’ , although not precisely the same in each case, are very similar, differing only in the length of the derivation, according to some specifiable formula. The respondent is required to exercise only minimal cooperative intelligence to see how the \textit{Sorites} derivation should be adjusted in length to fit the case in under discussion. But it could be that in each case there was a derivation, but not so similar that the respondent could be given simple-to-follow instructions how to tailor the basic model so as to fit the particular case. The Gödelian argument sketched in §2.2 of this chapter can be deployed with regard to any first-order system strong enough to include elementary number theory, but there is no set formula giving instructions how to do it in each case. Some non-minimal cooperative intelligence is required on the part of the respondent to see how the general Gödelian argument should be tailored to fit the particular case under discussion. Just as the argument from \textit{every} requires more cooperative intelligence than the argument from \textit{any},
Positive Universal Arguments

1. All. I have a proof of $(\forall x)F(x)$ and here it is.
2. Any. You choose $x$, and I will prove $F(x)$ by a standard procedure which will work for any other $x$ you might choose.
3. Every. You choose $n$, and I will prove $F(n)$ by a Sorites derivation which will work, with suitable adjustments, for any other $n$ you might choose.
4. Each. You choose $x$, and I will prove $F(x)$. You should be able to get the hang of my proof, and see how it could be altered to fit any other $x$ you might have chosen.

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Once dialogues (2), (3), or (4), have been concluded, you, if you are rational, will reckon that you cannot fault an All claim. So next time we meet, I claim All.

Whereas in these latter dialogues, I give you the first move, and then persuade you to concede, I now make the first move myself, defying anyone to controvert me. All is a challenge I make to all comers, whereas with Any, Every and Each, I let you have the first move, so that you can see that try as you will, you cannot evade defeat.

so the argument from each requires more cooperative intelligence than the argument from every.

The problem becomes tractable if we use the dialectical approach to cast these arguments in negative form. In the case of ‘all . . . ’, the negativity arises only from the nature of deductive proof. A formal derivation is a sequence of steps, each one of which must be allowed on pain of self-contradiction. In claiming ‘all . . . ’, I need to be able to produce a derivation which must be conceded on pain of self-contradiction. That is, if I say ‘all . . . ’, and you were to say ‘not all . . . ’, I could lead you step by step to inconsistency. ‘All . . . ’ is demonstrably ungainsayable.

With any, every and each the negative approach is much more significant. Instead of my saying that I have got a proof, whereupon you may challenge me to produce it, I challenge you to produce a counter-example, whereupon I show that it is not a counter-example in such a way that you realise that whatever purported counter-example you might propose, I should be able to refute your claim that it really was a counter-example. Provided you take
up the challenge, and name some particular purported counter-
example, I shall be able to prove you wrong.

If I claim 'any . . .', I challenge you to find a case that is not
ungainsayable. If you deny $F(x)$ thereby claiming $\neg F(x)$, I shall
show that $\neg F(x)$ leads to a self-contradiction, so that $F(x)$ is, as
I claimed, ungainsayable. And, as argued before, since my proof
is in a standard form, applicable without more ado to any case,
you can see (and if you cannot see, I can tell you) how to assure
yourself of the incontrovertibility of $F(x)$, whatever $x$ you choose.

The difficulty with the positive approach to the inference from
every to all was that the respondent had to make the adjustments
necessary for the proof to fit his particular case. There was no
upper bound to the length of proof that might be required, and
since proofs have to be finite, we feel queasy at the prospect of their
being indefinitely, even though not infinitely, long. Qualms are
avoided in the negative approach, since although we have an infinite
progression of numbers, for every individual number our argument
can be put in the form of a regress which is necessarily only finite:
I can prove my refutation of a purported counter-example for $n$,
provided I can prove it for $n-1$; and I can prove it for $n-1$ provided
I can prove it for $n-2$; and so on, until I come down to 0, where I
have already established it. The dialectical challenge reverses the
direction of the burden of proof. The burden of proof is still on
my shoulders, but instead of my taking on the impossible task of
scaling an infinite ascent and proving my claim for all of infinitely
many cases, I get you to propose to me the manageable task of
descending from any given stage and showing how well-grounded
my argument is there. And this I always can do, in a finite number
of steps. For although the natural numbers go on without ever
coming to an end, they do have a definite beginning. And the Rule
of Recursion secures its credit by trading on this fact.

The method of argument can be extended to transfinite num-
bers, since every transfinite ordinal can generate only a finite chain
descending ordinals. Transfinite set theory often appeals to the
Axiom of Choice, which can again be illuminatingly viewed in terms
of a dialogue, or alternatively, to the Axiom of Determinacy, which
is explicitly dialectical in form.

In the straightforward positive justification of the Gödelian
argument some significantly non-minimal cooperative intelligence
was required on the part of the respondent, and if this is not forth-
coming, it cannot be forced on an unwilling respondent. Hence
again the polemical value of casting the argument in the negative
form, where the respondent chooses some particular system, and I
point out its Achilles’ heel.
Negative Universal Arguments

1. All. If you deny $(\forall x)F(x)$ I shall show that $\neg(\forall x)F(x)$ leads to an inconsistency, so that $(\forall x)F(x)$ is unguainable.

2. Any. You choose $x$, and if you deny $F(x)$, I shall show that $\neg F(x)$ leads to an inconsistency. I shall show it in a standard way, which clearly would apply to any $x$ you might have chosen.

3. Every. You choose $n$, and if you deny $F(x)$, I shall show that $\neg F(n)$ leads to an inconsistency, by a step-by-step contrapositive Sorites argument, ending with a denial of $F(0)$, which was one of the premises originally given. You should be able to twig what is going on, and realise that whatever $F(n)$ you propose, I shall refute you, but if you do not, I can spell it out for you. If you cannot hoist it in, you can go on trying to produce a counter-example, and failing every time.

4. Each. You choose $x$, and if you deny $F(x)$, I shall show that $\neg F(x)$ leads to an inconsistency. You should be able to get the hang of my proof, and see how it could be altered to fit any other $x$ you might have chosen; but if you cannot, you can go on trying to produce a counter-example, and failing each time.

\section*{\textsection 2.7 Induction}

Induction perplexes philosophers. Inductive arguments are not valid deductive arguments, yet clearly possess some cogency. It is difficult to maintain a position of complete scepticism about them, but impossible to justify them by purely deductive means.

Many different characterizations of induction are given: the inference from the known to the unknown; from the past to the future; from the particular to the general; or from a certain number of particular instances that we have observed to another particular instance that we have not observed. These characterizations are not equivalent, nor does any constitute a definition. Not only do they pick on different aspects of inductive argument, but inductive arguments are of different types; sometimes posing different difficulties to the sceptic and requiring different sorts of justification. Nor is any one type adequately defined. Indeed, it is difficult to define induction. It is easy to begin, and lay it down as a necessary condition that inductive arguments are not deductive. An older generation of philosophers used to divide all argument into deductive and inductive, and define inductive arguments as
Reason and Reality

those that were not deductive. Hence the characterization of the Rule of Recursion, which was recognized as not being a simple deductive argument, as Mathematical Induction. Some philosophers would still stop there, though with a more adequate characterization of deduction, and offer, as a complete definition, that an inductive argument was a non-deductive one. But evaluative arguments, moral arguments, political arguments, legal arguments, historical arguments, literary arguments, philosophical arguments and theological arguments are nearly all non-deductive, but very far from being all inductive. Some of these would be excluded if we defined inductive arguments as empirical arguments, or as arguments about matters of fact, but our idea of the empirical is hazy, and the concept of a fact is systematically ambiguous, varying very much with context. Even if philosophy, theology, and literary criticism are deemed non-empirical and non-factual, history seems to be concerned with facts, and depends on empirical evidence, though few of its arguments conform to the canonical pattern of induction. As a rough criterion, we could say that inductive arguments are concerned with things rather than persons, and that anything evaluative or interpretative should be excluded. Some philosophers have attempted to exclude unwanted cases by stipulating that the conclusion of an inductive argument should be of the same logical type as the premises: if the premises are propositions about individual swans being white, so should the conclusion; from 'is' other 'is's may be derived inductively, but not 'ought's. Although types of inductive argument can be adequately defined in this way, others cannot: for example, the argument from particular instances to general laws, which was historically the first type of inductive argument to be distinguished. None of these definitions is satisfactory: like deduction, induction is fuzzy-edged. It is best to start with widely accepted standard cases, and recognize that the decision as to which other cases should be included is somewhat arbitrary, to be settled on grounds of convenience as much as anything else.

Let us start, then, with the simplest case, where we argue from particular to particular, and then extend it, in a natural but highly revealing way, to the argument from particular to general. In a simple induction we argue from a number of particular instances of a specified sort and having some further characteristic to some other particular instance’s having the same further characteristic. The argument can be displayed crudely and inadequately by the following schema:

This swan is white
That swan is white
A third swan is white

\[\vdots\]

A 256th swan is white

\[\therefore\]
The 257th swan is white.

We may call this type of inductive argument Inference to the Next Case. Other examples are the argument that the sun will rise tomorrow, and Russell’s chicken which on the basis of observation concluded that the farmer was coming to feed her on the day he was coming to wring her neck. Inference to the Next Case is the type of inference that Hume had in mind, when he sought to explain it in terms of a conditioned reflex. Admittedly, reflexes can be conditioned; but Hume’s contention that it is just a matter of habit is not plausible. For one thing, we sometimes cite reasons

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10 Some care is needed in expressing this inference, in order not to make it a tautology—days are often defined in terms of the sun’s rising, so that tomorrow would not be tomorrow unless the sun rose.

why a conclusion is to be believed, not just causes explaining how we have come to hold it; for another, we often jump to conclusions far more quickly than any habituation process could take place; often we bring to bear a whole lot of background information which justifies our taking a particular observation as indicative of the way things are, without needing to repeat it again and again.

**Inference to the Next Case**

Sun will rise tomorrow (note danger of tautology) the Next Swan will be White.


Is it just a matter of habit, as Hume holds?

(a) we give reasons, saying not only why we do, but why one should, accept the conclusion

(b) we jump to conclusions without waiting to be habituated to them.

Inference to the Next Case leads naturally to Inference to a Generalisation, or Inductive Generalisation, as it may be called. The schema of argument given above will yield conclusions not only about the 257th swan, but about the 258th, 259th, and indeed about any particular swan. Hence it is natural, much as in the case of the Rule of Recursion, to state the form of argument not as a schema that could be applied to yield a conclusion about any particular instance, but as a single argument yielding a conclusion about all instances, i.e. a conclusion in universal terms. This is the traditional form

```
This swan is white
That swan is white
A third swan is white
...
...
A 256th swan is white
therefore All swans are white.
```

The conclusion is of a different logical type from the premises. Its universality derived from the universality of reason. In simple induction we had a pattern of valid reasoning, which therefore applies in any particular case, and so is on universal application. And this is made explicit in the universal form.
As far as particular cases go, Inference to the Next Case and Inference to a Generalisation come to much the same thing. Granted the former, we can establish that something holds for each and every case, and granted the latter, we can obtain the particular case by one further step of deduction

\[ \text{The 257th swan is white.} \]

Nevertheless, the difference in logical type of the conclusion is of considerable philosophical importance, and brings to light covert assumptions in drawing the inference, as well as possible justifications of the whole pattern of argument.

The pattern of argument thus far displayed is defective. It is not good enough to argue

\[ \text{All swans are white} \]

\[ \text{This swan is white} \]
\[ \text{That swan is white} \]
\[ \text{A third swan is white} \]
\[ \vdots \]
\[ \text{A 256th swan is white} \]

\[ \text{All swans are white.} \]

The premises, as it happens, are all true, but I do not draw the conclusion because I have heard tell of black swans, some growing naturally in Australia, others more accessibly visible in Chartwell. In order to reach the conclusion I need a further premise

I have never seen, or heard of, a swan which was not white

This is expressed more traditionally

\[ \text{All swans I have ever seen were white} \]

\[ \text{All swans are white} \]

\[ \text{The next swan will be white} \]

in which the special force of the additional premise is played down. But this is a mistake. It plays down the two-sidedness of inductive inference, the importance for looking for arguments against the conclusion, and only accepting the conclusion after a reasonably sustained search has thrown up nothing substantial.\(^\text{12}\) The importance of the additional premise is brought out in Popper’s approach to inductive inference. Popper does not ask how the conclusion may be verified, but how it may be falsified, and since a

\(^{12}\) See previous section and §15.
single counter-instance, unless it could be explained away, would be enough to falsify it, Popper's main concern is to look for counter-instances, and to regard any universal proposition that has not been refuted as a candidate for truth. As soon as a black swan turns up, I abandon the claim that all swans are white: but until the existence of such a swan is brought to my notice, the law that all swans are white is a reasonable one; and therefore the critical issue is whether I have ever come across an un-white swan or not. Although Popper's account explains the importance of the additional premise, it does so at the cost of making all the others seem unnecessary. I do not need to have seen any white swans at all in order not to have refuted the universal proposition All swans are white. Exclusive emphasis on falsification does not accommodate our everyday belief that positive instances are as important as negative ones, and that a large number of positive instances does, in the absence of any negative ones, increase the evidence in favour of the conclusion. Nevertheless, even though it errs in neglecting the need for arguments in favour of any putative conclusion, Popper's account is valuable in stressing the importance of considering arguments against.

Although Inference to a Generalisation is inter-arguable with Inference to the Next Case, its conclusion is of a different logical type. 'The next swan is white' is not only a particular proposition, but a tensed one. 'Every swan is white', or, equivalently, 'All swans are white', is not only general, but tenseless; we can infer from 'Every swan is white' and 'Leda was a swan' the conclusion 'Leda was white'. Such an inference would not be valid if the 'is' of the first premise were a present-tense 'is'. It is, rather, an omnitemporal use of the verb 'to be' which is put into the present for lack of a better tense to put it into. Such a use of the present tense is sometimes indicated, following a suggestion of J.J.C. Smart, by italics. So we write 'Every swan is white' or 'All swans are white', to indicate that the grammatically present tense is being used in

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14 J.J.C. Smart, Philosophy and Scientific Realism, London, 1963, p.133. Smart uses italicised but otherwise grammatically correct present tenses; for reading aloud the grammatically incorrect use of the infinitive is clearer. See, more fully, Nicholas Rescher, "On the Logic of Chronological Propositions", Mind, 75, 1966, pp.75-76.
a logically tenseless way, or, better, use the grammatically incorrect ‘Every swan be white’ or ‘All swans be white’. Such a use is entirely unobjectionable. But it heightens the profile of induction. Induction does not merely argue from particular to particular in the ordinary tensed indicative mood, but from particular in the ordinary tensed indicative mood to general in a different, tenseless mood. The mood is clearly different, not only because it does not conjugate like the ordinary indicative mood, but because it yields counterfactual propositions, such as ‘If Zoe were a swan, she would be white’, which the ordinary indicative mood does not.

Inference to a General Case

The sun rises every day. All swans are white.

(a) Importance of absence of negative instances: inductive arguments of this type are essentially dialectical, two-sided: only if I have looked for negative evidence and failed to find it, am I entitled to draw a general conclusion.

(b) Inference to Next Case and Inference to a General Case are interderivable, but differ in logical status of the conclusion. Conclusion of former same logical type as premises (tensed indicative): conclusion of latter in tenseless present (be).

Often we go further still, and argue from particular premises not only to an omnitemporal generalisation in the tenseless present, but to a law of nature stating what must, under appropriate conditions, occur. It then becomes difficult to disallow, as also a species of inductive inference, arguments from actual instances to natural laws and from observed phenomena to unobserved entities. We argue from the regular whiteness of swans to a rule that they must be white, and from white appearances to a genetic make-up that accounts for them. Such inferences, although rejected by Hume, have commended themselves to scientists ever since. We seek generality, integration, unification and explanation in our account of the world, and it seems reasonable so to seek. Although quarks, psi-functions and wavelets all transcend the bounds of possible experience, we form some sort of concept of them, and succeed in saying things about them which can be significantly affirmed or denied. Nobody makes out that the Special Theory, the General Theory and Quantum Mechanics are plain sailing. They are difficult, and it is easy
Inference to a Natural Law

In any isolated system mass and energy must be conserved. The halogens can only have one valency bond.

(a) Importance of a wide variety of efforts to falsify the law:
only if I have tried hard to find negative instances and failed to find them, am I entitled to infer that I have come up against a law of nature.

(b) Importance of consilience: natural laws should mesh together.

Inference to a Natural Law yields a conclusion very obviously of a different logical type from premises. It shows us that reason can go from one mood to another: although we cannot *deduce* an ‘ought’ from an ‘is’, it may be possible to *derive*, or *infer*, it. This has consequences for metaphysics as well as for morals.

It shows that we can go, with reason’s aid, beyond the bounds of possible experience.

to be confused and talk nonsense about them. But it does not follow that rational argument about science is impossible, or that reason must acknowledge that such knowledge is too high for it, and it cannot attain unto it. The arguments Hume put forward for ruling out altogether knowledge of unobserved entities or explanations of the universe as a whole, would, if they were cogent, rule out all sub-atomic physics and cosmology. But, while many thinkers fear—or hope—they are cogent when deployed against metaphysics or natural theology, few seriously suppose they cast any aspersions on the reputability of modern science.

It is a moot point whether inferences leading to the acceptance of scientific theories should be called inductive. In recent years they have often been termed ‘inferences to the best explanation’. The term ‘abduction’ is sometimes used. The name indicates a difference of sanction. If having noticed a number of cases of people drinking hemlock and subsequently dying, I refuse to infer that if I drink hemlock I shall die, I shall pay for my scepticism with my life.
The difficulty of not reaching right conclusions about matters of fact, especially matters of future fact, is so great that even the most obdurate sceptic finds ways of not living down to his professions of ignorance. Omnitemporal generalisations are also useful, and the sceptic who disallows Inference to a Generalisation is depriving himself of much useful knowledge in a convenient and memorable form. Laws of nature and scientific theories can also be defended on grounds of utility, but the argument is more tenuous: many effective people lead happy and successful lives without having mastered Einstein’s General Theory, or Quantum Electrodynamics. Although some knowledge of science is in some circumstances useful, the real sanction against the sceptic who will not admit inferences to scientific theories is that he will suffer from avoidable ignorance. I want to know the nature of the universe and to understand the causes of things. Natural laws and scientific theories claim truth and offer understanding. They integrate and explain. Diverse phenomena are unified by being brought under a single principle, and a welter of confusing events are explained by means of a theory. It is because they unify and explain that we believe that the laws and theories are true. But if they are true, we want to know them. The same motive applies also, though to a lesser extent, to accepting Inference to a Generalisation, and even to Inference to the Next Case. We like to know the way things happen, quite apart from any utility. Few bird-watchers eat birds, or obtain any material benefit from knowing which species the birds in the garden belong to, but want to know all the same. Although our motives may be mixed, our methods are the same: we apply various principles, often unconsciously, to distinguish good inductive inferences from bad ones.

By contrast to the systematic way in which we can test different combinations of possible causal factors, there is no systematic test for explanatoriness. We have a number of general ideas of what possible explanations there can be, and are much readier to accept a putative causal generalisation or law of nature if we can see how it might be explained—if it fits in with what we already know about
the way things happen. Medical scientists are very reluctant to accept evidence in favour of homoeopathic remedies, and try hard to ascribe any indubitable cases to chance, because they do not see how homoeopathy could work. Consilience—the extent to which a new generalisation will fit in with what is already known—is a major factor in inductive inference, and one that cannot be reduced to a set of systematic rules. Nevertheless, all in all inductive inference is a rational activity, largely systematic, rule-governed, and we can properly speak of “inductive logic”, as it used to be called.\footnote{See further above, §2.1, n.2.}

We are not obliged to accept inductive arguments as we are deductive ones. Communication will not break down if I, having seen many white swans, refuse to infer that the next one will be white too. It is perfectly intelligible to maintain that the next sample of hydrogen cyanide will not be poisonous—but highly unwise. Without the aid of inductive inferences I shall not be able to anticipate the situations I shall have to face; nor ward off untoward outcomes of conditions about to obtain. I shall know less, if I do not allow myself to infer universal propositions, under suitable safeguards, from particular observations, and I shall understand less, if I do not go beyond the phenomena to the best explanation available.

\section*{§2.8 Practical Reasoning}

Once it is recognised that inductive inferences can lead from a tensed ‘was’ to a tenseless ‘is’ or ‘be’, it becomes hard to maintain as a matter of logical principle that we cannot derive an ‘ought’ from an ‘is’. Even deduction can lead us to conclusions not contained in the original premises. And in Inference to the Best Explanation we argue from premises of one sort to conclusions of a very different logical type. The conclusions of practical reasoning are of a different type again. They are action-guiding. And the premises are factual, characterizing the situation to which the action commended is a response. But if we reason at all about what to do, our conclusions must point towards some action as appropriate, and if our reasoning is to be relevant and effective it must be based on the facts of the case.

The particularism of practical reasoning seems to run counter to its rationality. Rationality is universal, and enjoins us to act only on that maxim through which we can at the same time will that it
should become a universal law.\textsuperscript{16} This would require that we had a stock of universal laws under which we could subsume any action indicated by practical reason. But any such set of universal precepts would be too coarse-grained to accommodate the unbounded subtlety of human affairs. To accommodate that, we need a more flexible canon of universalisability. Instead of requiring that there be some universal precept which covers the proposed action, and that all similar ones should be treated similarly, we should require only that if we propose to treat some apparently similar action differently, there should be some difference between them to justify the difference of treatment.\textsuperscript{17} Granted this different requirement of universalisability, we can have practical arguments being both rational and relevant to the particular case under consideration.

It is the particularity of the situations in which we have to act that requires our assessments to be holistic and gives rise to the two-sidedness of practical reasoning.\textsuperscript{18} However fully we have specified a situation, there always may be a further feature that entirely alters the complexion of the case; and since circumstances, too, alter cases, there always may be some extraneous circumstance that requires a re-assessment of our response with a further `but'. The fact that there is nearly always room for another `but' has often been taken to show that there are no valid arguments in practical reasoning. But what it actually shows is that there are few conclusive arguments. Many arguments are cogent in the absence of counter-considerations, and we often state them explicitly with this proviso, `other things being equal', \textit{a\`e\`ter\'is\ par\'\ib\'us}, "in the absence of special circumstances", `as a general rule', \textit{\\o\kappa\varepsilon\varepsilon\i\tau\o\nu\alpha\i\nu\nu\acute{i}}\ (\textit{hos\ epi\ to\ polu}).

But other things may not be equal. That a certain action will cause you pain is a good reason for not doing it, yet I could well go on to say `I am afraid this will hurt, but it is what I have to do': I might be a dentist, or a head-master, or an examiner, or a candid friend. But if I do conjoin `this will cause pain' and `this is what I ought to do', I need to explain why I ought to do it, in spite of its causing pain; perhaps because it will promote good

\begin{itemize}
\item \textsuperscript{17} See, more fully, J.R.Lucas, \textit{"The Lesbian Rule"}, \textit{Philosophy}, 30, 1956, pp.195-213.
\item \textsuperscript{18} See above, §1.5.
\end{itemize}
health, or teach you a lesson, or uphold the integrity of some public system of evaluation, or enable you to make rational decisions in the face of unpalatable facts. It is something that calls for further explanation. The explanation need not be a moral or benign one: it would not be unintelligible, though it would show me in a bad light, if I explained that I was a sadist, and liked causing pain, or a Nietzschean, and wanted to make my mark on the universe.

Often a consideration is not so much countered as over-ridden. My having promised to return a borrowed weapon is a good reason for doing so, but if the person I had borrowed it from has in the meantime gone mad, the obligation to return is over-ridden by concern for his, and other people's, safety. But that does not abrogate the original obligation. If for good reason I have had to break a promise, it is not as though the promise had never been made: on the contrary, I am subject to further obligations to make good to the person I let down the damage my broken promise caused. We need to distinguish cases where a putative obligation does not hold at all—as if I made the promise having been deceived by the person to whom it was made—and cases where it is simply over-ridden.¹⁹

Our decision will depend not only on the strength of the arguments on one side, but on the weakness of those on the other. For only when there are alternative courses under consideration can I decide between them. If you counsel flight, I shall re-examine the situation to see whether it would be better after all to flee than to give battle, and with these alternatives in view I can try and tell whether this is a sticking-it-out situation or a cutting-one's-losses one. If, alternatively, you counselled me to advance, making propitiatory gestures, I should need to re-examine the situation in a different way, to attempt to make a different discrimination. So, again, if you had urged me to go on the offensive myself and launch a pre-emptive attack. The alternatives offered determine the factors that are relevant. I do not have to—very often cannot—justify my own judgement absolutely, but only relatively: I am not bound to show that I ought to do this, full stop, but only that I ought to do this rather than that. Different factors are relevant for deciding between different pairs of alternatives. If you counselled flight, it is pertinent to point out that I cannot run very fast: but my

inability to run would be no argument against making propitiatory gestures, though the fact that my adversary was a cowardly bully would. Once the alternatives are fixed, we shall each look for features of the situation which will enable us to discriminate between its being a staying-and-sticking-it-out situation and its being a cutting-one’s-losses one. We may be able to: language, used by both of us on many occasions before our present argument, enables us to pick out many sorts of features which might be relevant. We can compare this situation, which we each read differently, with others where we have no disagreement. ‘But he looks very threatening’, you may say of the adversary, ‘Yes, but he is also anxious to secure his retreat’. I counter; but because we are using some other, and usually more general, classificatory scheme than our primary one, we can work round to an agreed description of the situation we are considering. We may still be unable to reach agreement about what to do: we may argue that he is looking round nervously; or you may suggest that he is looking round not in order to see how he may beat a hasty retreat, but because he is expecting his accomplices to be on their way to join him. In that case we could agree only that he was looking round rather a lot. Even that description is not immune to objection. You may say that these few casual glances are no more than anyone might cast behind him, to note the terrain covered or to admire the view. In some situations you might question whether the man came with any hostile intent at all, in others whether it even was a man that I saw.

There is no uniform agreed level of facts from which our arguments can start. Rather, what counts as a fact depends on the question in issue. Almost anything may be disputed; but invariably there are some facts not in dispute, and it could not be the case that almost everything was disputed, for then there would be no common language in which to carry on the dispute.

Although we often disagree how the balance should be struck in some particular case, we very largely agree about the relevant considerations. When it comes to making decisions in particular situations, it always may be the case that peculiar circumstances may make it impossible to do all the things that normally ought to be done; but that does not affect the general reckoning that in all ordinary circumstances they should indeed be done. In general the reason is a good reason for acting in a particular way, even though in the exceptional case the reasons on the other side are weightier. Apart from a few disputed issues—abortion, capital
punishment, sex—we all allow that in general promises should be kept, pain avoided, life preserved. If I have promised to do something, although in unusual circumstances I may meet your claim that I ought, therefore, to do it with a ‘but’—but I have got bronchitis and the doctor has told me to keep indoors—I cannot brush off your claim with a ‘so what’?. Not to recognise that a promise creates a prima facie obligation, or that a course of action might imperil life, is to put oneself outside the pale of moral discourse. Although no claim is incontrovertible, many are not to be overlooked. I can without inconsistency commend an action which will endanger life, cause pain, contravene a previous commitment, but if I do so, I need to explain myself, either in view of wholly exceptional circumstances or else by articulating some special moral view. Practical reasoning is not closed against the possibility of new situations or new insights, but the opinions of the many and the wise do to a large extent converge about what are the relevant considerations in the ordinary run of cases.20

There are many strands of argument within practical reasoning. The most primitive is the immediate decision what to do, to fight or flee, to eat or to court a mate. Much human reasoning is on this level, but we criticize the man who pursues only τὸ παράνομο ἰδίο (το paraon heihu), immediate pleasure, and responds only to immediate threats, and we think it more reasonable to be prudent, taking account of long-term future interests, and not only those close at hand. Reason, we hold, is not confined to present concerns, but to future ones too. But prudence alone is not enough. It is incoherent to have regard only to the future and not to the past, and it may be counter-productive to consider only myself, and not other people. Each of these points may be made with the aid of the Theory of Games, the Battle of the Sexes showing how I lay myself open to manipulation unless I take account of the past as well as the future, and the Prisoners’ Dilemma showing how we fare worse if we are selfish than if we consider others’ interests as well as our own.21

Practical reasoning is messy. It is easy to get it wrong. If we do, we may get away with it. Although wrong decisions can lead to disaster, there is no immediate, irresistible sanction against being unreasonable. The sanction is not the breakdown of communication, but, in the absence of dire consequences, just simply a failure to be sensible, accompanied by a loss of respect in the eyes of others, especially when some moral obligation has been disregarded.

20 See further below §3.3.
21 See further below, ch.14.
The Development of Normative Reason

2.9 Empathy and Other Minds

The two-sidedness of practical reasoning gives a key to our knowledge of other minds and our understanding of the humanities. Besides making up my mind about what I shall do, I can consider what I should do if circumstances were different; and although in the present circumstances I must over-ride and reject some considerations in accepting and acting on others, I can fully appreciate how I might in other circumstances act on them, and so I can appreciate also how you in your circumstances might act on them. Because I know what I shall do in the actual situation, I can know what I should do in hypothetical situations, and so understand what I might do if I were you. Empathy is possible because I experience in my own deliberation the conflict of argument and feel the force of factors inclining me to act in various ways. I never have murdered any one, but I have been tempted, and so can understand the minds of those who have found the temptation irresistible. Equally I can enter into the minds of historical agents or those portrayed in literature, and although sometimes their reasoning and reactions will be entirely opaque to me, often there will be enough resemblance between their situation and my actual or possible ones for their response to be one I can see the rationality of. I do not have to suppose, counterfactually and sometimes implausibly, that I would in the event respond in the same way, but only that I might—only that there would be some reasons for so acting, in the absence of weightier considerations against. And that supposition is one it is much easier to make. I can understand what makes other people tick because of the many-sidedness of what goes on in making up my own mind. The messiness of practical reasoning, and the many decisions it partially leads me to take, gives me a width of understanding I could never otherwise obtain, and a partial entrée into the minds of all sorts and conditions of men far beyond my actual ken.

Anger, fear, resentment, spite, greed, jealousy, gratitude, pity, love, awe, exaltation and joy commonly issue in actions and activities. Feelings are not, as too many philosophers have supposed, bare physical sensations, but are for the most part to be described in terms of what we want to do or would like to do. They are, largely, incipient actions, frustrated actions, or failed actions, and the concomitants of these. And therefore our common rationality as agents gives further and detailed support for our belief that other men are of like passions with ourselves. I think that you feel
angry because I know that I would be inclined to act angrily in like circumstances; and your subsequent actions constitute a further check on whether I am right or wrong. Thus, although it is neither logically impossible nor even emotionally impossible for you to overlook the slight, or alternatively to suppress the anger which you feel, nevertheless there are always cross-connections among different states of mind and between them and actions to make it unreasonable to deny that we have some sort of fellow feeling or empathy with others, or to suppose we might be always wrong in all our ascriptions of feeling to others. *Willst du die anden verstehen*, said Schiller, *blick in dein eigenes Herz*. If you want to understand others look into your own heart. Ποιθε ανεμπαθώ (*gnolhi seauton*), Know yourself, and then you will have insight into the minds of others too.

Insight, in popular, unprofessional philosophy, is important. Historians will say of a colleague that he has his facts right and his arguments are impervious to objection and his conclusions not demonstrably wrong, yet somehow he has not got the "feel" of his period, he has not worked his way "into" it. Thus, a reviewer can write that the author

has the rare quality of entering into the minds of those he is studying and seeing things from their point of view; the result in this case is that perhaps for the first time it is possible to understand the Aztecs and sympathize with them in their painful predicament... The book is one of the best ever written about the Aztecs: his portrait of their society is a triumph of scholarship, understanding and literary skill. ²²

On which Alan Richardson comments, "Each of these three words surely represents an essential feature of the historian's craft." ²³ In the same way, literary critics will allow that a student knows all his texts, is well acquainted with the historical background, is able to manipulate parallel passages, can explain all the allusions, can comment on linguistic points, and has intelligent views on the cruces, and yet has not "got inside" his author. Scientists, it is alleged, are often insensitive: nobody denies that they are very clever men, but

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they lack sympathy; they are well able to conduct difficult calculations and reach right conclusions about means to given ends, but they are unable to put themselves in other men's shoes, or sense what the human reaction will be. It is claimed for the humanities that they educate men's perceptiveness, and make them more sensitive in their dealings with other men. Education apart, different men are differently endowed with this faculty: some of our friends are, and always have been, very able; they will go far; but not all of them will notice much as they go; others are more sympathetic; they may be less clear-headed, unable to put out arguments clearly, and having a less good memory for facts and a less adequate command of argument, but nonetheless more perceptive in what they say.24

The untaught view is clear. Men have, besides the ability to draw conclusions from premises and to learn from experience, another, finer faculty, which enables them to "get inside". It is different from inductive reasoning both subjectively and in its operations. Subjectively, it feels different: the persons in question appear transparent, not opaque; one is unable to give as good reasons as one feels—one just knows; it is knowledge by acquaintance, not knowledge by description or by argumentation. It operates differently from induction: it does not generalise; we are quite clear that to the particular problem in mind a certain solution is the right one, but are not very ready to extend the solution to other cases; the role of evidence is unclear; we do not apply any canons of inductive inference; there seem to be no criteria of irrelevance; and often, very often, it operates when we should be unable to make any inductive inferences at all. It is as though each one of us had within him a well of singular hypotheticals, on which he was at times able to draw, and provide himself with immediate and complete solutions to certain of his problems. In ourselves we can recognize its happening: among others we can pick out those who are particularly gifted with it, those who have noses for human affairs. Sometimes it is insight into human character—Humane Insight, we might call it. Sometimes it is the ability to discard irrelevant, and fasten on important premises for an argument: this makes the good civil servant, the good historian. We value it so highly that we are constantly coining new names for it: sense, sensibility, sensitivity, being perceptive, intuition, insight, sympathy, empathy, understanding, verstehen, being en

24 Like Kitty Levin in §1.7 above.
Reason and Reality

Report, being *simpatico*, and being able to put oneself inside other people’s skins, all have been used at times to refer to it. Nor do we require it only of those engaged in practical affairs. Novelists, obviously, need to have it, and even philosophers are assessed according to, among other things, their understanding of human nature. We fault Hobbes’ political doctrines because his view of man is not true to human nature. And we say this not after having examined all men and found them different from what Hobbes portrays, but merely from knowing a few men and knowing from within ourselves what it is to be a man, and what are the loyalties which can win men and move human beings to action. The attraction of Freud’s doctrines lies not in the empirical evidence for them, which is often slender, but in their innate plausibility, which once Freud has expounded it, carries conviction because it corresponds to something in our own minds. We can see that this is how a man might respond, because it is how we might respond, even though we may have never responded in the fashion described, nor ever been in a situation comparable to that described. Freud’s views have been accepted because he has been able to strike chords in the hearts of his readers, not because he has been able to furnish adequate statistics to establish his case. Where he is recounting some particular case or describing some typical situation, his reconstructions of the workings of the unconscious mind ring true: but where he or his disciples start trying to establish his conclusions in the way in which a medical scientist would establish his, our incredulity sets in. His conclusions are not inductive conclusions, they are not based on the evidence of our senses, nor built up from repeated, but opaque, conjunctions of observed fact. Rather, they are new ways of seeing human motives and construing human behaviour, new insights brought up from the depths of Freud’s own mind. Little purpose is served by trying to assimilate this to ordinary inductive generalizations or by construing the actual experience Freud lived through himself and the hypothetical experience he could imagine for others, as a basis for an inductive generalisation in the way that our sense-experience is.

For the present we note that there seem to be inferential skills manifested by the historian and the literary critic, which do not fit into the account of inductive and scientific argument, but can be seen as stemming naturally from the two-sidedness of practical argument.25

§2.10 Reason

Reason is very different from what many philosophers have taken it to be. Even if we start with them, taking reason to be analytic deductive inference, we are led to a less restricted view of reason, as being typically two-sided, a dialogue between different people, conversing because they have some common objectives. Even deductive reason is not entirely analytic: and inductive arguments, and those of practical reasoning and its progeny, evidently lead to conclusions which were not implicit in the premises.

Looking back, we can now see the philosophers’ ideal of valid inference as a special case, in which dialogue has been collapsed into a monologue, where the only common objective they need have—and must share—is to be understood. Whereas ordinarily, as I ratiocinate, my friend (or my alter ego), butts in with objections and counter-considerations, in an analytic deductive argument there is no room for interposing any ‘but’s. Any attempt to gainsay anything established by an analytic deductive argument ends in self-contradiction. As I argue, I do not need to pause, to hear objections raised, because no objection can be consistently—and therefore intelligibly—raised. Anyone who attempts to controvert what I say is contradicting himself, and can be ruled out of court as not saying anything meaningful. Monological arguments are sometimes appropriate; in writing a book, for example, where the reader is, of necessity, in no position to interrupt. And monologous arguments are often attractive, since human nature is naturally inclined to brook no opposition. An “anti-deductivist” theme follows. Although deductive reason is the paradigm for coerciveness, it is not paradigmatic in other respects. We cannot adequately represent non-deductive inferences as deductions from additional major premises. Admittedly, sometimes, when a particular inference is in dispute, it may be helpful to articulate the rule of inference as a universal proposition, and if it proves acceptable, then the conclusion will follow deductively from it. But even in deductive logic, not all inferences can be so represented, as Lewis Carroll showed.\(^{26}\) Often also, it is unhelpful to pose the question in terms of the truth or falsity of a proposition—it makes less sense to ask whether Peano’s Fifth Postulate is true than to ask whether the Principle of Recursive Reasoning is cogent. Moreover, there are many inferences in the humanities which are too particular.

\(^{26}\) See above, §2.3.
to lend themselves to being formulated as a major premise, even though they are universalisable in a more flexible way. Reasoning is typically an open-textured dialogue in which the respondent can contribute, and the sort of contribution expected determines the structure of the dialogue. Therefore it cannot be adequately reconstructed into a deductive argument, which, since it is monologous, leaves no place for the respondent to join in. The solitary self-sufficiency of the deductivist thinker is purchased at the price of solipsistic vacuity. Reasoning is risky, and we do well to have candid friends who dare tell us that we are mistaken.

Most argument, though open-textured, is structured. As we have noted in §1.6, argument is fruitless, if we disagree about everything. Not only do we have to start from somewhere, but we have to have some aims and assumptions in common. Whereas simple deduction is subject to the one condition of being intelligible, usually we share a desire to know the truth, and often an aspiration to understand. Different aims and assumptions indicate different sorts of dialogue. Often in the course of one argument, we need for a time to confine the discussion to a particular type of dialogue for the sake of clarity, or in order to reach a resolution of a particular argument. It is good to be agreed about the facts of the case before embarking on interpreting them. We may need to work out the consequences of a particular hypothesis in order to test whether it is consistent with observations or falsified by them. We limit our shared commitment for the time being, in the hope of achieving some measure of agreement, before going on to more contentious matters. A serious argument is often composed of several sub-dialogues, each with its own standards of relevance and cogency. Unless we distinguish the way the different limbs are articulated into a coherent whole, we blunder, applying to one part canons only appropriate to another.

The sceptically minded may still be unwilling to go along with this account of reason. Thrasymachus may be forced to concede the cogency of simple deductive arguments, but can still refuse the invitation to enter into other men’s minds. What are the sanctions against unreasonableness? They differ. I can resist the full force of Gödel’s theorem by distinguishing the unprovability-within-the-system of the Gödelian formula of which there is a formally valid proof, from its being true, of which there is no formally valid proof, since there is no formal definition of the term ‘true’. This, which can be seen as a consequence of Gödel’s theorem, was established
independently by Tarski. He proved that the concept of truth cannot be defined in any adequate formal system, because if we add to such a system a term representing what we mean by our ordinary word ‘true’, we shall be led—again by an Epimenides argument—to a contradiction. So, when you point out the implication of denying that the Gödelian formula is true, I fail to follow your reasoning, professing not to know what ‘true’ means. And that is the sanction. I have divested myself of knowing what ‘true’ means: I have deprived myself of the concept of truth. Although syntactically I am in the clear—I have not broken any of the rules of the communication exercise—semantically I am self-mutilated—I am no longer a man of truth. The same sanction was invoked against a refusal to move from Sorites Arithmetic to the Rule of Recursion. In these two cases there are further, arcane sanctions. I can, without self-contradiction, deny the truth of the Gödelian formula; there is no inconsistency between it and the axioms of Elementary Number Theory. It follows that there is a model of the negation of the Gödelian formula and the axioms of Elementary Number Theory. It is a weird model, but it cannot be faulted on formal grounds. If I refuse to acknowledge the truth of the Gödelian formula, I cannot exclude weird models, and so cannot specify that the numbers I am talking about are the same as the numbers that you, and everybody else, are talking about. Somewhat similarly, if I refuse to accept the Rule of Recursion, although I can specify each individual number separately, I cannot talk about them all collectively. I could be guilty of “axioms ω-inconsistency”, alleging that although each natural number possessed some property, some did not. The sceptic, who will not accept the truth of the Gödelian formula, or the validity of the Rule of Recursion, is not guilty of any straightforward inconsistency, but will be talking at cross purposes when he argues with those who have a firm grasp on the nature of the natural numbers.

Inductive sceptics have difficulty in living down to their unbeliefs. If I refuse to anticipate future events, I am in for nasty surprises. Even if my animal instincts enable me to avoid disaster in the particular situations I find myself in, I am handicapped, if I cannot generalise the better to communicate to others and to

27 A. Tarski, “The Concept of Truth Formalised Languages” tr. J.H. Woodger in Logic, Semantics, Metamathematics, Oxford, 1956, Ch.VIII.

28 See above, this chapter, §2.3.
remember myself what experience has taught me. And the price of not inferring to the best explanation is not to have the best explanation. I can live without understanding why things happen as they do. It is my choice. But if I choose to do so, I am the sufferer.

In practical affairs, again, I shall be the chief sufferer if I do not use reason, though others may suffer too as a result of my imprudence, lack of consideration, lack of public spirit, or lack of commitment to anything outside myself. Traditionally, philosophers have drawn a sharp distinction between counsels of prudence and precepts of morality, and have thought the former unproblematic, while seeing great difficulties in showing why we ought to be moral. There is indeed a distinction, but it is not as clear-cut as has been made out, and the sanctions are not as sharply separated as commonly supposed. Others, as well as I, may suffer if I am imprudent, and I, as well as others, if I am selfish. In part it is a question of identity and identification: if I have no consideration for others, I debar myself from meaningful use of the first person plural, and isolate myself into the logical loneliness of Plato’s autistic autocrat; and if I am entirely insensitive to any obligation to any objective value, I cannot ground my agency, my actions, or my achievements in anything of greater worth than my own fleeting preferences. I do not have to enter into your concerns. Indeed, I economize on emotional drain, if I do not waste sympathy on you in your misfortunes, although in some cases the balance of advantage may go the other way—I shall gain more from your co-operation than I shall lose in bearing your troubles, but, whatever the balance of crude advantage, I lose out in knowledge and understanding if I make myself ignorant of the working of other men’s minds, reducing my range of sensibility and diminishing myself.

Sanctions not only underwrite cogency, but impose a strategy. If you are obdurate, and do not feel the force of my argument, I must manoeuvre you into a tight corner, where you will pay the penalty for your obduracy. Hence the pressure to formulate; hence also the importance of chains of derivations. If I give a derivation, I can challenge you to say where I am wrong, and you have to specify, and I can then concentrate on that point. Often, in mathematical argument especially, I can show you to be inconsistent. In physics, I may show that you would be having to deny some important symmetry, or other rational requirement. Often in physics and in other sciences too the cost of resisting reason is a loss of understanding. If you persist in not accepting evolution, you will
not attain the wide-ranging perspective on how living creatures are related to one another, which the theory of evolution offers, and will not understand how different species came to be: I shall therefore seek, perhaps from the geological record, telling examples of intermediate species, or structural similarities not easily explained by any other theory. At a more mundane level unpleasant consequences flow from not believing scientific laws and maxims of common sense; and in a different way from not cooperating with others, and treating them well. The experience of living, unshielded by parental care, in a small community is often the means of in-stilling common sense and common decency by exposing the foolish and fickle to inescapable peer-group pressure.

Normative reason has an edge. It arises from our being able to think wrong. I can think wrong, you can think wrong, and we want to convict others' wrong thinking of error. Equally, in view of similar desires on the part of others, we seek to make our own arguments invincible. In either case the strategy is polemical. Each side seeks to pin down its opponents' arguments, and search for weak spots, where a decisive victory can be gained; and, anticipating similar moves on the part of the other side, tries to formulate defences against hostile probing. But some reasonings have no sanctions and no strategy for securing agreement. I give my reasons for having acted as I did, and you may be able to share them, and accept them as telling, but if you do not feel their force, and do not accept them, your only loss is not to be able to understand why I acted as I did.

Sanctions
1. Analytic statements and First-order Logic: Failure to communicate.
2. Gödel's theorem: No grasp of the concept of truth; inadequate grasp of the concept of number.
3. Rule of Recursion: No grasp of the concept of truth; \( \omega \)-inconsistency; inadequate grasp of the concept of number.
4. Inference to the Next Case: Nasty surprises.
5. Inference to a Generalisation: No General Grasp.
6. Inference to Best Explanation: Inexplicability.
7. Practical Reason: No common sense
8. Humane Insight: No understanding of fellow men
The development of the idea of normative reason, starting from the minimal requirement for our communications to be intelligible, and moving through successive stages to a wide-ranging ability to enter into the minds of others, and to argue about what we, individually or collectively, ought to do, reveals a characteristic universality. If an argument is cogent on one occasion, it will be cogent on others too. Hence the move from *Sócrates* arguments to the Principle of Recursive Reasoning, and from Induction to the Next Case to Inductive Generalisation. In practical reasoning, especially in moral, political and judicial argument, a similar move is made, giving rise to principle of universalisability, cited in §2.8. But although often it is legitimate to argue that what is sauce for the goose is sauce for the gander, we had to recognise, at least where human beings are concerned, that one man's meat is another man's poison. Some principle of universalisability is still needed, but it needs to be a more flexible one, not only in moral discourse, but in explaining historical causes, and evaluating arguments in the humanities generally. Instead of positing some principle such that all cases falling under it are taken to be the same, we should require only that if some apparently similar case is taken to be different, a reason for the difference should be forthcoming.

If reason is universal, it can reason about reason itself, and from self-referential reasoning, it emerges that it is not possible to set bounds to reason, and in particular that metaphysical argument is not beyond the bounds of reason. Deductive argument gives rise not only to recursive arguments, but to Gödel's self-referential theorems; and inductive arguments merge into Inference to the Best Explanation, sometimes invoking entities beyond the bounds of possible experience. Practical reasoning leads on to empathy and moral argument, and they in turn lead to the humane insight of the humanities, and varieties of political, legal and judicial argument. Gödel's theorem shows that even with deductive argument, we cannot formalise completely; however far we formalise our rules of inference, there will still be some inference which is clearly valid but does not fall under any of the rules thus far formulated. Even deductive argument is fuzzy-edged, and the transition from one type of inductive inference to another shows that the same holds good for inductive arguments about matters of fact and their explanation; and counts against the contention that reason cannot lead on to the various styles of practical argument.
We can go further. The simple argument of the Logical Positivists against metaphysics does not work. If metaphysics could be ruled out by the Verification Principle, so too would the Verification Principle itself. Similarly, any claim to set a boundary to reason can be challenged. Clearly, if the bounds of reason are so tightly drawn as to exclude philosophical argument, the claim will exclude any possible justification of itself. But more generally, in order to determine the boundary exactly, it will need to specify what lies beyond it and is to be excluded, as well as what lies within it and is to be included. And if the boundary is really a boundary reason cannot overstep, reason will be precluded from stepping over it, to specify precisely what is to be excluded. Reason itself, then, is unbounded. We have a negative, self-referential argument, analogous to the negative, self-referential mathematical argument underlying Gödel’s theorem, against any claim that reason can be corralled within any antecedently set limit, and are led to a crucial conclusion. Reason is not “thin”.

Theses about Reason
1. Not all a priori arguments are analytic. For example, in mathematics, arguments by recursion: (point made by Poincaré): it is not straight inconsistent (though it is ω-inconsistent) to hold for each natural number, \( n \), that \( F(n) \), but to deny that \( (\forall n) F(n) \), that is, for all natural numbers \( F(n) \).
2. Not all deductive arguments are rule-bound, that is governed by some antecedently specified rule of inference. For example, Gödel’s theorem shows that the Gödelian sentence of a formal system is true, though this cannot be proved in the system. We need to know how to argue, not just know that certain rules of inference are allowed. [Achilles and the Tortoise]
3. Not all sound arguments are deductive. For example, inductive arguments.
4. Not all inductive arguments are of the same type. For example, some inductive arguments argue from particular propositions as premises to another particular proposition as conclusion (the sun has risen every day hitherto, so it will rise again tomorrow), others argue from particular propositions as premises to a general proposition as conclusion (the sun has risen every day hitherto, so it rises every day).
5. Not all arguments about the way the world is are simple inductive arguments. For example, many scientific arguments go
from observational premises to theories about entities that cannot themselves be observed.

6. Not all arguments go from premises to conclusions of the same logical type. For example those instanced in (4) and (5).

7. Some arguments go from factual premises to evaluative conclusions. For example, moral arguments; also intellectual arguments about the acceptability of conclusions of arguments.

8. Most arguments, apart from deductive arguments, are two-sided. The addition of further information may weaken, not strengthen, a conclusion; they are not monotonic, but are a matter of argument and counter-argument, objection and rebuttal. For example, induction (even though I have seen 256 swans, and they are all white, and it was reasonable on that evidence to infer that all swans are white, if I now come across a black swan, I can no longer assert that all swans are white). The point has long been known in practical reasoning, where the very word ‘deliberation’ suggests weighing considerations pro against considerations con.


10. Reason is creative.

11. Cumulative arguments.


13. Fallibility.

14. Self-reflective: metaphysical arguments often make use of this; as when we refute a metaphysical argument (e.g. the Verification Principle), on the grounds that it is sawing off the branch it is sitting on.